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## Outcoupling Properties of a Guided Atom Laser



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CHAPTER1

## Introduction

One of the fundamental principles of quantum mechanics is that there is a duality between particles and waves. Any particle with momentum $p$ also shows wave-like behavior with the de Broglie wavelength

$$
\begin{equation*}
\lambda_{d B}=\frac{h}{p}, \tag{1.1}
\end{equation*}
$$

where $h$ denotes Planck's constant. In analogy to optics with electromagnetic waves, many phenomena can be observed in matter-wave optics that could not be explained easily in terms of point-particle behavior.

It has already been seen in 1950 by Norman Ramsey that the wave-like nature of a particle can be of great value for precision measurements. He showed in [1] that the interrogation time (which can limit the measureable spectral resolution of a transition) of a particle interacting with an electromagnetic field can be independent of the actual interaction duration. This is because coherences between different components of the particle's wavefunction that were introduced by an interaction can persist over a long interrogation time. During a second interaction at the end of the interrogation time, an interferometer path can be closed and the evolution of the matter-waves can be read out. This idea of separately interacting fields is the underlying principle how the atomic clocks operate which currently serve as time standards.

During the 1980s several groups started to use laser light to cool down neutral or charged particles. In particular since 1989 atomic fountains 2 make it possible to produce clouds of cold atoms that follow a ballistic trajectory and are not subject to any external potential during their flight. With the availability of cold atomic samples soon many new types of atomic interferometers where proposed and implemented. Ramsey's original sequence was extended to allow precision measurements of quantities such as the local gravitational acceleration [3], gravity gradients[4] and fundamental constants such as the fine structure constant by determining the ratio of Planck's constant to the mass of a particle [5]. The Sagnac effect for matter-waves is used for atom interferometer-based gyroscopes [6].

But the field of atom optics is not restriced to precision measurements. For example, matter-waves can also be employed in atom lithography[7] to create structures whose size is not limited by the optical wavelength in use, as it would be the case for lithography with light. Matter-waves can be a useful tool to study fundamental physical effects such as Anderson localization [8] or quantum reflections from attractive potentials [9]. Also analogous systems to Fabry-Pérot cavities have been proposed for matter-waves [10. Also in analogy to conventional optics, multiple-wave mixing has
been observed with matter-waves [11. Atom-optical systems can sometimes even exhibit richer phenomenology than photon-optical ones because the involved particles can interact with each other and thus quickly cause non-linear behavior.

Except the multi-wave mixing, all the phenomena and methods mentioned above base on effects with single atoms. There is no coherence between the different atoms of a sample necessary. This is even the case for the interferometers, where the induced coherence is between different states of one atom but not between different atoms. Regarding the phase properties, the matter-waves used there are in a loose analogy comparable to the radiation emitted from an ordinary light bulb: there is no fixed phase relation between the individual particles and the intensity correlation functions show the bunching or anti-bunching behavior characteristic for their spin [12, 13].

About one decade after the first demonstrations of laser cooling another technique, called evaporative cooling was perfected to a level where it became possible to slow atoms down to low enough temperatures that an effect called Bose-Einstein condensation (BEC) occurs. The first experimental demonstrations of BEC in atomic vapors have been in 1995 in Boulder [14], at Rice university [15] and at MIT[16]. What a Bose-Einstein condensate is and how the methods involved in its creation work, will be explained in the background chapter of this thesis. One important property of a Bose-Einstein condensate is that the atoms in it all have the same wavefunction with a fixed phase relation.

When atoms are coupled out of the BEC in a way that does not destroy the coherence then it becomes possible to see interference effects between different atoms. The intensity correlation function of the outcoupled atoms is then flat without any features of bunching or anti-bunching [17]. Because of the analogy to the properties of the radiation emitted by an optical laser, these systems in which atoms are coherently coupled out of a condensate bear the name atom lasers.

An atom laser with a pulsed output has been first demonstrated short time after BECs became available [18]. The first atom laser with quasi-continuous output followed in 1999[19. With the same apparatus, is was also possible to outcouple two atom lasers from the same condensate and observe an interference pattern between them [20].

Apart from coherence, another characteristic of an optical laser is that the wavelength of the emitted light is within a very narrow band around a well defined center wavelength. But in contrast to photons, atoms have a rest mass and are thus subject to gravity. In the case of free falling atoms, gravitationally extracted from a condensate, this causes their de Broglie wavelength to be rapidly reduced.

The problem can be overcome by not letting the atoms fall freely after the extraction but to rather inject them into a waveguide. If the atoms are guided horizontally without any additional acceleration, their de Broglie wavelength stays constant.

## This Experiment

Our experiment is an evolution of the setup that first demonstrated a quasicontinous guided atom laser [21]. We create a BEC in an optical/magnetical hybrid trap. Atoms are continously outcoupled from the BEC and directly injected into
an atomic waveguide that is formed by the optical trapping potential. A horizontal geometry of the waveguide prevents atomic acceleration due to gravity. Propagation in the guide is quasi-one-dimensional.

In this thesis the experimental setup is described and measurements of the radiofrequency outcoupling resonance curve are presented. Furthermore transverse and longitudinal properties of the extracted matter-waves are studied.

