# Quantum State Control and Characterization in an Optical Lattice by 

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## Abstract

Quantum state control and characterization
in an optical lattice
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In this dissertation I present experimental work on the measurement and manipulation of the center-of-mass motion of laser-cooled atoms. The first experiment described demonstrates cooling of an atom cloud by 'delta-kick cooling'. A thermal cloud of atoms in a vacuum expands ballistically, generating correlations between position and momentum. An appropriate momentum kick, proportional to position, results in slowing down all the atoms in the cloud. Through this technique a cloud of atoms can be cooled by greater than a factor of 10, preserving phase-space density, but decreasing the number density of atoms.

By using laser-cooled atoms, it is also possible to confine atoms in potentials created by the AC-Stark shift of the atomic energy levels. Using interfering lasers to create the Stark shift, atoms are confined in a sinusoidal potential called an optical lattice. After preparing atoms in the lowest-energy band of the lattice, a spatial displacement can create coherent superpositions of many states of the potential. A combination of time delays and secondary displacements allows the measurement of the Q (Husimi) and W (Wigner) quasi-probability distributions, each of which completely characterizes the motional state of the atoms. Alternatively, a shallow lattice that only support two long-lived states can be used. The two-state system may be characterized with far fewer measurements, and furthermore, can be used as a model system for a qubit, a quantum representation of a single bit of information,
useful for quantum computation. We demonstrate reconstruction of the density matrix in the 2-state system. The two-state system has be further used to characterize the physical action of an operation. By preparing a complete set of input density matrices we perform quantum process tomography for the intrinsic decoherence of the lattice, and two operations that correspond to single qubit rotations.

This work is dedicated to my family. To my wife Renee, and to my kids Thea and Rune, for encouraging me and sticking by me when things got rough. And to my parents Nils and Barbro, for encouraging my curiosity and not complaining too much when my attempts to figure out 'how it works' resulted in numerous broken toys, appliances and power tools.

## Contents

List of Figures ..... viii
List of Tables ..... xi
1 Introduction ..... 1
2 Laser Cooling ..... 4
2.1 Laser Cooling and Trapping ..... 4
2.2 Interactions between light and matter ..... 5
2.3 Effects of Detuning in Real Atoms ..... 7
2.4 Doppler Cooling ..... 10
2.5 Sisyphus Cooling ..... 13
2.6 Corkscrew Molasses ..... 15
2.7 Magneto-Optic Trap ..... 17
2.8 Magnetic Trapping ..... 19
3 Delta-Kick Cooling ..... 21
3.1 Cooling Model ..... 23
3.1.1 Naive expectations- Linear gradient ..... 26
3.1.2 Rounding of the Potential ..... 26
3.2 Experimental results ..... 28
3.2.1 Delta-kick cooling on spin-polarized atoms ..... 29
3.2.2 Delta-Kicks from optical molasses ..... 34
4 Optical Lattices ..... 37
4.1 Dipole Force and a 1-D Lattice ..... 38
4.2 Band Structure ..... 39
4.3 Tilted lattices ..... 46
4.3.1 Wannier-Stark transitions ..... 50
4.4 Experimental Lattice Setup ..... 51
4.5 Lattice Controls ..... 54
4.5.1 The SRS function Generator ..... 54
5 Phase-Space Tomography ..... 56
5.1 Representations of Quantum Mechanics: Density Matrix and Phase- Space Distributions ..... 57
5.1.1 Wigner Function ..... 60
5.1.2 Husimi (Q) Distribution ..... 64
5.2 Measuring Q and W in a lattice ..... 66
5.2.1 Experimental Details: Lattice characterization ..... 70
5.2.2 Measuring State Populations ..... 70
5.2.3 State Preparation-Creating a ground state distribution ..... 74
5.2.4 Measuring Oscillation Frequencies ..... 77
5.2.5 Q of a coherent state ..... 79
5.2.6 Q (and pseudo-Wigner) Function for an Inverted Population ..... 81
6 State Tomography ..... 86
6.1 Density matrix reconstruction ..... 87
6.1.1 Measurements for Tomography ..... 88
6.1.2 The Experiment ..... 92
6.1.3 State Preparation ..... 94
6.1.4 Procedure ..... 95
6.2 Conclusion ..... 103
7 Process Tomography ..... 104
7.1 Superoperator Representations ..... 106
7.1.1 Operator-sum Representation: Kraus Operators ..... 107
7.1.2 Choi Matrix ..... 108
7.1.3 Bloch Sphere ..... 109
7.2 Experiment ..... 111
7.2.1 Superoperator for Decoherence ..... 112
7.2.2 Testing the superoperator: Predictability and Markov tests ..... 115
7.3 Superoperator for Resonant Coupling ..... 119
7.3.1 Future Directions ..... 126
8 Conclusions and loose ends ..... 128
Bibliography ..... 130
A The Rubidium atom: Properties and tables ..... 141
B Control Sequence: Timing and Synchronization ..... 143
C Table of Lattice Parameters ..... 146
D Characterization of the optical lattice ..... 149
D. 1 Adiabaticity: How slow is slow ..... 149
D. 2 Heating ..... 152
D. 3 Tunneling of excited state atoms ..... 153
D. 4 Displacement of the lattice ..... 154

## List of Figures

2.1 Energy level diagram for the Rb D 2 resonance. ..... 8
2.2 Relative coupling strengths in the $\mathrm{J}=1 / 2$ to $\mathrm{J}=3 / 2$ manifold. ..... 10
2.3 Clebsch-Gordon Couplings for a $\mathrm{J}=1 / 2$ to $\mathrm{J}=3 / 2$ transition ..... 14
2.4 Sisyphus Cooling Process ..... 15
2.5 Polarization rotation in a $\sigma^{+} \sigma^{-}$molasses ..... 16
2.6 Principle of Magneto-Optical Trapping ..... 17
3.1 Model for cooling with a harmonic pulse. ..... 25
3.2 Phase-space evolution with a linear gradient ..... 27
3.3 Shape of potential (red) and force (blue) in a conic section ..... 27
3.4 Delta-kick applied to spin-polarized atoms ..... 30
3.5 The ballistic expansion of spin-polarized atom clouds. ..... 32
3.6 Temperature reduction vs expansion ratio ..... 33
3.7 Final temperature over initial temperature as a function of the kicking strength. ..... 34
3.8 Free expansion and kicked atoms from a molasses. ..... 35
3.9 Cloud size vs. time for various kicks from molasses. ..... 36
4.1 Band Structures in a lattice ..... 41
4.2 Bloch wave functions (left) and probability densities(right) for $q=0$ and $q=\hbar k$ in the lowest band of a lattice with depth $\mathrm{U}=12 \mathrm{E}_{r e}$ ..... 42
4.3 Bloch wave functions and probability densities for $q=0$ and $q=\hbar k$ in the second lowest $(\mathrm{n}=1)$ band of a $12 E_{r e}$ deep lattice. ..... 43
4.4 Allowed energy vs well depth in recoil units and temperature. Shaded regions represent allowed energies. ..... 44
4.5 Bandwidth vs Well depth ..... 45
4.6 Band gap vs well depth ..... 45
4.7 Wannier wavefunctions in a lattice with depth $U=12 E_{r e}$ in a lattice shown for the two lowest energy states. ..... 47
4.8 Transition probability vs well depth in an optical lattice. $\mathrm{P}_{n}$ is the probability to tunnel from band $n$ to band $n+1$ ..... 49
4.9 Landau-Zener limited lifetime for atoms in an optical lattice. $\tau_{n}$ is the lifetime of atoms in the nth band. ..... 50
4.10 Optical setup for control and phase stability of lattice beams. ..... 53
5.1 Wannier states of an optical lattice. ..... 63
5.2 Gallery of several Wigner and Husimi Distributions ..... 67
5.3 Atom cloud after adiabatic release from lattice. ..... 73
5.4 Atom cloud after fast release from lattice. ..... 74
5.5 Sample procedures and images showing the state populations after thermally loading a lattice, and after filtering out the higher energy bands. ..... 75
5.6 Oscillation in ground state population observed in a Ramsey experiment. ..... 79
5.7 Measurement Procedure ..... 80
5.8 Q distributions for experiment and theory ..... 81
5.9 Q distribution for an inverted population ..... 83
5.10 Wigner Distribution for an inverted state ..... 84
5.11 Cuts through the measured(solid line) and theoretical(dashed line) dis- tributions. ..... 85
6.1 Band Structure for a lattice 18 recoils deep. The dashed line refers to the depth of the potential. ..... 93
6.2 Measurement Timing ..... 96
6.3 Sample images for reconstructing a density matrix. ..... 97
6.4 Profiles taken through the clouds after projecting the prepared state into 3 different bases. The red points correspond to the populations in the prepared state, blue to populations after a displacement and green to the populations after a time delay and displacement. The solid lines are fits to the curves. ..... 98
7.1 Bloch sphere for natural decoherence ..... 116
7.2 Matrix of disagreement between predicted superoperator and measured superoperator for 3 oscillations. The elements of the matrix show the number of standard deviations by which the Choi matrix elements disagree. ..... 118
7.3 Comparison of fits to Ramsey fringes: Exponential vs Gaussian decay. ..... 120
7.4 Position drive for resonant coupling ..... 121
7.5 Bloch sphere for driven coupling. ..... 124
7.6 Experimental Bloch sphere for cosine driven coupling. ..... 125
7.7 Experimental and theoretical bloch spheres for resonant coupling ..... 126
A. 1 Transition strengths for the Rb D2 resonance. ..... 142
B. 1 Circuit to synchronize the SRS function generator with LabView ..... 144
B. 2 Lattice control electronics . . . . . . . . . . . . . . . . . . . . . . . . 145
D. 1 Fraction of atoms measured in the ground state after filtering vs the slope of the potential ramp. . . . . . . . . . . . . . . . . . . . . . . . 151
D. 2 Heating of atoms in an optical lattice. . . . . . . . . . . . . . . . . . . 152
D. 3 Escape rate of excited state atoms from a shallow lattice . . . . . . . 154
D. 4 Theoretical coupling probabilities and measured state populations after lattice displacement . . . . . . . . . . . . . . . . . . . . . . . . . . . . 156

## List of Tables

4.1 Summary of SRS arbitrary waveform modulation capabilities ..... 55
5.1 Characteristics of Q,W and P quasi-probability distributions ..... 60
5.2 Energy spacings and lifetime of atoms in a $60 E_{r e}$ deep lattice ..... 77
5.3 Raw measurements for a Q distribution ..... 80
6.1 Projections for density matrix reconstruction and resulting information gained ..... 89
6.2 Displacements and rotations to achieve measurement basis ..... 91
6.3 Measurement results and their relation to the density matrix ..... 92
6.4 Mean band energy, Band gap, band width and lifetime of the energy bands in the tilted lattice. ..... 93
6.5 Raw measurements used for density matrix reconstruction ..... 99
6.6 sample density matrix reconstruction extracted from raw data. Num- bers listed in brackets are the uncertainty in the value. Populations are measured to an accuracy of 0.02 and coherences with an accuracy of 0.04 . ..... 99
6.7 Reconstructed density matrices with statistical uncertainties. ..... 100
6.8 Comparison of the measured density matrices to the theoretical expec- tation, given the prepared ground state. The similarity between the experimental and theoretical density matrices is quantified using the measure of fidelity. ..... 102
7.1 Measured density matrices for the input states (left) and the output states after one period in the lattice(right). ..... 113
7.2 Measured and predicted density matrices after two periods. ..... 117
7.3 Measured and predicted density matrices for a resonant coupling op- eration. ..... 123
C. 1 Mean band energy, band width and band gap in units of $1 / \mathrm{s}$ for various well depths. ..... 147
C. 2 Mean band energy, band width and band gap in units of recoil energies. Also given in column two is the well depth in microKelvins. . . . . . 147
C. 3 Landau-Zener transition probabilities and lifetimes for a tilted lattice under gravity.148

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## Chapter 1

## Introduction

In 1975, Hänsch and Schawlow[1] and Wineland and Dehmelt[2] independently laid down the foundation for the laser-cooling of atoms and ions. Their proposals were realized 10 years later by Steve Chu[3], who demonstrated laser-cooling of sodium. Some careful measurements revealed a surprising result. The atoms could be lasercooled to a temperature nearly two orders of magnitude lower than predicted by the theory $[4]$. Soon after, Dalibard and Cohen-Tannoudji provided new theoretical models for cooling that included the multi-level nature of the atom and explained the observed temperatures[5]. In 1997 the Nobel prize in physics was awarded to Claude Cohen-Tannoudji, Steve Chu and Bill Phillips for their contributions to the field of laser-cooling. The success of laser-cooling soon allowed the realization of another long standing goal, Bose-Einstein Condensation (BEC) of an atomic vapour. A mere four years later, the Nobel prize went to Eric Cornell, Carl Wieman and Wolfgang Ketterle for their work on BEC of alkali gases.

As it became increasingly possible to cool atoms to extremely low temperatures it was realized that the deBroglie wavelength of these cooled atoms could become as large, or larger than the wavelength of light, leading to the beginning of the field of atom optics. In atom optics, we look for methods to treat a beam of atoms much as we could a beam of light. Experiments have demonstrated atoms bouncing
off of optical mirrors [6, 7], atom beam collimation and lensing[8], diffraction [9, 10] and even holography $[11,12]$. Numerous experiments have demonstrated the coherence properties of atoms by performing atom interferometry with thermal clouds and with Bose-Einstein Condensates. These experiments lead to the development of a continuous, coherent matter wave, an atom laser[13, 14, 15].

In this dissertation, I discuss phase-space manipulation and measurement of Ru bidium atoms in a vapour-cell atom trap. The background for these experiments is presented in the first chapter, covering cooling techniques and dipole-force potentials created by interfering laser beams.

The following section discusses classical manipulation in phase-space to cool an atom cloud. The phase-space evolution of atom clouds may be manipulated by pulsed magnetic gradients to either cool (collimate) an atom cloud, or to cause it to focus at a later time. In the process of cooling the atom cloud we find a feature that is much smaller than the original atom cloud, possibly revealing some existing structure within the cloud following the cooling process.

The next several chapters discuss measurement and manipulation of the atoms in a quantum sense. Atoms are trapped in a one-dimensional optical lattice, a periodic potential created by interfering a pair of laser beams. Atoms trapped in the lattice behave much as electrons do in a crystal. The optical lattice may be time-dependently modulated much faster than the atom can move. The variety of controls available allows for a very flexible system that can be used to study a wide variety of problems.

Measurement of the motional quantum state is performed by measuring the Husimi and Wigner distributions, quantum mechanical phase-space representations. The phase-space distribution of atoms trapped in the lattice is measured by displacing the lattice followed by measurements of the state populations. In the case of lowdimensional systems the quantum state can often be determined by several carefully chosen measurements. Once it is possible to characterize the quantum state the atoms may be used to characterize a process or operation. By subjecting a variety
of input states to some process, the complete action of the process may be measured, including effects such as loss, decoherence and errors. Such a measurement allows the characterization of a 'real world' device, from which point it can be compared to the desired operation. In this manner the fidelity of a quantum gate, a building block of a quantum computer, could be determined, and error-correcting codes developed.

## Chapter 2

## Laser Cooling

### 2.1 Laser Cooling and Trapping

As the technology of laser cooling has matured it has become a workhorse of atomic physics. Through laser-cooling it is possible to effectively eliminate Doppler broadening, enabling better spectroscopy. It has also enabled much longer interaction times, allowing atomic clocks[16] to gain several orders of magnitude in accuracy.

In the last several years Bose-Einstein Condensation[17, 18, 19] (BEC) of an atomic vapour has been achieved by numerous groups around the world. A BEC is a coherent state of matter where all the atoms (bosons) are in the quantum mechanical ground state of the system. BEC is the state of matter that produces the effects of superconductivity and superfluidity. BEC in an atomic vapour provides another way to study such effects without a crystal medium (superconductivity), and in the absence of non-condensate atoms. Likewise it is possible to modify the potential the atoms are in, changing the way the BEC evolves, allowing the creation of vortices[20, 21], solitons[22, 23] and superpositions[24] of BECs.

### 2.2 Interactions between light and matter

This dissertation focuses on laser cooling of Rubidium. Rubidium is an alkali atom, and therefore has a relatively simple atomic structure with only a single electron in its outer orbital, making it very hydrogen-like. All the laser-cooling methods discussed here work by taking advantage of the interaction of the Rb atom with an electromagnetic field. Some cooling methods are the result of absorbing momentum from incoming laser beams, while other methods are the result of energy shifts of the ground state.

An atom in a time-varying electric field $\vec{E}$ gains an induced dipole moment $\vec{p}$ that oscillates at the driving frequency. The amplitude of the dipole moment is related to the electric field by

$$
\begin{equation*}
\vec{p}=\alpha \vec{E}, \tag{2.1}
\end{equation*}
$$

where $\alpha$ is the complex polarizability, which is dependent on the driving frequency. The in-phase part of the polarizability leads to a dipole potential

$$
\begin{equation*}
U_{d i p}=-\frac{1}{2}\langle\vec{p} \cdot \vec{E}\rangle=-\frac{1}{2 \epsilon_{0} c} \operatorname{Re}(\alpha) I \tag{2.2}
\end{equation*}
$$

where $I=\frac{1}{2} \epsilon_{0} c|E|^{2}$ is the laser field intensity, $\epsilon_{0}$ is the permittivity of free space, $c$ is the speed of light and the factor of $1 / 2$ takes into account the fact that the dipole is induced.

The out-of-phase part of the polarizability corresponds to the scattering of photons. The scattering rate (i.e. the rate of change of energy/the energy of a photon with frequency $\omega$ ) is

$$
\begin{equation*}
\Gamma_{s c}=-\frac{\langle\dot{p} \cdot E\rangle}{\hbar \omega}=-\frac{1}{\hbar \epsilon_{0} c} \operatorname{Im}(\alpha) I . \tag{2.3}
\end{equation*}
$$

The polarizability can be easily determined in the classical Lorentz model (re-
markably close to the quantum result for alkali atoms) to be

$$
\begin{equation*}
\alpha=\frac{e^{2}}{m_{e}} \frac{1}{\omega_{0}^{2}-\omega^{2}-i \omega \Gamma_{\omega}}, \tag{2.4}
\end{equation*}
$$

where $e$ is the charge of an electron, $m_{e}$ is the mass of an electron, $\omega_{0}$ is the resonant frequency and $\Gamma_{\omega}$ is the frequency dependent damping rate.

Replacing the frequency dependent damping rate,

$$
\begin{equation*}
\Gamma_{\omega}=\frac{e^{2} \omega^{2}}{6 \pi \epsilon_{0} m_{e} c^{3}} \tag{2.5}
\end{equation*}
$$

the polarizability is then found to be

$$
\begin{equation*}
\alpha=\frac{6 \pi \epsilon_{0} c^{3}}{\omega_{0}^{2}} \frac{\Gamma_{0}}{\omega_{0}^{2}-\omega^{2}-i \Gamma_{0}\left(\omega^{3} / \omega_{0}^{2}\right)} . \tag{2.6}
\end{equation*}
$$

Using a semi-classical treatment we can treat the atom as a two-level system interacting with a classical field, we obtain the same result, with the damping rate being determined by the dipole matrix element between the ground $|g\rangle$ and excited $|e\rangle$ states,

$$
\begin{equation*}
\left.\Gamma_{0}=\frac{\omega_{0}^{3}}{3 \pi \epsilon_{0} \hbar c^{3}}|\langle e| \mu| g\right\rangle\left.\right|^{2}, \tag{2.7}
\end{equation*}
$$

where $\mu=-e r$ is the dipole operator. Using the semi-classical model, the dipole potential and scattering rate are

$$
\begin{gather*}
U_{d i p}=\frac{3 \pi c^{2}}{2 \omega_{0}^{3}}\left(\frac{\Gamma_{0}}{\omega-\omega_{0}}+\frac{\Gamma_{0}}{\omega-\omega_{0}}\right) I \approx \frac{3 \pi c^{2}}{2 \omega_{0}^{3}} \frac{\Gamma_{0}}{\Delta} I,  \tag{2.8}\\
\Gamma_{s c}=\frac{3 \pi c^{2}}{2 \hbar \omega_{0}^{3}} \frac{\omega^{3}}{\omega_{0^{3}}}\left(\frac{\Gamma_{0}}{\omega-\omega_{0}}+\frac{\Gamma_{0}}{\omega-\omega_{0}}\right)^{2} I \approx \frac{3 \pi c^{2}}{2 \hbar \omega_{0}^{3}}\left(\frac{\Gamma_{0}}{\Delta}\right)^{2} I, \tag{2.9}
\end{gather*}
$$

where $\Delta=\omega-\omega_{0}$ is the detuning from resonance.
Far from resonance the polarizability is predominately real. If the detuning is positive then the atomic dipole lags the electric field by a phase $\pi$ and the energy
shift of the atom is positive. If the detuning is negative then the dipole is in-phase with the field, resulting in a negative energy shift.

In the far-detuned limit, the scattering rate and the energy shift are simply related by

$$
\begin{equation*}
\hbar \Gamma_{s c}=\frac{\Gamma_{0}}{\Delta} U_{d i p} \tag{2.10}
\end{equation*}
$$

If the light field is spatially dependent, then the energy shift is also spatially dependent. The change of energy with position acts as a force upon the atom,

$$
\begin{equation*}
\vec{F}_{d i p}=-\frac{d U_{d i p}}{d r} \widehat{r} . \tag{2.11}
\end{equation*}
$$

This force is generally referred to as a dipole force, due to its origin with the dipole interaction. If the detuning is negative (red detuned), then the energy shift is also negative, resulting in an attractive potential. If, on the other hand, the detuning is positive (blue-detuned) then the atom will be repelled from regions of high intensity. If the atom has lower kinetic energy than the size of the light shift, then it is possible to confine an atom using the dipole potential.

The simplest dipole trap that can be made is simply a focused, red-detuned laser beam. Such a trap was first demonstrated by Steve Chu et al. [3] to trap laser cooled sodium. Since the beam has a Gaussian intensity distribution, the atom responds as though it is in a Gaussian shaped potential. In the direction transverse to the beam propagation, the energy changes rapidly over the beam radius, creating tight confinement. The longitudinal direction has weaker confinement since the intensity decreases more slowly over the Rayleigh range.

### 2.3 Effects of Detuning in Real Atoms

The energy shift discussed so far is valid for a 2-level atom. Rubidium has a much more complicated structure than a 2-level atom. In the absence of external fields,


Figure 2.1: Energy level diagram for the Rb D2 resonance.
the ground state of rubidium- 85 has a hyperfine structure with total spin F that takes a value of 2 or 3 . Each hyperfine level has $2 F+1$ degenerate magnetic sublevels(see Appendix 1), ranging from $m_{F}=-F$ to $F$. The atom has excited states with $F^{\prime}$ ranging from 1 to 4 , each with $2 F^{\prime}+1$ magnetic sublevels. Figure 2.1 is the energy level diagram for the $\mathrm{D} 2(\mathrm{~J}=1 / 2$ to $\mathrm{J}=3 / 2)$ transition in Rb .

The energy shift of a particular ground state $\left|g_{i}\right\rangle$ is dependent on the coupling to all of the available excited states $\left|e_{j}\right\rangle$. The dipole matrix element between a pair of these states is

$$
\begin{equation*}
\mu_{i j}=\left\langle e_{i}\right| \mu\left|g_{j}\right\rangle=c_{i j}|\mu| \tag{2.12}
\end{equation*}
$$

where $|\mu|$ is the reduced matrix element and $c_{i j}$ is the Clebsch-Gordon coefficient, the
strength of the coupling between the states. Figure A. 1 in appendix A shows the relative transition strengths for the D 2 resonance in Rb . To determine the actual energy shift, we need to take into account the contributions to all the excited states to which the atom may couple.

$$
\begin{equation*}
\Delta E_{i}=\frac{3 \pi c^{2}}{2 \omega_{0}^{3}} \Gamma_{0} I \sum_{j} \frac{c_{i j}^{2}}{\Delta_{i j}} . \tag{2.13}
\end{equation*}
$$

At large detuning the relative energy shift for each sub-level is proportional to the Clebsch-Gordon coefficients. For small detunings the atom can have vastly different couplings to different excited states since it may be very close to resonance with some excited states and far from others. It may even have different signs of the detuning for different excited states.

If the detuning is much larger than the excited state hyperfine splitting, then the individual excited states are not resolved. In a linearly polarized field each magnetic sub-level then experiences the same energy shift. If the detuning is larger than the fine-structure splitting and the laser polarization is linear, then the dipole potential is given by the two-level atom result.

In the experiments described herein we use a lattice where the lasers are detuned 30 GHz from the D 2 resonance. 30 GHz is much smaller than the fine structure splitting of 7000 GHz , but larger than the hyperfine splitting of 3 GHz . We can therefore treat the atoms using their spin-orbit interaction.

In this detuning regime we ignore the nuclear spin and only consider contributions from the orbital and spin angular momentum. The nearest transition to the lattice frequency is the $5 S_{1 / 2}-5 P_{3 / 2}$ transition, that corresponds to transitions from $|L=0, S=1 / 2\rangle$ to the $|L=1, S=1 / 2\rangle$ manifold.

Figure 2.2 shows the transitions between the $\mathrm{J}=1 / 2$ and $\mathrm{J}=3 / 2$ manifolds, along with the relative transition strengths. The optical lattice is created using a pair of parallel linearly polarized beams. Therefore at all points in the lattice the local polarization is linear. The linear polarized light can be decomposed into an equal


Figure 2.2: Relative coupling strengths in the $\mathrm{J}=1 / 2$ to $\mathrm{J}=3 / 2$ manifold.
superposition of left-circular and right-circular beams. The atom experiences the same energy shift regardless of the magnetic ground state the atom is in. This is also true if the lattice is composed of $\pi$-polarized light.

### 2.4 Doppler Cooling

The original proposals for laser-cooling[1, 2] rely on the fact that moving atoms see laser beams Doppler shifted to different frequencies. The velocity-dependent cooling process came to be known as Doppler cooling for this reason. In lasercooling experiments the initial cooling mechanism is Doppler cooling, lowering the velocity of atoms from hundreds of meters per second down to several centimeters per second.

To begin, consider a two-level atom in a resonant laser beam travelling in the $+z$ direction. An atom initially in the ground state can absorb a photon from the beam. The absorption process promotes the atom up to the excited state and simultaneously changes the momentum of the atom by an amount equal to the recoil momentum $\vec{p}_{\text {rec }}=\hbar \vec{k}_{l}$, where $\hbar$ is Planck's constant and $\vec{k}_{l}$ is the wavevector of the incident laser photon. The corresponding kinetic energy of a single recoil is called the recoil energy $E_{r}=\left(\hbar k_{l}\right)^{2} / 2 m$. Following the excitation, the atom must decay back to the ground state by emission of a photon. The spontaneous emission of the photon is in a random direction, with the atom recoiling in the opposite direction from the emitted
photon. After $N$ such absorption and emission events the momentum kicks from the absorption events add coherently so the transferred momentum from absorption is equal to $\left\langle\Delta \vec{p}_{\text {absorb }}\right\rangle=N \vec{p}_{\text {rec }}$. The spontaneous emission events on the other hand are all in random directions such that the average momentum change is zero, i.e. $\left\langle\Delta \vec{p}_{\text {emiss }}\right\rangle=\overrightarrow{0}$.

Doppler cooling an atom (in 1-D) requires two beams travelling in opposing directions. If the two beams are of equal intensity and exactly on resonance, then an atom will be equally likely to absorb a photon from either beam regardless of the velocity of the atom. The atom is Doppler shifted equally far from resonance with either beam, so its probability of absorbing a photon from either beam decreases equally with detuning. To cool the atom requires some way to cause the atom to preferentially absorb a photon from the beam that opposes its motion. This may be easily accomplished by tuning the laser beams a little bit below resonance. The atom then sees the beam that it co-propagates with Doppler shifted to an even lower frequency, while the counter-propagating beam is seen at a higher frequency, therefore closer to resonance. The atom will then tend to absorb more photons from the counter-propagating laser beam than the co-propagating beam, therefore it gradually slows down.

The scattering rate for an atom is given by

$$
\begin{equation*}
\Gamma_{s c a t}=\frac{\gamma_{o}}{2} \frac{I}{I_{o}} \frac{1}{1+\frac{I}{I_{o}}+\left(\frac{2 \Delta}{\gamma_{o}}\right)^{2}}, \tag{2.14}
\end{equation*}
$$

where $\gamma_{o}$ is the natural linewidth, $I$ is the intensity, $I_{o}$ is the saturation intensity (the intensity at which the population difference is $1 / 2$ ) and $\Delta$ is the detuning from resonance.

