PHY132 Practicals Day 10
Student Guide

Summer 2009

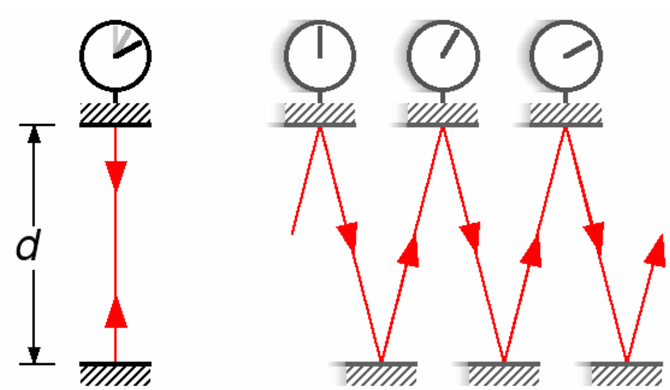
Concepts of today's Module

- Light clocks
- Time dilation
- Length contraction

Relativity Module, Activity 15

A thought-experiment, sometimes called a Gedanken experiment, is an experiment that you can imagine in order to test or explore theories in physics or other fields. Typically, thought-experiments might be very inconvenient or practically impossible to set up in real life, and you might have no intention of actually setting up the experiment. Nevertheless, thought-experiments can be very helpful in testing and discussing theories.

One famous thought experiment is the "Light-Clock". A light clock is made up of two parallel mirrors, separated by a vacuum and held at a fixed distance of d, as shown in the figure. A short pulse of light bounces between the mirrors. Each time the light pulse reflects off the top mirror, the clock "ticks". The time between ticks for a stationary light clock then is the time for a round-trip of the light pulse: $t = 2d/c$, where c is the speed of light.



One of the most fundamental and surprising principles of Einstein's Theory of Relativity is "**light travels at speed *c* in all inertial reference frames.**" Here an inertial reference frame is just one that is not accelerating.

If the light clock is moving toward the right at speed v, the time between ticks is longer, because the light pulse must travel along the diagonal. This time-dilation, or slowing of time, can be computed using the Pythagorean theorem. This is done on page 1157 of Knight Physics for Scientists and Engineers 2nd Edition, and there is a nice applet showing this derivation at http://physics.ucsc.edu/~snof/Tutorial/ .

A. Please open this tutorial with your browser. Click on the [1] to view the #1 Tutorial. It should take about 2 minutes to go through this tutorial and see the derivation of equation 37.22 from Knight: $\gamma = \dfrac{1}{\sqrt{1 - v^2 / c^2}}$. At the end of the tutorial, it says "Type a number 1 through 7 to set the speed of the clock, then click 'Play' to watch it." When you click 1, what is the value of γ? What is the corresponding value of *v*? Using a stopwatch, measure the round-trip time of reflections of the pulse from the top mirror.

B. When you click 5, what is the value of γ? What is the corresponding value of *v*? Using a stopwatch, measure the round-trip time of reflections of the pulse from the top mirror.

Relativity Module, Activity 3

For this activity, assume Toronto and Montreal are exactly 500 km apart in the Earth frame of reference. Assume that Kingston is exactly half way between Toronto and Montreal. They are all, of course, stationary relative to each other and the Earth. Ignore any effects due to the Earth's rotation on its axis. Ignore any effects due to the Earth's gravitational field. Assume the surface of the Earth is flat. Assume that the speed of light in the air is exactly equal to c.

Note that if any object is moving relative to you, its length along the direction of motion is $L = L_0 \sqrt{1 - v^2 / c^2}$, where L_0 is the "rest-length" of the object. For everyday speeds, L is almost exactly equal to L_0, which is why we don't normally notice length-contraction. However, for speeds approaching the speed of light, $L < L_0$, and as the speed of an object approaches c, its length approaches zero!