## Outline of this thesis

In Chapter 2 the theoretical background necessary to understand the experiment will be given. Also some of the techniques that we use to obtain a condensate and to create the guided atom laser are explained. Chapter 3 describes the experimental setup that we are using to create the BEC in the hybrid trap. The actual guided atom laser along with studies of its outcoupling properties is presented in Chapter 4. A new cooling laser system that will be used for the next generation of the experiment is described in Chapter 55. In the conclusion (Chapter 6), the properties of our guided atom laser are summarized and an outlook on necessary future studies and possible applications is given.

## CHAPTER 2

## Background

An atom laser from a Bose-Einstein condensate is a rather complicated system to theoretically describe and to experimentally implement. In this chapter the aspects most important for the understanding of our experiment are presented. In the first section Bose-Einstein condensates are introduced and some of their basic properties are discussed. The remaining sections deal with the experimental methods we are using to obtain a condensate and create the guided atom laser.

### 2.1 Bose-Einstein Condensation

This section is devoted to a brief introduction to the phenomenom called BoseEinstein condensation (BEC) including an overview of the condensate properties that are most relevant for this thesis. Two comprehensive review articles on the topic are [22] and [23].

A result of quantum mechanics is that particles with integer spin (called bosons) tend to "stick together". If a certain mode in a system is already occupied then extra bosons that are added to the system preferably go into the same mode. This is the effect that causes the photons emitted from the gain medium in an ordinary laser to go all into the same cavity mode. In a dilute atomic gas at room temperature such effects are not observed because the average interatomic distance $d$ is large compared to the size of the particles. Another way to say this is that each individual atom occupies its own mode (characterized by its wavefunction and its position) and these modes do not overlap. In this case the atoms can be treated classically as point-like particles.

Things are different if a gas of bosonic particles is cooled down to very low temperature. When the velocity of the atoms is reduced, their de Broglie wavelength $\lambda_{d B}$ becomes longer and at some point one has to give up the classical treatment and rather has to think of the atoms in terms of wavepackets instead of point-particles. The longer its de Broglie wavelength is, the larger is the volume of the mode occupied by an atom. If the sample is cooled down to sufficiently low temperatures, then the de Broglie wavelength can even become larger than the average interatomic distance. At this point the modes of the atoms start to overlap and quantum effects set on. For an illustration see Fig. 2.1.

In the case of bosons what happens is that different atoms start occupying the same mode. Instead of having many individual wavefunctions, they then have one common macroscopic wavefunction, with all atoms being in phase. The onset of this


Figure 2.1: Onset of Bose-Einstein condensation
(left) Particles at high temperature can be treated classically. (right) At low tamperature, quantum effects become more pronounced. When the de Broglie wavelength $\lambda_{d B}$ becomes comparable to the interatomic distance $d$, the wavefunctions start to overlap.
is called Bose-Einstein condensation.
For a gas of $N$ particles uniformly distributed in a volume $V$, the density is $n=N / V$. The relation between the thermal de Broglie wavelength

$$
\begin{equation*}
\lambda_{d B}=\frac{h}{\sqrt{2 \pi m k_{B} T}} \tag{2.1}
\end{equation*}
$$

and the average interparticle distance $d=n^{-1 / 3}$ can be expressed in terms of the phase-space density

$$
\begin{equation*}
D:=n \lambda_{d B}^{3} \tag{2.2}
\end{equation*}
$$

Above, $h$ is Planck's constant, $m$ is the mass of the involved particles, $k_{B}$ is the Boltzmann constant and $T$ is the temperature of the gas. The higher $D$ is, the more do the individual wavefunctions overlap. The exact value at which BEC sets on can be calculated as [24]

$$
\begin{equation*}
D=g_{3 / 2}(1)=2.612 \ldots \tag{2.3}
\end{equation*}
$$

With

$$
\begin{equation*}
g_{k}(x):=\sum_{n=1}^{\infty} \frac{x^{n}}{n^{k}} . \tag{2.4}
\end{equation*}
$$

When practically creating a BEC, the atomic cloud is trapped and manipulated in an external potential. For a certain number of atoms in a given potential, one can define a critical temperature $T_{c}$ at which the phase transition occurs. Using (2.1) it is possible to rewrite (2.3) for the case of a uniform gas as

$$
\begin{equation*}
k_{B} T_{c}=\frac{2 \pi \hbar^{2}}{m}\left(\frac{n}{g_{3 / 2}(1)}\right)^{2 / 3} \tag{2.5}
\end{equation*}
$$

For atoms confined in a 3-D harmonic trap with trapping frequencies $\omega_{i}, i=x, y, z$ the critical temperature is modified due to the different density of states compared to the free case. The condition then reads [22]

$$
\begin{equation*}
k_{B} T_{c}=\hbar \bar{\omega}\left(\frac{N}{g_{3}(1)}\right)^{1 / 3} \approx 0.94 \hbar \bar{\omega} N^{1 / 3} \tag{2.6}
\end{equation*}
$$

Here $\bar{\omega}=\left(\omega_{x} \cdot \omega_{y} \cdot \omega_{z}\right)^{1 / 3}$ is the geometrically averaged oscillator frequency. Translated into phase-space density this gives a result similar to (2.3) but with the uniform density in definition (2.2) replaced by the density at the center of the trap.