By taking a time-average of the momentum transferred to the atom, one arrives at a force

$$
\begin{equation*}
\langle\vec{F}\rangle=\frac{\langle\Delta \vec{p}\rangle}{\langle\Delta t\rangle}=\langle\Delta \vec{p}\rangle \Gamma_{s c a t} \tag{2.15}
\end{equation*}
$$

For simplicity, we consider the limit where $\left(\frac{2 \Delta^{\prime}}{\gamma}\right)^{2} \gg \frac{I}{I_{o}}$ i.e. the normalized detuning from resonance is larger than the saturation parameter $\frac{I}{I_{o}}=S$. The scattering rate can then be simplified to

$$
\begin{equation*}
\Gamma_{s c a t}=\frac{\gamma}{2} \frac{S}{4 \Delta^{2} / \gamma^{2}}=\frac{\gamma^{3} S}{8 \Delta^{2}} \tag{2.16}
\end{equation*}
$$

Cooling is possible if there are a pair of beams with opposite directions of propagation.
We generalize the detuning by considering the Doppler shift of a moving atom,

$$
\begin{equation*}
\Delta^{\prime}=\Delta+\omega_{o} v_{a} / c=\Delta+\overrightarrow{k_{l}} \cdot \vec{v}_{a} \tag{2.17}
\end{equation*}
$$

where $\Delta$ is the detuning of the laser from resonance and $\vec{v}_{a}$ is the velocity of the atom. The time-averaged force on an atom in a pair of counter-propagating beams, where $\vec{k}_{l 2}=-\vec{k}_{l 1}$, expanded around zero velocity is given by

$$
\begin{align*}
\vec{F} & =\frac{\hbar \vec{k}_{l 1}}{m} \Gamma_{\text {scat } 1}+\frac{\hbar \vec{k}_{l 2}}{m} \Gamma_{\text {scat } 2}  \tag{2.18}\\
& =\frac{\hbar \vec{k}_{l 1}}{m} \frac{\gamma^{3} S}{8\left(\Delta+\vec{k}_{l 1} \cdot \vec{v}\right)^{2}}+\frac{\hbar \vec{k}_{l 2}}{m} \frac{\gamma^{3} S}{8\left(\Delta+\vec{k}_{l 2} \cdot \vec{v}\right)^{2}}  \tag{2.19}\\
& \approx-\frac{\hbar k_{l 1}^{2}}{m} \frac{\gamma^{3} S}{8 \Delta^{2}} \frac{4}{\Delta} \vec{v}  \tag{2.20}\\
& =-\beta \vec{v}, \tag{2.21}
\end{align*}
$$

where $\beta=\frac{\hbar k_{l 1}^{2}}{m} \frac{\gamma^{3} S}{8 \Delta^{2}} \frac{4}{\Delta}$ is a damping constant. The time-averaged force acting on the particle is always in a direction opposing the motion of the atom. This is a viscous force, which prompted the naming of this mechanism as optical molasses. The discussion so far has focussed solely on the result of stimulated absorption of photons from the laser beam. In reality the atom must always emit a photon after each absorption event. When the atom spontaneously emits, it does so with no preference for the direction. After many absorption/emission events the average momentum change due to emission is zero. While the emission doesn't lead to a change in velocity, it does lead to heating of the atom, with the heating rate being equal to
$\langle\dot{T}\rangle=\Gamma_{\text {scat }} \frac{(\hbar k)^{2}}{2 m}$. The temperature (the mean kinetic energy) of the atom cloud is determined by the balance between the heating rate and the cooling rate. The minimum achievable temperature through this technique is called the Doppler limit. It is given by $k_{B} T_{\text {Doppler }}=\hbar \gamma / 2$ at the optimal detuning of $\Delta=-\gamma / 2$, resulting in a Doppler temperature of 140 microKelvin for rubidium.

When this result was experimentally tested it was found that the actual temperatures achieved were nearly two orders of magnitude lower[4] than the Doppler limit. It turns out that the two-level atom approximation is too simplistic and the internal level structure of the atom creates new cooling mechanism

### 2.5 Sisyphus Cooling

The usual model used for sub-Doppler cooling is that of Sisyphus cooling[5], also called a lin $\perp$ lin molasses. This technique obtained its name for its analogy to the mythical figure Sisyphus, who was doomed to forever push a boulder up a hill only to have it suddenly transported back to the bottom. Sisyphus cooling comes about in a system where the molasses beams in 1D have orthogonal linear polarizations. We can decompose the linear polarizations into a superposition of left-circular and right-circular polarizations. We find that the two laser beams create an interference pattern such that the polarization of the electric field varies through space,

$$
\begin{equation*}
\vec{E}(z)=E_{0}\left(e^{i(k z-\omega t)} \widehat{x}+e^{i(-k z-\omega t)} \widehat{y}\right) . \tag{2.22}
\end{equation*}
$$

When we evaluate the polarization at a number of points we find that the polarization varies extremely rapidly in space, going from horizontal, to left circular to vertical to right circular within half a wavelength as seen in Figure 2.4. An atom in an electric field has a Stark shift (sometimes called a light shift) of its energies. In the weak field limit considered here the shift of the energy is proportional to the intensity of the electric field and is also dependent on the polarization and the magnetic sub-
level of the atom. Figure 2.3 shows the relative coupling strengths for a transition from a $J=1 / 2$ to $J^{\prime}=3 / 2$ state. The size of the Stark shift is proportional to the square of the Clebsch-Gordon coefficient.


Figure 2.3: Clebsch-Gordon Couplings for a $\mathrm{J}=1 / 2$ to $\mathrm{J}=3 / 2$ transition

An atom that remains in one magnetic sub-level has a rapid change of its light shift over a half-wavelength of travel. The resulting light shift potentials are given in Figure 2.4 along with the model of cooling. Consider an atom with $m_{j}=\frac{-1}{2}$ that travels on the potential described by the solid line. At $z=\lambda / 8$, the atom is in its lowest energy state, trapped in a well with $\sigma^{-}$polarization. If the atom absorbs a $\sigma^{-}$ photon it is excited to a state $m_{j}^{\prime}=-3 / 2$, from which it can only decay to $m_{j}=\frac{-1}{2}$ with no change in energy. If, on the other hand the atoms moves to a position b , then the local light polarization is mostly $\sigma^{+}$. If the atom absorbs a photon it is excited to a state $m_{j}=\frac{1}{2}$, from which it can decay to $m_{j}=\frac{1}{2}$ or $m_{j}=\frac{-1}{2}$. If it decays to $m_{j}=\frac{-1}{2}$ then no change in energy takes place; if on the other hand the atoms decays to $m_{j}=\frac{1}{2}$ (point c) then it loses energy, which is carried off by the photon. This cooling process can continue until the atom is no longer able to traverse the barrier between the wells. The final energy of the atom is then on the order of the well depth. Once the well depth becomes as small as a recoil energy then the process breaks down as the spontaneous emission imparts kinetic energy to the atom.


Figure 2.4: Sisyphus Cooling Process

Typically, an atom cloud can be cooled to a temperature on the order of $10 E_{r}$, or about $6 \mu K$ for rubidium.

### 2.6 Corkscrew Molasses

The technique used to initially cool and trap atoms is the Magneto-Optical Trap (see section 2.7), which uses $\sigma^{+}$and $\sigma^{-}$polarized beams. In practice few groups will opt to cool the atoms further using Sisyphus cooling since it requires lasers with different polarizations than are used for the MOT. It turns out that the circularly polarized beams can cool atoms nearly as well as Sisyphus cooling, although the cooling process is very different.

The interference of the circularly polarized beams causes the polarization to be linear everywhere, but it rotates as a function of position, undergoing a full rotation every half wavelength as shown in Figure 2.5.


Figure 2.5: Polarization rotation in a $\sigma^{+} \sigma^{-}$molasses

The simplest model to illustrate the cooling process is a $J=1$ to $J^{\prime}=2$ transition, first discussed by Dalibard and Cohen-Tannoudji[5]. The energy level shift for an atom with $m_{j}=0$ is larger than the energy shift for the $m_{j}= \pm 1$ states. Due to optical pumping from the laser, the magnetic sub-level populations of a stationary atom eventually approach a steady-state distribution that has the highest population in the $m_{j}=0$ state and equal populations in the the states $m_{j}= \pm 1$.

If the atom is moving then the quantization axis rotates as a function of time, and the atom must be optically pumped for the population distribution to follow the quantization axis. The population distribution then lags the steady state distribution that is appropriate to the local polarization. It turns out that an atom travelling to the right will have a larger population in the $m_{j}=-1$ state than in the $m_{j}=+1$ state due to the lag. The atoms then has a greater probability to absorb a photon from the $\sigma^{-}$beam, thereby damping its velocity. This cooling process is unlike Sisyphus cooling since the energies are independent of position, but similar to Doppler cooling since the atom will likely absorb photons from the beam that opposes its motion. It has a cooling limit similar to the limit of Sisyphus cooling.

### 2.7 Magneto-Optic Trap

The techniques described so far all have to do with cooling atoms, reaching temperatures as low as several microKelvins. These techniques, however, are independent of position and can not spatially trap or localize atoms. A spatial restoring force can be added to the system by adding a magnetic field gradient[25]. Consider a $J=0$ to $J^{\prime}=1$ transition with atoms in a 1-D corkscrew molasses with an additional magnetic field gradient such that the x-component of the magnetic field is $\overrightarrow{B_{x}}=B^{\prime} \vec{x}$. The magnetic field Zeeman-shifts the energy levels of the atoms according to, $\Delta E=-\vec{\mu} \cdot \vec{B}=m_{j} g_{j} \mu_{B}|B|$, where $\mu_{B}$ is the Bohr magneton.


Figure 2.6: Principle of Magneto-Optical Trapping

Figure 2.21 shows the shift in the energies as a function of position. The lasers used for cooling are detuned below resonance by several linewidths. On the far right of the origin the $\sigma^{+}$polarized laser is much closer to resonance with the transition from $\left|J=0, m_{j}=0\right\rangle$ to $\left|J=1, m_{j}=1\right\rangle$ than the $\sigma^{-}$laser is with the $\left|J=1, m_{j}=-1\right\rangle$ transition. The atom is then much more likely to absorb a photon from the right-
travelling $\sigma^{+}$polarized laser. The atom will then get a momentum kick $\hbar \vec{k}$ that will push it towards the origin. Averaging over many absorption events, this acts like a linear restoring force on the atom.

A simple model of this mechanism can be added to the Doppler cooling model by modifying the detuning to include a spatially-dependent detuning resulting from the Zeeman shift:

$$
\begin{equation*}
\Gamma_{s c a t}=\frac{\gamma_{o}}{2} \frac{I}{I_{o}} \frac{1}{1+\frac{I}{I_{o}}+\left(\frac{2(\Delta+\vec{k} \cdot \vec{v}+\vec{\mu} \cdot \vec{B}}{\gamma_{o}}\right)^{2}} . \tag{2.23}
\end{equation*}
$$

After following a similar treatment as we performed for 2.21 we arrive at a formula for the time-averaged force

$$
\begin{equation*}
\vec{F}=-\beta \vec{v}-\eta \vec{x}, \tag{2.24}
\end{equation*}
$$

where $\eta$ is a spring constant. This equation has the form of a damped oscillator, and we would expect all the atoms to come to rest at the origin. Much like the result for Doppler cooling, this only equation includes effects of absorption, and excludes the diffusive effect of spontaneous emission. With a standard MOT, atoms can typically be trapped in a cloud with an rms size of about 1 millimeter. The temperature of the atom cloud will be around 50 to $70 \mu K$, several times hotter than a molasses, but colder than achievable by just Doppler cooling. The temperature reflects the fact that some $\sigma^{+} \sigma^{-}$molasses cooling does occur in a MOT, but is not as effective as a pure molasses because of the added magnetic fields that Zeeman-shift the energies.

The cooling/trapping mechanism here is easily extended to 3D by using a pair of anti-Helmholtz coils (a Helmholtz configuration, but with the currents rotating in opposite directions.) The field produced by a pair of these coils oriented along the z-axis can be expanded near the origin (the midpoint between the coils) as

$$
\begin{equation*}
\vec{B}(x, y, z)=\frac{B^{\prime}}{2}(-x \widehat{x}-y \widehat{y}+2 z \widehat{z}) . \tag{2.25}
\end{equation*}
$$

The field along the coil axis has twice the gradient as the other two dimensions. Typical gradients used for cooling and trapping are on the order of 10 Gauss $/ \mathrm{cm}$. This magneto-optical trapping process is at the heart of all laser-cooling experiments as it combines both spatial trapping and provides cooling through the built-in polarization rotation molasses.

### 2.8 Magnetic Trapping

Trapping using a MOT is a dynamic situation where the atom constantly absorbs photons in order to remain trapped. Sometimes, it is desirable to trap the atoms in the absence of light, which can be accomplished with a magnetic field.

The interaction of an atom with a magnetic field shifts the energy of the atom according to

$$
\begin{align*}
E & =-\vec{\mu} \cdot \vec{B}  \tag{2.26}\\
& =-g_{F} \mu_{B} \vec{F} \cdot \vec{B}  \tag{2.27}\\
& =-g_{F} m_{F} \mu_{B}|B| \tag{2.28}
\end{align*}
$$

where $g_{F}$ is the gyromagnetic factor, $\mu_{B}$ is the Bohr magneton and $m_{F}$ is the projection of the spin $\vec{F}$ along the magnetic field direction. In a quadrapole trap atoms that are aligned with the local magnetic field will move toward the origin (the middle of the axis between the coils). However, the atom will typically be moving in 3 dimensions, and as such will see the direction of the magnetic field change with position. The velocity of the atom is slow enough that the spin of the atom can adiabatically follow the direction of the magnetic field, thereby remaining aligned. The potential experienced by the atoms is then:

$$
\begin{align*}
U(x, y, z) & =-g_{F} \mu_{B} \vec{F} \cdot \vec{B}  \tag{2.29}\\
& =\frac{-g_{F} m_{F} \mu_{B} B^{\prime}}{2} \sqrt{x^{2}+y^{2}+4 z^{2}} \tag{2.30}
\end{align*}
$$

An atom in a particular sub-level will move toward either high (high field seeker) or low (low field seeker) magnetic fields, depending upon the sign of the gyromagnetic factor and m . For an atom to be trapped it must be in a low-field seeking state. Atoms in high-field seeking states are quickly ejected from the trap.

A problem arises if the atom travels through the origin of the magnetic trap. The magnetic field becomes zero at the origin and the atomic spin can no longer adiabatically follow the magnetic field. The atom can then undergo an unwanted 'Majorana flip' and end up in a high-field seeking state, thereby becoming ejected from the trap.

Practical considerations place constraints on the magnetic field gradients that can be used. Gravity adds an extra potential gradient in the vertical dimension and must be compensated for by the magnetic gradient. For rubidium atoms this sets a lower limit to the usable gradient to trap a doubly polarized $\left(F=3, m_{F}=3\right)$ atom to be $15 \mathrm{G} / \mathrm{cm}$ in the vertical direction.

## Chapter 3

## Delta-Kick Cooling

Advances in laser cooling and atom optics have led both to the ability to directly observe new physical effects (e.g., in connection with weakly interacting Bose gases) and to develop new, potentially applicable technologies (such as atom-wave gravimetry). There has been a great deal of interest in paving the way for future developments by studying atom optical components such as mirrors, lenses, and beam-splitters $[6,7,26,27,28]$. Often, these components themselves rely on new and interesting physical effects, which is one of the explanations for the self-sustaining excitement in this field. In this chapter, we show experimentally how pulsed magnetic fields may be used in situ as atom lenses, and discuss their use for cooling and for Stern-Gerlach measurements.

Many interesting studies of the quantum motion of atoms should be possible in one-dimensional systems[29, 30, 31]. For this reason, we are investigating methods of achieving low one-dimensional temperatures; while the lowest temperatures ever achieved have been obtained using evaporative cooling, this method is expensive in terms of atom number. To cool in one dimension alone, techniques such as velocity selection can reach the same temperatures while retaining a greater fraction of the atoms. In addition, "adiabatic" cooling methods such as "delta-kick cooling" are suitable for many experiments in which phase-space density is not a consideration.

In a scheme based loosely on Ammann and Christensen's "delta-kick cooling" proposal[32], also similar to independent proposals by Chu et al., [33] and Summhammer et al.[34] , we have used time-dependent magnetic forces to cool atoms in one dimension. While it is well-known[35] that time-independent conservative potentials cannot increase phase-space density (it has recently been shown that adiabatically changing the shape of the potential can increase phase-space density [36, 37]), temperature may nevertheless be reduced at the expense of giving up spatial density. We have achieved temperatures below 700 nK (a factor of 10 below the temperature of our molasses), and much lower temperatures are possible in principle. The technique is also easily generalizable to three dimensions. We have observed unexpected effects when these magnetic kicks are applied to atoms from an optical molasses. They appear to suggest that within an optical molasses, there is a correlation between atomic spin and position and/or velocity. We believe that in addition to the usefulness of magnetic kicks for cooling and focusing, this Stern-Gerlach-like effect will prove to be an important probe of the atomic spin in laser-cooled atom clouds.

In the original "delta-kick cooling" proposal, Ammann and Christensen suggested an approach in which an atomic wavepacket prepared in a minimum-uncertainty state within an optical lattice is first allowed to expand freely for a short period to allow position to become correlated with momentum. Application of a position-dependent force from a pulsed standing wave can then be used to reduce the mean momentum of the atoms. Near the bottom of the potential well the atoms experience an essentially harmonic potential. As the atoms expand beyond this harmonic region the proposed cooling process breaks down. Instead of a sinusoidal potential, we consider a true harmonic potential, easily generated with magnetic field coils. This variant is not only easier to understand and to implement; it is also immune to the original proposal's cooling limit. Although our implementation does a better job of cooling, the original proposal has advantages for increasing phase-space density. Cooling in a periodic potential decreases the momentum distribution, while not giving up spatial density,
resulting in an increase in phase-space density[38].

### 3.1 Cooling Model

Consider an ensemble of laser-cooled atoms initially confined within a small region of size $r_{o}$ with mean thermal velocity $v_{o}$. At time $t=0$ the trap is turned off and atoms are allowed to freely expand away from the trap center. After a time $t_{\mathrm{f}}$, long compared to $r_{o} / v_{o}$, the atoms are located at positions essentially given by

$$
\begin{equation*}
\vec{x}_{a}=\vec{v}_{a} t_{\mathrm{f}}, \tag{3.1}
\end{equation*}
$$

where $\vec{x}_{a}$ and $\vec{v}_{a}$ refer to the position and velocity of an individual atom. Application of a harmonic potential,

$$
\begin{equation*}
U(x)=\frac{1}{2} m \omega^{2} x^{2} \tag{3.2}
\end{equation*}
$$

for a short time will apply an impulse to the atoms proportional to the position;

$$
\begin{equation*}
\Delta \vec{p}_{a}=-m \omega^{2} \vec{x}_{a} t_{\mathbf{k}} \tag{3.3}
\end{equation*}
$$

where $t_{\mathrm{k}}$ is the duration of the kick. If this impulse is chosen to be equal to $-m \vec{v}_{a} \approx$ $-m \vec{x}_{a} / t_{f}$, then the atom in question is stopped. Thus for $t_{\mathrm{f}} t_{\mathrm{k}}=1 / \omega^{2}$, all atoms will essentially be brought to rest.

In reality several factors place limits on the achievable temperature. At time $t=0$ the atoms are not all located at the center of the trap, but instead have some distribution of initial positions. As the ratio between the final size to initial size increases, the correlation between position and momentum improves, allowing a greater degree of cooling. At any finite time, the correlations will be imperfect, preventing the cooling from being optimal. Practical considerations may limit the time for which the atoms can be allowed to expand.

After free expansion, the typical atom is at a position on the order of

$$
\begin{equation*}
r_{f}=\sqrt{r_{0}^{2}+\left(v_{0} t\right)^{2}} \tag{3.4}
\end{equation*}
$$

and has a velocity on the order of $v_{0}$. Each individual velocity class has a spread in position given by the initial size $r_{0}$ of the cloud. A transferred impulse proportional to displacement, and designed to cancel out the mean velocity of $v_{0}$ at a typical distance of $r_{f}$, will therefore transfer a random velocity on the order of $v_{0} r_{0} / r_{f}$ to each velocity group of atoms. Indeed, a more careful phase-space treatment given below shows clearly that the rms velocity of the kicked atoms decreases by a factor of $r_{0} / r_{f}$, leading to a temperature reduction of

$$
\begin{equation*}
\left(r_{0} / r_{f}\right)^{2} . \tag{3.5}
\end{equation*}
$$

In other words, this technique is equivalent to adiabatic expansion. At a practical level, however, it has the advantage that there is no adiabatic criterion to meet. To cool a cloud by a factor of $N$ would require a time of $\sqrt{N} \tau_{0}$ where $\tau_{0}$ is the period of a harmonic-oscillator potential matched to the atom cloud (i.e., the cloud radius divided by the atoms' rms velocity). A similar adiabatic expansion would need to be accomplished slowly relative to the instantaneous period ( $N \tau_{0}$ by the end of the expansion).

The harmonic-kick cooling process can be readily understood in a phase-space picture. Figure 3.1 shows the evolution of an atomic cloud in phase space from the initial state at the start of expansion, to after free expansion and finally, the distribution after application of a kick. We start with a phase-space distribution characterized by the widths of the distribution in momentum and position. Free expansion stretches the distribution in position, but has no effect on momentum. The effect of a harmonic kick is to rotate the distribution in phase space. If the duration of the kick is chosen correctly, this rotates the distribution back onto the position axis, thereby lowering the temperature. Note that this does not require a true


Figure 3.1: Model for cooling with a harmonic pulse.
"delta-kick" in the sense of a very short duration; over any time-scale the effect of a harmonic-oscillator potential is to rotate the phase-space distribution. By Liouville's theorem, the area in phase space is conserved both during free expansion and during the kick, yielding

$$
\begin{equation*}
v_{f}=v_{0} x_{0} / x_{f} \tag{3.6}
\end{equation*}
$$

The longer the cloud is allowed to undergo free expansion before the kick, the narrower the final distribution is in momentum, resulting in lower temperatures.

Harmonic potentials are not unique in their ability to cool atoms. The original delta-kick cooling proposal[32] relied on sinusoidal potentials. Unfortunately, that effect was only productive insofar as the sinusoid could be approximated harmonically, placing a severe constraint on the degree of cooling possible. We have observed that a pulsed quadrupole potential may also be used to cool atoms. Although this technique cannot match the ideal harmonic kicks, it is worth studying the effects of such field configurations, which are in more widespread use and can more easily generate strong forces. In 1D, a quadrupole field exerts a fixed impulse toward the center of the potential. If the atoms have been allowed to expand significantly, most are moving away from the center, and a kick chosen to be equal to the mean thermal
velocity concentrates the atoms near $\mathrm{v}=0$ (in a highly non-thermal, nearly flat-topped distribution). In 1D, the mean kinetic energy may be reduced by up to a factor of 6 in this way.

### 3.1.1 Naive expectations- Linear gradient

The delta-kick is implemented by applying a short duration pulse from the quadrupole field coils. The magnitude of the magnetic field near the origin is

$$
\begin{equation*}
|B(x, y, z)|=\frac{B^{\prime}}{2} \sqrt{x^{2}+y^{2}+4 z^{2}} \tag{3.7}
\end{equation*}
$$

If we only consider the motion of the atom along the z -direction, then it seems reasonable to set $x=0$ and $y=0$. The magnitude of the magnetic field is then

$$
\begin{equation*}
|B(z)|=B^{\prime}|z| \tag{3.8}
\end{equation*}
$$

This is a linear potential gradient, with a resulting force that is constant in magnitude and changes sign over the origin. For a spin polarized atom this force is $\vec{F}_{z}=-\mu B^{\prime} \widehat{z}$ where $\mu$ is the magnetic moment of the atom. An optimal kick would then cause a shift (or shear) in the phase-space distribution that would center the distribution on the momentum axis as shown in Figure 3.2

Cooling with this form of potential could, at best, reduce the mean kinetic energy of the cloud by a factor of 6 . The resulting clouds would have a distinctly non-thermal velocity distribution.

### 3.1.2 Rounding of the Potential

Simulations for a true 3D quadrupole potential show even greater cooling due to the "rounding" of the potential energy surface when transverse excursions are taken into account. However, thermal expansion is not the only cause of transverse excursions. At sufficiently late times, gravity causes the entire cloud to fall below the


Figure 3.2: Phase-space evolution with a linear gradient.


Figure 3.3: Shape of potential (red) and force (blue) in a conic section
apex of the conical quadrupole potential. The transverse potential seen by the cloud thus looks like a conic section, e.g., a parabola.

Near the origin, the force is linear with position, as shown in Figure 3.3. The longer the cloud is allowed to fall before applying the quadrupole field, the larger the linear region of the force, implying greater cooling.

Thus even in a quadrupole potential, transverse cooling may be accomplished by an effective harmonic force. Cooling is predominantly along the coil axis but some cooling is achieved along the other two directions. At appropriately chosen times, this force may be much larger than the force we could create by reversing the current
in one of our MOT coils to produce a three-dimensional harmonic potential. The kick created by an anti-Helmholtz coil pair has the additional advantage that unlike a quadratic field gradient, the quadrupole field vanishes at the center of the trap. This reduces the danger of stray effects due to large bias fields as the coils are turned off.

The experimental data presented all concern one-dimensional cooling using quadrupole kicks. However, these techniques can also be generalized to provide cooling in all three directions. Harmonic kicks can easily be generalized to 3D by application of successive kicks along the three Cartesian axes. With quadrupole kicks, perfect spherical symmetry cannot be achieved, but may be approximated using a similar approach.

### 3.2 Experimental results

Our MOT coils are oriented such that the coil axis is horizontal and defines the z-axis of the trap. The coils are made of 200 turns of wire, with a coil radius of 4 cm and the coils are separated by 8 cm . With the currents in opposing directions as used in a MOT or a quadrupole kick, the coils can produce gradients up to 180 $\mathrm{G} / \mathrm{cm}$ given our present maximum current of 18 A . These same coils can alternatively be used for generation of a harmonic potential. The coil separation is twice the separation required for a Helmholtz configuration, leading to a nonvanishing $d^{2} B / d z^{2}$. By reversing the direction of current in one of the coils we can achieve harmonic fields with a second derivative of about $60 \mathrm{G} / \mathrm{cm}^{2}$, corresponding to a trap frequency of about 60 Hz . The coils can be switched on and off in $200 \mu \mathrm{~s}$, residual fields falling to $1 \%$ in 1 ms .

We start by cooling and trapping ${ }^{85} \mathrm{Rb}$ atoms within our MOT using a field gradient of $20 \mathrm{G} / \mathrm{cm}$, and 40 mW of total power in trapping beams with 2 cm diameter. The trapping beam is detuned 10 MHz to the red of the $\mathrm{D} 2(F=3 \Rightarrow F=4)$ tran-
sition at 780 nm . We cool and trap about $10^{8}$ atoms in our MOT with a diameter of 0.5 mm . We further cool the atoms to about $6 \mu \mathrm{~K}$ in 1 millisecond of $\sigma^{+} \sigma^{-}$optical molasses detuned 34 MHz to the red of the $F=3 \Rightarrow F=4$ transition.

We then turn off all light and magnetic fields to allow the atoms to undergo free expansion. After a time of 9 to 15 ms we apply a short ( 3 ms ) pulse of the quadrupole field. The resulting force is directed towards the origin, mostly along the symmetry axis of the coils, and applies an impulse to the atoms opposing their direction of motion. The atoms ballistically expand for some time after which we apply a short ( 1 ms ) burst of resonant light, causing the atoms to fluoresce. The spatial distribution of the atoms is captured on a CCD camera. The size of the cloud is determined by fitting it to a Gaussian distribution. The temperature of the resulting cloud is determined by time-of-flight imaging. A series of images is taken as a function of time after the kick, and the temperature is extracted from the expansion curve.

### 3.2.1 Delta-kick cooling on spin-polarized atoms

The existence of multiple spin levels in atoms released from an optical molasses implies that different atoms experience different magnetic potentials, making spatial distributions of unpolarized atoms difficult to interpret. For this reason we select the atoms in a doubly polarized state, $F=3, m_{F}=3$, by capturing the atoms within a weak magnetic trap that is unable to hold atoms with $m_{F}<3$ against gravity.

We release the doubly polarized atoms from the magnetic trap after 200 ms and allow the $m_{F}=3$ atoms to expand freely before application of the quadrupole magnetic field.

Figure 3.4 shows fluorescence images of the atoms under free expansion and after application of a delta kick. The atoms released from the magnetic trap have a temperature of $7.5 \mu K$ with an rms velocity of $2.7 \mathrm{~cm} / \mathrm{s}$. The atoms are first allowed to undergo free expansion for a time of 11 ms and then have a kick applied for 3 ms .


Figure 3.4: Delta-kick applied to spin-polarized atoms

For atoms along the symmetry axis, this 3-ms kick imparts a change in velocity of $\delta v \equiv \mu_{B} t_{k} \frac{d B}{d z} / m=3 \mathrm{~cm} / \mathrm{s}$. After application of the kick, the temperature of the cloud along the coil axis has decreased by a factor of about 6 , from $7.5 \mu \mathrm{~K}$ down to $1.2 \mu \mathrm{~K}$. Expansion curves for a cloud are shown in Figure 3.5 for several values of the kick strengths $\delta v$. The thick solid line shows the free expansion of the cloud. The thin solid line, dotted line and dash-dot line represent kicks of $2.4 \mathrm{~cm} / \mathrm{s}, 3.6 \mathrm{~cm} / \mathrm{s}$, and $4.8 \mathrm{~cm} / \mathrm{s}$ respectively. For a kick of $2.4 \mathrm{~cm} / \mathrm{s}$, near the original rms velocity of the atoms, we observe optimal cooling. Over 20 ms , essentially no expansion of the atom cloud is seen, and fits indicate a temperature below 700 nK . For stronger kicks, the atoms are impelled back to the center of the cloud, where they come to a focus. As the strength of the kick increases, the atoms come to a smaller focus (consequently heating the atom cloud), until the strongest kick strength results in a cloud hotter than the cloud from the original magnetic trap. This demonstrates the utility of the effect as an atom lens. Extrapolating backwards along each trajectory, we find that the trajectories appear to intersect at time $\mathrm{t}=0$, as expected.

At times so short such that the radius of the cloud has not increased significantly, the positions and momenta of the atoms are not very well correlated and the kick does little to cool the atoms. In Figure 3.6 we show results (solid circles) and simulations (dashed curve) for the cooling ratio versus the ratio of final size to initial size. This ratio is a measure of how long the atoms are allowed to freely expand before the kick is applied. For each time, the kick strength is optimized to reach the lowest temperature, independently in the simulations and in the experiment. The temperatures obtained experimentally were lower than the initial temperature by as much as a factor of 10 , even at times when the simple simulations (which neglect gravity) predicted only a factor of 5 . The solid line is the result of a simulation that includes gravity.

As explained earlier, the inclusion of gravity causes the atoms to fall below the center of the potential, to positions where they experience a transverse harmonic potential, and hence more effective cooling. However, the magnitude of the transverse


Figure 3.5: The ballistic expansion of spin-polarized atom clouds.