A powerful searchlight is in Toronto, pointed towards Montreal. A second searchlight is in Montreal, pointed towards Toronto. Initially the two searchlights are turned off. Assume that both searchlights are visible from each other and from any point between Toronto and Montreal.

A. You are in a very fast bullet-train traveling from Toronto to Montreal at 0.8 c relative to the Earth. [This is a thought-experiment; if your train was actually moving this fast in real-life you could get from Toronto to Montreal in 2 milliseconds!] For you, what is the distance between Toronto and Montreal? For you what is the distance between Toronto and Kingston? [Note that in your own personal reference frame, you and the train are stationary, and it is the earth and all the cities on it that are moving at 0.8 c.]
B. The two searchlights in Toronto and Montreal are quickly turned on and off, both emitting quick flashes of light. **For you, the two flashes were emitted simultaneously.** Imagine you were just leaving Toronto when the searchlight in Toronto is turned on. You see the flash from the searchlight in Toronto instantaneously. Sketch the positions of the two flashes of light a very brief moment after they are emitted, indicating their speeds and the distance between

them for you. How long should it take before you see the flash from Montreal? Be sure to clearly indicate how you arrived at your answer.

C. Another member of your Team took an earlier train, also traveling from Toronto to Montreal at 0.8 c. What is your Teammate's speed relative to you? Imagine your Teammate is just over Kingston when the flashes are emitted. Are the two flashes of light emitted simultaneously for your Teammate? Will he/she see the flashes simultaneously? If yes, how long after the flashes will he/she see them? If no, which flash will he/she see first and by how much? You may find it useful to add your Teammate to your sketch from Part B.
D. Your Instructor is in Kingston, and is stationary relative to Kingston and the Earth. What is your Instructor's speed relative to you? Will your Instructor see the flashes from the two searchlights of Part B simultaneously? If no, which flash will he/she see first and by how much as measured by you? Explain. You may find it useful to add your Instructor to the sketch from Parts B and C.
E. Imagine your Instructor has an ipod that will begin playing music when it receives a flash of light, and quits when it receives a second flash of light. In the instructor's reference frame, stationary relative to the Earth, does the ipod turn on, and if so, for how long? Is there any music? In a train's reference frame, moving at 0.8 c, does the ipod turn on, and if so, for how long? Is there any music?

Course Concepts *Relativity Module, Activity 6*

Here is another thought-experiment. Imagine we have a 120 m long car and a 100 m long garage, both as measured at rest relative to the car and the garage. We will assume the garage has both a front door and a back door, making it possible for the car to drive straight through the garage and out the other side.

A. Make a scale-diagram in your notebook of two rectangles: one representing the car, and one representing the garage, both at rest. Use a scale of 1 cm in your notebook represents 50 m in real life. When parked, does this car fit in the garage?

Instead of buying a smaller car, you might be able to squeeze this huge car into the garage by driving it extremely fast! If the car is driving towards the garage at 70% of the speed of light, its length will be contracted.

B. Assume the car is moving at 0.7 c in a direction parallel to its length. How long will the car be in the frame of reference of the garage? Make a scale-diagram in your notebook of two rectangles representing the car and garage, from the **garage** reference frame. Does the car fit in the garage?

But if we are riding along with the car, the car is at rest relative to us, so its length is not contracted. In fact, in the car's reference frame, the garage is actually moving in the opposite direction at 70% the speed of light. Therefore, the garage should be length contracted.

C. According to an observer sitting in the car, the garage is moving towards them at 0.7 c. How long will the garage be in the frame of reference of the car? Make a scale-diagram in your notebook of two rectangles representing the car and garage, from the **car** reference frame. Does the car fit in the garage?

There is a nice applet available at http://www.physics.uq.edu.au/people/mcintyre/applets/relativity/relativity.html .