The interaction between particles in a BEC can be described by replacing the the actual potential between atoms at positions $\mathbf{r}$ and $\mathbf{r}^{\prime}$ by an effective mean-field potential of the form

$$
\begin{equation*}
V_{M F}\left(\mathbf{r}-\mathbf{r}^{\prime}\right)=g \delta\left(\mathbf{r}-\mathbf{r}^{\prime}\right), \tag{2.7}
\end{equation*}
$$

where $\delta(\mathbf{r})$ is the Dirac delta function. With the s-wave scattering length $a$, the coupling strength

$$
\begin{equation*}
g=\frac{4 \pi \hbar^{2} a}{m} \tag{2.8}
\end{equation*}
$$

The wavefunction $\Phi(\mathbf{r}, t)$ of the condensate in an external potential $V_{e x t}$ then obeys the Gross-Pitaevskii (GP) equation

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t} \Phi(\mathbf{r}, t)=\left(-\frac{\hbar^{2} \nabla^{2}}{2 m}+V_{e x t}(\mathbf{r})+g|\Phi(\mathbf{r}, t)|^{2}\right) \Phi(\mathbf{r}, t) \tag{2.9}
\end{equation*}
$$

Two conditions need to be fulfilled for the GP equation to be valid. The first one is that the number of atoms in the condensate has to be much larger than 1 . The other condition is that the gas has to be so dilute that $d \gg a$.

When dealing with time-independent problems, the wavefunction can be written as

$$
\begin{equation*}
\Phi(\mathbf{r}, t)=e^{-i \frac{\mu}{\hbar} t} \varphi(\mathbf{r}) \tag{2.10}
\end{equation*}
$$

where we have introduced the chemical potential $\mu$ as the condensate energy per particle. The spatial part of the wavefunction $\varphi(\mathbf{r})$ then obeys the time-independent GP equation

$$
\begin{equation*}
\mu \varphi(\mathbf{r})=\left(-\frac{\hbar^{2} \nabla^{2}}{2 m}+V_{e x t}(\mathbf{r})+g|\varphi(\mathbf{r})|^{2}\right) \varphi(\mathbf{r}) . \tag{2.11}
\end{equation*}
$$

The total interaction energy $E_{\text {int }}=g N n\left(\right.$ since $\left.n(\mathbf{r}, t)=|\Phi(\mathbf{r}, t)|^{2}\right)$ can be much larger than the total kinetic energy $E_{k i n}$, which is of the order $N \hbar \bar{\omega} \mathbb{1}$. In these cases of $E_{\text {int }} \gg E_{k i n}$, the kinetic energy term $\left(\propto \nabla^{2}\right)$ in (2.9) can be dropped. This neglection is called the Thomas-Fermi approximation.

[^0]

Figure 2.2:

## 1-D Thomas-Fermi density profile

The blue (dotted) curve shows the the Gaussian ground state in a harmonic trap. The Thomas-Fermi wavefunction of a condensate (red, solid) in the same trap is much wider due to the interaction between the atoms. In case of a harmonic trap the density profile is an inverse parabola. Both curves have the same normalization.

In the Thomas-Fermi approximation, the time-independent Gross-Pitaevskii equation becomes purely algebraical and is solved by

$$
\varphi_{T F}(\mathbf{r})= \begin{cases}\sqrt{\frac{\mu-V_{e x t}(\mathbf{r})}{g}} & \text { if } \mu \geq V_{\text {ext }}(\mathbf{r})  \tag{2.12}\\ 0 & \text { else },\end{cases}
$$

which is called the Thomas-Fermi wavefunction of the condensate. Fig. 2.2 shows the density $\left|\varphi_{T F}(\mathbf{r})\right|^{2}$ for a condensate of $10^{6}{ }^{87} R b$ atoms in a harmonic trap with oscillation frequency $\omega=280 \mathrm{~Hz}$. For comparison also the Gaussian ground state solution of the harmonic oscillator is shown.

The chemical potential is found by normalizing the Thomas-Fermi wavefunction to N . In a 3-D harmonic oscillator potential with geometrically averaged frequency $\bar{\omega}$, one finds [22]

$$
\begin{equation*}
\mu=\frac{\hbar \bar{\omega}}{2}\left(15 \frac{a N}{l}\right)^{2 / 5} \tag{2.13}
\end{equation*}
$$

where $l=\sqrt{\hbar / m \bar{\omega}}$ is the characteristic length of the ground state wavefunction.

### 2.2 Laser Cooling

Every BEC experiment with an atomic vapor starts by laser cooling of the sample. Various techniques have been developed in this field since the first proposals to cool matter with laser radiation. In this section those techniques relevant for our experiment are introduced.

### 2.2.1 Doppler Cooling

When atoms absorb or emit a photon then energy and momentum have to be conserved. Energy conservation expresses itself in the way that the process can only take place if the photon energy equals the energy difference between the atomic state
before and after the event. To fulfill momentum conservation the atom recoils in order to compensate for the photon's momentum. In case of absorption of a photon at frequency $\omega$ the atom receives a momentum kick of magnitude $\hbar \omega / c$, where $c$ is the speed of light. The kick is in the direction which the photon had originally. In the case of emission, momentum conservation is fulfilled by the atom recoiling in the opposite direction of the photon.