Figure 3.6: Temperature reduction vs expansion ratio
force also decreases as the atoms fall below the center of the quadrupole potential. For this reason, simulations predict that the lowest temperatures will be reached using a somewhat stronger field than would be expected in the absence of gravity. Figure 3.7 shows the cooling ratio as a function of quadrupole kick strength (in units of the rms thermal velocity $v_{0}$, which is expected to yield ideal cooling in the absence of gravity). As predicted, the best cooling ratios are achieved for higher kick strengths than in the naïve model. The solid line shows the optimal cooling ratio for the given kick strength. The data points show a lot of scatter, since the cooling ratio depends on the initial conditions of the cloud as well as the kick strength.

Furthermore, the solid curve in Figure 3.6 shows the simulations after gravity is included. Much lower temperatures are clearly obtained; with certain parameters, cooling by as much as a factor of 30 is predicted at long times.


Figure 3.7: Final temperature over initial temperature as a function of the kicking strength.

### 3.2.2 Delta-Kicks from optical molasses

Kicks have also been performed upon atoms from an optical molasses. Since there are 7 different spin states in the $\mathrm{F}=3$ ground state of Rb , we expect the different states to undergo different amounts of cooling or heating. We therefore expect to see a multimodal distribution, in which each spin component expands at a different rate following the kick. Figure 3.8 shows the results of a kick on atoms released from optical molasses. The upper set of expansion images shows free expansion and the lower set displays the atom cloud after a kick that was designed to cancel out the thermal velocity for $m_{F}=3$ atoms. After the kick, the cloud separates as expected into a bi-modal distribution consisting of a very cold central stripe and a broader, hotter background. Surprisingly, even in the absence of focusing, the central stripe is significantly smaller than the size of the atom cloud at the time of kick, as seen clearly in Figure 3.9. We believe this may indicate a pre-existing correlation between an


Figure 3.8: Free expansion and kicked atoms from a molasses.
atom's spin state and its position and/or velocity in optical molasses. For instance, atoms in a molasses may tend to be in low-helicity states, implying that atoms with $m_{z}=3$ have their velocities primarily in the $x-y$ plane.

The use of pulsed field gradients for Stern-Gerlach-like analysis of atom clouds appears to be a very promising new probe for such phenomena.

Through the use of these kicks we have achieved temperatures as low as 700 nK both on atoms released from a magnetic trap and on a subset of the atoms released from an optical molasses. Cooling ratios as large as a factor of 10 have been observed. The effect of gravity may be used to enhance cooling, but also limits this technique for certain applications. Since the atoms begin to fall due to gravity, they are no longer


Figure 3.9: Cloud size vs. time for various kicks from molasses.
located at the origin of the potential, and cannot be recaptured without heating. The gain in cooling may still be useful for experiments in which the atoms are allowed to fall for some distance, such as atomic mirrors, lensing and atom deposition systems. Along these lines Maréchal et al.[39] have studied a system where inhomogeneous magnetic potentials separate the spin components of a cloud and provide cooling as the cloud falls a distance of about a meter. The addition of a 1D optical lattice or anti-gravity field will allow a larger free-expansion time without having the atoms fall from the center of the trap, to obtain cooling greater than a factor of 10.

This technique is easily extendable to harmonic potentials and to three dimensions. Moreover, it should prove useful as an atom lens and for Stern-Gerlach studies of atom clouds. Already, it suggests a previously unnoticed dependence of the center-of-mass motion of optically cooled atoms on the magnetic quantum number

## Chapter 4

## Optical Lattices

By interfering sets of laser beams, it is possible to create sinusoidal interference patterns, leading to sinusoidal potentials. The term optical lattice is applied to such a structure because it can trap and organize atoms in a crystal-like structure. The behaviour of an atom in an optical lattice is analagous to the behaviour of an electron in a solid-state crystal. An optical lattice has several distinct differences from a solid state crystal. The optical lattice has a lattice spacing on the order of the wavelength of light instead of the interatomic spacing of angstroms. The depth of the potential may be easily modulated on a timescale fast compared to the motion of the atom. The shape of the potential may be readily modified using external fields, while remaining defect-free. In fact, many groups have used optical lattices to demonstrate effects predicted for solid-state physics, such as Bloch oscillations[40] and Wannier-Stark ladders[41], that had been observed in solid state superlattices[42, 43] only several years earlier.

An optical lattice may be easily created using inexpensive, commercially available diode lasers. The potential depth of the lattice is inversely proportional to the detuning of the laser from the atomic resonance. It would seem that the strongest optical lattice could be achieved by detuning very close to the atomic resonance, but this increases the probability for an atom to absorb and spontaneously emit a photon.

Unfortunately, the limiting factor in most optical lattices is laser power, so a balance must be achieved between acceptable scattering rates and well depth.

### 4.1 Dipole Force and a 1-D Lattice

The dipole force experienced by an atom in an electric field is proportional to the gradient of the intensity, which in turn is limited by the characteristic distance scale of the potential. Using laser beams, the smallest structures we can create are by interfering a pair of counter-propagating laser beams. The interference pattern created in this method is given by $\sin ^{2}\left(k_{L} x\right)$ where $k_{L}$ is the wave-vector of the beams. The interference pattern is periodic with a spacing of $\lambda / 2$. If the phases of the beams creating the lattice are stable, then the position of the lattice will also be stable in space. The atoms feel a spatially varying light shift from the interference pattern. If the detuning of the lasers is negative, then the atoms can be trapped at the anti-nodes of the interference, and at the nodes if the detuning is positive. The potential (for large detuning) is then given by

$$
\begin{equation*}
V(x)=U_{0} \sin ^{2}\left(k_{L} x\right) \tag{4.1}
\end{equation*}
$$

The trap frequency of the lattice can be found by approximating the lattice around the potential minima by an expansion. Expanding near $x=0$, we get

$$
\begin{equation*}
V(x) \approx U_{o} k_{L}^{2} x^{2} \tag{4.2}
\end{equation*}
$$

Using this harmonic approximation, we find

$$
\begin{equation*}
V(x)=\frac{m}{2} \omega^{2} x^{2}=U_{o} k_{L}^{2} x^{2} . \tag{4.3}
\end{equation*}
$$

Recall that the recoil energy (kinetic energy imparted to a stationary atom by absorbing one photon) is given by

$$
\begin{equation*}
E_{r}=\hbar^{2} k_{L}^{2} / 2 m=\hbar \omega_{r}=\frac{k_{B} T_{r}}{2} \tag{4.4}
\end{equation*}
$$

where $\omega_{r}=2 \pi \times 3.92 k H z$ is the recoil frequency, and $T_{r}=360 \mathrm{nK}$ is the recoil temperature. Substituting for $k_{L}^{2}$ in equation 4.3 we find the characteristic trap frequency

$$
\begin{equation*}
\omega=2 \omega_{r} \sqrt{\frac{U_{0}}{E_{r}}} \tag{4.5}
\end{equation*}
$$

In the experiments described in this dissertation, the lattices are not created with counter-propagating beams. Instead, the lattice is created by interfering lasers that have a 49.6 degree angle between the propagation directions. The interference pattern created has the same intensity as a lattice of counter-propagating beams, but the spatial period of the lattice isn't given by the wavenumber of the laser, but by its projection onto the plane orthogonal to the plane bisecting the wave-vectors. The lasers used here are $\pm 24.8$ degrees away from horizontal, the magnitude of the resulting effective k -vector is $k_{e}=k_{L} \sin (\theta / 2)$, where $\theta$ is the full angle between the beams. The resulting potential is then proportional to $\sin ^{2}\left(k_{e} z\right)$, which has a periodicity of $\frac{\lambda_{e}}{2}=\frac{\lambda_{L}}{2 \sin (\theta / 2)}=0.93$ microns. The effective recoil frequency for this lattice may be determined in a similar way, giving $\omega_{r e}=\omega_{r} \sin ^{2}(\theta / 2)=2 \pi \times 685 \mathrm{~Hz}$.

### 4.2 Band Structure

To get a more complete understanding of the energy levels in a lattice it is necessary to determine the band structure using techniques developed for solid state theory. The Hamiltonian used to determine the eigenstates is given by

$$
\begin{equation*}
H \Psi=\left(-\frac{\hbar^{2}}{2 m} \frac{d^{2}}{d x^{2}}+U_{0} \sin ^{2}(k x)\right) \Psi=E \Psi \tag{4.6}
\end{equation*}
$$

Bloch's theorem states that when the Schrödinger equation has a potential that is periodic with period L , then the solutions to Schrödinger's equation are described by wavefunctions

$$
\begin{equation*}
\Psi_{n, q}(x)=e^{i q x} u_{n, q}(x) \tag{4.7}
\end{equation*}
$$

that have the form of a plane wave multiplied by a periodic function

$$
\begin{equation*}
u_{n, q}(x)=u_{n, q}(x+L), \tag{4.8}
\end{equation*}
$$

that is also periodic with period $L$. The plane waves are characterized by parameters $q$ called the quasi-momentum and labeled by a band index $n$. For an infinite lattice the parameter $q$ is continuous within a range defining one energy band. The energies are periodic in $q$, such that

$$
\begin{equation*}
E_{n}(q)=E_{n}(q+k) \tag{4.9}
\end{equation*}
$$

The solutions $\Psi_{n, q}(x)$ are called Bloch states, which are delocalized states spread out across the entire lattice.

Solutions to the Hamiltonian can be obtained by diagonalizing a tridiagonal matrix resulting from a Fourier series expansion of the wavefunction and potential. The eigenvalues of the tridiagonal matrix correspond to the eigenenergies of the Hamiltonian and the eigenvectors are the Fourier expansion of the wavefunction.

The band structure (energy vs quasi-momentum) for several lattice depths is given in Figure 4.1. The energy of the state is plotted versus the quasi-momentum $q$ in the first Brillouin zone. With a zero potential depth lattice (a) we recover the free particle band structure folded into the first Brillioun zone. With a non-zero potential we find band-gaps that increase with potential depth. For deep lattices the lower lying levels become increasingly flat and the width of the band gap approaches $\hbar \omega$. The anharmonicity of the lattice approaches $E_{r e}=\hbar \omega_{r e}$, such that the mean energy of the $n t h$ band is approximately

$$
\begin{equation*}
E_{n}=\hbar \omega / 2+n \hbar \omega-\frac{n^{2}}{2} \hbar \omega_{r e} . \tag{4.10}
\end{equation*}
$$

Figure 4.2 shows the Bloch wavefunctions and probability densities for a particle in the lowest energy band of the lattice. The plot shows the $q=0$ and $q=\hbar k$ Bloch waves of a lattice with depth $U_{0}=12 E_{r e}$ deep lattice. The probability densities


Figure 4.1: Band Structures in a lattice


Figure 4.2: Bloch wave functions (left) and probability densities(right) for $q=0$ and $q=\hbar k$ in the lowest band of a lattice with depth $\mathrm{U}=12 \mathrm{E}_{\text {re }}$.
for these wavefunctions are nearly identical for the lowest energy band, reflecting the flatness of the band.

Figure 4.3 shows the Bloch wavefunctions and probability densities for the second lowest energy band $(\mathrm{n}=1)$ of the same potential. Both of the wavefunctions have a node at the origin just as the first excited state of a harmonic oscillator does.

If we are only concerned with determining the maximum and minimum energies of each band then we can use the Mathieu equation, which is well known in solid-state theory. The Hamiltonian may be rewritten by a transformation of variables to get the Mathieu equation,

$$
\begin{equation*}
y^{\prime \prime}+(a-2 q \cos (2 z)) y=0 . \tag{4.11}
\end{equation*}
$$

There are characteristic solutions of the Mathieu equation that give the maximum and minimum energies of a band for a given potential depth, which can be easily calculated with Mathematica.

Figure 4.4 shows the band energies as a function of the lattice depth in units of


Figure 4.3: Bloch wave functions and probability densities for $q=0$ and $q=\hbar k$ in the second lowest $(\mathrm{n}=1)$ band of a $12 E_{r e}$ deep lattice.
recoil energies (upper graph) and temperature (lower graph). In recoil energies the graphs are universal. The solid black line is a line of slope 1 that corresponds to the depth of the well. If an entire band has energy less than the well depth then it is referred to as a bound band. Figures 4.5 and 4.6 show the width of the energy bands and the band gap as a function of the lattice depth.

The eigenstates of the lattice are Bloch states which are delocalized states. Delocalized wavefunctions do not always provide for an intuitive understanding of the resulting physics. Another representation that can be used, sometimes more intuitive, is a Wannier state description. A Wannier state is a localized state that can be constructed by specific superpositions of Bloch states. The Wannier states are not eigenstates of the Hamiltonian. An atom confined to a single lattice site can tunnel out into the neighbouring wells, becoming delocalized.

Wannier states are well-localized states forming a complete orthonormal basis. The Wannier states can be formed by a specific superposition of Bloch waves. Equa-


Figure 4.4: Allowed energy vs well depth in recoil units and temperature. Shaded regions represent allowed energies.


Figure 4.5: Bandwidth vs Well depth


Figure 4.6: Band gap vs well depth
tions 4.12 and 4.13 demonstrate how to obtain Wannier states $\Phi_{n, \ell}$, with an index $\ell$ to label the well site, from Bloch states and vice-versa. Care must be taken to ensure the Bloch waves are added with the correct phase[44] in order to get the exponentially localized Wannier state.

$$
\begin{align*}
\Phi_{n, \ell}(z) & =\frac{1}{k} \int_{-k / 2}^{k / 2} d q \Psi_{n, q}(z) e^{-i q \ell L}  \tag{4.12}\\
\Psi_{n, q}(z) & =\frac{1}{k} \sum_{\ell=-\infty}^{\infty} \Phi_{n, \ell}(z-\ell L) e^{-i q \ell L} . \tag{4.13}
\end{align*}
$$

In the limit of tightly confining potentials, when the lowest energy bands are nearly flat, the Wannier states closely approximate the localized states of a harmonic oscillator potential. Figure 4.7 shows the two lowest energy Wannier state wavefunctions(red) in a lattice with a well depth of $12 E_{r e}$. The ground and the first excited states are shown in red as well as the corresponding wavefunction for a harmonic oscillator (blue) with the corresponding trap strength, as expressed by equation 4.5. Even at this relatively shallow well depth the lower energy Wannier states already closely match the harmonic oscillator wavefunctions. The Wannier states have longer tails than the harmonic oscillator states.

### 4.3 Tilted lattices

The lattice used in this experiment is oriented vertically, so gravity plays an important role. Gravity adds an additional term to the potential, resulting in a total potential

$$
\begin{equation*}
V(z)=U_{0} \sin ^{2}\left(k_{e} z\right)+m g z \tag{4.14}
\end{equation*}
$$

where $m$ is the mass of the atom and $g$ is the acceleration due to gravity. The extra term serves to add a tilt creating a so-called 'washboard potential'. A similar tilt results any time there is a force acting in the direction of the lattice, which includes


Figure 4.7: Wannier wavefunctions in a lattice with depth $U=12 E_{r e}$ in a lattice shown for the two lowest energy states.
effects such as adding a linear electric field, a magnetic field gradient or accelerating a lattice (achieved by varying the frequency difference between the lasers comprising the lattice).

In the case of a vertical lattice gravity adds an energy shift between neighbouring lattice sites that is given by $m g a$, where $a$ is the lattice constant. We can equate this with a temperature difference of $T_{g r}=2 m g a / k_{B}$ per lattice constant. Basis states for such a potential are Wannier-Bloch states or Wannier-Stark states. Wannier-Bloch states are delocalized states similar to the Bloch states of an untilted lattice. WannierStark states are well localized states analagous to the Wannier states. Neither of these two representations are eigenstates since there is always some probability to tunnel into a near-continuum.

Astonishingly, the application of a constant force does not cause the atoms to undergo constant acceleration. Instead, the particles will undergo oscillations in their momentum. The application of a force causes the quasi-momentum $q$ of the atom to increase until it reaches the Brillioun zone edge when $q=\hbar k$. At this momentum the atom satisfies the Bragg condition and is Bragg diffracted, losing $2 \hbar k$ of momentum, and being transported to the other zone edge where $q=-\hbar k$. This type of oscillation
is termed a Bloch oscillation. The period of the Bloch oscillation, $\tau_{B}$, is the time required for the atoms to traverse the Brillioun zone with width $2 k_{L}$.

$$
\begin{equation*}
\tau_{B}=\frac{2 \hbar k_{e}}{m g}=\frac{2 v_{r e}}{g} \tag{4.15}
\end{equation*}
$$

The lattice used in this dissertation has an effective recoil velocity $v_{r e}=v_{r} \sin \left(24.8^{\circ}\right)=$ $2.54 \mathrm{~mm} / \mathrm{s}$, leading to a Bloch period of $514 \mu \mathrm{~s}$, or a frequency of 1944 Hz .

If the atom is subjected to a large force, then it may not always undergo Bloch oscillations. The atom will have some probability to tunnel from one band to the next higher band by a Landau-Zener transition. An extreme case (classical limit) can be considered when the acceleration is so large that the lattice no longer has any potential minima. The possibility for Landau-Zener tunneling occurs well before we reach this limit. The atom is most likely to tunnel when it is near the band edge (or band center for odd numbered bands) where the periodic potential has created an avoided crossing. The probability for the atom to move along the diabatic potential at an avoided crossing was given by Zener[45] in 1932,

$$
\begin{equation*}
P_{L Z}=\exp \left(-\frac{\pi}{2 \hbar} \frac{E_{g}^{2}}{\frac{d}{d t}\left(E_{1}-E_{2}\right)}\right) \tag{4.16}
\end{equation*}
$$

where $E_{g}$ is the energy difference between the perturbed levels, and $E_{1}$ and $E_{2}$ are the unperturbed energy eigenvalues. For a free particle $\frac{d}{d t}\left(E_{1}-E_{2}\right)=2 n a \hbar k_{L}$, where $n$ is the band index. The atom is most likely to tunnel when the band gap is smallest, i.e. when $q=\hbar k$ or $q=0$.

We then arrive at a simplified formula for the probability of a Landau-Zener transition is

$$
\begin{equation*}
P_{L Z}=\exp \left(\frac{-a_{c}}{a}\right) \tag{4.17}
\end{equation*}
$$

where $a$ is the acceleration and $a_{c}$, the critical acceleration is

$$
\begin{equation*}
a_{c}=-\frac{\pi E_{g}^{2}}{4 \hbar^{2} k_{e} n} \tag{4.18}
\end{equation*}
$$

The tunneling probability under gravitational acceleration is then


Figure 4.8: Transition probability vs well depth in an optical lattice. $\mathrm{P}_{n}$ is the probability to tunnel from band $n$ to band $n+1$

$$
\begin{align*}
P_{L Z} & =\exp \left(-\frac{\pi E_{g}^{2}}{4 \hbar^{2} k_{e} n g}\right)  \tag{4.19}\\
& =\exp \left(-.439 \frac{E_{g}^{2}}{E_{r e}^{2} n}\right) . \tag{4.20}
\end{align*}
$$

Figure 4.8 shows the Landau-Zener tunneling probability for the three lowest bands in our lattice. The lifetime for an atom in a given band in the lattice can then be easily computed from the tunneling probability and the Bloch oscillation period to be

$$
\begin{equation*}
\tau=\frac{-\tau_{B}}{\ln \left(1-P_{L Z}\right)} \tag{4.21}
\end{equation*}
$$

Figure 4.9 shows the lifetime versus well depth for the same lattice.
Atoms in unbound bands of the lattice have nearly unit probability to undergo Landau-Zener tunneling. An atom in a band that is just marginally bound has a lifetime on the order of one Bloch period. All the unbound atoms (and possibly


Figure 4.9: Landau-Zener limited lifetime for atoms in an optical lattice. $\tau_{n}$ is the lifetime of atoms in the nth band.
any weakly bound atoms) will be lost from the lattice almost immediately. The remaining atoms will have negligible probability to tunnel to another band, locking in the band populations. By decreasing the lattice depth different bands can become unbound at different times, causing atoms to become lost from the lattice at different times, allowing the band occupation to be measured. Another way to measure the band populations is to increase the force acting on the lattice instead of decreasing the depth of the lattice.

### 4.3.1 Wannier-Stark transitions

In addition to Bloch oscillations and Landau-Zener transitions there are other effects possible in a lattice under the influence of a force, such as Wannier-Stark transitions. A Wannier-Stark transition is a transition where an atom jumps over to a neighbouring well while simultaneously changing vibrational state. This transition can occur when the system is perturbed at a frequency that corresponds to the en-
ergy difference between neighbouring wells[41, 46]. If the energy difference between neighbouring vibrational states is equal to the well-to-well energy difference, then the atom may resonantly tunnel from one well to another. If the lattice is deep enough then the energy level in one lattice can be resonant with a state multiple wells over.

Wannier-Stark transitions are not studied in this work but are mentioned because the frequency required for a Wannier-Stark transition in the lattice used here is equal to the Bloch frequency, i.e., $m g a / h=\tau_{B}=1944 \mathrm{~Hz}$, which is close to the typical frequencies used in the lattice.

### 4.4 Experimental Lattice Setup

Figure 4.10 shows the setup for the optical lattice. The master oscillator is a TUI Optics grating-stabilized diode laser that generates 12 mW of power. The laser has a linewidth of about 1 MHz and is detuned 30 GHz from the D 2 transition at 780 nm. The beam from the TUI is sent into a SDL (now JDSU) MOPA travelling wave amplifier. The amplifier puts out a beam with up to 450 mW of power with nearly the same spectral properties as the seed beam, but poor spatial characteristics. The output spectrum is predominantly the same as the seed beam, but may have some extra light in the form of amplified spontaneous emission(ASE). The ASE extends over a range of roughly 100 nm , centered at 730 nm . The total power in the ASE can be as large as 25 mW , when not being seeded.

The beam from the MOPA goes through a half-wave plate to give it equal amplitudes of horizontally and vertically polarized light. The horizontal and vertical components are separated at a polarizing beam splitter (PBS), with each beam going through an acousto-optic modulator (AOM). The horizontal beam is frequency shifted by 80 MHz and the vertical beam is shifted by 80 MHz , but with control over the phase. The first order beams are recombined on another PBS and then sent through a spatial filter before being sent up to the MOT.

Spatial filtering is necessary to generate a beam with flat wavefronts and no spatial intensity fluctuations. The spatial filter is comprised of two lenses and a 25 micron pinhole which serves to transmit only a $\mathrm{TEM}_{00}$ mode. The laser beam exiting the MOPA has very poor spatial characteristics with numerous higher order modes. The process of filtering the beam results in a $60 \%$ drop in available power.

The beams are overlapped with orthogonal polarizations before sending them to the MOT to prevent vibrations from causing random phase shifts between the two beams. Near the MOT the polarization components are separated by another PBS. The PBS transmits horizontal polarization with good extinction (1000:1) but does poorly with vertical polarization. As much as $2 \%$ of the reflected beam actually comes from the horizontal component. Another PBS is inserted into this path to eliminate this polarization contamination ( $2 \%$ intensity from the wrong beam can create distorted wells as it can create a separate, stationary interference pattern with $14 \%$ of the depth of the desired lattice). Finally, the polarization of one beam is rotated so that the beams have parallel polarizations when they interfere in the MOT.

The AOM's are driven at 80 MHz , with the signal being derived from an SRS function generator. The SRS generates a 10 MHz clock that is frequency doubled 3 times to reach 80 MHz , which drives one of the AOM's. A synthesized waveform is also generated at 20 MHz , that is doubled twice to reach 80 MHz to drive the second AOM. The SRS allows control over the phase and frequency of the synthesized waveform. By shifting the phase of the synthesized waveform we can shift the phase of the RF driving the second AOM. This phase shift is imprinted onto the beam going through the AOM, which then causes the lattice to shift by a distance $d=a \frac{\theta}{2 \pi}$, where $a$ is the lattice constant and $\theta$ is the phase shift on the beam. Alternatively, the frequency generated by the SRS may be modulated. On changing the frequency of one lattice beam relative to the other, the lattice picks up a constant velocity. In


Figure 4.10: Optical setup for control and phase stability of lattice beams.
the lab frame the lattice moves at a velocity $v=\delta f a$, where $\delta f$ is the frequency difference. In the frame of the lattice, this is the velocity at which the atom sees both beams Doppler shifted to the same frequency. By continually changing the lattice frequency the lattice may be accelerated. The acceleration is only accurate when averaging over a time larger than the timestep, as the velocity of the lattice is actually changing in a stepwise fashion.

### 4.5 Lattice Controls

### 4.5.1 The SRS function Generator

Control over the lattice is achieved by programming the SRS in a mode referred to as arbitrary modulation. In this mode, the SRS applies small programmable modulations to an existing waveform. The waveform is internally generated by the SRS and has a set frequency, amplitude and phase. The SRS has three different arbitrary modulation modes, allowing control over one of frequency, phase or amplitude. Only one of these parameters may be controlled in one modulation cycle. Although phase and frequency cannot be modulated simultaneously, phase changes can be effected via short duration frequency changes and vice-versa. The SRS also does not allow for continuous control of the phase or frequency, only step-wise updates. The rate at which updates occur is decided before programming the SRS and may not be changed during a cycle. Reprogramming the SRS with a new program takes about 5 seconds via GPIB control from a computer. The following table shows the controllable parameters for each mode of operation. The time-step is the minimum amount of time between updates. The update time is the time it takes for the SRS to change the phase, or frequency, of its output. The final row is the number of points that may be programmed into the SRS.

Once the SRS has been programmed it immediately begins generating the wave-

| Mode | Phase-Control | Frequency Control |
| :--- | :--- | :--- |
| Smallest Time Step | $0.25 \mu \mathrm{~s}$ | $1.0 \mu \mathrm{~s}$ |
| Update time | $0.25 \mu \mathrm{~s}$ | $0.5 \mu \mathrm{~s}$ |
| Number of Points | 4000 | 1500 |

Table 4.1: Summary of SRS arbitrary waveform modulation capabilities
form and does not have any triggering capability. It does, however have a reference output (called modulation output) on the back that is proportional to the signal currently being output. In phase control mode the reference outputs a voltage

$$
\begin{equation*}
V_{r e f}=2.5+\frac{\Delta \phi}{\max (\Delta \phi)} \tag{4.22}
\end{equation*}
$$

where $\Delta \phi$ is the difference in phase between the last update and the current update, and $\max (\Delta \phi)$ is the largest phase change in the cycle. In frequency control mode the reference voltage is similar

$$
\begin{equation*}
V_{r e f}=5 \frac{f-f_{\min }}{f_{\max }-f_{\min }}, \tag{4.23}
\end{equation*}
$$

where $f$ is the current frequency, $f_{\min }$ is the smallest frequency in the cycle and $f_{\max }$ is the largest frequency in the cycle. The SRS is synchronized with LabView with the aid of an external trigger circuit, shown in Figure B.1. LabView is programmed to pause during the execution of a laser cooling cycle until it receives a trigger signal from the SRS. The pause occurs at the end of the MOT phase, before optical molasses. The timing between the start of the SRS cycle and LabView continuing may be adjusted by up to $500 \mu s$. A block diagram of the experimental control system (and planned improvements) are shown in Appendix A.

## Chapter 5

## Phase-Space Tomography

Tomography is a method of determining the structure of an object by a reconstruction based upon data which integrates over many directions. Tomography is used in medical diagnostics to reconstruct the structure of a 3D object using a large number of 2 D images acquired from a variety of directions. A common example of tomography is a medical CAT scan, or MRI (magnetic resonance imaging). In these measurements a scan is performed that integrates some quantity (i.e. density, radioactivity or number of 'tagged particles') over the scanning direction. After acquiring scans over many different directions an inverse transformation can be applied that returns the internal structure of the object.

Quantum tomography likewise aims to determine the structure of an object, this time a wavefunction. The wavefunction can be determined using tomographic techniques, or by direct measurement of an observable after a large number of unitary interactions. Both these methods result in the reconstruction of the quantum state. While the second method is not a tomographic technique, it is commonly referred to a tomography, presumably because they have the same result. Tomography of a quantum state has recently been accomplished by tomographic means to measure the Wigner distribution[47] for a photon in a Fock state[48], for helium atoms[49] and for a dissociating molecule[50]. By means of direct sampling the Wigner function has
been measured for trapped ions[51, 52] and for photons[53].
In this section we focus on tomography of the motional state of a quantum system. In this chapter we first look at the most general of cases, i.e. measuring phase-space distributions that allow for the reconstruction of the quantum state via the Wigner [47] and Husimi [54] distributions. In the following chapter we examine the specific case where only 2 states are populated. In this limiting case we find the system may be characterized with fewer measurements. The 2 state system is of particular interest because it can be used to represent a single quantum bit (a qubit), the fundamental building block of a quantum computer.

### 5.1 Representations of Quantum Mechanics: Density Matrix and Phase-Space Distributions

In any quantum system it is often desirable to have complete information about the state of the system. Some properties are often readily available, such as expectation values for energy, position, momentum and spin. This isn't always enough information to be able to predict and/or understand the state of the system. We therefore try to find ways to get complete information about a system, and find convenient representations for this information.

The quantum state of a system can be expressed in numerous forms. The density matrix is a complete description of a system as it allows for statistical mixtures of states, which cannot be represented by a wavefunction. The density matrix (also called the density operator, but in this dissertation we use the term density matrix) is defined as a probabilistic sum of outer-products of state vectors

$$
\begin{equation*}
\rho=\sum_{i} p_{i}\left|\Psi_{i}\right\rangle\left\langle\Psi_{i}\right|, \tag{5.1}
\end{equation*}
$$

where $p_{i}$ is the probability that the state $\left|\Psi_{i}\right\rangle$ was prepared. The density operator
has the properties that it is Hermitian and that

$$
\begin{equation*}
\operatorname{Tr}(\rho)=1, \tag{5.2}
\end{equation*}
$$

where $\operatorname{Tr}$ is the trace operation. Having a trace equal to one is equivalent to having a normalized quantum state, with total probability of 1 to find the system in some state. The trace operation is defined as

$$
\begin{equation*}
\operatorname{Tr}(\rho)=\sum_{k}\langle k| \rho_{i}|k\rangle, \tag{5.3}
\end{equation*}
$$

where $\{|k\rangle\}$ is any orthonormal basis. The numerical result is independent of the choice of basis.