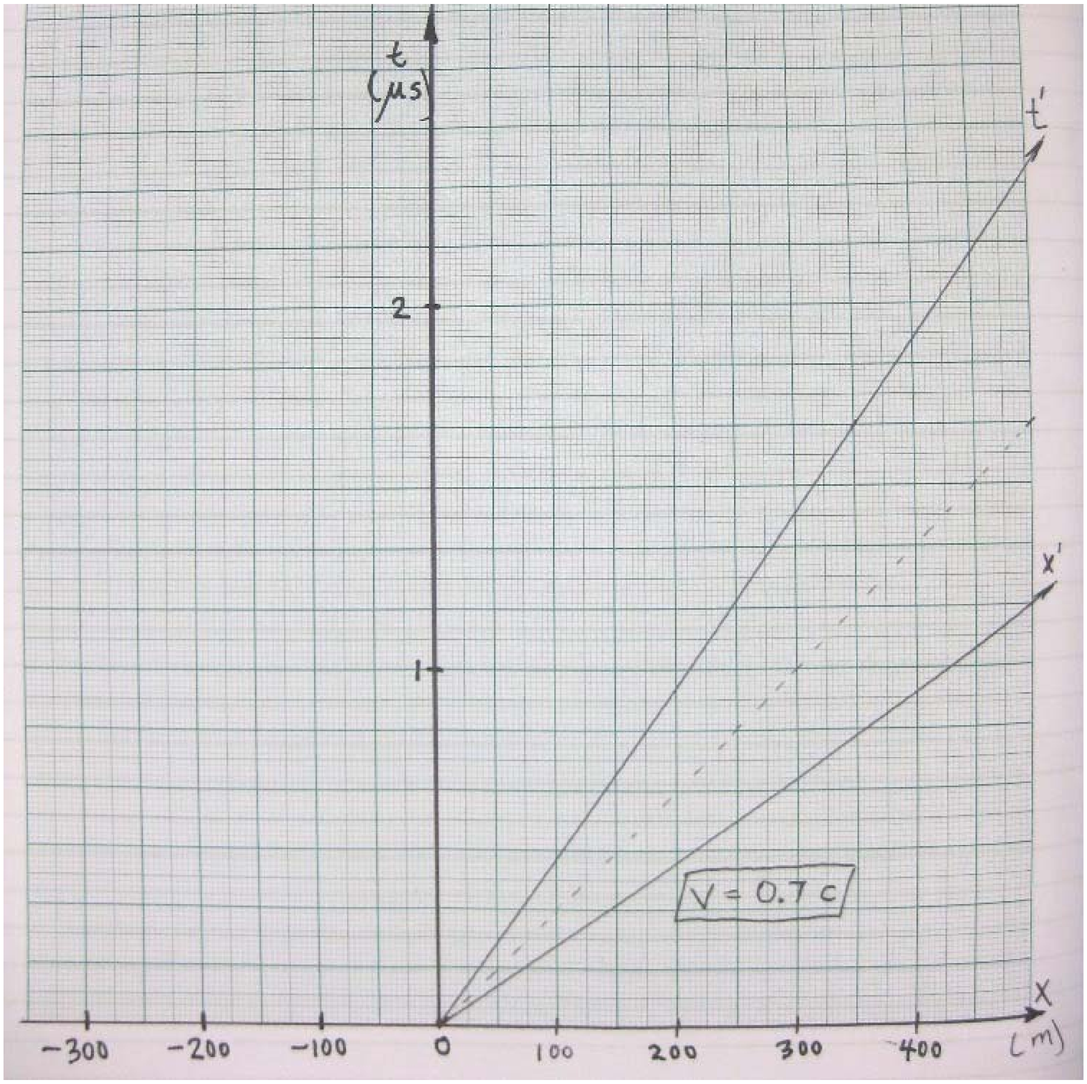
When you first open the page, or reload it, the blue car is pointed toward the red garage, with its center a distance of 200 m to the left of the center of the garage. The origin is defined to be the center of the garage, and the car is 120 m long when at rest, so the car's front begins at a position of $x = -140$ m, and the car's back begins at a position of $x = -260$ m.