The most simple, yet very powerful form of laser cooling relies on this momentum conservation. If atoms preferably absorb photons from the direction in which they are moving and subsequently re-emit them into random directions, then their velocity gets reduced after many cycles. This is because the kicks due to emission average out as consequence of the randomness regarding the photon direction ${ }^{2}$,

The aim of most laser cooling techniques is to create the above scenario in which photons are preferably absorbed from the direction of atomic motion. In fact it can already be enough to send a laser against the direction of motion of an atomic sample. If the laser at frequency $\omega_{L}$ is red detuned with respect to an atomic resonance at frequency $\omega_{0}$ then the Doppler shift

$$
\begin{equation*}
\Delta \omega=\mathbf{k} \cdot \mathbf{v} \tag{2.14}
\end{equation*}
$$

with photon wavevector $\mathbf{k}$ and atomic velocity $\mathbf{v}$, brings the laser closer to resonance if the atom is moving towards the beam. If the velocity is such that $\Delta \omega$ equals the detuning then the light is exactly on resonance in the atomic rest frame and

$$
\begin{equation*}
\omega_{0}=\omega_{L}-\mathbf{k} \cdot \mathbf{v} \tag{2.15}
\end{equation*}
$$

For a two-level atom with transition linewidth $\Gamma$ moving in a laser beam close to resonance, the steady-state excited state population ${ }^{3}$ can be calculated to be [25]

$$
\begin{equation*}
\rho_{2}(\mathbf{v})=\frac{1}{2} \cdot \frac{s}{1+s+4\left(\frac{\omega_{L}-\omega_{0}-\mathbf{k} \cdot \mathbf{v}}{\Gamma}\right)^{2}}, \tag{2.16}
\end{equation*}
$$

where the saturation parameter is defined as

$$
\begin{equation*}
s:=2 \Omega^{2} / \Gamma^{2} . \tag{2.17}
\end{equation*}
$$

$\Omega=\mathbf{d} \cdot \mathbf{E} / \hbar$ is the Rabi frequency of the light's electric field component $\mathbf{E}$ interacting with the transition dipole moment $\mathbf{d}$.

Multiplying $\rho_{2}(v)$ with the excited state decay rate $\Gamma$ gives the rate at which photons are absorbed. Furthermore multiplying with the momentum $\hbar k$ transferred in each absorption process one obtains the force exerted on the atom by the laser beam:

$$
\begin{equation*}
F_{\text {Doppler }}(v)=\hbar k \Gamma \rho_{2}(v) . \tag{2.18}
\end{equation*}
$$

[^1]Figure 2.3:


## Doppler cooling force

Force exerted by the individual beams (red and blue) and their sum (black) as function of velocity. Since the individual forces have opposite sign the sum can be close to linear for the right choice of detuning (i.e. distance between their extrema). Here the case of red detuning by $\Gamma / 2$ is shown.

If the light beam is red detuned then the force has its maximum when the atom moves towards the beam. If a second beam is applied with same frequency but opposite direction than the first one, it also exerts a force on the atom. But this time the force has opposite direction due to the opposite photon momentum and also has its extremum at exactly the opposite velocity than the first beam. The two forces are plotted in Fig. 2.3 along with their sum that acts on the atom. The slope of the resulting force can be close to linear over a velocity range of the order $\Gamma / k$ if the detuning is chosen to be around $\Gamma / 2$.

The atomic velocity is damped by the two beams because the Doppler shift always brings the atom closer to resonance with the beam counterpropagating its motion. The resulting disbalance in the absorption rate from the two beams gives rise to the net force. The effect of the light is in this case comparable to that of a viscous medium. For this reason the beam configuration is said to form an optical molasses.

When comparing the heating due to spontaneous emission cycles with the damping due to the cooling force one finds a lowest temperature that can be reached by Doppler cooling. It is found at a red detuning $\delta:=\omega_{L}-\omega_{0}=-\Gamma / 2$. The temperature is then the Doppler temperature $T_{\text {Doppler }}$. With the Boltzmann constant $k_{B}$

$$
\begin{equation*}
k_{B} T_{\text {Doppler }}=\frac{\hbar \Gamma}{2} . \tag{2.19}
\end{equation*}
$$

In case of cooling ${ }^{87} R b$ and cooling on the $D_{2}$-line $T_{\text {Doppler }}=146 \mu \mathrm{~K}$.
The first proposal of an optical molasses was by Hänsch and Schawlow in 1975 [26]. The first actual implementation was in the group of Steve Chu in 1985 [27].

### 2.2.2 Zeeman Slowing

It is sometimes useful to have a beam of slow atoms passing from one region of the experimental setup to another. For example such a beam can stem from an atomic oven source and point towards a ultra-high vacuum region where the atoms
are then trapped and manipulated. Getting a beam with low transverse atomic velocity spread can be performed rather easily by filtering with two apertures at some distance from each other. Only atoms with low tranverse velocity can pass through both apertures. Slowing the atoms down in the longitudinal direction is more difficult because standard beam techniques developed for charged particles or particles with a strong permanent magnetic moment are not suitable for table-top experiments with neutral particles at low energy.

Fortunately it is possible to combine the effect of the Doppler shift and the Zeeman shift to slow atoms down longitudinally to velocities that are controllable by the local magnetic field strength. Consider an atomic beam that counterpropagates a red detuned laser beam. At some longitudinal velocity $v$ the resonance condition (2.15) is fulfilled. Atoms around this velocity then absorb photons from the laser light and slow down. But for a fixed laser frequency and atomic resonance, the width $\Delta v$ of this velocity class depends linearly on the linewidth of the transition in use:

$$
\begin{equation*}
\Delta v=\frac{c \Gamma}{\omega_{L}} . \tag{2.20}
\end{equation*}
$$

This capture range is much smaller than the maximum longitudinal velocities of atoms coming from an oven. If one now intends to stop these atoms or slow them down to low velocities it is necessary to either change the laser frequency $\omega_{L}$ or the atomic resonance frequency $\omega_{0}$ to keep condition (2.15) fulfilled as the atoms slow down.