A measure of how pure a quantum state or ensemble is called the purity.

$$
\begin{equation*}
\text { Purity }=\operatorname{Tr}\left(\rho^{2}\right) . \tag{5.4}
\end{equation*}
$$

A pure state, $\rho=|\Psi\rangle\langle\Psi|$, has the property that $\rho^{2}=\rho$. The purity of a pure state is then equal to one.

The expectation value of any operator is easy to calculate with the density matrix. If we have an observable $M$, then the expectation value of the observable is

$$
\begin{align*}
\langle M\rangle & =\operatorname{Tr}(\rho M)  \tag{5.5}\\
& =\operatorname{Tr}(M \rho) . \tag{5.6}
\end{align*}
$$

There are other ways to represent the complete information about a quantum state, related to a wavefunction, that contain all the same information as a density matrix. Representations based upon a quantum picture of phase space are included in these classes. In classical mechanics the state of a particle in one dimension is represented by a point in phase space, and the motion of a particle in one dimension
can be completely described by a trajectory in phase-space. In a phase-space plot the horizontal axis corresponds to position and the vertical axis correspond to momentum. From the phase-space diagram one can gain information of the type of potential in which the particle travels. For example, in a harmonic oscillator particles travel in circular orbits, i.e. at some instant they are at rest on the position axis and slowly pick up momentum as they move toward the origin, gradually tracing out a circle. At each instant in time the particle has a definite position and momentum.

In quantum mechanics there is no direct analog to a phase-space diagram. A quantum particle does not have a precisely defined momentum or position. By measuring the momentum or position, one can make the position, or momentum well known, but would necessarily make the conjugate variable less known, a direct consequence of Heisenberg uncertainty. Therefore, in phase space, a quantum particle can not be represented by a point, but at best, a circle of area $\hbar / 2$, the limit of measurement enforced by Heisenberg uncertainty. One can then map out the probability for a particle to be in a particular region of phase-space. Since the wavefunction can be complex, care must be taken in how the information is represented.

There are several well-known phase-space distributions called the Wigner[47], Husimi[54] and the Glauber-Sudarshan $[55,56]$ distributions, often referred to as W, Q and P distributions respectively. The distributions are all related to one another, but they can have dramatically different behaviors. Of these distributions, the Wigner function is most commonly used as it has a number of useful properties. Given the Wigner function, the marginal distributions can be obtained with a single integral. The marginal distributions are positive definite, but the Wigner function itself need not be positive. The Wigner function also has the result that orthogonal states have orthogonal Wigner distributions. The Q distribution in contrast is always positive, but is not as convenient to extract information from. The marginal distibutions cannot be simply extracted from Q, and orthogonal states are not orthogonal in the Q distribution. The P distribution isn't necessarily positive, and it may have singu-

|  | Q | W | P |
| :--- | :--- | :--- | :--- |
| real | yes | yes | yes |
| positive | yes | no | no |
| singular | no | no | yes |
| marginals | no | yes | no |
| orthogonality | no | yes | yes |

Table 5.1: Characteristics of $\mathrm{Q}, \mathrm{W}$ and P quasi-probability distributions
larities. It is a distribution where orthogonal states have orthogonal P distributions, but the marginal distributions are not simply determined.

Table 5.1 summarizes some key features of the distributions. In this thesis we focus on the Q and W distributions as they are readily measured in an experiment.

### 5.1.1 Wigner Function

The Wigner distribution[47] is the unique probability distribution that provides a simple procedure to find the marginal distributions as well as the expectation value of any measurement based on the variables characterizing it. The Wigner function can be measured for any system either by tomographic or sampled data.

The Wigner distribution, first proposed by Wigner in 1934 is

$$
\begin{equation*}
W(x, p)=\frac{1}{2 \pi} \int_{-\infty}^{\infty} \Psi\left(x+\frac{q}{2}\right) \Psi^{*}\left(x-\frac{q}{2}\right) e^{i q p / \hbar} d q \tag{5.7}
\end{equation*}
$$

for a pure state. For a mixed state it can be written using density matrix notation, and is given by

$$
\begin{equation*}
W(x, p)=\frac{1}{2 \pi} \int_{-\infty}^{\infty}\left\langle x+\frac{q}{2}\right| \rho\left|x-\frac{q}{2}\right\rangle e^{i q p / \hbar} d q \tag{5.8}
\end{equation*}
$$

The Wigner function is a phase-space representation that is real everywhere, but not necessarily positive. This implies that it may contain negative values, making its interpretation as a probability function incorrect. The Wigner distribution is
therefore usually referred to as a quasi-probability distribution. The marginal distributions can be easily obtained from the Wigner distribution by integrating over one of the directions.

$$
\begin{align*}
|\Psi(x)|^{2} & =\int_{-\infty}^{\infty} W(x, p) d p  \tag{5.9}\\
|\widetilde{\Psi}(p)|^{2} & =\int_{-\infty}^{\infty} W(x, p) d x \tag{5.10}
\end{align*}
$$

The Wigner distribution is also useful for obtaining expectation values. If we have a symmetrically ordered operator $\widehat{A}$, with a phase-space representation of the operator, $A(x, p)$, then the expectation value is given by an integral over all phase-space.

$$
\begin{equation*}
\langle A\rangle=\int A(x, p) W(x, p) d x d p \tag{5.11}
\end{equation*}
$$

Measurement of the Wigner function can be a difficult task in a quantum system, partly because we are unable to directly measure the wavefunction, and also because in most systems, the range over which $x$ and $p$ extend cannot be resolved experimentally. In certain systems the Wigner function has properties that allow measurement to be performed through clever manipulations of the state prior to measurement of some observable. This is often possible when dealing with symmetric potentials.

We consider a system where we have a set of orthogonal eigenstates $\left\{\Theta_{i}\right\}$. Each state $\Phi_{i}$ is also an eigenstate of the parity operator $\widehat{\mathcal{P}}$

$$
\begin{equation*}
\widehat{\mathcal{P}} \Theta_{n}(x)=\Theta_{n}(-x)=\lambda_{n} \Theta(x), \tag{5.12}
\end{equation*}
$$

where $\lambda_{n}= \pm 1$ is the eigenvalue of the parity operator acting on the state.
We consider an arbitrary pure quantum state $\Psi(x)$,

$$
\begin{equation*}
\Psi(x)=\sum_{i=0}^{\infty} c_{i} \Theta_{i}(x) . \tag{5.13}
\end{equation*}
$$

The Wigner function at the origin can then be evaluated

$$
\begin{align*}
W(0,0) & =\frac{1}{2 \pi} \int_{-\infty}^{\infty} \Psi\left(\frac{q}{2}\right) \Psi^{*}\left(\frac{-q}{2}\right) e^{0} d q  \tag{5.14}\\
& =\frac{1}{2 \pi} \int_{-\infty}^{\infty} \sum_{i=0}^{\infty} c_{i} \Theta_{i}\left(\frac{q}{2}\right) \sum_{j=0}^{\infty} c_{j}^{*} \Theta_{j}^{*}\left(\frac{-q}{2}\right) d q  \tag{5.15}\\
& =\frac{1}{2 \pi} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \int_{-\infty}^{\infty} c_{i} \Theta_{i}\left(\frac{q}{2}\right) c_{j}^{*} \Theta_{j}^{*}\left(\frac{-q}{2}\right) d q  \tag{5.16}\\
& =\frac{1}{2 \pi} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \int_{-\infty}^{\infty} c_{i} \Theta_{i}\left(\frac{q}{2}\right) c_{j}^{*} \lambda_{j} \Theta_{j}^{*}\left(\frac{q}{2}\right) d q  \tag{5.17}\\
& =\frac{1}{2 \pi} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{i} c_{j}^{*} \lambda_{j} \int_{-\infty}^{\infty} \Theta_{i}\left(\frac{q}{2}\right) \Theta_{j}^{*}\left(\frac{q}{2}\right) d q  \tag{5.18}\\
& =\frac{1}{2 \pi} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{i} c_{j}^{*} \lambda_{j} 2 \delta_{i j}  \tag{5.19}\\
& =\frac{1}{\pi} \sum_{i=0}^{\infty}\left|c_{i}\right|^{2} \lambda_{i}  \tag{5.20}\\
& =\frac{1}{\pi}\left(\left|c_{0}\right|^{2}-\left|c_{1}\right|^{2}+\left|c_{2}\right|^{2}-\left|c_{3}\right|^{2}+\ldots\right) . \tag{5.21}
\end{align*}
$$

For mixed states, we use the density matrix notation and the Wigner function at the origin is simply

$$
\begin{equation*}
W(0,0)=\frac{1}{\pi} \sum_{i=0}^{\infty} \lambda_{i} \rho_{i i}, \tag{5.22}
\end{equation*}
$$

where $\rho_{i i}$ is a diagonal element of a density matrix. When the eigenstates of the system have definite parity, the Wigner function at the origin may be easily determined by its populations, independent of its coherences. Figure 5.1 shows the two lowest energy Wannier states for an optical lattice of depth $U_{0}=18 E_{r e}$. The Wannier states shown have no imaginary component. The dashed line is the optical lattice potential and the solid lines are the wavefunctions. The lowest energy state has even parity and the next lowest energy state has odd parity. The Wigner function at the origin for a lattice can then be represented by equation 5.22.

What we really desire is to measure the Wigner function at all locations in phasespace, not just at the origin. One method to do this is to perform a transformation

## Wannier States




Figure 5.1: The first two Wannier states of an optical lattice that is 18 recoil energies deep. The left plots $\mathrm{n}=0$ Wannier state and the right plot shows the $\mathrm{n}=1$ Wannier states. The dashed line is the potential.
that takes a point in phase-space and translates it onto the origin. A state, $\Psi(x)$, operated on with an arbitrary displacement in position of $x^{\prime}$ becomes

$$
\begin{align*}
\widehat{D}\left(x^{\prime}\right) \Psi(x) & =e^{i x^{\prime} \widehat{P} / h} \Psi(x)  \tag{5.23}\\
& =\Psi\left(x-x^{\prime}\right) \tag{5.24}
\end{align*}
$$

where $\widehat{D}$ is the displacement operator, $\widehat{P}$ is the momentum operator and $x^{\prime}$ is the magnitude of the shift in position.

In a harmonic oscillator the displacement operator is given by

$$
\begin{equation*}
\widehat{D}(\alpha)=\exp \left(\alpha \widehat{a}^{\dagger}-\alpha^{*} \widehat{a}\right) \tag{5.25}
\end{equation*}
$$

where $\widehat{a}^{\dagger}$ and $\widehat{a}$ are the usual creation and annihilation operators and $\alpha$ is the complex amplitude of the coherent state. Operating on a ground state with the displacement $\widehat{D}(\alpha)$ creates the state $|\alpha\rangle$. The displacement operator can translate any point in phase-space onto any other point, such as the origin. We act on the unknown state
with a large number of different $\alpha$, translating different parts of the distribution onto the origin, followed by a measurement of the state populations. The Wigner function is then reconstructed using equation 5.22.

In practice, we are able to perform displacements in position by displacing the trapping potential. The displacements have coefficents $\alpha$ that are real since they are translations along the position axis. It is still necessary to do translations along the momentum axis. In a harmonic oscillator this can be easily accomplished by a time delay, which corresponds to a rotation in phase space. The rotation operator is given by

$$
\begin{equation*}
\widehat{R}(\theta)=\exp \left(-i \widehat{a}^{\dagger} \widehat{a} \theta\right) \tag{5.26}
\end{equation*}
$$

where $\theta=\omega t$. The combination of time delays (rotation) and position displacements allows for arbitrary displacements in phase space. This technique has previously been demonstrated to work for an ion in a harmonic trap[51, 52]. The work discussed in this thesis is performed in the same manner. Future work will demonstrate a true displacement in position and momentum simultaneously, thereby eliminating the need for a time delay, or rotation. For anharmonic potentials, the time delay can cause spreading in phase-space as well as rotation.

### 5.1.2 Husimi (Q) Distribution

The Husimi Q distribution is a distribution in which the probability for a particle to be at a particular location in phase-space is given by its overlap with a coherent state centered at the same location in phase-space. The mathematical expression for the Q function is given by

$$
\begin{equation*}
Q(\alpha)=\frac{1}{\pi} \operatorname{Tr}(\rho|\alpha\rangle\langle\alpha|)=\frac{1}{\pi}\langle\alpha| \rho|\alpha\rangle, \tag{5.27}
\end{equation*}
$$

where $|\alpha\rangle$ is a coherent state and $\rho$ is the density matrix. The Q distribution is everywhere non-negative, as is the classical phase-space distribution. The Q
distribution is equivalent to a Wigner function smoothed by a minimum-uncertainty Gaussian. This smoothing results in a distribution where the smallest features have a size scale on the order of $\hbar$, thereby washing out small scale features such as interferences.

Similar to the Wigner function, measurement of the Q distribution is not always straightforward. Measurement requires a method to determine the overlap of the unknown state with a large number of coherent states. Fortunately, some simple systems provide for easy transformations from equation 5.27 to the form

$$
\begin{equation*}
Q(\alpha)=\frac{1}{\pi}\langle 0| \widetilde{\rho}|0\rangle \tag{5.28}
\end{equation*}
$$

where

$$
\begin{equation*}
\widetilde{\rho}=D(\alpha) \rho D^{\dagger}(\alpha) . \tag{5.29}
\end{equation*}
$$

Here, $D(\alpha)$ is a unitary transformation that moves a coherent state $|\alpha\rangle$ onto the origin. This can be accomplished just as with a Wigner function using a combination of spatial displacement operators and a phase-shifting operation..

There is a definite relationship between the Wigner distribution and the Husimi distribution. By analogy with equation 5.28 we can define a parameter

$$
\begin{equation*}
Q_{n}(\alpha)=\langle n| \widetilde{\rho}|n\rangle \tag{5.30}
\end{equation*}
$$

Then the Wigner function may be defined in these terms by the simple sum,

$$
\begin{equation*}
W(\alpha)=\frac{1}{\pi} \sum_{n=0}^{\infty}(-1)^{n} Q_{n}(\alpha) \tag{5.31}
\end{equation*}
$$

while the Q distribution is

$$
\begin{equation*}
Q=\frac{Q_{0}}{\pi} . \tag{5.32}
\end{equation*}
$$

Both the Wigner and Husimi distribution are complete, meaning all information about the state can be extracted from them. However, the phase information
in the Husimi distribution is locked into the analytic smoothing and can be determined from the derivatives of the Husimi distribution. This then means that small errors in the determination of the Husimi distribution can cause large errors in the extraction of the density matrix.

Given one of the distributions, the other distributions can be determined from them by Gaussian convolution (or deconvolution)[57].

$$
\begin{aligned}
& Q\left(\alpha, \alpha^{*}\right)=\int d^{2} \beta W\left(\beta, \beta^{*}\right) \frac{2}{\pi} e^{-2|\alpha-\beta|^{2}} . \\
& W\left(\alpha, \alpha^{*}\right)=\int d^{2} \beta Q\left(\beta, \beta^{*}\right) \frac{-2}{\pi} e^{2|\alpha-\beta|^{2}} .
\end{aligned}
$$

Figure 5.2 shows Wigner and Husimi distributions for several Fock states and a superposition state in a harmonic oscillator. Also shown are the distributions for a delta-function in position that clearly shows that a Husimi distribution is a Gaussian convolution of the Wigner function. Note that the Q distribution has a non-zero component at the origin only for states that have some population in the ground state.

### 5.2 Measuring $Q$ and $W$ in a lattice

The tools suggested to measure the Wigner and Husimi distributions include measurements of the state populations, displacements in position and rotations in phasespace effected by a time delay.

The Q distribution is defined as the projection of the unknown state onto coherent states. In a harmonic oscillator the ground state is a coherent state and we can project onto any displaced ground state, $|\langle\Psi \mid \alpha\rangle|^{2}$, where $|\alpha\rangle=\widehat{D}|0\rangle$ and $|0\rangle$ is the ground state of a harmonic oscillator. However, in an optical lattice we measure the populations in the ground state of the lattice, not a harmonic oscillator. The measured value is $\left|\left\langle\Psi \mid \alpha_{L}\right\rangle\right|^{2}$ where $\left|\alpha_{L}\right\rangle=\widehat{D}\left|\Phi_{0}\right\rangle$ and $\left|\Phi_{0}\right\rangle$ is a ground state Wanier


Figure 5.2: Gallery of several Wigner and Husimi Distributions
function. The deviation between our measurement and a true Husimi distribution is characterized by

$$
\xi(d)=\left\langle\Phi_{0}\right| \widehat{D}|0\rangle-\left\langle\Phi_{0}\right| \widehat{D}\left|\Phi_{0}\right\rangle
$$

for a displacement of magnitude $d$. In a deep lattice with a depth of $60 E_{r e}$ the ground state Wannier function is well approximated by a ground state harmonic oscillator wavefunction, and the maximum deviation is $\xi(0)=0.012$. For such deep lattices the Husimi distribution and our measured function differ by less than the uncertainty in our measurements.

Spatial displacement of the state is accomplished by displacing the trapping potential by changing the relative phase between the lasers that create the lattice. The phase-shift is generated by the SRS function generator, which is programmed to 1 part in $2^{12}$. The phase-shift is limited by the vertical resolution of the SRS. The SRS has 12 bit vertical resolution of its full scale output of 10 volts, which corresponds to 2.5 mV resolution. The output waveform is 2.0 V in amplitude, so the phase shift is accurate to $0.1 \%$. The phase shift however, is in the 20 MHz output, and becomes 1 part in 250 after quadrupling the frequency. This uncertainty in the displacement is much smaller than the uncertainty in the measurements and may be safely ignored.

To create displacements in momentum it is necessary to use a time delay to allow rotation in phase space. In general, each state evolves at a rate determined by the energy of the state, so the phase evolution is

$$
\begin{equation*}
\widehat{R}\left(\theta_{n}\right)=\exp \left(-i E_{n} t / \hbar\right) \tag{5.33}
\end{equation*}
$$

When the energies of the states are equally spaced, as in a harmonic oscillator, then a time delay corresponds to pure rotation in phase space. If the state has only 2 frequency components then pure rotation once again occurs.

In a lattice this result doesn't hold true. For deep lattices, the energies are well approximated by an anharmonic equation

$$
\begin{equation*}
E_{n}=E_{0}+n E^{\prime}-\frac{n^{2}}{2} E^{\prime \prime} \tag{5.34}
\end{equation*}
$$

where $E_{0}=\hbar \omega / 2$ is the ground state energy, $E^{\prime}=\hbar \omega$ and the anharmonicity $E^{\prime \prime}=$ $\hbar \omega_{r e}$ is the effective recoil energy of the lattice. A superposition state evolving in a lattice may then have a change in the shape of the state in phase-space as well as undergoing rotation. In addition to the anharmonicity, each band has an associated bandwidth. For bands deep in the well, the band width can be negligible compared to the anharmonicity, while bands near the top of the wells can have bandwidths nearly as large as the band gap.

In this experiment we perform an ensemble measurement using several million atoms in a cloud that is about a millimeter in size trapped in a lattice composed of laser beams with an rms width of 3 mm . In the Wannier picture each atom in the lattice can be described by a vibrational state $n$ and a well index $\ell$. An atom in an arbitrary state of the lattice is then

$$
|\Psi\rangle=\sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty} c_{n} c_{\ell}\left|\Psi_{n, \ell}\right\rangle,
$$

and the corresponding density matrix is

$$
\rho=|\Psi\rangle\langle\Psi| .
$$

Our measurements are insensitive to the long range order of the lattice. The measurements are insensitive to the quasi-momentum degree of freedom and to the lattice site where an atom may be localized. We do obtain information about which band the atom occupied, effectively measuring the reduced density matrix describing the vibrational state, averaging over all lattice sites.

$$
\rho_{n}=T r_{\ell}(\rho)
$$

### 5.2.1 Experimental Details: Lattice characterization

The experiment begins by cooling and trapping ${ }^{85} \mathrm{Rb}$ atoms in a vapour cell MOT. The MOT phase is on for 1.5 seconds to load roughly 50 million atoms into the MOT. The magnetic gradient for the MOT is turned off and the detuning of the laser cooling beams is increased to cool the atoms to a temperature of about 8 microKelvins with an optical molasses. The optical lattice is loaded with atoms during the molasses cooling phase. If the lattice is turned on after the atoms have been cooled, then roughly half the number of atoms will be located near the top of a potential well, and will gain a significant amount of potential energy. This increase in potential energy makes it very unlikely, if not impossible, to trap any atom that starts near the top of a well. If the lattice is turned on during the optical molasses phase, then the atoms will localize near the minima of the wells, and will be more likely to become trapped.

The depth of the lattice is adjustable, but is chosen to be 60 recoil energies deep for the first experiment. The optical lattice beams have an rms radius of 3 millimeters. The power in the central 1 mm then varies by less than $6 \%$ from the center of the atom cloud to the edges.

After cooling the atoms in an optical molasses for 4 milliseconds the molasses beams are turned off and the atoms are subject only to the optical lattice potential. When initially loaded in this manner the lattice typically has atoms occupying many different bands.

### 5.2.2 Measuring State Populations

In order to measure phase-space distributions using the techniques outlined above we require a method to measure the band populations. Measurement of vibrational state populations has been accomplished by adiabatically decreasing the depth of the optical lattice[58] after loading it with a BEC[59, 60]. This technique works well if only a single quasimomentum is populated since it relies on mapping band
populations into the free-particle Brillioun zones. For our work we require a method to measure state populations for arbitrary quasi-momenta and when the atoms are subject to an another force (gravity).

Measurement of the state populations is performed by adiabatically decreasing the depth of the lattice potential. The potential depth must be decreased adiabatically to prevent transitions from occurring between bands. For a process to be adiabatic requires that the timescale for the external perturbation is much slower than the timescale for the internal dynamics of the system. To satisfy adiabaticity we must have

$$
\begin{equation*}
\frac{|d \omega / d t|}{\omega}=\epsilon \omega \tag{5.35}
\end{equation*}
$$

where $\epsilon \ll 1$ for all timescales. This adiabaticity is least satisfied when the potential is decreased quickly, and also when the potential is weak. In the course of these experiments the potential is decreased by at most $6 \mathrm{E}_{r e}$ per millisecond, which also happens to occur when ramping down to the shallowest optical lattice we use. With these conditions we find that the maximum value is $\epsilon=0.021$, thereby satisfying the adiabaticity criterion. An experimental characterization of the adiabaticity is performed in Appendix D. 1 by varying the time over which the potential is decreased.

The time scale of the ramp down in this experiment varies from 2 ms to 60 ms . With a long ramp duration atoms in each band of the lattice tunnel out of the lattice as the band energy approaches the well depth. In this manner atoms in each energy band tunnel out of the lattice at different times, with the atoms in the highest energy bands escaping first. Once an atom becomes unbound it quickly accelerates under gravity, causing each energy band to be mapped into a different region of space. The spatial distribution of the atoms is recorded by applying resonant laser light for 1 ms and imaging the fluorescence on a CCD camera placed to the side of the vacuum chamber, with the upwards direction on the image corresponding to the upward direction in the lab. The beams used are the optical molasses beams, but the frequency of the beams has been adjusted to be resonant with stationary atoms. Each measurement
is performed 9 times, capturing one image on the CCD camera for each measurement. We perform averaging of the images to get 3 final images. The images can contain background light from fluorescence of untrapped atoms and scattering of laser light off of the glass surfaces of the glass vacuum cell. An extra image is obtained (also averaged), that has an optical molasses but no lattice, that is used to subtract the background from the images. A sample image, after background subtraction, is shown in Figure 5.3a. The populations of each band are determined by integrating the light signal over a region of interest. Note in the Figure that atoms at the edges of the cloud fall out of the lattice before atoms in the center, resulting in an umbrellashaped cloud for each energy band. This results from the weakening of the lattice potential as one moves toward the edges. As the band edge approaches the top of the well the tunneling probability gradually increases until it approaches unity when the band becomes unbound. The spatial distribution of the cloud in the vertical direction is then no longer Gaussian. Figure 5.3b shows a sample vertical profile under this form of adiabatic decrease. Each cloud is broader than the separation between states, but 3 peaks can be clearly resolved. The relative populations are measured by fitting the cloud profile to a sum of Gaussian distributions. The area under each Gaussian is proportional to the number of atoms in the state.

If only 2 energy states are bound in the lattice then the state populations may be more easily measured by a faster, but still adiabatic, ramp down. The depth of the lattice may be quickly (in 3 ms ) ramped down from the initial depth,with 2 bound bands, to an intermediate depth with 1 bound band. The depth can then be held constant for 10 ms , whereupon the lattice is quickly turned off and fluorescence imaging is performed after a 20 ms delay. In this manner atoms in band $N=1$ fall to a position of $z=-4 m m$, atoms in $N=0$ end up at $z=-2 m m$, allowing each band population to be easily determined via fits to Gaussian profiles. If a displacement is performed when there is some population in the excited state, then some fraction of the atoms will be transferred into unbound states of the lattice. These atoms


Figure 5.3: Atom cloud after adiabatic release from lattice.


Figure 5.4: Atom cloud after fast release from lattice.
immediately start to accelerate under gravity, and end up at a position below the $\mathrm{N}=1$ atoms. These atoms must be counted as well to ensure proper normalization of the populations. Figure 5.4a shows a sample fluorescence image with step-wise potential decrease and Figure 5.4b shows the corresponding profile demonstrating curves that are well fit by a Gaussian. The fractional population in each band is determined by from the ratio of the area under the curve representing the desired state to the area under all the curves. The fractional populations have a typical statistical uncertainty of 0.02 .

### 5.2.3 State Preparation-Creating a ground state distribution

Before we begin to measure a phase-space distribution we prepare the atoms in a single state of the lattice. The first step of this preparation is to eject any atoms that are in the excited state and leave behind a sample of atoms that are all in the ground state. Atoms in the excited states of the lattice are filtered out by adiabatically decreasing the depth of the lattice until it only supports a single bound state. This occurs at a depth of roughly $9 E_{r e}$. At this lower potential depth all states have a
decreased lifetime due to Landau-Zener tunneling. The Landau-Zener limited lifetime in the ground state is 250 ms , and in the excited state it is about 1.0 ms (see Appendix D.3). The lattice is held at this depth for 3 to 5 milliseconds, long enough for excited state atoms to escape, but not too many of the ground state atoms. Figure 5.5 shows the well depth as a function of time for a typical run and sample CCD images before and after filtering.

$\mathbf{t}$ (ms)



Figure 5.5: Sample procedures and images showing the state populations after thermally loading a lattice, and after filtering out the higher energy bands.

A sample of atoms prepared in the ground vibrational state will be the starting point for many of the experiments that will be discussed in this dissertation. The filtering step outlined above is used to prepare the ground state, but it isn't perfect. Upon measuring the state populations we typically find $90 \%$ of the atoms are in the ground state and $10 \%$ are in the excited state. We consider several possible reasons for this to occur. The time scale of the changes in the lattice potential could be too fast, causing ground state atoms to under Landau-Zener transitions to the excited state. In Appendix D. 1 we characterize the adiabaticity of the changes in the
potential. We find that if the lattice depth changes too quickly, then some atoms can change vibrational state. So long as the potential changes slower than $12 E_{r e} / m s$ we find that the changes are adiabatic. In the experiment we modulate the potential by at most $4 E_{r e} / m s$, and therefore expect all changes in the potential depth to be adiabatic.

Another possible explanation is that the atoms could be subject to heating in the period between preparation of the ground state and measurement of the populations (see Appendix D.2). We do find some small amount of heating, but it is not enough to explain the discrepancy. The heating was only measured at a single well depth. It is possible that at some intermediate well depth that the atoms could have a large heating rate.

In Appendix D.3. we measure the tunneling rate of excited state atoms out of the trap at a depth of $9 \mu K$. We find that the excited state atoms escape with a lifetime of 1.2 ms . We typically hold the lattice at a low depth for 3 ms , so we might not expect to have lost all the excited state atoms. However when we hold the lattice at a low depth for 20 ms , we still find about $10 \%$ of the atoms in the excited state.

Alternatively, imperfections in the lattice potential may be the source of the imperfect ground state creation. The optical lattice beams travel through several glass surfaces and can get unwaned interference fringes. The fringes create areas of the lattice that are shallower than expected, and other areas that are deeper than expected. If the lattice has different well depths in different locations, then it can be difficult to cause the excited state population to tunnel out of the lattice without losing too many of the ground state atoms.

Another possible explanation is Wannier-Stark transitions between neighbouring wells. At a lattice depth of about $6 E_{r e}$ the ground state in a particular well is degenerate with the energy of a neighbouring excited state. Atoms can then tunnel to the neighbouring well, gaining a vibrational quanta of energy. To accurately determine the cause of the excited state population we may require an alternative

| $n$ | $E_{n} / h$ | $\left(E_{n}-E_{n+1}\right) / h$ | $\Delta E_{n} / h$ | Lifetime $(\mathrm{ms})$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 5204 | 10026 | 0.01 | $\infty$ |
| 1 | 15230 | 9200 | 0.49 | $\infty$ |
| 2 | 24430 | 8200 | 10.9 | $3.3 \cdot 10^{7}$ |
| 3 | 32630 | 6930 | 132 | 2360 |
| 4 | 39555 | 6040 | 803 | 2.54 |
| 5 | 45650 | 6098 | 2291 | 0.219 |

Table 5.2: Energy spacings and lifetime of atoms in a $60 E_{r e}$ deep lattice
method to load atoms into the ground state, such as Raman sideband cooling[61, 62], or another method to probe the state populations.

### 5.2.4 Measuring Oscillation Frequencies

The oscillation period of the lattice is determined by a Ramsey style experiment. After preparing a sample of ground state atoms the optical lattice is displaced by 155 nm . The displacement projects the ground state into a superposition of states of the new lattice. A time delay is added to allow the relative phase of each state to evolve. A final displacement, undoing the initial displacement, is performed to couple the states once again. The population in the ground state is then measured and plotted as a function of time delay. Figure 5.6 shows a typical experimental run with a well depth of 60 recoil energies. We see a clear signature of fringes, but with an exponential decay of the signal. The fringes have a period of 96 microseconds, implying a well depth of $60 E_{r e}(\approx 4.0 \mu \mathrm{~K})$. The decay time is observed to be about 300 microseconds.