D. Use the slider to adjust the speed of the car to be 0.7 c. The simulation shrinks the blue car, showing the instantaneous situation as observed by an observer at rest relative to the garage. Start the simulation, then pause it just after the car passes through the garage. A space-time diagram is plotted in the frame of reference of the garage. The blue lines show the front and back of the car, the red lines show the front and back of the garage. The ct axis defines where x=0, and it is parallel to the line for any object that is stationary in the garage frame. The x axis defines where t=0, and is parallel to the line connecting any simultaneous events in the garage frame. Note where the back of the car enters the garage: call this event A. Note where the front of the car exits the garage: call this event B. Which event has a greater value of t as measured in the garage frame, A or B? So which event happened first? Does that mean the car was entirely in the garage at some time?

E. Make a careful space-time diagram in your notebook in the frame of reference of the garage. x is plotted on the horizontal axis, and t is plotted on the vertical axis, as shown in the photo on the next page.

   Use a horizontal scale of 1 cm on the page = 50 m in real life. Use a vertical scale of 6 cm on the page = 1 μs of time. Place the origin at the bottom of the graph, and leave 10 cm of space to the right of the origin (corresponding to 500 m). A dashed line extending from the origin up and to the right at a 45° angle represents a beam of light.

   The t' axis represents the position of the origin in the frame of reference of the car which is traveling at 0.7 c. It has a slope of 10/7 on this scale. The t' axis is parallel to the line for any object that is stationary in the car frame. The x' axis defines where t'=0, and is parallel to the line connecting any simultaneous events in the car frame. The x' axis has a slope of 7/10 on this scale.