In a Zeeman slower a magnetic field is applied to change the atomic resonance frequency by making use of the Zeeman effect. The field is oriented along the beam direction and the field strength is varied over the distance that the atoms travel. By choice of the laser frequency and the magnetic field strength at the beginning of the slower one determines the maximum velocity of atoms that are slowed down. The field strength is then reduced along the beam line. This way atoms are kept in resonance as they are being slowed down while moving along. The final velocity is then determined by the final value of the magnetic field.

The first Zeeman slower was operated in the group of Bill Phillips [28, 29]. They can even be used to stop the beam[30]. They are often employed as atomic sources when differential vacuum systems are used.

### 2.2.3 Magneto-Optical Trapping

The first magneto-optical trap (MOT) was built in the group of Steve Chu at Bell Labs in 1987 [31]. In a MOT an optical molasses is overlapped with a spatially inhomogeneous magnetic field. For appropriately chosen parameters, atoms are then not only cooled but also trapped at the zero-point of the magnetic field. This allows for accumulation of very large numbers of atoms. To understand the principle of a MOT it is sufficient to restrict oneself to one dimension. The extension to three dimensions is straight-forward.

The most simple case to understand magneto-optical trapping is that for an atom with a ground state that has total angular momentum $F=0$ and excited state


Figure 2.4: Magneto-optical trapping principle
A magnetic field is applied along the $x$-direction with magnitude $B(x) \propto x$ around the origin. The energies of the $m_{F}= \pm 1$ excited state levels then also depend on the position due to the linear Zeeman effect. The atom is initially in the $m_{F}=0$ ground state and the applied light is red detuned. Light with $\sigma^{-}$-polarization is then closer to resonance on the positive side of the $x$-axis and light with $\sigma^{+}$polarization is closer to resonance on the negative side. The $\sigma^{+}$light is applied in the positive $x$ direction and $\sigma^{-}$light in the negative one. Atoms moving away from the origin then absorb more atoms from the beam counterpropagating them, than from the copropagating one. Thus there is a net force towards the origin.
angular momentum $F=1$. Let the atom be initially in the ground state. In the presence of a magnetic field that points in the $x$-direction and depends linearly on position, $m_{F} \neq 0$ excited states are Zeeman shifted by the field (see Fig. 2.4). When the excited state has a positive Landé factor, then the $\left|F=1, m_{F}=-1\right\rangle$ excited state level has lower energy at positions of positive field than the $|1,+1\rangle$ level. Now apply red detuned beams of an optical molasses with $\sigma^{+}$-polarization in the direction of positive field and $\sigma^{-}$-polarization in the opposite ond. The Zeeman shift brings atoms on the positive field side closer to resonance with the $\sigma^{-}$-beam and vice versa with the $\sigma^{+}$-beam on the negative field side. Therefore in addition to the Doppler cooling effect of the molasses formed by the counterpropagating beams, atoms on the right hand side in Fig. 2.4 will also absorb more photons from the beam pointing to the left than from the other one. The inverse applies for atoms on the left hand side. Hence the atoms always feel a net force to the center where the magnetic field is zero and the detuning for $\sigma^{+}$- and $\sigma^{-}$-light is the same.

[^2]Depending on the level structure of the species in use it can happen that not only the cooling transition is excited but also off-resonant transitions to other states occur. The following spontaneous emission process can leave the atoms in a final state that does not couple to the applied light field. In this case the atoms get lost for the cooling process. In order to bring these atoms back into the game one can use a repumping beam to pump the atoms back into the ground state of the cooling transition. As an example consider rubidium- 87 with the level diagram shown in Appendix C. When the $D_{2}, F=2 \rightarrow F^{\prime}=3$ transition is used for cooling with a red detuning smaller but roughly comparable to the upper level hyperfine splitting then off-resonant excitations of the $F^{\prime}=2$ state are possible. From there the atoms have a $50 \%$ probability to fall into the $F=1$ ground state. But since the ground state hyperfine splitting is very large compared to the excited state splitting, this state is dark with respect to the $F=2 \rightarrow F^{\prime}=3$ cooling light ${ }^{5}$. One can then apply $F=1 \rightarrow F^{\prime}=2$ repumping light to bring the atoms back into the cycle.

When the cloud in a MOT becomes increasingly dense then there is also an increasing probability that a photon which was emitted by one atom is reabsorbed by another one. This causes the atoms to repell each other and counteracts the force that compresses the cloud to the trap center. At some point the internal radiation pressure equals the radiation pressure exerted by the MOT beams and the density as well as the temperature have reached limits. In the case of a system where repumping light is used this limit can be overcome to a certain extend by applying the repumping light not actually to the entire sample but only to the outer part. There the atoms are hotter and less dense than in the center, so cooling and compression are still happening in those places. In contrast, in the central part where the atoms are already cold they are pumped into the dark state and remain there due to the lack of repumping. But since these atoms are then not cycling any more, they also do not contribute to the internal radiation pressure, allowing for a higher final density and lower final temperature. Such a system with the cloud center in a dark state is called dark MOT.