At a depth of $60 E_{r e}$ the lattice supports 5 bound bands. Table 5.2 lists the mean energy, energy separation, bandwidth and Landau-Zener limited lifetime for atoms in the bound bands in a lattice with a potential depth of 60 recoil energies.

The observed decay cannot be attributed to the anharmonicity, or the widths of the bands alone. The anharmonicity alone would not cause a complete decay of the
signal, and the bandwidth wouldn't cause the signal to decay for several milliseconds. A decaying signal such as we observe is possible if we have inhomogeneous well depths in the lattice. The number of periods observed is consistent with the number of oscillations observed in related work studying decay of wave-packet oscillations in optical lattices[63, 64, 65].

The optical lattice beams have a width of 3 mm , implying a $6 \%$ variation over the central mm where the atoms are initially trapped. The $6 \%$ intensity variation would cause a $3 \%$ change in the oscillation frequency, which would lead to decay over roughly 30 oscillations ( 3 ms ). But, the atoms sits in the lattice for roughly 30 ms before the displacements and time delays begin. An atom cloud with an initial size of 1.0 mm and a transverse temperature of $7 \mu K$ would expand to a size of 1.3 mm . The atoms then see a variation in the potential of up to $10 \%$, which leads to a decay time of 20 periods, far longer than the three periods observed. Other factors can increase the inhomogeneity, such as distorted phase-fronts in the laser beams, which would distort the shape of the lattice, and spatial intensity fluctuations from multiple reflections in the cuvette walls.

In Figure 5.6 we also see a phase-advance at long times. We suspect that this advance may be the result of different decay times for different frequency components. For example, if the resulting signal were a result of inhomogeneous well depths, then the lower frequency components, which correspond to lower well depths, have wider bands and would dephase faster than the high frequency components which have narrower bands. After the lower frequency components have dephased, the higher frequency components would still be present, resulting in a phase-advance.

The fast decay of the oscillations implies that the coherence between the bands are quickly decaying. This fast decay can cause errors in the measurement of the phase-space distribution since the state in phase space doesn't undergo pure rotation, but instead changes shape as well. To keep errors to a minimum we perform all of our measurements within the first half oscillation. During this time the coherences


Figure 5.6: Oscillation in ground state population observed in a Ramsey experiment.
should have decayed by at most $15 \%$. Averaging over all the measurements, we expect about $7 \%$ decoherence of the state.

### 5.2.5 Q of a coherent state

The Q distribution is measured for a displaced ground state in an optical lattice that is generated by displacing the lattice after preparation of a ground state. In the harmonic oscillator approximation, a displacement of the trap by a distance $d$ creates a coherent state with

$$
\begin{equation*}
|\alpha|=d \sqrt{\frac{m \omega}{2 \hbar}} \tag{5.36}
\end{equation*}
$$

The displacement used is 155 nm , which creates a coherent state $|\alpha=1.0\rangle$ in the harmonic limit. In this limit we would expect to find $37 \%$ of the atoms in the ground state. We actually find about $59 \%$ in the ground state after creating the state. If


Figure 5.7: Measurement Procedure

| Delay $\backslash$ Displacement | -90 | -75 | -60 | -45 | -30 | -15 | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0 \mu S$ | .74 | .89 | .91 | .89 | .82 | .75 | .61 | .36 | .39 | .12 | .06 | .1 | .03 |
| 10 | .60 | .73 | .80 | .82 | .73 | .66 | .56 | .40 | .40 | .38 | .12 | .14 | .05 |
| 20 | .29 | .30 | .75 | .63 | .53 | .63 | .58 | .41 | .31 | .42 | .22 | .14 | .09 |
| 30 | .12 | .17 | .67 | .29 | .50 | .58 | .59 | .65 | .37 | .61 | .51 | .48 | .40 |
| 40 | .05 | .05 | .15 | .25 | .44 | .46 | .67 | .77 | .46 | .77 | .66 | .53 | .72 |
| 50 | .05 | .07 | .33 | .35 | .47 | .59 | .52 | .80 | .69 | .85 | .78 | .65 | .64 |

Table 5.3: Raw measurements for a Q distribution
we had created a coherent state, this would be more consistent with an amplitude $|\alpha|=0.70$.

Measurement proceeds as follows (see Figure 5.7). After preparing the state, we wait some time $t_{\text {rot }}$ to allow the state to rotate in phase-space. The time delay is kept to less than half a period to reduce broadening effects. The rotation is followed by displacement, where the displacements range from -250 nm to +250 nm , allowing measurement of the Q distribution in this region of phase-space. The population in the ground state is then measured for each combination of delay and displacement. The Q distribution is measured for 6 different rotations (time delays) and 13 different spatial displacements. The table below shows the measured ground state population for each combination of time and displacement.


Figure 5.8: Q distributions for experiment and theory

A linear interpolation is used to determine the value between data points. The resulting Q distribution, in normalized units is shown in Figure 5.8a. We have already noted that the displacement is inconsistent with the harmonic approximation. Figure 5.8a shows the theoretical Q distributions for a superposition of ground and excited states with a ground state population corresponding to our measurement, while 5.8 b shows the distribution coherent state $|\alpha=0.7\rangle$.

The experimental data shows similarity to both of the theoretical harmonic oscillator Q -functions. The peak value in the experimental Q function is 0.91 (before smoothing) which is located midway between the peaks of the 2 component superposition and the 'coherent state'.

### 5.2.6 Q (and pseudo-Wigner) Function for an Inverted Population

We now continue with measuring the Q function for an inverted state. A state with an inverted population distribution is created in a very deep lattice that supports 6 bound states. In this lattice a ground state is prepared using the state-filtering
procedure outlined above. The filtering is immediately followed by resonantly shaking the lattice for 3 periods. The shaking excites many atoms into higher energy states of the lattice. The lattice depth is subsequently decreased until the lattice only supports 2 bound states. After waiting sufficiently long that all the unbound atoms have escaped we measure the state populations to be 0.37 and 0.63 for the ground and excited states respectively. The oscillation frequency of the lattice is determined to be 180 microseconds by a Ramsey experiment at the same potential depth. An additional 3 ms delay is added after the state filtering to ensure that the sample has completely decohered. The lack of coherence is observed by displacing the lattice and finding very little dependence of the state populations on the delay before displacement.

Measurements are performed for time delays over the first half-period, with the delays ranging from 10 to $100 \mu s$, in steps of $10 \mu s$. At each time delay we perform a displacement, ranging from -250 nm to +250 nm in 15 steps. Each measurement consists of 3 images, each of which is an average of 3 images. The measured points are placed in a square matrix that is $30 \times 30$. Using only the ground state population we first find the Husimi distribution from the data points. The lattice used here is shallower than the lattice used in the measurement of a 'coherent' state, so we expect larger deviations from the Husimi distribution. The lattice depth for this experiment is $22 E_{r e}$ and as such the ground state has longer tails than the minimum-uncertainty coherent state. The maximum deviation $\xi$ between the measured function and the Husimi distribution is 0.03 that occurs at the origin.

Figure 5.9a shows the experimentally measured distribution that had $37 \%$ of the atoms in the ground state and $63 \%$ in the excited state. 5.9 b shows the theoretical distribution for a statistical mixture with $37 \%$ of the atoms in the ground state and $63 \%$ in the excited state for a harmonic oscillator.

The Wigner function is determined by the populations in the number states. In this experiment the populations in the unbound states were unable to be resolved,


Figure 5.9: Q distribution for an inverted population
but the populations in the ground and excited states were resolved. Figure 5.10 shows the smoothed distribution of data points corresponding to $\left|c_{0}\right|^{2}-\left|c_{1}\right|^{2}=Q_{0}-Q_{1}$ for the various displacements while 5.10 b shows the theoretical distribution in a harmonic oscillator with a mixed state with the same populations as observed in the lattice.

There is a clear signature of a negative value at the origin, which is characteristic of a large population in odd numbered Fock states. The cylindrical symmetry of the distribution points to a mixture of states with minimal coherences. Cuts through the center of the distribution are shown in Figure 5.11. The curves shown are cuts along the momentum axis, where the solid curve is the experimental distribution while the dashed curve is theoretical distribution. Near the origin the experimental and harmonic distributions agree very well, with the same depth and width to the central, negative valued feature. Farther out, near $|x|$ or $|p|=1$ the distributions do not agree as well. The theoretical distribution has a peak height of 0.2 , while the experimental distribution peaks at a height of 0.14 . This can be partly explained by the shape of the potential in the lattice. The lattice has a sinusoidal shape where the slope of the potential decreases near the top of the well. This softening of the potential makes the higher energy states broader than the equivalent harmonic oscillator eigenstates.


Figure 5.10: Wigner Distribution for an inverted state

If the excited state is broader in the lattice than in the harmonic oscillator, then the distribution $Q_{1}(\alpha)$ would be broader as well.


Figure 5.11: Cuts through the measured(solid line) and theoretical(dashed line) distributions.

## Chapter 6

## State Tomography

In this chapter we limit ourselves to the situation where the atom has significant amplitudes to be in only 2 different states. This situation is engineered by choosing an optical lattice that only has 2 bound states, corresponding to the 2 lowest energy states. All higher energy states are unbound and atoms in these states are ejected from the lattice by gravity. We identify the lowest energy band of the lattice as a ground state $|0\rangle$ and the next lowest energy band as an excited state $|1\rangle$. The states $|0\rangle$ and $|1\rangle$ may be identified as the simplest implementation of a quantum bit of information, in which the logical qubit is identical to the physical qubit. The atomic density in the lattice is low enough that we can ignore interactions between atoms and consider each atom as undergoing evolution determined only by its own quantum state and its interaction with the beams comprising the optical lattice. Individual atoms cannot be identified in this experiment, implying that we must perform an ensemble measurement. Likewise the spacing of the lattice is on the order of the wavelength of light making spatial resolution of the atoms impossible. Information about the quantum motional state can be obtained by measuring the probability for the atoms to be in the ground state or excited state after various operations.

### 6.1 Density matrix reconstruction

Determination of the quantum state of a system is a field of growing interest. Knowledge of the density matrix tells us everything there is to know about the quantum state. Density matrix reconstruction has been performed to measure the density matrix for large angular momentum atoms[66], for photons [67, 68], hydrogen-helium collisions[50] and molecular vibrational states[69]. Recent advances have demonstrated how a quantum system may be used for cryptography[70, 71], communication and computing $[72,73,74]$. All of these advances, and in particular quantum computing, have the requirement of accurate determination of the quantum system at some time.

In this chapter state tomography is performed for the motional state of atoms trapped in a lattice that supports two bound states. An arbitrary state of this system, whether it is a pure state or a statistical mixture, can be represented using a density matrix. The density matrix for a 2 state system is

$$
\rho=\left[\begin{array}{ll}
\rho_{11} & \rho_{12}  \tag{6.1}\\
\rho_{21} & \rho_{22}
\end{array}\right]
$$

where the diagonals $\rho_{11}$ and $\rho_{22}$ correspond to the populations in the ground and excited states respectively, and the off-diagonals $\rho_{12}$ and $\rho_{21}$ correspond to the coherences. Since the density matrix is Hermitian we can characterize it with 3 real parameters. These 3 parameters may be determined by measuring the overlap of the unknown state with a set of known states. Experimental characterization of this kind was first performed for the polarization state of light by Stokes[75] in 1852. To measure the Stokes parameters for a beam of light the beam is sent through various settings of waveplates and polarizers, with the transmitted power measured for each setting. The waveplates/polarizer are in turn set to transmit horizontal polarization $|H\rangle$, vertical $|V\rangle$, the diagonal $|D\rangle=\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle$ ) and right hand circular $|R\rangle=\frac{1}{\sqrt{2}}(|H\rangle+i|V\rangle)$. If the total power is known before measurement, then one
of the horizontal or vertical settings may be omitted. If the total power isn't known then 4 measurements will be required to provide normalization. By measuring all of these projections the polarization state of the light is completely determined.

In an optical lattice a similar set of measurements may be performed. A convenient choice of parameters would be equivalent to those used in measurement of the Stokes parameters, i.e., measurement by projecting onto the states $\{|0\rangle,|1\rangle$, $|D\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ and $\left.|R\rangle=\frac{1}{\sqrt{2}}(|0\rangle+i|1\rangle)\right\}$. We then need to find the right set of tools to project onto these states in a lattice.

### 6.1.1 Measurements for Tomography

A pure state spanning a 2-dimensional Hilbert space can be expressed as

$$
\begin{equation*}
|\Psi\rangle=c_{0}|0\rangle+c_{1}|1\rangle . \tag{6.2}
\end{equation*}
$$

Complete determination of this state is possible if the coefficients $c_{0}$ and $c_{1}$ can be measured. The coefficients are in general complex, but we are free to multiply the state by an arbitrary phase such that $c_{0}$ is real and positive, leaving $c_{1}$ to be a complex valued number. The magnitudes of $c_{0}$ and $c_{1}$ can be determined by measuring the projections $|\langle 0 \mid \Psi\rangle|^{2}$ and $|\langle 1 \mid \Psi\rangle|^{2}$. These projections can be measured if the population in the ground and excited states can be measured. The relative phase between $c_{0}$ and $c_{1}$ can be determined by projecting the state $|\Psi\rangle$ onto a pair of superposition states such as $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle+i|1\rangle)$. If the state is pure, then projecting onto just one of these states suffices.

We use a density matrix treatment of the projections since we are dealing with an ensemble of systems that is unlikely to be in a pure state. The equivalent projections and the results, which completely identify the density matrix, are listed in the Table 6.1.

Projecting onto equal superpositions is not a necessary requirement of this mea-

| Projection | Result |
| :---: | :---: |
| $\langle 0\| \rho\|0\rangle$ | $\rho_{11}$ |
| $\langle 1\| \rho\|1\rangle$ | $\rho_{22}$ |
| $\left\langle\frac{1}{\sqrt{2}}(\|0\rangle+\|1\rangle)\right.$ | $\rho\left\|\frac{1}{\sqrt{2}}(\|0\rangle+\|1\rangle)\right\rangle$ |
| $\left\langle\frac{1}{\sqrt{2}}(\|0\rangle+i\|1\rangle)\right.$ | $\rho \frac{1}{2}+\frac{1}{2}\left(\rho_{12}+\rho_{12}^{*}\right)=\frac{1}{2}+\operatorname{Re}\left(\rho_{12}\right)$ |

Table 6.1: Projections for density matrix reconstruction and resulting information gained
surement, but projecting onto a complete set of states is required. In this experiment we perform state tomography by measuring the projection of the density matrix onto the set of states $\Phi_{1 . .4}=\{|0\rangle,|1\rangle,|D\rangle$ and $|R\rangle\}$. The states presented here are not an orthonormal set, but are sufficient for state tomography[76]. The first two states are an orthogonal pair, but the latter two states are not. The latter states are

$$
\begin{align*}
|D\rangle & =a|0\rangle+b|1\rangle  \tag{6.3}\\
|R\rangle & =a|0\rangle+i b|1\rangle \tag{6.4}
\end{align*}
$$

where $a^{2}+b^{2}=1$ and $a$ and $b$ are real.
Each measurement is an evaluation of the projection $\langle\Phi| \rho|\Phi\rangle$. If we can find a unitary process such that $|\Phi\rangle=\widehat{U}|0\rangle$, then the projections may be written as $m=\langle 0| \widehat{U}^{\dagger} \rho \widehat{U}|0\rangle$. Therefore instead of measuring the projection of the unknown state onto a set of known states, we can instead measure the ground state population after transforming the unknown state. What is then required is finding the operation that transforms state $|\Phi\rangle$ into state $|0\rangle$. As previously discussed in connection with phase-space distributions, this transformation can be achieved using a combination of spatial displacement and time delays.

Displacement of a ground state creates a new state that is a superposition of a number of other states. This is well known in a harmonic oscillator where the
coherent state,

$$
\begin{equation*}
|\alpha\rangle=e^{-|\alpha|^{2} / 2} \sum \frac{\alpha^{n}}{n!}\left(\widehat{a}^{\dagger}\right)^{n}|0\rangle, \tag{6.5}
\end{equation*}
$$

is a displacement of the ground state, i.e. $|\alpha\rangle=\widehat{D}(\alpha)|0\rangle$ where $\widehat{D}(\alpha)=\exp \left(\alpha \widehat{a}^{\dagger}-\right.$ $\left.\alpha^{*} \widehat{a}\right)$. If the translation is only in position, then the parameter $\alpha$ is real. In an optical lattice we expect the displacement operation to have a similar effect (to create a superposition of states) but we cannot expect a superposition state with the same amplitudes as in equation 6.5. Displacement of the state in the lattice is achieved by displacing the trapping potential. We consider a displacement operator of the form

$$
\begin{align*}
\widehat{D}|0\rangle & =\sum_{n=0}^{\infty} c_{n 0}|n\rangle  \tag{6.6}\\
\widehat{D}|1\rangle & =\sum_{n=0}^{\infty} c_{n 1}|n\rangle \tag{6.7}
\end{align*}
$$

where $c_{n m}$ is the inner product between the state $m$ of the initial lattice and state $n$ of the displaced lattice. We use small displacements (typically 116 nm ) such that the inner product between the ground state and the unbound states is negligible. We directly measure the coefficients $c_{00}, c_{10}, c_{01}, c_{11}$ and neglect all others. If the displacement is small enough, then $\left|c_{00}\right|^{2}+\left|c_{10}\right|^{2} \approx 1$, but $\left|c_{01}\right|^{2}+\left|c_{11}\right|^{2} \leq 1$, reflecting the possibility that atoms in the excited state may be coupled to unbound states and become lost. The displacements used here are in position, so we expect all the coefficients in $\widehat{D}$ to be real. The operator $\widehat{D}$ can be characterized by preparing a ground state and performing the desired displacement. A subsequent measurement of the ground and excited state populations will give the magnitudes of the couplings $c_{00}$ and $c_{01}$. Due to the symmetry of the states, $c_{10}=-c_{01}$. The parameter $c_{11}$ could also be determined, but it is not vital to the reconstruction of the state.

Since a ground state is projected into a superposition of ground and excited states only, we can redefine the coefficients $c_{00}$ and $c_{10}$ in terms of an angle $\gamma$, such that $c_{00}=\cos \gamma$, and $c_{10}=\sin \gamma$. The displaced ground state corresponds to the state

| State | Band Basis | Displacement (nm) | Rotation |
| :---: | :---: | :---: | :---: |
| $\left\|\Phi_{1}\right\rangle$ | $\|0\rangle$ | 0 | 0 |
| $\left\|\Phi_{2}\right\rangle$ | $\|1\rangle$ | 0 | 0 |
| $\left\|\Phi_{3}\right\rangle$ | $\cos \gamma\|0\rangle+\sin \gamma\|1\rangle$ | 250 | 0 |
| $\left\|\Phi_{4}\right\rangle$ | $\cos \gamma\|0\rangle-i \sin \gamma\|1\rangle$ | 250 | $\pi / 4$ |

Table 6.2: Displacements and rotations to achieve measurement basis
$|D\rangle$ because it has a real value for the coherence between the ground and excited states. In addition to creating superpositions with real coherences, we also require superpositions that have an imaginary coherence. In order to change the relative phase between the ground and excited states, we use a phase-shift operator

$$
\widehat{R}(\theta)=\left[\begin{array}{cc}
1 & 0  \tag{6.8}\\
0 & e^{-i \theta}
\end{array}\right]
$$

where $\theta=\left(E_{1}-E_{0}\right) t / \hbar$. The phase-shift operator is merely a time delay, causing the states to pick up a relative phase.

The set of states which the density matrix is projected onto is determined by the states that can be produced subject to the restrictions placed on the displacement. The following table gives the projection states and their decomposition in the vibrational state basis, and the displacement magnitude and rotations

Projection onto the states $\left|\Phi_{3}\right\rangle$ and $\left|\Phi_{4}\right\rangle$ are accomplished by operating on the unknown state with the operation that rotates $\left|\Phi_{3}\right\rangle$ (or $\left|\Phi_{4}\right\rangle$ ) onto the ground state, followed by measurement of the ground state population. The measurement for states $\left|\Phi_{3,4}\right\rangle$ can then be stated

$$
\begin{equation*}
m_{3,4}=\langle 0| U^{\dagger} \rho U|0\rangle \tag{6.9}
\end{equation*}
$$

where $U$ is the required operation.
After performing the necessary displacements and time delays on the unknown state we then project onto the ground state and record a number of values $m_{1 . .4}$,

| $\|\Phi\rangle$ | Measurement Result |
| :---: | :---: |
| $\|0\rangle$ | $m_{1}=\rho_{11}$ |
| $\|1\rangle$ | $m_{2}=\rho_{22}$ |
| $\|D\rangle$ | $m_{3}=\cos ^{2} \gamma \rho_{11}+\sin ^{2} \gamma \rho_{22}-2 \sin \gamma \cos \gamma \operatorname{Re}\left(\rho_{12}\right)$ |
| $\|R\rangle$ | $m_{4}=\cos ^{2} \gamma \rho_{11}+\sin ^{2} \gamma \rho_{22}+2 \sin \gamma \cos \gamma \operatorname{Im}\left(\rho_{12}\right)$ |

Table 6.3: Measurement results and their relation to the density matrix
given in the following table.
Using the value of $\gamma$ and the results of the four measurements, the 2 x 2 density matrix may be reconstructed.

### 6.1.2 The Experiment

State tomography is performed using Rubidium- 85 atoms in an optical lattice. Rubidium atoms are first cooled in an standard vapour-cell magneto-optical trap, with a magnetic gradient of $10 \mathrm{G} / \mathrm{cm}$. Following the collection/cooling stage we further cool the atoms with a $\sigma^{+} \sigma^{-}$molasses to a temperature of 6 to 10 microKelvin with about 50 million atoms remaining in a cloud with an rms size of 600 to 1000 microns. The optical lattice beams are turned on during the molasses stage to maximize the number of atoms trapped in the bound states of the lattice. The optical lattice beams have an rms radius of 3 millimeters. The power in the center 1 mm then varies by less than $6 \%$ from the center of the atom cloud to the edges. The depth of the trap is chosen to be 18 recoil energies. At this potential depth the lattice supports two bound states and the mean energy difference between the ground and excited states is

$$
\begin{equation*}
\left\langle E_{1}\right\rangle-\left\langle E_{0}\right\rangle=\hbar 2 \pi \cdot 5.0 \mathrm{kHz} \tag{6.10}
\end{equation*}
$$

This is a convenient depth as the period of oscillation is $200 \mu s$, which is easily addressed by the SRS function generator. A quarter period $(50 \mu s)$ is also easily addressed by the SRS function generator at this well depth. The band structure for

| $\mathbf{n}$ | $\left\langle E_{n}\right\rangle / h$ | $\left(\left\langle E_{n}\right\rangle-\left\langle E_{n-1}\right\rangle\right) / \hbar$ | $\Delta E_{n}$ | Lifetime $(\mathrm{ms})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 2721 |  | 4.9 | $3 \cdot 10^{9}$ |
| 1 | 7721 | 5000 | 109 | 70.8 |
| 2 | 11786 | 4064 | 714 | 0.56 |
| 3 | 15627 | 3841 | 1860 | 0.11 |

Table 6.4: Mean band energy, Band gap, band width and lifetime of the energy bands in the tilted lattice.
this lattice is given by Figure6.1.


Figure 6.1: Band Structure for a lattice 18 recoils deep. The dashed line refers to the depth of the potential.

The following table list some properties of the four lowest energy bands of the lattice. The third band is almost bound, but not completely, and experiences a large escape rate. The Landau-Zener limited lifetime of atoms in the third band of the lattice is less than a millisecond, so atoms in the third band will be lost from the lattice and be indistinguishable from atoms that are in unbound states. Only atoms that are in the two lowest energy bands have an appreciable lifetime.

The well depth of the lattice is typically less than the temperature of the atom
cloud and holds less than $10 \%$ of the initial atoms. The remaining atoms are in unbound states of the lattice. Atoms from the MOT that are not in bound states of the lattice experience large Landau-Zener tunneling probabilities and follow freeparticle trajectories, falling under the influence of gravity. Within 500 microseconds they have gained too much energy to become trapped by any spatial translation of the lattice, and within 14 milliseconds they have fallen 1 mm , becoming spatially separated from the atoms confined in the lattice. The atoms that remain trapped in the lattice are in the two bound states.

### 6.1.3 State Preparation

Several different states must be prepared in order to test the effectiveness of the state tomography procedure. The states we used for testing are a ground state, an excited state, and 2 coherent states that differ in relative phase by $\pi / 2$.

The ground state is prepared by filtering out the higher energy states in the lattice. This is accomplished by adiabatically decreasing the depth of the lattice until it only supports a single bound state. This occurs at a depth of roughly $9 E_{r e}$. At this lower potential depth all states have a decreased lifetime due to Landau-Zener tunneling. The Landau-Zener limited lifetime of the ground state is 250 ms , and in the excited state it is 0.9 ms . The lattice is held at this depth for 3 milliseconds, long enough for excited state atoms to escape, but not enough time to lose many of the ground state atoms. Figure 5.5 shows a sample CCD image before and after filtering.

Superposition states are created by displacing the optical lattice after preparing the atoms in a ground state. Displacing the atoms projects the ground state into a linear superposition of the states of the displaced lattice. The displacement used in this step is typically larger than the displacement used in the measuring stage. This is done to obtain a superposition with a more equal weighting between the states than could be achieved with the measurement displacement. Unfortunately this also has the effect of creating a measurable population in the unbound states which
must be accounted for. The remaining atoms are in a superposition of the ground and excited states. The relative populations are measured by using the adiabatic ramp-down method. Immediately after displacing the lattice, both states have real amplitudes. By waiting a quarter period of oscillation the excited state will have an imaginary amplitude.

Preparation of a purely excited state has not yet been achieved. We instead produce a mixed state with population in both the ground and excited states, but no coherence between them. The mixed state is produced in one of two ways. The easiest way to prepare a mixed state is to simply remove the filtering step, in which case the lattice is thermally loaded. The second method is to create a superposition state and then wait several milliseconds to allow the states to decohere.

### 6.1.4 Procedure

The optical lattice is turned on during the molasses stage so atoms may be cooled into the lattice. The lattice has a small potential depth, such that it only contains two bound states, with a maximum energy of $1.2 \mu \mathrm{~K}$. At this shallow depth, the lattice contains about $10 \%$ of the atoms that began in the MOT. We have a delay of 10 ms to allow all unbound atoms to begin falling before we filter the lattice to obtain a sample with only ground state atoms. State preparation (by displacements and time delay) follows immediately after ground state preparation. The measuring phase immediately follows the preparation of states. Figure 6.2 shows a typical procedure showing the well depth and the lattice displacements as a function of time.

At a well depth of $1.2 \mu K$ the energy difference between the ground and excited states is $\Delta E=h \cdot 5.0 \mathrm{kHz}$. The set of input states is prepared as discussed in the previous section.

A state $\rho$, corresponding to one of the state preparations, is prepared after creating a sample of ground state atoms. For an unknown state $\rho$, we perform projections onto the 4 states $\left\{\Phi_{1 . .4}\right\}$. At the beginning and end of the data run we characterize


Figure 6.2: Measurement Timing
the displacement operator by displacing the ground state. The state $\rho$ is prepared 9 times for each projection measurement and the populations are recorded on a CCD camera after operating on the state. Profiles taken through the center of the cloud are extracted and fit to a sum of Gaussians. The area under each Gaussian is integrated and the ground state population is determined from the ratio of the area under the curve representing the ground state to the area under all 3 curves.

Figure 6.3 shows a typical set of images used to reconstruct a coherent state. In each image, the dark regions correspond to atoms. Three groups of atoms are seen in each image. The upper group of atoms corresponds to atoms that were in the ground state of the lattice. The second group corresponds to atoms that were in the excited state, and the lowest group corresponds to atoms in unbound states. The images 6.3a-c are of atoms with identical preparations, but with different measurement projections. Figure 6.3a shows the image obtained when we project onto the ground and excited states. The groupings of atoms, from top to bottom, are proportional to $\rho_{11}, \rho_{22}$ and unbound atoms respectively. The unbound atoms are generated during the state


Figure 6.3: Sample images for reconstructing a density matrix. From top to bottom, the number of atoms in each cloud is proportional to ground state atoms, excited state atoms and unbound atoms. Image a) shows the projection onto the vibrational states. Images b) and c) show the resulting clouds after projecting onto superposition states with real (b) and imaginary (c) coherences.
creation process. Figure 6.3 b is obtained after performing a displacement to project onto the state $\left|\Phi_{3}\right\rangle=\cos \gamma|0\rangle+\sin \gamma|1\rangle$. The number of atoms in the top grouping is proportional to $\cos ^{2} \gamma \rho_{11}+\sin ^{2} \gamma \rho_{22}-2 \sin \gamma \cos \gamma \operatorname{Re}\left(\rho_{12}\right)$. The third image, Figure 6.3 c show the image obtained after projecting onto $\left|\Phi_{4}\right\rangle$. In this case number of atoms in the upper group is proportional to $\cos ^{2} \gamma \rho_{11}+\sin ^{2} \gamma \rho_{22}+2 \sin \gamma \cos \gamma \operatorname{Im}\left(\rho_{12}\right)$.

We now proceed with the reconstruction of a density matrix. Here we shall follow the steps from images to reconstructed density matrices for an initial state created by displacing a ground state. Profiles taken through the center of the clouds are shown in Figure 6.4. All the profiles result from a step-wise decrease of the lattice potential. In each of the curves there are three prominent peaks corresponding to (left to right) ground state atoms, excited state atoms and lost (escaped) atoms. The solid lines are fits to the profiles, where each fit is to the sum of three Gaussians plus a slope and an offset. The red curve corresponds to a measurement of the state populations


Figure 6.4: Profiles taken through the clouds after projecting the prepared state into 3 different bases. The red points correspond to the populations in the prepared state, blue to populations after a displacement and green to the populations after a time delay and displacement. The solid lines are fits to the curves.
after state preparation. The blue and green curves correspond to measuring the state populations after a displacement immediately after state preparation(blue) and after an additional quarter period time delay(green). The red curve has a noticeable population in the escaped atoms, which occurs during state preparation, but not during the measurement. The blue and green curves have larger populations in the escaped cloud that comes from the state preparation as well as from performing a displacement when there is population in the excited state.