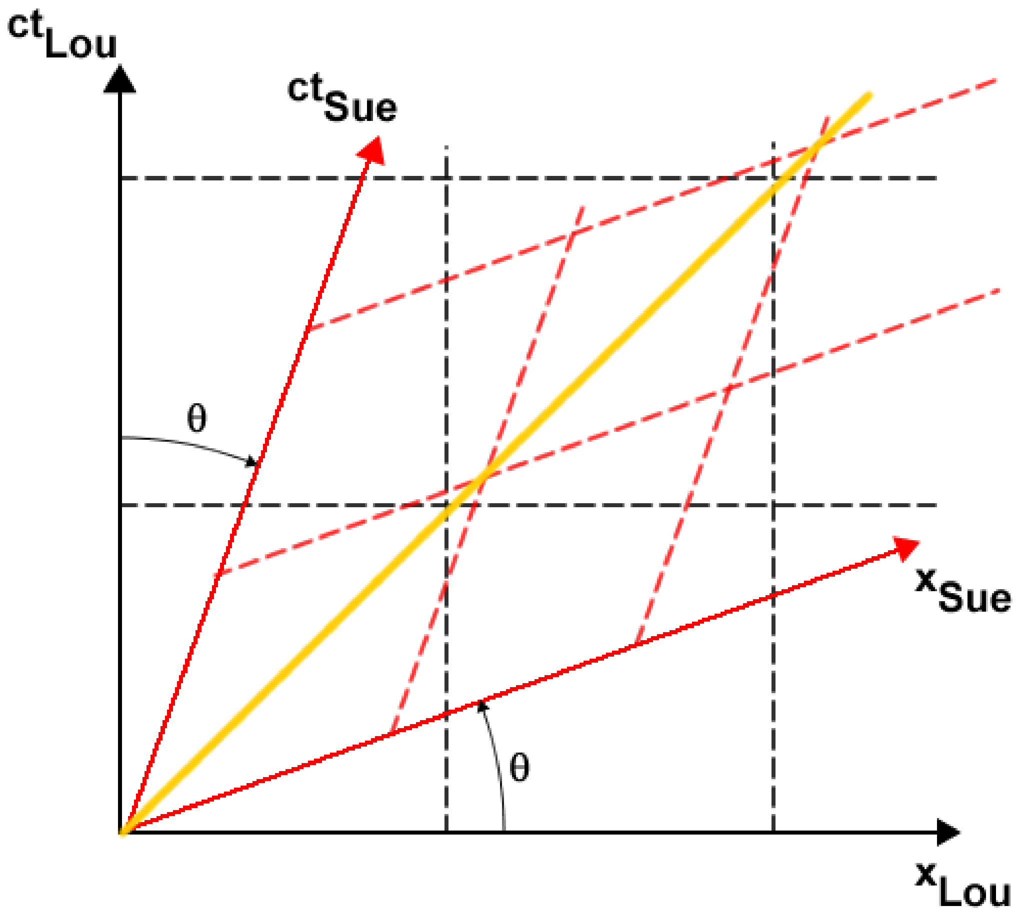


F. As was done in the applet, add and label lines that show the front and back of the car, and lines that show the front and back of the garage. Note where the back of the car enters the garage: label this event A. Note where the front of the car exits the garage: label this event B.

Add lines that pass through A and B and are parallel to the *x′* axis (with slopes of 7/10). Note where these lines pass the *t′* axis. In the car frame, events that lie along these lines are simultaneous. Which event has a greater value of *t′* as measured in the car frame, A or B? So which event happened first? Does that mean the car was entirely in the garage at some time?

If you have time: Activity 8

Sue and Lou are moving relative to each other at speed *v*. We can relate their spacetime diagrams as shown.



In the above diagram:

$$\tan(\theta) = \frac{v}{c}$$

The axes are rotated relative to each other, but they are rotating in opposite directions. Also shown in yellow is the worldline for light traveling at *c* relative to both Sue and Lou.

You will be using this diagram below, and may find it useful to print a few copies of it to staple into your lab book.

A. Explain how the diagram shows that the speed of light is the same value for both Sue and Lou.
B. Place two dots on the diagram representing the positions and times of two events that are simultaneous for Lou. Are the two events simultaneous for Sue? If no, which event occurs first? Place two more dots on the diagram representing the

positions and times of two events that are simultaneous for Sue. Are the two events simultaneous for Lou? If no, which event occurs first?

C. Sketch the worldline of an object that is stationary relative to Lou. What is the direction of motion of the object relative to Sue? Explain.

D. Sketch the worldline of an object moving at some speed $u < c$ relative to Lou. Show geometrically that the object is moving at speed $u' < c$ relative to Sue.

Last revision to this write-up: July 10, 2009 by Jason Harlow.

The Relativity Module Student Guide was written by David M. Harrison, Dept. of Physics, Univ. of Toronto in January 2009. Except for Activity 15, which was written by Jason Harlow in July 2009.

Activity 3 is based on Rachel E. Scherr, Peter S. Shaffer, and Stamatis Vokos, "The challenge of changing deeply held student beliefs about the relativity of simultaneity," American Journal of Physics **70** (12), December 2002, 1238 – 1248.

The applet used in Activity 6 was written by Patrick Leung and Tim McIntyre at The University of Queensland, 2007.

David learned about the geometric approach to Special Relativity of Activity 8 from Edwin F. Taylor and John Archibald Wheeler, **Spacetime Physics** (W.H. Freeman, 1963). This classic is highly recommended.

The image in Relativity Module Activity 15 was downloaded on July 10, 2009 from http://commons.wikimedia.org/wiki/File:Light-clock.png and was created by Michael Schmid on October 18, 2005.

The applet used in Activity 15 was developed at the University of California at Santa Cruz (UCSC). Devin Kelly-Sneed programmed the applets and built this site as an undergraduate Computer Science student at UCSC. Joel Primack is a Professor of Physics at UCSC. He originally designed the programs that make up Einstein's Rocket in the early 1980s with the help of Eric Eckert, who programmed them in 6502 assembly language.