### 2.2.4 Sub-Doppler Cooling in Optical Molasses

To understand the cooling techniques described above it was sufficient to model the atom as a system with only one ground state. But much to their surprise several groups doing early work on laser cooling of atoms found the final temperatures of their clouds in an optical molasses to be actually below the Doppler limit (2.19) [32]. This was due to cooling mechanisms present for certain beam configurations and magnetic backgrounds. These effects arise as a consequence of the atoms having more than only two internal states, in particular having more than only one ground state energy level. Two distinct mechanism where identified and studied thoroughly in [33, 34] and [35]. One is called Sisyphus cooling and the other one is polarization gradient cooling. Their qualitative principle of operation is explained in the following.

[^3]

Figure 2.5:

> Relative oscillator strengths for $$
\text { an } F=1 / 2 \rightarrow F^{\prime}=3 / 2
$$ transition

Calculated from Clebsh-Gordan coefficients for $\pi$ (red), $\sigma^{+}$ (green) and $\sigma^{-}$(blue) polarized light.

### 2.2.4.1 Sisyphus Cooling

When two laser beams with the same frequency and amplitude but orthogonal linear polarization counterpropagate, they add up to a field with spatially varying polarization. If the field is linearly polarized at the origin of the propagation axis then it is circularly polarized at point $\lambda / 8$. At $\lambda / 4$ the field is linearly polarized again but orthogonal to the polarization at the origin. In between, its polarization is elliptical. Over the distance of one wavelength the polarization makes two rotations around the Poincaré sphere.

The most simple case where Sisyphus cooling can occur is for an atom with an $F=1 / 2$ ground state and an $F^{\prime}=3 / 2$ excited state. Consider the case of light red detuned from the $F=1 / 2 \rightarrow F=3 / 2$ transition. The ground state energies in presence of the field are reduced due to the light shift (AC Stark shift). The magnitude of the light shift for different polarizations of the radiation field and for the different ground states scales with the oscillator strength of the respective transition. The relative oscillator strengths are shown in Fig. 2.5. For example at a position where the field has $\sigma^{+}$-polarization, the light shift of the $m_{F}=+1 / 2$ ground state is three times larger than the shift of the $m_{F}=-1 / 2$ ground state. At a distance $\lambda / 4$ away the situation is inversed. Thus the ground state energy levels vary sinusoidally as function of position as depicted in Fig 2.6.

At low light intensities (Rabi frequency $\Omega \ll \Gamma$ ) the optical pumping rate satisfies $\Gamma_{p} \ll \Gamma$. This means the timescale associated with the pumping $\tau_{p}=\Gamma_{p}^{-1}$ is long. Consequently an atom that is at the bottom of a valley at some point and moves along the $x$-axis climbs up quite some part of the potential hill ahead of it before being pumped in the other ground state. In the new state it again climbs up until it is pumped back into the first state and so on (see Fig. 2.6). This effect is dubbed Sisyphus cooling with reference to the Greek myth. Since the atom is climbing up more time than it is going down, it is effectively decelerated. The deceleration is strongest if the velocity $v$ fulfills the condition $v \tau_{p} \approx \lambda / 4$. This means that the velocity is such that the atom climbs up a full potential hill before being pumped back into a valley. Sisyphus cooling therefore operates in a velocity range where $k v \approx \Gamma_{p}$, much lower than the Doppler cooling range for which $k v_{\text {Doppler }} \approx \Gamma$. On the other hand a calculation of the cooling force [35] shows that it scales as $\hbar k^{2} \delta / \Gamma$ with


Figure 2.6: Sisyphus cooling $m_{F}=+1 / 2$ (solid) and $m_{F}=-1 / 2$ (dashed) ground state energies at different positions.
$\delta$ being the laser detuning. This means that the cooling force in Sisyphus cooling is maximal for negative detuning with $|\delta| \ll \Gamma$. In contrast the Doppler cooling force is maximal for detuning $\delta=-\Gamma / 2$. This allows to reach lower final temperatures with Sisyphus cooling than reached by Doppler cooling. When operating at large detuning, also effects due to multiple scattering of photons are reduced.

### 2.2.4.2 Polarization Gradient Cooling

For atoms with angular momenta least $F=1$ for the ground state and $F=2$ for the excited state, there is also another mechanism called polarization gradient cooling that can be present in optical molasses. An intuitive picture to understand it is the following.

Consider two counterpropagating light beams with same amplitude and frequency but opposite circular polarizations. The resulting light field has linear polarization at any time but the direction of polarization rotates as a function of position (see Fig. 2.7). Let the laser power be low (Rabi frequency $\Omega \ll \Gamma$ ) and the frequency be red detuned with respect to the atomic $F=1 \rightarrow F^{\prime}=2$ transition. We take the polarization of the light's electric field component at points $z$ along the beam direction as quantization axis. Looking at the relative oscillator strengths for this transition in Fig. 2.8, we see that with the $\pi$-polarized pumping light, the population in steady-state is highest for the $m_{F}=0$ ground state with $m_{F}= \pm 1$ each being


Figure 2.7: Effective light polarization in $\sigma^{+}-\sigma^{-}$configuration
Two counterpropagating beams with $\sigma^{+}$and $\sigma^{-}$polarization and same amplitude create a light field that has constant amplitude along the beam axis. The polarization of the resulting field is linear and does not vary in time but the axis of polarization rotates around the beam axis over the distance of one wavelength.