The fractional area under each Gaussian is tabulated in Table 6.5 along with the standard deviation. The table also includes the characterization of the displacement operator. The second table uses the values to find the results of the measurement $m_{1} \ldots m_{4}$, and these are used to reconstruct the density matrix.

Atoms lost during the preparation of the state are not counted in the normalization. While we have chosen a small displacement such that the coupling between the

| Operation | ground state <br> fraction | excited state <br> fraction | escaped <br> fraction |
| :---: | :---: | :---: | :---: |
| Prepare G.S. | $.915(16)$ | $.085(16)$ | 0 |
| Displace G.S. | $.707(5)$ | $.238(5)$ | $0.055(5)$ |
| Prepared state | $.626(14)$ | $.281(7)$ | $.093(7)$ |
| Displaced state | $.545(20)$ | $.310(2)$ | $.146(21)$ |
| delayed and displaced state | $.217(3)$ | $.310(15)$ | $.472(15)$ |

Table 6.5: Raw measurements used for density matrix reconstruction

| Displacement | $\cos \gamma=.866(17)$ | $\sin \gamma=.500(22)$ |
| :---: | :---: | :---: |
| State Prep (G.S.) | $m_{1=\frac{.626}{.626+.281}}$ | $\rho_{11}=.690(19)$ |
| State Prep (E.S.) | $m_{2}=\frac{.281}{.626+.281}$ | $\rho_{22}=.310(17)$ |
| Displace | $m_{3=\frac{.545}{.526+.281}}=.601$ | $\operatorname{Re}\left(\rho_{12}\right)=\frac{m_{3}-\left(\cos ^{2} \gamma \rho_{11}+\sin ^{2} \gamma \rho_{22}\right)}{-2 \sin \gamma \cos \gamma}=-0.01(4)$ |
| Delay and Displace | $m_{4}=\frac{.217}{.626+.281}=.239$ | $\operatorname{Im}\left(\rho_{12}\right)=\frac{m 4-\left(\cos ^{2} \gamma \rho_{11}+\sin ^{2} \gamma \rho_{22}\right.}{2 \sin \gamma \cos \gamma}=-0.41$ (4) |

Table 6.6: sample density matrix reconstruction extracted from raw data. Numbers listed in brackets are the uncertainty in the value. Populations are measured to an accuracy of 0.02 and coherences with an accuracy of 0.04 .
ground state and unbound states is negligible the coupling between the excited state and unbound states is not small. When we prepare a coherent state we use a larger displacement than we have used for the measurement. This large displacement has a significant coupling to the unbound states, and we must be careful not to include atoms lost during preparation of the state in the normalization. The number of atoms remaining in the ground and excited states is normalized against the number in those states after preparation of the state, but without further displacements of the lattice.

The numbers listed above for the reconstruction include the statistical errors due to fluctuations in the state populations. There are several possible sources of systematic errors. The laser used to create the optical lattice is a grating-stabilized diode laser that operates 30 GHz away from resonance, far from any resonance to which it could be locked. Therefore the laser frequency may drift and occasionally mode-hop. A mode-hop in the diode laser causes a frequency shift of at least 8 GHz .

| Attempted State | Experimentally Reconstructed <br> Density Matrix |  |  |
| :---: | :---: | :---: | :---: |
| $\|0\rangle\langle 0\|$ | $\left[\begin{array}{cc}0.90 & -0.01+i 0.02 \\ -0.01-i 0.02 & 0.10\end{array}\right.$ | Purity |  |
| $P_{0}\|0\rangle\langle 0\|+P_{1}\|1\rangle\langle 1\|$ |  | 0.60 $0.01+0.02 i$ <br> $0.01-0.02 i$ 0.40 | $0.828(26)$ |
| $(a\|0\rangle+b\|1\rangle)(a\langle 0\|+b\langle 1\|)$ |  | 0.69 $0.41+0.01 i$ <br> $0.41-0.01 i$ 0.31 | $0.528(8)$ |
| $(a\|0\rangle+i b\|1\rangle)(a\langle 0\|-i b\langle 1\|)$ |  | 0.69 $0.01-0.35 i$ <br> $0.01+0.35 i$ 0.31 | $0.916(68)$ |

Table 6.7: Reconstructed density matrices with statistical uncertainties.

This will cause a change in the depth of the optical lattice and also in the oscillation frequency. This frequency shift can then lead to errors in reconstructing the coherences, partly due to a change in the displacement operator and due to a change in the phase evolution. Errors due to mode hops are minimized by monitoring the effects of the displacement operator. At the beginning and end of each data set (for a particular state preparation) we characterize the displacement operator as outlined in the earlier sections. If the detuning of the laser has changed then the displacement operator is significantly changed as well.

The purity of the reconstructed state is limited by the purity with which we can prepare a ground state. Typically after attempting to prepare a ground state, we have about $90 \%$ of the atoms in the ground state and $10 \%$ in the excited state. The purity (calculated as $\operatorname{Tr}\left(\rho^{2}\right)$ ) of this state is 0.82 . The purity of the state will not be improved by any unitary transformation (such as the displacement operator.)

State tomography is tested on a number of different prepared states. We prepare a ground state, a mixed state, and 2 different coherent states. Tables 6.7 shows the reconstructed density matrix for a set of different initial states along with the purity of the measured state. Table 6.8 then compares the measured density matrix to the theoretical expectation.

The starting point for the creation of these states (with the exception of the
mixed state) is preparation of the ground state. The experimentally measured state does have some population in the excited state and some small coherence, but the coherences are zero, within error. The purity of the prepared ground state is calculated to be $0.828(26)$. Any unitary transformation doesn't change the purity of the state, so we would expect this to be the upper limit for the purity of the other state preparations.

The mixed state, prepared by displacement and a time delay long enough for decoherence to occur, shows a nearly equal mixture of ground and excited state populations. The coherences are again within a standard deviation of zero. The purity is calculated to be $0.528(8)$ which is close to the lower limit of 0.5 for a twodimensional system.

The first coherent state that is measured is created by displacing a ground state. No time evolution is allowed before measurement. The displaced state should have only real coherences. The measured state has mostly real coherences, with a small imaginary part consistent with zero. The purity is found to be 0.916(68), larger than the purity of the ground state! The measured purity has a large uncertainty and is just barely consistent with the purity of the ground state. The purity of the state may not be a result of statistical deviations, it may be a real effect. A larger purity can result when we consider only a subsystem of a larger system. The displacement operation is a unitary transformation but this is followed by a projective measurement after losing all the atoms in the unbound states of the lattice. In this manner the purity of a subsystem can be larger than the whole system.

The second coherent state measured adds a quarter-period time delay after displacement. This time delay turns the in-phase amplitudes into amplitudes $\pi / 4$ out of phase with one another. In measurement we find that the phase is predominantly imaginary with a small real part consistent with zero. The purity is determined to be $0.824(59)$, consistent with the purity of the ground state. However, this state is just the previous state with some extra time evolution. Therefore we could expect to

| Experimental Density Matrix | Theoretical | Fidelity |
| :---: | :---: | :---: |
| $\begin{array}{cc}0.90 & -0.01+i 0.02 \\ -0.01-i 0.02 & 0.10\end{array}$ | $\begin{array}{cc}0.90 & 0 \\ 0 & 0.10\end{array}$ | 0.9998 |
| 0.60 $0.01+0.02 i$ <br> $0.01-0.02 i$ 0.40 | $\begin{array}{cc}0.56 & 0 \\ 0 & 0.44\end{array}$ | 0.9989 |
| 0.69 $0.41+0.01 i$ <br> $0.41-0.01 i$ 0.31 | $\begin{array}{ll}0.66 & 0.41 \\ 0.41 & 0.34\end{array}$ | 0.9992 |
| 0.69 $0.01-0.35 i$ <br> $0.01+0.35 i$ 0.31 | $\begin{array}{cc}0.66 & -0.37 i \\ 0.37 i & 0.34\end{array}$ | 0.9993 |

Table 6.8: Comparison of the measured density matrices to the theoretical expectation, given the prepared ground state. The similarity between the experimental and theoretical density matrices is quantified using the measure of fidelity.
see a large purity once again. But, we see from Figure 5.6 that the system is subject to a significant amount of decoherence (that is studied in the next chapter) which leads to a decreased purity.

Table 6.8 compares the measured density matrices to the theoretical expectation. The expected density matrices are calculated using displaced Bloch waves assuming that the initial state is a mixed state with populations 0.90 and 0.10 for ground and excited states respectively. Decoherence is taken into account for the state that is created with the help of a quarter period time delay.

We quantify the agreement between the states by calculating the quantum fidelity[77]. The fidelity is a measure of how identical two density matrices are. The fidelity is defined

$$
\begin{equation*}
\mathcal{F}\left(\rho_{1}, \rho_{2}\right)=\operatorname{Tr} \sqrt{\sqrt{\rho_{1}} \rho_{2} \sqrt{\rho_{1}}}, \tag{6.11}
\end{equation*}
$$

where $\rho_{1}$ and $\rho_{2}$ are the density matrices being compared. The fidelity has a value of 1 for identical pure states and a value of 0 for orthogonal states. The fidelity between the measured and expected states has a minimum value of 0.999 , showing excellent agreement.

### 6.2 Conclusion

We have successfully demonstrated reconstruction of the density matrix of the vibrational states in an optical lattice. While this work is demonstrated for a fairly simple 2-level system we can imagine more complicated systems. Using a deeper lattice we can study higher dimensional systems. Using a higher dimensional system we have several options available. For example, if we consider a 4 dimensional system we could use states 0 and 1 as one qubit and then states 2 and 3 as another qubit, thereby using the motional state of the atom as 2 qubits of information. We can also attempt to define the system in a different manner, using all 4 levels to define one qubit, a situation where one logical qubit is created out of two physical qubits. Definitions such as this allow us to find decoherence free sub-spaces, or they can serve as copies of qubits to enable error correction.

State tomography can also be used to study the Hamiltonian acting on a system. By preparing and measuring the system both before and after some operation we can characterize the operation. If this is performed for enough different input states then this becomes quantum process tomography, i.e., tomography of an operation. The following chapter deals with exactly this type of tomography.

## Chapter 7

## Process Tomography

Quantum computing is a growing field that will have a large impact on many other fields. A major hurdle to be overcome is the extreme sensitivity of a quantum computer to noise, be it from decoherence or inperfect operations[78, 79]. A quantum computer[80] promises to solve certain problems that are intractable on a classical computer, such as factoring large numbers[81] and simulation of quantum dynamics[72]. A quantum computer can in general be created using single-qubit rotations and a 2-qubit entangling operation[82, 83]. Experimental characterization of both the decoherence and the operations will be required in order to successfully build a quantum computer. Different implementations of quantum computing are susceptible to different types of errors so error correction needs to be tailored to the physical implementation.

Numerous systems have been proposed for quantum computing, ranging from SQUIDs to quantum dots to NMR to solid state devices and to trapped ions or atoms. In this chapter we demonstrate motional state quantum process tomography in an optical lattice. An optical lattice is a useful testing ground for quantum computing $[84]$ as it provides many controllable parameters, such as energy spacings, coupling strengths and timescales. By modifying the polarizations of the beams comprising the lattice one can create pairs of overlapping lattices, where the electronic
state of the atom determines which lattice the atom sees, thereby entangling the position of the atom with the state of the atom[64, 85]. Recent proposals demonstrate how atoms in a lattice may be individually addressed[86, 87, 88, 89], and methods to perform 2-qubit operations $[90,91]$ and procedures to initialize a lattice with unity occupation using a BEC[92] or thermal cloud[89]. By varying the depth of the wells it is possible to study decoherence due to tunneling and inhomogeneous broadening. By varying the detuning of the lattice the homogenous broadening due to scattering can be studied.

In this chapter we characterize several operations on the motional state of atoms in an optical lattice. To characterize an arbitrary operation we use quantum process tomography [93, 74], or QPT. QPT is a procedure by which an unknown 'blackbox' may be fully characterized. The result of QPT reveals the superoperator that governs the evolution of the density matrix for the operation. QPT has recently been demonstrated using spins in a NMR system[94], the polarization of single photons[95] and for a singlet state filter for photon pairs[96]. The experimentally determined superoperator, $\mathcal{E}$, is a positive linear mapping from input density matrices to output density matrices. From the superoperator one can determine the types of errors that occur and develop procedures to reduce or eliminate errors, without requiring $a$ priori knowledge of the underlying physical mechanism causing the errors[97, 98, 99]. Once the nature of the decoherence is understood we can then take steps to reduce decoherence using techniques such as "bang-bang" operations[100] or decoherence-free subspaces[101].

Quantum process tomography is accomplished by subjecting a complete set of density matrices to an operation and subsequently measuring the resulting density matrices. Due to the linearity of quantum mechanics, QPT of a process on an N dimensional system can be achieved by sending in $N^{2}$ input states, or as has recently been shown, one could use a single entangled state with a larger Hilbert space as the input[102, 95].

In this chapter we perform QPT using the motional states of atoms trapped in an optical lattice. The motional dynamics of atoms in optical lattices has previously been studied in connection with wavepacket oscillations and revivals [63, 64, 103, 104, 85]. Many experiments to date focus on measuring the position or momentum distributions and characterizing the damping of oscillations. Information is obtained about the atoms by measuring the momentum distribution directly in free expansion, or by measuring changes in the power balance of the lattice beams [105, 63]. These techniques are useful for characterizing decoherence and how it changes under the influence of external fields. Unlike these results, the experiment reported on in this chapter can identify the what is happening to the state to cause the decay of the wave-packet oscillations. Through QPT we can determine if the decay is due to heating, population relaxation, or dephasing.

In this chapter we perform quantum process tomography using the motional states of atoms trapped in the potential wells of a 1-D optical lattice. State tomography is performed on atoms in the lattice using the techniques outlined in the previous two chapters. As discussed in those chapters our measurements are insensitive to the long-range degrees of freedom, i.e., quasi-momentum or well index. Therefore our measurements comprise of measuring the occupation of energy bands or vibrational levels. Using QPT we characterize the intrinsic decoherence of the system as well as two driving operations. The accuracy of the superoperator is tested by quantifying how well it predicts the states resulting from a second application of the same operation.

### 7.1 Superoperator Representations

The superoperator can be expressed in a number of ways, although generically it is usually written

$$
\begin{equation*}
\rho^{\prime}=\mathcal{E}(\rho), \tag{7.1}
\end{equation*}
$$

where $\mathcal{E}$ is the superoperator, which reflects information about changes in state populations, coherences and also losses in the system, and therefore does not necessarily conserve the trace of the density matrix. The new density matrix must have a trace equal to unity, so the new matrix must be normalized to obtain a density matrix. In this dissertation we discuss the Kraus operator-sum representation, the Choi matrix representation and a graphical Bloch sphere representation.

### 7.1.1 Operator-sum Representation: Kraus Operators

A quantum operation can be represented in a form known as an operator-sum representation. This is an expression of the quantum operation in terms of operators that act on the Hilbert space of the system. In this representation the superoperator is

$$
\begin{equation*}
\mathcal{E}(\rho)=\sum_{i=1}^{N^{2}} \widehat{K}_{i} \rho \widehat{K}_{i}^{\dagger} \tag{7.2}
\end{equation*}
$$

where $\widehat{K}_{i}$ is an operator on the Hilbert space of the system and $N$ is the dimensionality of the Hilbert space. The operators $\left\{\widehat{K}_{i}\right\}$ are known as operational elements, often referred to as Kraus operators[106]. The Kraus operators are subject to a constraint

$$
\begin{equation*}
\sum \widehat{K}_{i}^{\dagger} \widehat{K}_{i}=I \tag{7.3}
\end{equation*}
$$

where $I$ is the identity operator. Determining the Kraus operators acting on the system from a set of input/output density matrices is not a simple matter, a situation that is compounded when experimental uncertainties are present. Even for a simple 2 x 2 density matrix the Kraus operators have 16 independent parameters. The set of Kraus operators $\left\{\widehat{K}_{i}\right\}$ are not unique[107, 108]. A different set of operators can be obtained by decomposing $\mathcal{E}$ over a different basis. The action of a Kraus operator $\widehat{K}_{i}$ is to turn a pure state $\rho$ into a pure state $\widehat{K}_{i} \rho \widehat{K}_{i}^{\dagger}$.

### 7.1.2 Choi Matrix

The Choi matrix [107, 108] is an $N^{2}$ by $N^{2}$ matrix representing a linear map of how each element of the density matrix evolves under an operation.

The Choi matrix is given by

$$
\begin{equation*}
C=\sum_{i, j=1}^{N}|i\rangle\langle j| \otimes \mathcal{E}(|i\rangle\langle j|), \tag{7.4}
\end{equation*}
$$

where $|i\rangle\langle j|$ is an outer product of basis states and $\mathcal{E}(|i\rangle\langle j|)$ is the superoperator acting on the outer-product. Note that $|i\rangle\langle j|$ is not in general, a density matrix, but can always be written as a linear combination of density matrices. Since $\mathcal{E}(|i\rangle\langle j|)$ is an $N \times N$ matrix, the Choi matrix is then an $N^{2}$ by $N^{2}$ matrix. The Choi matrix is a positive, Hermitian matrix, and must therefore preserve the positivity of any positive input matrix, such as a density matrix. The density matrix after an operation can evaluated as

$$
\begin{equation*}
\rho^{o u t}=\sum_{i, j=1}^{N} C_{i, j} \rho_{i, j}^{i n} \tag{7.5}
\end{equation*}
$$

where $C_{i, j}=\mathcal{E}(|i\rangle\langle j|)$ is the $\mathrm{i}, \mathrm{jth} 2 \mathrm{x} 2$ submatrix of $C$ and $\rho_{i, j}^{i n}$ is the $i, j$ th element of the input density matrix $\rho^{i n}$. The Choi matrix is a natural choice to use for sampled data where the input and corresponding output density matrices are measured. We can write the outer product $|i\rangle\langle j|$ as a linear combination of the input density matrices with complex coefficients $c_{i}$.

$$
\begin{equation*}
|i\rangle\langle j|=\sum_{i=1}^{N} c_{i} \rho_{i} . \tag{7.6}
\end{equation*}
$$

We can then we determine the superoperator acting on $|i\rangle\langle j|$ simply as

$$
\begin{equation*}
\mathcal{E}(|i\rangle\langle j|)=\sum_{i=1}^{N} c_{i} \rho_{i}^{\prime}, \tag{7.7}
\end{equation*}
$$

using the coefficients $c_{i}$ from equation 7.6.

The Choi matrix has the extra advantage that it can be used to determine the canonical Kraus operators (i.e. a set of mutually orthogonal operators). The Kraus operators are related to the eigenvectors of the Choi matrix

$$
\begin{equation*}
\widehat{K}_{i}=\sqrt{\kappa_{i}} \bar{k}_{i}, \tag{7.8}
\end{equation*}
$$

where $\kappa_{i}$ is an eigenvalue and $\bar{k}_{i}$ is the ith eigenvector written in matrix format (filling columns first.) Since the Choi matrix is positive and Hermitian we can use a Cholesky decomposition[109] to find the best-fit Choi matrix to sampled data. Using the Cholesky decomposition we can write a positive, Hermitian matrix $C$ as

$$
\begin{equation*}
C=A^{\dagger} A \tag{7.9}
\end{equation*}
$$

where $A$ is an upper triangular matrix.

### 7.1.3 Bloch Sphere

The Bloch sphere can be used to show the effect of a superoperator on a $2 \times 2$ density matrix. The Bloch sphere is a vector representation of a two dimensional state. The surface of the Bloch sphere represents all possible pure states with the top (bottom) pole representing a pure ground (excited) state and the x and y axis referring to equal superpositions with real and imaginary coherences respectively. By operating on a Bloch sphere with a superoperator, all points on the sphere are mapped into new points, generating a disturbed sphere.

For a 2 dimensional system the density matrix is a $2 \times 2$ matrix. It can be expanded in a basis of spin matrices $\left\{\mathbf{1}, \sigma_{1}, \sigma_{2}, \sigma_{3}\right\}$. Since the $\sigma_{i}$ are traceless the coefficient of 1 must be $1 / 2$. The density matrix can then be defined in terms of a vector $\vec{P}$, as

$$
\begin{align*}
\rho(P) & =\frac{1}{2}(\mathbf{1}+\vec{P} \cdot \vec{\sigma})  \tag{7.10}\\
& =\frac{1}{2}\left(\mathbf{1}+P_{1} \sigma_{1}+P_{2} \sigma_{2}+P_{3} \sigma_{3}\right)  \tag{7.11}\\
& =1 / 2\left[\begin{array}{cc}
1+P_{3} & P_{1}-i P_{2} \\
P_{1}+i P_{2} & 1-P_{3}
\end{array}\right] . \tag{7.12}
\end{align*}
$$

The set of density matrices with $|P|=1$ are pure states, located on the surface of a Bloch sphere of radius 1. Non-pure states have $|P|<1$ and are located inside the Bloch sphere.

Representing a pure state by

$$
|\psi(\theta, \phi)\rangle=\left[\begin{array}{c}
e^{-i \phi / 2} \cos \left(\frac{\theta}{2}\right)  \tag{7.13}\\
e^{i \phi / 2} \sin \left(\frac{\theta}{2}\right)
\end{array}\right]
$$

we can define a density matrix in terms of $\theta$ and $\phi$ as

$$
\begin{align*}
\rho(\theta, \phi) & =|\psi(\theta, \phi)\rangle\langle\psi(\theta, \phi)|  \tag{7.14}\\
& =\left[\begin{array}{cc}
\cos ^{2}\left(\frac{\theta}{2}\right) & \cos \left(\frac{\theta}{2}\right) \sin \left(\frac{\theta}{2}\right) e^{-i \phi} \\
\cos \left(\frac{\theta}{2}\right) \sin \left(\frac{\theta}{2}\right) e^{i \phi} & \sin ^{2}\left(\frac{\theta}{2}\right)
\end{array}\right]  \tag{7.15}\\
& =\frac{1}{2} \mathbf{1}+\left[\begin{array}{cc}
\cos (\theta) & \sin (\theta) e^{-i \phi} \\
\sin (\theta) e^{i \phi} & \sin (\theta)
\end{array}\right]  \tag{7.16}\\
& =\frac{1}{2} \mathbf{1}+\frac{1}{2}(\widehat{\mathbf{n}} \cdot \overrightarrow{\boldsymbol{\sigma}}) . \tag{7.17}
\end{align*}
$$

where $\widehat{n}=(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, and $\overrightarrow{\boldsymbol{\sigma}}$ is the coherence vector. This parametrization of the density matrix allows for simple representation of the density matrix as a vector in a 3 dimensional space.

The Bloch sphere can be used to show the effect of a superoperator. Under the effect of an operation an initial coherence vector $\overrightarrow{\boldsymbol{\sigma}}$ evolves to a new vector $\vec{\sigma}^{\prime}$ given by[80]

$$
\begin{equation*}
\vec{\sigma}^{\prime}=m \vec{\sigma}+\vec{c} \tag{7.18}
\end{equation*}
$$

where $m$ is a $3 x 3$ matrix that governs the deformation and rotation of the sphere and the vector $\vec{c}$ is the displacement of the center of the sphere. Once we know how 4 points on the Bloch sphere move after an operation, then we know the entire operation.

### 7.2 Experiment

In this experiment we perform quantum process tomography using the motional states of atoms trapped in a 1-D optical lattice. We examine processes which are independent of the electronic state of the atom and are only dependent upon the motional states of the atom. We use a shallow 1-D lattice that only supports 2 bound bands, which we label as ground $(|0\rangle)$ and excited $(|1\rangle)$. The lattice is vertically oriented, causing all atoms in higher energy, classically unbound states to quickly fall out of the lattice and become spatially separated from atoms that remain in bound states of the lattice.

We begin by cooling and trapping Rubidium- 85 atoms in a standard vapour cell MOT to a temperature of $7 \mu K$. During the optical molasses stage we turn on an optical lattice in a vertical orientation. The optical lattice is created by interfering two laser beams that are detuned 30 GHz below the Rb D 2 resonance at 780.03 nm . The scattering rate from the lattice beams is on the order of 4 Hz , which is insignificant on the timescale of the experiment. The depth of the lattice is controlled by the intensity and detuning of the beams, and is chosen to be $18 E_{r e}$, where $E_{r e}=\hbar \cdot 2 \pi \cdot 685 \mathrm{~Hz}$ is the effective recoil energy of the lattice, at which depth it contains two bound states. The energy separation between the states is $\hbar \cdot 2 \pi \cdot 5.0 \mathrm{kHz}$. Each beam travels through an acousto-optic modulator, each of which is driven by a function generator, providing control of the relative phase between the lattice beams. By modulating the
phase of one of the lasers the lattice may be displaced by up to a lattice spacing within 250 nanoseconds. The beams are superposed after the acousto-optic modulators, and co-propagate to the vacuum chamber with orthogonal polarizations, to reduce phase fluctuations between the beams which would impress noise on the lattice. The beams are separated on a polarizing beam splitter near the MOT. The polarization of one beam is then rotated such that the beams have parallel polarizations in the MOT. The beams have an angle of 50 degrees between them, creating an optical lattice with a lattice constant of $L=0.93$ microns. The lattice beams have an rms radius of 3 mm , creating an optical lattice with a Gaussian distribution of well depths.

After preparing a ground state in the lattice we perform a Ramsey style experiment as discussed in chapter 5. The optical lattice is displaced in order to couple the ground and excited states. A time delay is introduced to allow the phase between the ground and excited states to evolve. This is followed by another displacement, undoing the first displacement, to once again couple the states. The new ground state population is measured and plotted versus the time delay. Figure 5.6 shows a sample plot of the ground state population vs time delay. A clear sinusoidal signal is observed with a decaying amplitude that is attributed to decoherence. The trap frequency is obtained by fitting a decaying cosine to the measured points.

### 7.2.1 Superoperator for Decoherence

The system we are working with already has a significant amount of decoherence. It is then natural to perform process tomography on the intrinsic decoherence of the system. The process is chosen to be a single period of free evolution in the optical lattice. We also find the superoperator for 3 periods of free evolution in the lattice. By comparing the superoperator for both single period and three period data, we can test the Markov approximation, i.e., that the bath has no memory.

Process tomography proceeds by creating a complete set of input density matrices, composed of a ground state, a mixed state, and a pair of superposition states.

| Input states | Output states after 1 period |
| :---: | :---: |
| $\begin{array}{cc}0.90 & -0.01+0.02 i \\ -0.01-0.02 i & 0.10\end{array}$ | $\begin{array}{cc}0.90 & -0.01 \\ -0.01 & 0.10\end{array}$ |
| 0.60 $0.01+0.02 i$ <br> $0.01-0.02 i$ 0.40 | 0.60 $0.05-0.02 i$ <br> $0.05+0.02 i$ 0.40 |
| 0.69 $0.41+0.01 i$ <br> $0.41-0.01 i$ 0.31 | 0.69 $0.28-0.01 i$ <br> $0.28+0.01 i$ 0.31 |
| 0.69 $0.01-0.35 i$ <br> $0.01+0.35 i$ 0.31 | $\begin{array}{cc}0.69 & 0-0.26 i \\ 0+0.26 i & 0.31\end{array}$ |

Table 7.1: Measured density matrices for the input states (left) and the output states after one period in the lattice(right).

State tomography is performed on the input states and the states after a spending a period in the lattice. State preparation and tomography proceed according to the procedure outlined in chapter 6 . Table 7.1 shows measured input and output density matrices. The density matrices are given to two decimal places because the statistical uncertainty is 0.02 on the populations and 0.05 on the coherences. We can see that in the case of a ground and mixed state we still find some coherence, but this is within error of zero.

The superoperator is found in the Choi representation by a maximum likelihood fit. The Choi matrix is constrained to be a positive definite matrix by using a Cholesky decomposition[109], i.e., $C=A^{\dagger} A$, where $A$ is an upper-diagonal matrix. The fitting routine proceeds by initializing a guess to the parameters and computing the Chi-squared value for the guess.

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{4} \sum_{j, k=1}^{2} \frac{\left(\rho_{j, k}^{i}-\rho_{j, k}^{i} C_{j, k}\right)^{2}}{\sigma_{i, j}^{2}} \tag{7.19}
\end{equation*}
$$

where $i$ labels the density matrix, $j$ and $k$ label the element of the density matrix and $\sigma_{i, j}$ is the uncertainty in the measured element. The routine was programmed in Mathematica by our post-doc Morgan Mitchell, and finds the best fit by a method of steepest decent. The uncertainty in the superoperator is found by a Monte-Carlo technique where the input and output density matrices are Gaussian distributed about
the measured data set. The standard deviation of the Monte-Carlo set of 50 superoperators is computed and is used as the standard deviation of the superoperator.

The Choi matrix is then (to 2 decimal places)

$$
\begin{align*}
C_{\text {decoh }}= & {\left[\begin{array}{cccc}
0.99 & -0.01 i & 0 & .64-0.03 i \\
0.01 i & .01 & 0 & 0 \\
0 & 0 & 0.04 & 0.05-0.04 i \\
.64+0.03 i & 0 & 0.05+0.04 i & .98
\end{array}\right] }  \tag{7.20}\\
& \pm\left[\begin{array}{cccc}
0.02 & 0.04+0.04 i & 0.02+0.03 I & .09+.12 i \\
0.04-0.04 i & .01 & 0.01+0.01 i & 0.03+0.03 i \\
0.02-0.02 I & 0.01+0.01 i & .03 & 0.08+0.08 i \\
.09-.12 i & 0.03-0.03 i & 0.08-0.04 i & 0.05
\end{array}\right] \tag{7.21}
\end{align*}
$$

We can immediately see that the Choi matrix is dominated by the four corner elements. The corners are the elements that map density-matrix elements into themselves. The diagonal corners are nearly one, reflecting the fact that the populations are unchanged by the process. The off-diagonal corners, which map coherences into themselves, have a value of 0.64 , reflecting a $36 \%$ loss of coherence per oscillation, in agreement with the decoherence we observe in the Ramsey fringes. The remaining elements of the Choi matrix are within one standard deviation of zero.

The Kraus operators may be determined from the Choi matrix. Using the eigenvalues and eigenvectors, we find the Kraus operators to be

$$
\begin{array}{ll}
\widehat{K}_{1}=\left[\begin{array}{cc}
0.90 & 0.03-i 0.03 \\
0 & 0.90+i 0.04
\end{array}\right] & \widehat{K}_{2}=\left[\begin{array}{cc}
0.41 & -0.08+i 0.05 \\
i 0.01 & -0.41-i 0.02
\end{array}\right] .  \tag{7.22}\\
\widehat{K}_{3}=\left[\begin{array}{cc}
0.02 & -0.06 \\
0.15-i 0.10 & -0.06
\end{array}\right] & \widehat{K}_{4}=\left[\begin{array}{cc}
0.01 & i 0.01 \\
-0.06+i 0.10 & 0
\end{array}\right] .
\end{array}
$$

We check the constraint 7.3 and find that

$$
\sum_{i} \widehat{K}_{i}^{\dagger} \widehat{K}_{i}=\left[\begin{array}{cc}
1.02 & -0.02  \tag{7.23}\\
-0.02 & 1.00
\end{array}\right] \approx \widehat{I}
$$

To find the dominant terms of the Kraus operators, we rewrite the Kraus operators by extracting the largest Pauli matrix components. We can then write the first two Kraus operators as $\widehat{K}_{1}=0.90 \widehat{I}+\widehat{R}_{1}$ and $\widehat{K}_{2}=0.41 \widehat{\sigma}_{z}+\widehat{R}_{2}$ where $\widehat{R}_{1}$ and $\widehat{R}_{2}$ are the remainders. The other two Kraus operators and the remainders have a small contribution to the superoperator. The remaining components all have small values of $\operatorname{Tr}\left(\widehat{B}_{i}^{\dagger} \widehat{B}_{i}\right) \leq 0.03$ for all $\widehat{B}=\left\{\widehat{R}_{1}, \widehat{R}_{2}, \widehat{K}_{3}, \widehat{K}_{4}\right\}$ compared to 1.62 and 0.334 for $\widehat{K}_{1}$ and $\widehat{K}_{2}$ respectively. The operator sum representation for this measurement is then approximately

$$
\begin{equation*}
\rho^{\prime}=0.90^{2} \widehat{I} \rho \widehat{I}+0.41^{2} \widehat{\sigma}_{z} \rho \widehat{\sigma}_{z} \tag{7.24}
\end{equation*}
$$

which is exactly the form that one would expect to use to model decoherence that appears as a loss of phase information due to tunneling or dephasing. The populations are unaffected but the coherences drop to $64 \%$ of their previous value.

The Bloch sphere is a visually appealing representation. Figure 7.1 shows the initial Bloch sphere and the Bloch sphere after one period in the lattice. The sphere has contracted in the horizontal plane, representing a decrease in the coherence. The experimental Bloch sphere also has some residual rotation, but this is within the uncertainty of the superoperator.

### 7.2.2 Testing the superoperator: Predictability and Markov tests

The veracity of the superoperator is tested by quantifying its predictive power for multiple applications of the same operation. We test the superoperator using two methods. We first compare the measured states after three operations to the


Figure 7.1: Bloch sphere for natural decoherence
predicted state using the measure of fidelity[77]. The fidelity is not always a good measure of how similar mixed states are to one another. We therefore also find the trace distance [110] $\mathcal{D}$ (also called the Kolgomorov distance) between the states, where

$$
\begin{equation*}
\mathcal{D}\left(\rho_{1}, \rho_{2}\right)=\frac{1}{2} \operatorname{Tr}\left|\rho_{1}-\rho_{2}\right| \tag{7.25}
\end{equation*}
$$

and $|X|=\sqrt{X^{\dagger} X}$ for a matrix $X$. The distance $\mathcal{D}$ takes a value of 0 for identical states and a value 1 for orthogonal states. The trace distance is equal to one-half the Euclidean distance between two states on the Bloch sphere. Table 7.2 shows the measured and predicted states after 3 periods along with the computed fidelity and trace distance.

In addition to tests of measured states, we can also test the Markov approximation for the superoperator. The Markov approximation is an assumption that the bath that the system interacts with has no memory. This means that the future evolution of the system is governed by the state of the system now and the Hamiltonian acting upon it, i.e., the history of the system doesn't affect future evolution. If the Markov

| Measured | predicted | Fidelity | Distance |
| :---: | :---: | :---: | :---: |
| $\begin{array}{cc} \hline \hline 0.90 & -0.01-0.01 i \\ -0.01+0.01 i & 0.10 \end{array}$ | $\begin{array}{cc}.87 & 0.01 i \\ -0.01 i & .13\end{array}$ | 0.998 | 0.04 |
| $\begin{array}{cc} 0.60 & -0.03-0.10 i \\ -0.03+0.10 i & 0.40 \end{array}$ | 0.62 $0.05+0.01 i$ <br> $0.05-0.01 i$ 0.38 | 0.990 | 0.14 |
| $\begin{array}{cc}0.69 & 0.17-0.02 i \\ 0.17+0.02 i & 0.31\end{array}$ | $\begin{array}{ll}0.69 & 0.17 \\ 0.17 & 0.31\end{array}$ | 0.999 | 0.02 |
| $\left[\begin{array}{cc}0.69 & -0.02-0.17 i \\ -0.02+0.17 i & 0.31\end{array}\right]$ | 0.69 $0.04-0.12 i$ <br> $0.04+0.12 i$ 0.31 | 0.997 | 0.08 |

Table 7.2: Measured and predicted density matrices after two periods.
approximation doesn't hold then some information has been moved from the system into the bath and can affect how the system evolves.

To test the Markov approximation with superoperators we measure the superoperator for 1 and 3 periods of evolution in the lattice, which we label $\mathcal{E}_{1}$ and $\mathcal{E}_{3}$. If the Markov approximation is valid then we expect to find that

$$
\mathcal{E}_{3}=\mathcal{E}_{1}^{3} .
$$

The superoperator for 1 period, $\mathcal{E}_{1}$, is given by equation 7.20 . We find the superoperator for three periods of evolution using the prepared input states and the output states after 3 periods, from table 7.2. The experimental Choi matrix for 3 oscillations is

$$
\mathcal{E}_{3}=\left[\begin{array}{cccc}
.99 & -0.01 i & -0.01 & 0.40+0.01 i  \tag{7.26}\\
0.01 i & 0.01 & 0 & -0.01-0.01 i \\
-0.01 & 0 & 0.05 & -0.03-0.12 i \\
0.40-0.01 i & -0.01+0.01 i & -.03+0.12 i & 0.98
\end{array}\right]
$$

We calculate $\mathcal{E}_{1}^{3}$ and its uncertainty using the same Monte-Carlo technique that was used for the 1 period superoperator. We then find $\mathcal{E}_{1}^{3}$ to be


Figure 7.2: Matrix of disagreement between predicted superoperator and measured superoperator for 3 oscillations. The elements of the matrix show the number of standard deviations by which the Choi matrix elements disagree.

$$
\mathcal{E}_{1}^{3}=\left[\begin{array}{cccc}
0.95 & -0.01 & -0.03-0.02 i & 0.32-0.01 i  \tag{7.27}\\
-0.01 & 0.05 & -0.01 & -0.01-0.01 i \\
-0.03+0.02 i & -0.01 & 0.17 & 0.14 \\
0.32+0.01 i & -0.01+0.01 i & 0.14 & 0.90
\end{array}\right]
$$

By inspection we see that the off-diagonal corners (which specifies how coherences map into themselves) do not agree with one another. The measured superoperator $\mathcal{E}_{3}$ has corners with value ${ }^{\sim} 0.40$ while the calculated superoperator $\mathcal{E}_{1}^{3}$ has value a 0.32 . This is a significant deviation that might indicate non-Markovian evolution. Figure 7.2 plots the number of standard deviations each element of $\mathcal{E}_{3}$ and $\mathcal{E}_{1}^{3}$ disagree with one another.

We find disagreement on the two elements that neighbour the off diagonals in the $C_{22}$ submatrix and in the lower diagonal corner of the $C_{11}$ matrix. The off diagonals terms in the $C_{22}$ submatrix signify how population in the excited state is
turned into coherences. These elements disagree by 1 standard deviation. This may be a result of the relatively large coherence measured for the case of a mixed state after 3 oscillations. The lower diagonal corner of the $C_{11}$ matrix signifies how ground state population turns into excited state population and it disagrees by 1.2 standard deviations. We additionally measured larger coherences after 3 oscillations than we would expect if the system evolved under Markovian evolution, although it is within error. If the states evolve consistently with the Markov approximation then decoherence would be exponential in character. The results shown here indicate that the evolution of the system may be non-Markovian, but that after 3 periods it is inconclusive. If our system was dominated by inhomogeneous broadening however, we can expect the decoherence of the system to follow a Gaussian dependence of coherence with time. At late times the non-Markovian evolution becomes apparent in the Ramsey fringes, as shown in Figure 7.3. We fit the Ramsey fringes to a decaying cosine were the decay is either exponential or Gaussian in character. The $\chi^{2}$ value for the exponential and Gaussian fits are 145 and 95 respectively (for 90 data points). This shows that when we include all the data that the system is inconsistent with Markovian evolution. However, for short times exponential decay is indistinguishable from Gaussian decay. If we fit only the first 3 fringes we find $\chi^{2}$ values of 70 for both the exponential and Gaussian decay models. Therefore we expect to be able to treat the system as Markovian for the first several oscillations.

### 7.3 Superoperator for Resonant Coupling

In order to perform any kind of computation with a system of qubits, single qubit rotations are a requirement. In this section we attempt to resonant coupling the vibrational levels of the atoms to effect a single qubit rotation. To resonantly couple the ground and excited states, the position of the lattice is oscillated at the resonant frequency. The first coupling tested is to drive the lattice with a sine wave with a


Figure 7.3: Comparison of Guassian and exponential decay for Ramsey fringes. The upper data set is fit to an exponentially decaying cosine. The lower data set is a copy of the upper set and has been displaced vertically for clarity. The lower data set is fit to a decaying cosine with Gaussian decay. The $\chi^{2}$ value for the exponential and Gaussian fits are 145 and 96 respectively.


Figure 7.4: Position drive for resonant coupling
small displacement of 63 nm . The relative phase of the lasers is controlled by our SRS function generator, preventing continuous control of the position of the lattice. The position of the lattice is updated every 10 microseconds, with a total of 20 steps in a single period. Figure 7.4 shows the displacement as a function of time for the actual experimental drive.

A set of states representing a ground state, a mixed state, and 2 coherent states are prepared as outlined in the previous chapter. State tomography is performed on the input and output state to determine the density matrices. The resulting Choi matrix is

$$
\begin{align*}
C_{\sin }= & {\left[\begin{array}{cccc}
0.91 & -0.22+0.07 i & 0.26-0.06 i & 0.42-0.33 I \\
-0.22-0.07 i & 0.10 & -0.06+0.02 i & -0.10+0.11 i \\
0.26+0.06 i & -0.06-0.02 i & 0.22 & 0.19-0.06 i \\
0.42+0.33 I & -0.10-0.11 i & 0.19+0.06 i & 0.53
\end{array}\right] }  \tag{7.28}\\
& \pm\left[\begin{array}{cccc}
0.03 & 0.03+0.03 i & 0.05+0.05 i & 0.10+0.10 i \\
0.03+0.03 i & 0.02 & 0.04+0.05 i & 0.03+0.03 i \\
0.05+0.05 i & 0.04+0.05 i & 0.16 & 0.10+0.11 i \\
0.10+0.10 i & 0.03+0.03 i & 0.10+0.11 i & 0.13
\end{array}\right] \tag{7.29}
\end{align*}
$$

The resulting Kraus operators are

$$
\begin{array}{cc}
K_{1}=\left[\begin{array}{cc}
0.93 & 0.31+0.09 i \\
-0.24-0.04 i & 0.50+0.41 i
\end{array}\right] & K_{2}=\left[\begin{array}{cc}
-0.18+0.13 i & 0.13+0.10 i \\
0.11+0.12 i & 0.29
\end{array}\right] .  \tag{7.30}\\
K_{3}=\left[\begin{array}{cc}
-0.05-0.04 i & 0.30 \\
-0.04+0.02 i & -0.02
\end{array}\right] & K_{4}=\left[\begin{array}{cc}
0.02-0.03 i & 0.01 i \\
0.11 & -0.02+0.03 i
\end{array}\right] .
\end{array}
$$

In this case we find that the Kraus operators do not fit the constraint $\sum_{i} \widehat{K}_{i}^{\dagger} \widehat{K}_{i}=\widehat{I}$. We instead find

$$
\sum_{i} \widehat{K}_{i}^{\dagger} \widehat{K}_{i}=\left[\begin{array}{cc}
1.02 & 0.16+.02 I  \tag{7.31}\\
0.16-0.02 I & 0.73
\end{array}\right]
$$

The superoperator does not fit the constraint because only part of the system is measured. The operation tested does not couple only the two lowest energy states, it also couples to higher energy states. The measured superoperator however, only describes the evolution of the two lowest energy, bound states. If an atom is coupled from a bound state into an unbound state, then this appears as a loss of atoms, when they are actually moving into part of the system that isn't being measured. From the measured superoperator we find that at most $28 \%$ of the atoms move from the bound states of the lattice into unbound states.

| Measured 2 oscillations | Predicted 2 oscillations | Fidelity $\mathcal{F}$ | Distance $\mathcal{D}$ |
| :---: | :---: | :---: | :---: |
| $\left[\begin{array}{cc} 0.67 & 0.25-0.05 i \\ 0.25+0.05 I & 0.26 \end{array}\right]$ | $\left[\begin{array}{cc} 0.62 & 0.27-.06 i \\ 0.27+.06 i & 0.28 \end{array}\right]$ | 0.997 | 0.04 |
| $\begin{array}{cc}0.57 & 0.04-0.03 i \\ 0.04+0.03 i & 0.27\end{array}$ | $\begin{array}{cc}0.52 & 0.17-0.30 i \\ 0.17+0.30 i & 0.30\end{array}$ | 0.969 | 0.17 |
| 0.22 $0.17+0.12 i$ <br> $0.17+0.12 i$ 0.42 | $\begin{array}{cc}0.25 & 0.22-.01 i \\ 0.22+.01 i & 0.37\end{array}$ | 0.948 | 0.22 |
| 0.47 $0.14-0.14 i$ <br> $0.14-0.14 i$ 0.28 | $\begin{array}{cc}0.46 & 0.18-0.15 i \\ 0.18+0.15 i & 0.25\end{array}$ | 0.976 | 0.08 |

Table 7.3: Measured and predicted density matrices for a resonant coupling operation.

Provided that the coupling of atoms from unbound states of the lattice to bound states of the lattice is neglible then the measured superoperator is the projection of the complete superoperator (the superoperator describing the entire system, that includes bound and unbound atoms) onto the 2 x 2 subspace describing the bound states. The initial states used in this experiment have few atoms in the unbound states, and at most $28 \%$ of them can be transferred into the bound states, so we do not expect them to have a significant effect on the superoperator.

We test the accuracy of the superoperator by comparing the predicted states for 2 oscillations with the measured states after 2 oscillations. The following table lists the measured density matrices after two oscillations and the prediction of two applications of the superoperator to the initial states. We then quantify how well a reproduction we have accomplished by calculating the quantum fidelity and trace distance for each pair of density matrices. Note that the matrices listed are not normalized, although the normalized matrices are used for calculating the fidelity and distance.

It is difficult to use the superoperators in the Choi or Kraus representations to get intuitive information about the process. We therefore also use the Bloch sphere representation to show the evolution of the 2-state system. Since the Bloch sphere is defined for 2-state normalized density matrices we normalize the measured density
matrices before calculating a superoperator for the operation.
The Bloch sphere picture shows clearly the effect of the superoperator. Figure 7.5 shows the experimental Bloch sphere after one oscillation. Figure a) shows the disturbed Bloch sphere and Figure b shows the projection onto the x-z plane. The Bloch sphere has a 35.5 degree rotation about the $y$-axis.


Figure 7.5: Bloch sphere for resonant coupling with sine drive. Part a) shows the Bloch sphere, while part b) shows the projection onto the y-axis.

The superoperator should reflect the phase of the driving field. A second driving operation is attempted by using a displaced cosine drive to test whether this dependence is seen. The driving field used is $x(t)=x_{m}\left(1-\cos \left(\omega_{o} t\right)\right)$ for a single period with $x_{m}=64 \mathrm{~nm}$. The resulting Bloch sphere is shown in Figure 7.6 along with its projection onto the $y-z$ plane. The rotation of the Bloch sphere is 36.4 degrees about the x -axis, $\pi / 2$ out of phase with the sine drive as expected.

A numerical simulation is performed in order to test the resonant coupling process. The simulation consists of a harmonic oscillator subject to displacements, time delays and the decoherence measured in the previous experiment. An initial density matrix


Figure 7.6: a)Experimental Bloch sphere for a cosine drive. b) Projection of the Bloch sphere onto the y-z plane.
is input to the simulation and a new density matrix is calculated after each time step of 10 microseconds,

$$
\begin{equation*}
\rho_{n+1}=0.91^{2} \widehat{I} \widehat{R} \widehat{D}_{n} \rho_{n} \widehat{D}_{n}^{\dagger} \widehat{R}^{\dagger} \widehat{I}+.41^{2} \widehat{\sigma}_{z} \widehat{R} \widehat{D}_{n} \rho_{n} \widehat{D}_{n}^{\dagger} \widehat{R}^{\dagger} \widehat{\sigma}_{z}^{\dagger} . \tag{7.32}
\end{equation*}
$$

The operators $\widehat{R}$ are rotations of $1 / 20$ of a period and $\widehat{D}_{n}$ is the displacement of the nth step. The magnitudes of the displacements used in the theoretical model are identical to those used in the experiment, however, the theoretical model uses the displacement operators that are well known from harmonic oscillator theory. An orthonormal set of density matrices is input to the simulation and combined with the resulting output density matrices we can generate a Bloch sphere representation for the theoretical model. Figure 7.7 shows the experimental and theoretical Bloch spheres. They show qualitative agreement between one another, and the theoretical Bloch sphere has a rotation of 35.0 degrees about the $y$-axis, similar to the 35.5 degree rotation measured for the experimental superoperator.

The two Bloch spheres show qualitative agreement with one another. We can see some differences, mainly that the experimental superoperator is compressed compared


Figure 7.7: Experimental and theoretical bloch spheres for resonant coupling
to the theoretical Bloch sphere. Nevertheless we would not expect perfect agreement since the model is a harmonic oscillator approximation for a shallow lattice.

### 7.3.1 Future Directions

Following a successful demonstration of measurement of a superoperator, we next desire to use the superoperator to determine more properties of the system, and use the superoperator to 'correct' for errors in an operation. To this end we are working toward increasing the coherence time of the system. If the decay of the coherences was due to dephasing caused by spatial inhomogeneities of the potential then it should be possible to re-phase the atoms at a time $T$ by applying a $\pi$-pulse at time $T / 2$. It may also be possible to find a continuous modulation that can have the same effect.

The superoperator also allows the tests of several assumptions put into many systems. Decoherence in an analytical model is often assumed to be Markovian, meaning that the bath has no memory. When dealing with complex systems, and reducing it
down to a simple model, this assumption will often fail. Measuring the superoperator for a number of different time will reveal any existing non-Markovian effects. Some form of active or passive feedback on the lattice may then allow us to reduce the non-Markovian nature of the system.

## Chapter 8

## Conclusions and loose ends

In this thesis atoms are loaded into optical lattices, and the motional state of the atoms is manipulated and measured. In particular the density matrix is measured for atoms loaded into a lattice that supports two bound states. The techniques discussed here can be extended to a higher dimensional system. By using a deeper lattice a system comprised of four motional states can be constructed. With a larger number of states available for the qubit, we gain some options regarding how data is encoded. We can imagine using the two lower energy levels as one qubit, and the two higher energy levels as another qubit. Alternatively the extra energy levels can be used for error-correction or to create one logical qubit using two physical qubits, possibly creating a system less sensitive to decoherence, i.e., a decoherence-free subspace.

Using the superoperator we can find how well a given operation matches a desired operation. This information can then be used to 'tweak' the operation until it matches the desired operation. In the experiments described here we observed a large amount of decoherence. To eliminate the decoherence we are attempting to create a $\pi$-pulse. If the decoherence in our system is dominated by inhomogeneous well depths then atoms in each well have different oscillation frequencies, causing the rate of phase-evolution to vary across the atom cloud. By applying a $\pi$-pulse we switch the roles of ground and excited states. This has the effect of reversing the
phase-evolution of the states, causing all the atoms to rephase at a time $T$, if a $\pi$-pulse was applied at time $T / 2$. A successful pulse can also be used in the implementation of a decoherence-free system by Bang-Bang pulses.

Future work will focus on using the techniques of state tomography and process tomography to develop tailored error correction. A learning algorithm will be used to inhibit decoherence by time-dependent control of the position and intensity of the lattice beams. The techniques can also be used as a probe into the dynamics of many optical lattice systems. We need not limit ourselves to using the vibrational states for information. We can encode information into the electronic levels of the atoms as well as the vibrational levels. In this manner each atom can be used to encode two qubits of information. Using a pair of lasers to effect Raman transitions, the vibrational and electronic levels can be coupled to create two-qubit gates.

The techniques used here can also be applied to colder, more dense systems where interactions between atoms become very important. For example, beginning with a Bose-Einstein Condensate, an optical lattice can be loaded in such a way that all the atoms are in the quantum mechanical ground state of a lattice, i.e. a $q=0$ state. Using the same techniques studied here we can characterize operations when only a single quasi-momenta is occupied. By shifting the velocity of the lattice different quasi-momentum states can be probed. Furthermore, beyond some critical well depth, the system undergoes a phase-transition into a Mott insulator state, changing the dynamics of the system.