Figure 2.8:
Relative oscillator strengths for an $F=1 \rightarrow F^{\prime}=2$ transition

Calculated from Clebsh-Gordan coefficients for $\pi$ (red), $\sigma^{+}$ (green) and $\sigma^{-}$(blue) polarized light.
equally populated ${ }^{6}$
But when the atom moves along the $z$ direction with velocity $v$ then the laser polarization axis rotates with respect to the atomic alignment at a frequency $k v$ (again, see Fig. 2.7). Now optical pumping tends to redistribute the atomic population between the $m_{F}$ states to have the atomic alignment follow the new quantization axis. In the case of low intensity this pumping process is rather slow at a rate $\Gamma_{p} \ll \Gamma$. For long pumping times $\tau_{p}=\Gamma_{p}^{-1}$ the alignment cannot follow the quantization axis adiabatically. According to Larmor's theorem the effect of the rotating quantization axis is then like that of an inertial magnetic field along the direction of flight. The magnitude of the field is such that the Larmor frequency associated with it equals the frequency of rotation $k v$. This inertial field now causes the atomic alignment to have a non-vanishing component in the direction of flight.

In the case of an atom moving along the $z$-axis in Fig. 2.7, the $z$-component of the alignment is positive causing the atom to absorb more photons from the coun-

[^4]terpropagating $\sigma^{+}$-beam than from the $\sigma^{-}$-beam due to the six time larger oscillator strength for $\sigma^{+}$-transitions starting from the $m_{F}=+1$ state (see Fig. 2.8). The inverse argument applies when the atom is moving in the opposite direction.

In order to actually compute the resulting damping force it is necessary to calculate the $z$-component of the atomic alignment as a function of velocity $v$. Here only an outline of the calculation shall be given. The full theory can be found in [35].

It is useful to transform from the atomic rest frame into a frame that moves along with the atom and rotates with the local laser field. This transformation is performed by the operator

$$
\begin{equation*}
T(t)=\exp \left(-i k v L_{z} t / \hbar\right) \tag{2.21}
\end{equation*}
$$

with $L_{z}$ being the angular momentum operator generating rotations around the $z$-axis [36]. Since the transformation $T(t)$ has an explicit time-dependence, the Hamiltonian $H^{\prime}(t)$ in the rotating system does not only contain the transformed $T(t) H(t) T^{+}(t)$ of the original Hamiltonian $H(t)$ but also an additional term

$$
\begin{equation*}
i \hbar\left[\frac{\mathrm{~d} T(t)}{\mathrm{d} t}\right] T^{+}(t)=k v L_{z} . \tag{2.22}
\end{equation*}
$$

This means that in the moving rotating frame the problem is equivalent to the one of an atom interacting with a field of fixed polarization but subject to an additional inertial term

$$
\begin{equation*}
V_{\text {inert }}=k v L_{z}, \tag{2.23}
\end{equation*}
$$

as expected from Larmor's theorem. For small $k v$ this term can now be treated perturbatively. Calculating the perturbed eigenstates to first order and computing the expectation value $\left\langle F_{z}\right\rangle$ using the steady state populations one finds it to be nonvanishing. More precisely, it is found to be proportional to the ratio $k v / \Delta$, where $\Delta$ is the AC Stark shift that the $m_{F}=0$ ground state experiences due to the light field.
$\left\langle F_{z}\right\rangle$ is proportional to the ratio of the population in the $m_{F}=+1$ state to the population in the $m_{F}=-1$ state, when taking the $z$-direction as quantization axis. For $v$ positive, the $m_{F}=+1$ state is more populated than the $m_{F}=-1$ state. Looking at the relative oscillator strength for $\sigma^{+}$- and $\sigma^{-}$-polarized light in Fig. 2.8 one sees that an atom in the $m_{F}=+1$ state has a six times larger probability to absorb a $\sigma^{+}$-photon than to absorb a $\sigma^{-}$-photon. But since the $\sigma^{+}$-beam is the one that counterpropagates the atomic motion, the result is that the atom is more probable to be slowed down due to $\sigma^{+}$-absorption than it is to be accelerated by the $\sigma^{-}$-beam. We conclude that the net force $F_{P G}(v)$ on the atom is proportional to the population imbalance $\left\langle F_{z}\right\rangle$. The force is also proportional to the pump rate $\Gamma_{p}$ and to the momentum $\hbar k$ transferred in each absorption process. Putting all together we find

$$
\begin{equation*}
F_{P G}(v) \propto \hbar k \Gamma_{p} \frac{k v}{\Delta} . \tag{2.24}
\end{equation*}
$$

Owing to the low intensity, the light shift $|\Delta| \ll \Gamma$. Therefore the velocity range $v_{P G} \approx \Delta / k$ in which polarization gradient cooling happens is much smaller than the velocity range $v_{\text {Doppler }} \approx \Gamma / k$ in which Doppler cooling is working. The sum of both


Figure 2.9:
Cooling force for polarization gradient cooling

The figure shows the sum of the Doppler cooling effect and the effect due to the polarization gradient around zero velocity. $\Omega=$ $\Gamma / 4$ and $\delta=-\Gamma / 2$. From [35].
the force due to Doppler cooling and due to polarization gradient cooling is shown in Fig. 2.9. The cooling force due to polarization gradient cooling is stronger around $v=0$ than the force coming from Doppler cooling. This allows to reach temperatures below $T_{\text {Doppler }}$.

When operating an optical molasses with polarization gradient cooling care has to be taken that there is no magnetic background field that overlaps the inertial term (2.23). In the presence of a magnetic field along the $z$-axis the atomic motion is not decelerated towards $v=0$ but to a velocity $v_{0}$ for which $k v_{0}=\omega_{\text {Larmor }}$ with $\omega_{\text {Larmor }}$ being the Larmor frequency associated with the field.