## Bibliography

[1] T. W. Hansch and A. L. Schawlow, Opt. Commun. 13, 68 (1975).
[2] D. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20, 637 (1975).
[3] S. Chu, L. Hollberg, J. E. Bjorkholm, A. Cable, and A. Ashkin, "Threedimensional viscous confinement and cooling of atoms by resonance radiation pressure," Phys. Rev. Lett. 55, 48 (1985).
[4] P. D. Lett, R. N. Watts, C. I. Westbrook, W. D. Phillips, P. Gould, and H. Metcalf, "Observation of atoms laser cooled below the Doppler limit," Phys. Rev. Lett. 61, 169 (1988).
[5] J. Dalibard and C. Cohen-Tannoudji, "Laser cooling below the Doppler limit by polarization gradients:simple theoretical models," J. Opt. Soc. Am. B 6, 2058 (1989).
[6] C. Aminoff, A. Steane, P. Bouyer, P.Desboilles, J. Dalibard, and C. CohenTannoudji, "Cesium atoms bouncing in a stable gravitational cavity," Phys. Rev. Lett. 71, 3083 (1993).
[7] T. Roach, H. Abele, M. Boshier, H. Grossman, K. Zetie, , and E. Hinds, "Realization of a magnetic mirror for atoms," Phys. Rev. Lett. 75, 629 (1995).
[8] T. Sleator, T. Pfau, V. Balykin, and J. Mlynek, "Imaging and focusing of an
atomic-beam with a large-period standing light-wave," Appl. Phys. B 54, 375 (1992).
[9] E. M. Rasel, M. K. Oberthaler, H. Batelaan, J. Schmiedmayer, , and A. Zeilinger, "Atom Wave Interferometry with Diffraction Gratings of Light," phys. Rev. Lett. 75, 2633 (1995).
[10] J. D. M. Arndt, P. Szriftgiser and A. M. Steane, "Atom optics in the time domain," Phys. Rev. A 53, 3369 (1996).
[11] M. Morinaga, M. Yasuda, T. Kishimoto, and F. Shimizu, "Holographic Manipulation of a cold atomic beam," Phys. Rev. Lett. 77, 802 (1996).
[12] F. Shimizu, "Atom holography," Adv. Atom. Mol. Opt. Physics 42, 73 (2000).
[13] M. O. Mewes, M. R. Andrews, D. Kurn, D. Durfee, C. Townsend, and W. Ketterle, "Output coupler for Bose-Einstein condensed atoms," Phys. Rev. Lett. 78, 582 (1997).
[14] E. Hagley, L. Deng, M. Kozuma, J. Wen, K. Helmerson, S. Rolston, and W. Phillips, "A well-collimated Quasi-continuous atom laser," Science 283, 1706 (1999).
[15] I. Bloch, T. W. Hansch, and T. Esslinger, "An Atom Laser with a cw Output Coupler," Phys. Rev. Lett. 82, 3008 (1999).
[16] M. Kasevich, E. Riis, S. Chu, and R. D. Voe, "RF spectroscopy in an atomic fountain," Phys. Rev. Lett. 63, 612-615 (1989).
[17] M. H. Anderson, J. R. Ensher, M. Matthews, and C. E. W. E. A. Cornell, "Observation of Bose-Einstein condensation in a dilute atomic vapor," Science 269, 198 (1995).
[18] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, "Bose-Einstein condensation in a gas of sodium atoms," Phys. Rev. Lett. 75, 3969 (1995).
[19] C. C. Bradley, C. A. Sackett, and R. G. Hulet, "Bose-Einstein condensation of lithium: Observation of limited condensate number," Phys. Rev. Lett. 78, 985 (1997).
[20] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, "Vortices in a Bose-Einstein Condensate," Phys. Rev. Lett. 83, 2498 (1999).
[21] K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, "Vortex Formation in a stirred BEC," Phys. Rev. Lett. 84, 806 (2000).
[22] S. Burger, K. Bongs, S. Dettmer, W. Ertmer, and K. Sengstock, "Dark solitons in BEC," Phys. Rev. Lett. 83, 5198 (1999).
[23] K. Strecker, G. Partridge, A. Truscott, and R. Hulet, "Bright Matter Wave Solitons in Bose-Einstein Condensates," New Journal of Physics 5, 73 (2003).
[24] M. Andrews, C. Townsend, H. Miesner, D. Durfee, D. Kurn, and W. Ketterle, "Observation of interference between two Bose-Einstein condensates," Science 275, 637 (1997).
[25] E. Raab, M.Prentiss, A. Cable, S. Chu, and D. E. Pritchard, "Trapping of neutral sodium atoms with radiation pressure," Phys. Rev. Lett. 59, 2631 (1987).
[26] T. Pfau, C. Kurtsiefer, C. Adams, and J. Mlynek, "Magneto-optical beamsplitter for atoms," Phys. Rev. Lett. 71, 3427 (1993).
[27] J. Lawall and M. Prentiss, "Demonstration of a novel atomic beam splitter," Phys. Rev. Lett. 72, 993 (1994).
[28] H. Hinderthur, A. Pautz, V. Rieger, F. Ruschewitz, J. Peng, K. Sengstock, and W. Ertmer, Phys. Rev. A 56, 2085 (1997).
[29] F. Moore, J. Robinson, C. Bharucha, B. Sundaram, and M. Raizen, "Atom optics realization of the quantum delta-kicked rotor," Phys. Rev. Lett. 75, 4598 (1995).
[30] C. Bharucha, K. Madison, P. Morrow, S. Wilkinson, B. Sundaram, and M. Raizen, Phys. Rev. A. 55, 857 (1997).
[31] A. Steinberg, Superlattices and Microstructures 23, 823 (1998).
[32] H. Ammann and N. Christensen, "Delta Kick Cooling: A new method for cooling atoms," Phys. Rev. Lett. 78, 2088 (1997).
[33] S. Chu, E. Bjorkholm, A. Ashkin, J. Gordon, and L. Hollberg, Opt. Lett. 73, 11 (1986).
[34] J. Summhammer, L. Niel, and H. Rauch, Rauch, Z. Phys. B. 62, 269 (1986).
[35] W. Ketterle and D. E. Pritchard, "Atom cooling by time dependent potentials," Phys. Rev. A. 46, 4051 (1992).
[36] P. Pinkse, A. Mosk, M. Weidemüller, M. Reynolds, T. Hijmans, and J. Walraven, "Adiabatically changing the phase-space density of a trapped Bose gas," Phys. Rev. Lett. 78, 990 (1997).
[37] D. Stamper-Kurn, M. Andrews, A. Chikkatur, S. Inouye, H. Miesner, and W. Ketterle, "Reversible formation of a Bose-Einstein Condensate," Phys. Rev. Lett. 81, 2194 (1998).
[38] A. Kastberg, W. Phillips, S. Rolston, R. Spreeuw, and P. Jessen, "Adiabatic cooling of Cesium to 700 nK in an optical lattice," Phys. Rev. Lett. 74, 1542 (1995).
[39] E. Maréchal, S. Guibal, J.-L. Bossennec, M.-P. Gorza, R. Barbé, J.-C. Keller, and O. Gorcei, "Longitudinal Stern-Gerlach effect for slow cesium atoms," European Physical Journal D 2, 195 (1998).
[40] M. B. Daham, E. Peik, J. Reichel, Y. Castin, and C. Salomon, "Bloch oscillations of atoms in an optical potential," Phys. Rev. Lett. 76, 4508 (1996).
[41] S. R. Wilkinson, C. F. Bharucha, K. Madison, Q. Niu, and M. G. Raizen, "Observation of Atomic Wannier-Stark Ladders in an Accelerating Optical Potential," Phys. Rev. Lett. 76, 4512 (1996).
[42] M. Dignam and J. Sipe, "Exciton Stark ladder in $G a A s / G a_{1-x} A l_{x} A s$ superlattices," Phys. Rev. Lett. 64, 1797 (1990).
[43] P. Leisching, P. H. Bolivar, W. Beck, Y. Daibi, F. Bruggeman, R. Schwedler, H. Kurz, K. Leo, and K. Kohler, "Bloch oscillations of excitonic wave packets in semiconductor," Phys. Rev. B 52, 14389 (1994).
[44] W. Kohn, "Analytic properties of Bloch waves and Wannier functions," Phys. Rev. 115, 809 (1959).
[45] C. Zener, "Non-adiabatic crossing of energy levels," Proc. R. Soc. London A 137, 696 (1932).
[46] K. W. Madison, M. C. Fischer, and M. G. Raizen, "Observation of the WannierStark fan and the fractional ladder in an accelerating optical lattice.," Phys. Rev. A 60, R1767 (1999).
[47] E. P. Wigner, "On the quantum correction for thermodynamic equilibrium," Phys. Rev. 40, 749 (1932).
[48] A. I. Lvovsky, H. Hansen, T. Aichele, O. Benson, J. Mlynek, and S. Schiller, "Quantum State Reconstruction of the Single-Photon Fock State," Phys. Rev. Lett. 87, 050402 (2001).
[49] C. Kurtsiefer, T. Pfau, and J. Mlynek, "Measurement of the wigner function of an ensemble of helium atoms," Nature 386, 150 (1997).
[50] E. Skovsen, H. Stapelfeldt, S. Juhl, and K. Molmer, "quantum state tomography of dissociating molecules," Phys. Rev. Lett. 91, 090406 (2003).
[51] J. F. Poyatos, R. Walser, J. I. Cirac, P. Zoller, and R. Blatt, "Motion Tomography of a single trapped Ion," Phys. Rev. A. 53, 1966 (1996).
[52] D. Liebfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, "Experimental determination of the motional quantum state of a trapped atom," Phys. Rev. Lett. 77, 4281 (1996).
[53] K. Banaszek, C. Radzewicz, K. Wodkiewicz, and J. S. Krasinski, "Direct measurement of the Wigner function by photon counting," Phys. Rev. A. 60, 674 (1999).
[54] K. Husimi, Proc. Phys. Math. Soc. Jpn 22, 264 (1940).
[55] R. Glauber, "Photon Correlations," Phys. Rev. Lett. 10, 84 (1963).
[56] E. Sudarshan, "Equivalence of Semiclassical and Quantum Mechanical Descriptions of Statistical Light Beams," Phys. Rev. Lett. 10, 277 (1963).
[57] G. D'Ariano, M. G. A. Paris, and M. F. Sacchi, "Quantum Tomography," quantph /0302028 (2003).
[58] A. Kastberg, W. D. Phillips, S. L. Rolston, R. J. C. Spreeuw, and P. S. Jessen, "Adiabatic cooling of cesium to 700 nK in an optical lattice," Phys. Rev. Lett. 74, 1542 (1995).
[59] M. Greiner, I. Bloch, O. Mandel, T. Hansch, and T. Esslinger, "exploring phase coherence in a 2D lattice of Bose-Einstein condensates," Phys. Rev. Lett. 87, 160405 (2001).
[60] O. Mandel, M. Greiner, A. Widera, T. Rom, T. Hansch, and I. Bloch, "Coherent transport of atoms in a spin dependent potential," quant-ph/0301169 (2003).
[61] E. Hamman, D. L. Haycock, G. Klose, P. H. Pax, I. H. Deutsch, and P. S. Jessen, "Resolved-Sideband Raman Cooling to the Ground State of an Optical Lattice," Phys. Rev. Lett. 80, 4149 (1998).
[62] V. Vuletić, C. Chin, A. J. Kerman, and S. Chu, "Degenerate Raman Sideband Cooling of Trapped Cesium Atoms at Very High Atomic Densities," Phys. Rev. Lett. 81, 5768-5771 (1998).
[63] G. Raithel, W. D. Phillips, and S. L. Rolston, "Collapse and Revivals of Wave Packets in Optical Lattices," Phys. Rev. Lett. 81, 3615 (1998).
[64] D. L. Haycock, P. M. Alsing, J. Grondalski, I. H. Deutsch, and P. S. Jessen, "Mesoscopic Quantum Coherence in an Optical Lattice," Phys. Rev. Lett. 85, 3365 (2000).
[65] B. K. Teo, J. R. Guest, and G. Raithel, "Tunneling resonances and coherence in an optical lattice," Phys. Rev. Lett. 88, 173001 (2002).
[66] G. Klose, G. Smith, and P. Jessen, "Measuring the Quantum state of a Large Angular Momentum," Phys. Rev. Lett. 86, 4721 (2001).
[67] D. Smithey, M. Beck, M. Raymer, and A. Faridani, "Measurement of the Wigner distribution and density matrix of a light mode using optical homodyne tomography," Phys. Rev. Lett. 70, 1244 (1993).
[68] K. L. Pregnell and D. T. Pegg, "Measuring the elements of the optical density matrix," Phys. Rev. A 66, 013810 (2002).
[69] T. J. Dunn, I. Walmsley, and S. Mukamel, "Experimental determination of the quantum-mechanical state of a molecular vibrational mode using flourescence tomography," Phys. Rev. Lett. 74, 884 (1995).
[70] C. H. Bennet and G. Brassard, "Quantum Cryptography: Public Key Distribution and Coin Tossing," Proceedings of the IEEE international Conference on Computers, Systems and signal Processing, bangalore, India, B. bederson an H. walther, eds. pp. 175-179 (IEEE, New York 1984) (1984).
[71] A. K. Ekert, "Quantum cryptography based on Bell's theorem," Phys. Rev. Lett. 67, 661 (1991).
[72] R. P. Feynman, "Simulating physics with computers," Int. J. of Th. Phys. 21, 467 (1982).
[73] D. Deutsch, "quantum Theory, the Church-Turing principle and the universal quantum computer," Proc. R. Soc. Lond. A 400, 97 (1985).
[74] I. L. Chuang and M. A. Nielsen, "Quantum process Tomography," J. Mod. Opt. 44, 2455 (1997).
[75] G. C. Stokes, Trans. Cambridge Philos. Soc. 9, 399 (1852).
[76] R. T. Thew, K. Nemoto, A. G. White, and W. J. Munro, "Qudit quantum-state tomography," Phys. Rev. A 66, 012303 (2002).
[77] B. Schumacher, "Quantum Coding," Phys. Rev. A. 51, 2738 (1995).
[78] E. Knill, R. Laflamme, and W. Zurek, "Accuracy Threshold for quantum computation," e-print quant-ph/9610001 (1996).
[79] D. Aharonov and M. Ben-Or, "Fault-Tolerant quantum computation with constant error," e-print quant-ph/9611025 (1996).
[80] Nielson and Chuang, Quantum Computation and Quantum Information (Cambridge University Press, 2000).
[81] P. W. Shor, "Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a quantum computer," quant-ph/9508027 (1995).
[82] D. Deutsch, A. Barenco, and A. Ekert, Proc. R. Soc. London ser. A 449, 669 (1995).
[83] S. Lloyd, Phys. Rev. Lett. 75, 346 (1996).
[84] J. K. Pachos and P. L. Knight, "Quantum Computation with a OneDimensional Optical Lattice," arXiv quant-ph/0301084 (2003).
[85] B. K. Teo, J. R. Guest, and G. Raithel, "Tunneling Resonances and Coherence in an Optical Lattice," Phys. Rev. Lett. 88, 17301 (2002).
[86] L. You and M. S. Chapman, "Quantum entanglement using atomic spins," Phys. Rev. A. 62, 052303 (2000).
[87] R. Scheunemann, F. Cataliotti, T. Hansch, and M. Weitz, "Resolving and addressing atoms in individual sites of a C02-laser optical lattice," Phys. Rev. A. 62, 051801 (2000).
[88] S. Peil, J. V. Porto, B. L. Tolra, J. M. Obrecht, B. E. King, M. Subbotin, S. L. Rolston, and W. D. Phillips, "Patterned loading of a BEC into an optical lattice," Phys. Rev. A 67, 051603 (2003).
[89] J. Vala, A. Thapliyal, S. Myrgren, U. Vazirani, D. S. Weiss, and K. Whaley, "Perfect Initialization of a quantum computer of neutral atoms in an optical latttice of large lattice constant," arXiv:quant-ph/0307085 (2003).
[90] H. J. Briegel, T. Calarco, D. Jaksch, J. I. Cirac, and P. Zoller, "Quantum computing with neutral atoms," Journal of Modern Optics 47, 415 (2000).
[91] G. K. Brennen, I. H. Deutsch, and C. J. Williams, "Quantum Logic for Trapped Atoms via Molecular hyperfine Interactions," Phys. Rev. A 65, 022313 (2002).
[92] M. Greiner, O. Mandel, E. T, T. Hansch, and I. Bloch, "Collapse and revival of the matter wave field of a Bose-Einstein condensate," Nature 419, 51-54 (2002).
[93] J. F. Poyatos, J. I. Cirac, and P. Zoller, "Complete Characterization of a Quantum Process: The Two-Bit Quantum Gate," Phys. Rev. Lett 78, 390 (1997).
[94] A. M. Childs, I. L. chuang, and D. W. Leung, "Realization of quantum process tomography in NMR," Phys. Rev. A. 64, 012314-1 (2001).
[95] J. B. Altepeter, D. Branning, E. Jeffrey, T. C. Wei, P. G. Kwiat, R. T. Thew, J. L. O'Brien, M. A. Nielson, and A. G. White, "Ancilla-assisted quantum process tomography," arXiv quant-ph, 0303038 (2003).
[96] M. Mitchell, C. Ellenor, and A. M. Steinberg, "Quantum Process Tomography of a Singlet State Filter," To appear in Phys. Rev. Lett. (available at quantph/0305001) .
[97] E. Knill and R. Laflamme, "Theory of Error-Correcting codes," Phys. Rev. A 55, 900 (1997).
[98] D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. Zurek, T. Havel, and S. S. Somaroo, "Experimental Quantum Error Correction," Phys. Rev. Lett. 81, 2152 (1998).
[99] M. Pravia, N. Boulant, J. Emerson, E. Fortunato, T. Havel, D. G. Cory, and A. Farid, "Robust Control of Quantum Information," e-print quant-ph/0307062 (2003).
[100] L. Viola and S. Lloyd, "dynamical suppresion of decoherence in two-state quantum systems," Phys. Rev. A 58, 2733 (1998).
[101] L. Wu and D. Lidar, "Creating decoherence free subspaces using strong and false pulses," Phys. Rev. Lett. 88, 207902 (2002).
[102] G. M. D'Ariano and P. L. Presti, "Quantum Tomography for measuring Experimentally the Matrix Elements of an Arbitrary Operation," Phys. Rev. Lett 86, 4195 (2001).
[103] O. Morsch, P. Jones, and D. R. Meacher, "Coherent Transients in optical lattices," Phys. Rev. A. 61, 023410 (2000).
[104] F. Buchkremer, R. Dumke, H. Levsen, G. Birkl, and W. Etmer, "Wave Packet Echoes in the motion of trapped atoms," Phys. Rev. Lett. 85, 3121 (2000).
[105] A. Görlitz, M. Weidemüller, T. W. Hänsch1, and A. Hemmerich, "Observing the Position Spread of Atomic Wave Packets," Phys. Rev. Lett. 78, 2096 (1997).
[106] K. Kraus, States, Effects and Operations (Springer-Verlag, Berlin, 1983).
[107] M. D. Choi, Linear Algebra, Appl. 10, 285 (1975).
[108] D. Leung, "Choi's Proof and Quantum Process Tomography," arXiv:quantph/020119 (2002).
[109] J. E. Gentle, Numerical Linear Algebra for Applications in Statistics (SpringerVerlag, Berlin, 1998).
[110] M. Hilley, "Non-classical distance in quantum optics," Phys. Rev. A. 37, 725 (1987).

## Appendix A

## The Rubidium atom: Properties and tables

Rubidium is the 37 th element in the periodic table. It has 2 naturally occurring isotopes, with atomic masses of 85 ( $72 \%$ abundancy) and 87 ( $28 \%$ abundancy). It is an alkali atom, having only a single electron in its outer 5S orbital, and all the inner orbitals are closed. The experiments in this thesis are all performed with Rb 85 , and we focus on this element.

| Property |  |  |  |
| :--- | :--- | :--- | :--- |
| mass | 84.9118 au | $1.4099 \cdot 10^{-25} \mathrm{~kg}$ |  |
| Nuclear Spin | $\mathrm{I}=5 / 2$ |  |  |
| Wavelength D2 transition | 780.24 nm | 3.846 E 14 Hz | 1.589 eV |
| Natural linewidth (D2) | 5.89 MHz |  |  |
| Ground state Hyper-Fine Splitting | 3.036 GHz |  |  |
| Recoil Frequency | 3.86 kHz |  |  |
| Recoil Velocity | $0.602 \mathrm{~cm} / \mathrm{s}$ |  |  |
| Saturation Intensity | $1.6 \mathrm{~mW} / \mathrm{cm}^{\wedge} 2$ |  |  |
| Doppler Temperature | $140 \mu \mathrm{~K}$ |  |  |
| Recoil Temperature | $0.370 \mu \mathrm{~K}$ |  |  |

Figure A. 1 shows the relative transition strengths for the D2 resonance in Rb. The strengths (proportional to the square of the Clebsch-Gordon coefficients) are normalized to make all the transitions strengths integers. The upper plot shows the $\sigma^{+}$transitions and the lower plot shows all the $\pi$-polarized transitions. For $\sigma^{-}$


Figure A.1: Relative transition strengths for a) $\backslash$ sigma + polarized light and b) $\backslash \mathrm{Pi}$ polarized light. The strength is proportional to the square of the Clebsch-Gordon coefficient for the transition.
polarized light use the strengths from the $\sigma^{+}$chart, replacing $m$ with $-m$.

## Appendix B

## Control Sequence: Timing and Synchronization

The timing of the various laser beam intensities, frequencies and phase shifts must all be carefully synchronized. The majority of the experiment is run by Labview, which interfaces the computer with the current controls, acousto-optic modulators (AOM) and electromagnets. However, Labview is unable to directly control the optical lattice on the required timescales. The optical lattice is operated with the help of a SRS arbitrary waveform generator. In the arbitrary modulation mode the SRS has no triggering capability, so we must trigger Labview off of the SRS to properly synchronize the MOT with optical lattice. Figure B. 1 shows the triggering circuit. The has an adjustable delay (up to $200 \mu s$ ) that allows fine-tuning of the synchronization of Labview and the SRS.

Figure B. 2 shows the control sequence involved in a typical run of the experiment. Labview is initially used to program the SRS 1 in arbitrary phase modulation mode. Upon completion of the programming the SRS begins to output its waveform in a continuous cycle. Labview then proceeds to create a MOT and capture atoms. After collecting atoms for about 1 second Labview pauses and waits for a trigger signal from SRS 1. The trigger signal signifies the beginning of an SRS cycle. From this time on


Figure B.1: Circuit to synchronize the SRS function generator with LabView

Labview and the SRS are synchronized with one another. Two outputs are obtained from the SRS. These are a 10 MHz sinewave output that is frequency doubled three times, and a 20 MHz sinewave that is frequency doubled twice. Phase shifts are written onto the 20 MHz output. The 80 MHz signals each go through an RF switch that can also serve to attenuate the RF power. The signals then go through an RF amplifier to obtain 1 Watt of RF power to drive the AOMs. The intensity of the optical lattice is controlled by Labview, which programs the current sent to the MOPA optical amplifier.

The second iteration of the experiment (shown in dotted lines) adds the capability to modulate the intensity of the lattice beams on a microsecond time scale. To accomplish this feat we program another SRS with an arbitrary waveform. The waveform is triggerable, and takes its trigger from SRS 1. The output of SRS 2 is a simply a DC signal that changes as a function of time. This signal is applied to the RF switches allowing control of the RF power, thereby modulating the amount of optical power that is diffracted in the AOM.


Figure B.2: Control electronics for the optical lattice. The dashed line shows electronics that will be added for future experiments. The red lines represent the laser output of the MOPA.

## Appendix C

## Table of Lattice Parameters

This chapter is a collection of several tables that characterize the optical lattice. The first table C. 1 list the energies, band widths and mean energy separation for the three lowest energy bands in an optical lattice for a variety of well depths. The well depth is listed in units of recoil energies and the lattice characteristics in frequency units.

The following table repeats the information from table C. 1 but all energies are list in recoil units.

The last table, C. 3 lists the Landau-Zener tunneling probability $(P)$ for atoms in the three lowest energy bands of a lattice of depth U while also subject to gravity. Also listed is the lifetime $(\tau)$ in ms for the same states and well depths.

| $\mathbf{U}\left(\mathbf{E}_{r e}\right)$ | $\frac{\left\langle E_{0}\right\rangle}{h}$ | $\frac{\left\langle E_{1}\right\rangle}{h}$ | $\frac{\left\langle E_{2}\right\rangle}{h}$ | $\frac{\Delta E_{0}}{h}$ | $\frac{\Delta E_{1}}{h}$ | $\frac{\Delta E_{2}}{h}$ | $\frac{E_{1 \text { min }}-E_{0 \text { max }}}{h}$ | $\frac{E_{2 \text { min }}-E_{1 \text { max }}}{h}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 342.5 | 1712 | 4452 | 343 | 1028 | 1712 | 0 | 0 |
| 2.5 | 905 | 2753 | 5369 | 177 | 820 | 1660 | 425 | 64.3 |
| 5 | 1336 | 3731 | 6377 | 90.5 | 632 | 1548 | 836 | 232 |
| 10 | 1976 | 5476 | 8493 | 26.3 | 341.3 | 1224 | 1566 | 726 |
| 15 | 2468 | 6948 | 10585 | 8.93 | 169.6 | 894 | 2151 | 1286 |
| 18 | 2722 | 7724 | 11789 | 4.96 | 110 | 717 | 2443 | 1618 |
| 20 | 2880 | 8205 | 12565 | 3.41 | 82 | 612 | 2619 | 1832 |
| 30 | 3571 | 10303 | 16087 | 0.63 | 20.8 | 248 | 3355 | 2757 |
| 45 | 4417 | 12857 | 20424 | 0.075 | 3.20 | 56.0 | 4218 | 3754 |
| 60 | 5128 | 15001 | 24041 | .011 | 0.60 | 13.23 | 4936 | 4513 |

Table C.1: Mean band energy, band width and band gap in units of $1 / \mathrm{s}$ for various well depths.

| $\mathrm{U}\left(E_{\text {rec }}\right)$ | $\mathrm{U}(\mu K)$ | $\left\langle E_{0}\right\rangle$ | $\left\langle E_{1}\right\rangle$ | $\left\langle E_{2}\right\rangle$ | $\Delta E_{0}$ | $\Delta E_{1}$ | $\Delta E_{2}$ | $\left\langle E_{1}\right\rangle-\left\langle E_{0}\right\rangle$ | $\left\langle E_{2}\right\rangle-\left\langle E_{1}\right\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0.50 | 2.50 | 6.50 | 0.50 | 1.50 | 2.50 | 0 | 0 |
| 2.5 | 0.164 | 1.32 | 4.02 | 7.84 | 0.259 | 1.20 | 2.43 | 0.62 | 0.007 |
| 5 | 0.329 | 1.95 | 5.45 | 9.31 | 0.13 | 0.92 | 2.26 | 1.22 | 0.027 |
| 10 | 0.658 | 2.88 | 7.99 | 12.40 | 0.038 | 0.50 | 1.79 | 2.28 | 0.085 |
| 15 | 0.987 | 3.60 | 10.14 | 15.45 | 0.013 | 0.25 | 1.31 | 3.14 | 0.15 |
| 18 | 1.18 | 3.97 | 11.28 | 17.21 | .0072 | 0.161 | 1.05 | 3.57 | 0.19 |
| 20 | 1.32 | 4.21 | 11.98 | 18.34 | $5.0 \mathrm{E}-3$ | 0.12 | 0.89 | 3.82 | 0.21 |
| 30 | 1.97 | 5.21 | 15.04 | 23.5 | $9.3 \mathrm{E}-4$ | 0.030 | 0.36 | 4.89 | 0.32 |
| 45 | 2.96 | 6.44 | 18.77 | 29.82 | $1.1 \mathrm{E}-4$ | $4.7 \mathrm{E}-3$ | 0.08 | 6.16 | 0.44 |
| 60 | 3.95 | 7.49 | 21.90 | 35.10 | $1.7 \mathrm{E}-5$ | $8.8 \mathrm{E}-4$ | 0.19 | 7.21 | 0.53 |

Table C.2: Mean band energy, band width and band gap in units of recoil energies. Also given in column two is the well depth in microKelvins.

| $\mathbf{U}$ | $P_{0}$ | $P_{1}$ | $P_{2}$ | $\tau_{0}(m s)$ | $\tau_{1}$ | $\tau_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 2.5 | 0.508 | 0.992 | 0.999992 | 0.73 | 0.107 | 0.044 |
| 5 | 0.073 | 0.904 | 0.9995 | 6.80 | 0.22 | 0.068 |
| 10 | $1.05 \mathrm{E}-3$ | 0.373 | 0.974 | 4933 | 1.11 | 0.143 |
| 15 | $3.10 \mathrm{E}-8$ | 0.046 | 0.808 | 1.6 E 7 | 11.14 | 0.315 |
| 18 | $\sim 0$ | 0.0075 | 0.609 | 2.5 E 9 | 68.9 | 0.553 |
| 20 | $\sim 0$ | 0.0019 | 0.458 | $\sim \infty$ | 274 | 0.85 |
| 30 | $\sim 0$ | $6.8 \mathrm{E}-7$ | 0.028 | $\sim \infty$ | $\sim \infty$ | 17.95 |
| 45 | $\sim 0$ | $\sim 0$ | $1.7 \mathrm{E}-5$ | $\sim \infty$ | $\sim \infty$ | 3.01 E 4 |
| 60 | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim \infty$ | $\sim \infty$ | $\sim \infty$ |

Table C.3: Landau-Zener transition probabilities and lifetimes for a tilted lattice under gravity.

## Appendix D

## Characterization of the optical

## lattice

In this section we perform several characterizations of the optical lattice. The optical lattice is modulated in order to measure the population in the bands of the lattice, and to manipulate the states of atoms in the lattice. Populations are measured by slowly decreasing the depth of the optical lattice, mapping different bands into different regions of space. The potential must be changed adiabatically to prevent transitions between different states. When we attempt to create a ground state we typically find that about $90 \%$ of the atom are in the ground state while $10 \%$ are in the excited state. We try to find the cause of the excited state population by characterizing the optical lattice in terms of adiabaticity, loss (tunneling rates) and heating. We also characterize the displacement operation in order to compare our theoretical displacements to the theoretical expectations.

## D. 1 Adiabaticity: How slow is slow

For a process to be adiabatic requires that the timescale for the external perturbation is much slower than the timescale for the internal dynamics of the system. The
adiabaticity criterion for an atom to move from the lowest energy band to a higher one is[40]

$$
\begin{equation*}
\left.\left|\langle n, q| \frac{d}{d t}\right| 0, q\right\rangle|\ll| E_{n}^{q}-E_{0}^{q} \mid / \hbar . \tag{D.1}
\end{equation*}
$$

The adiabaticity criterion is least satisfied near the edges of the Brillouin zone. The shallowest well depths used during the course of these experiments is $6 E_{r e}$. At this depth the energy separation is $\left|E_{n}^{q}-E_{0}^{q}\right| \approx 3 E_{r e}$. The condition D. 1 then becomes

$$
\begin{equation*}
\frac{d}{d t} U_{0} / E_{r e} \ll 24 \sqrt{2} E_{r e} / \hbar \sim 1.5 \cdot 10^{5} / \mathrm{s} \tag{D.2}
\end{equation*}
$$

The fastest potential decrease used in our experiment in on the order of $12 \mathrm{E}_{r e}$ over a time of 2 ms . Using these timescales and potential depths, the LHS of equation D. 2 is about 4000, much less than the RHS, thereby satisfying the condition. At larger potential depths this criterion is more easily met since the energy separations become larger. The LHS equals the RHS if we decrease the potential over a time of $80 \mu s$.

Figure D. 1 show the fraction of atoms found in the ground state as a function of the slope of the ramp after attempting to filter the ground state. The test was performed using a lattice that is $18 E_{r e}$ deep. The filtering stage was performed by decreasing the optical lattice to a depth of $9 E_{r e}$, holding the depth constant for 3 ms , and then increasing the depth back to the initial $18 E_{r e}$. The population measurement stage is performed stepwise using the same well depth and time. The rate at which the well depth is modulated is changed, keeping the total time of the experiment constant as well as the time between the filtering and measuring. For slower changes in the potential, i.e., slower than $12 E_{r e}$ over 0.5 ms , we find the ground state fraction is essentially constant at a value of 0.85 . This implies that the limiting factor in producing (or measuring) ground states is not the timescale of the potential changes, which have been kept at slopes of $10^{4} E_{r e} / s$.


Figure D.1: Fraction of atoms measured in the ground state after filtering vs the slope of the potential ramp.


Figure D.2: The fraction of atoms measured in the ground state is plotted against the time the atoms spend in the lattice after being prepared in a ground state. The solid line in a linear fit to the data with a slope of $0.005 / \mathrm{ms}$ and an intercept of 0.90

## D. 2 Heating

Another possibility to explain the excited state population is heating. We test this conjecture by varying the time between preparation of the ground state and the measurement of the state populations. The changes of the potential depth are kept in the adiabatic regime. Figure D. 2 shows the measured ground state fraction versus time. The solid line is a linear fit to the data points and has a slope of 0.005 and an intercept of 0.90 . It is apparent that a long delay between preparation and measurement will cause a decrease in the number of atoms found in the ground state. However the intercept of 0.90 clearly signifies that heating cannot be the only reason we find atoms in the excited state. During the course of a normal experimental run the time between preparation and measurement is typically kept to 10 ms which is required to get clear separation of the vibrational states.

## D. 3 Tunneling of excited state atoms

Another possible reason for the imperfect state preparation is for our filtering time (the duration the lattice is held at a weak potential) to be too short. We measure the loss rate of excited state atoms from the lattice by varying the duration that the lattice is held at a weak depth of $9.0 E_{r e}$. An adiabatic ramp is not used since the atoms would have a significant tunneling rates before the lattice gets to its lowest potential depth, and tunneling continues at a large rate as we begin to increase the lattice. For this measurement we use a diabatic decrease of the well depth instead of the adiabatic decrease. This is done to get a better measure of the tunneling rate at a given well depth. Figure D. 3 show the fraction of ground state atoms as a function of the time held in a shallow lattice of depth $9.0 E_{r e}$. Without filtering the atoms had a distribution of $62 \%$ of the atoms in the ground state and $38 \%$ in the excited state. The solid line in the plot is an exponential fit to the data. The time constant of the fit is 1.2 ms whereas the expected lifetime is 0.9 ms . A time constant of 1.2 ms would be expected for a lattice depth of $10.2 E_{r e}$. After dwelling at the shallow lattice depth for 4 ms we expect that almost all the atoms that will tunnel out of the lattice will have tunnelled out. Note that the ground state population doesn't exceed 0.88 , although this does include two diabatic changes of the lattice potential.

Creation and measurement of a state wherein all the atoms are in a ground state has not been accomplished. Typically we find $10 \%$ of the atoms in the excited state after attempting to prepare a ground state. We cannot attribute finding atoms in the ground state being a result of insufficient time during filtering or to non-adiabatic transitions due to abrupt changes in the potential depth. The duration between the preparation of the ground state and measurement of the populations does influence the measured value, if the time is less than 3 ms . Our measurements are performed by holding the potential low for 3 ms , so this still does not explain the discrepancy.


Figure D.3: Fraction of ground state atoms is plotted as a function of the time the lattice is held at a shallow depth. The solid line is a fit to the data and has an exponential time constant of 1.21 ms .

## D. 4 Displacement of the lattice

Transferring atoms between different vibrational states of the lattice is accomplished by displacing the lattice potential by a controlled shift in the relative phase between the lattice laser beams. On displacing the optical lattice we change the basis states. The wavefunction of the atoms in the new basis has different populations and coherences than in the undisplaced lattice. We characterize the displacement of the lattice by preparing a ground state distribution (or as well as it may be prepared) and displacing the lattice. A measurement of the populations follows the displacement. Figure D. 4 shows the theoretical coupling probabilities between ground and excited states, i.e., $|\langle 0| \widehat{D}(x)| 0\rangle\left.\right|^{2}$ and $\left.|\langle 0| \widehat{D}(x)| 0\right\rangle\left.\right|^{2}$ and $\left.|\langle 0| \widehat{D}(x)| 0\right\rangle\left.\right|^{2}$, labelled as $\left|c_{00}\right|^{2},\left|c_{10}\right|^{2}$ and $\left|c_{11}\right|^{2}$ respectively, where $\widehat{D}(x)$.is the displacement operator and $x$ is the size of the displacement. The probabilities are computed using Bloch waves for a lattice that is $18 E_{r e}$ deep. The calculation is carried out for 20 different quasimomenta equally spaced across the Brillouin zone. Figure D.4b shows the measured ground and excited state probabilities as a function of displacement for an initial
mixed with $89.1 \%$ of the atoms in the ground state and $10.9 \%$ of the atoms in the excited state. The solid lines are the result of a calculation using the theoretical coupling coefficients from part a) with a mixed input state with the same populations as initially prepared in the experiment. The coherences are assumed to be zero. The measured data points agree very well with the theoretical line, suggesting that the oscillation frequency is a good method of characterizing the depth of the optical lattice.


Figure D.4: Figure a): Theoretical overlap integrals between displaced and undisplaced lattice. $|c 00|$ is coupling from ground state to ground state. $|c 10|$ is coupling from ground state to excited state and $|\mathrm{C} 11|$ is from excited state to excited state. b) Measured ground state fraction (circles) and excited state fraction (hollow diamonds) as a function of displacement. The solid lines are theoretical curves for an initial mixed state with no coherences.