For all the cooling techniques described until this point the atom had a photon emitted into a random direction at the end of one cooling cycle. Since this emission causes the atom to recoil with momentum $\hbar k$ there is a lowest possible temperature limit that can be reached with these techniques. This is the temperature that corresponds to the kinetic energy of the atom moving with velocity $\hbar k / m$. It is called the recoil temperature $T_{\text {Recoil }}$ and

$$
\begin{equation*}
k_{B} T_{\text {Recoil }}=\frac{\hbar^{2} k^{2}}{m} . \tag{2.25}
\end{equation*}
$$

For the case of rubidium- 87 and light on the $D_{2}$-line $T_{\text {Recoil }}=362 \mathrm{nK}$.
There have also been cooling schemes developed that allow for final temperatures even below the recoil limit. One of them is velocity-selective coherent population trapping [37] where atoms are velocity-selectively pumped into a coherent superposition of states which does not couple to the light field. Another scheme [38] uses stimulated Raman transitions between ground state hyperfine levels separated by energy $E_{H F S}$. When the stimulating beams have opposite direction then the net transferred momentum in such a process is $E_{H F S} / c \ll \hbar k$.

Yet another technique that can be implemented in one, two and three dimensions and can also be applied to trapped atoms is described in the following subsection.

Figure 2.10:

## Degenerate Raman Sideband Cooling

Atoms in vibrational levels $\nu>0$ are Raman coupled to neighboring $m_{F}$ states. Optical pumping with $\sigma^{+}$and $\pi$ light transfers population to $m_{F}=F$. The spontaneous emission of the pumping process tends to preserve the vibrational quantum number (see text). Atoms cycle several times until they decay into the $|F, 0\rangle$ state which is dark with respect to both the Raman and the pump light. Figure adapted from 39.

### 2.2.5 Degenerate Raman Sideband Cooling

A particularly elegant method to cool atoms below the recoil limit and at the same time even increase the phase-space density is the so called degenerate Raman sideband cooling.

Consider atoms confined in an optical lattice. If the potential wells are sufficiently deep and the atoms cold enough then the particles are localized to individual lattice sites. Within these lattice sites they initially occupy high vibrational levels $\nu$. Aim of the cooling technique is now to transfer the atoms to lower vibrational levels and to thereby reduce their kinetic energy. The idea how to do this is to apply a magnetic field that lifts the degeneracy of the different $m_{F}$ levels of the $F$ state in which the atoms are. When the field has exactly the strength so that the Zeeman shift between neighboring $m_{F}$ states equals the energy difference between the lattice's vibrational levels, then the states $\left|m_{F}=m, \nu=n\right\rangle$ and $|m \pm 1, n \pm 1\rangle$ are degenerate (see Fig. 2.10). For an appropriate choice of beam directions and polarizations of the light that creates the lattice it is possible that these lattice beams induce Raman transitions between the degenerate states.

When now slightly elliptical polarized light with a strong $\sigma^{+}$and a weak $\pi$ component resonant to an $F \rightarrow F^{\prime}=F-1$ transition is applied, then population is transferred to the $m_{F}=F$ Zeeman state. In the Lamb-Dicke regime where the vibrational level splitting of the lattice is larger than the photon recoil energy, the vibrational quantum number is conserved during the optical pumping process. Once the population is pumped to the $m_{F}=F$ end of the Zeeman ladder, only Raman transitions $\left|m_{F}=F, n\right\rangle \rightarrow\left|m_{F}=F-1, n-1\right\rangle$ and $\left|m_{F}=F-1, n-1\right\rangle \rightarrow\left|m_{F}=F-2, n-2\right\rangle$ are followed by optical pumping. Therefore in each cycle the vibrational evergy is
reduced by one or two quanta, but is never increased. Once the atoms are in the lowest vibrational level of the $m_{F}=F$ Zeeman state they cannot undergo any more Raman transitions because of the lack of a neighboring state with same energy. Since $\left|m_{F}=F, \nu=0\right\rangle$ is also dark with respect to the pump light the atoms then cannot undergo any further transitions.

Once all population is in the dark state, the atoms can be released from the lattice by adiabatically lowering the light intensity and thereby the depth of the confining potential. This also reduces the vibrational ground state energy and consequently the kinetic energy of the atoms in this state. Cooling then ceases only when the de Broglie wavelength of the atoms becomes so long that they are no longer localized in the potential wells. In practice temperatures below the recoil limit are achieved this way with almost no loss of atoms. Since the density stays constant, the cooling even increases the phase-space density of the sample.

Degenerate Raman sideband cooling in optical lattices has been demonstrated experimentally in 1D [40, 2D [41] and in 3D [42]. In each of these experiments a magnetic field was applied to lift the degeneracy of the neighboring magnetic sublevels, leaving the $m_{F}=F$ state in the dark. Even though phase-space density is increased by this cooling method, quantum degeneracy has not been reached with it.

It has also been proposed to use the AC Stark effect to lift the degeneracy of the the $m_{F}$ states [43]. In this case the dark state in which the population is accumulated has $m_{F}=0$. This is advantageous for many precision measurements where $m_{F}=$ $0 \rightarrow m_{F}=0$ transitions are used $\sqrt{7}$.

### 2.3 Ioffe-Pritchard trap

An alkali atom in a state with total angular momentum $F$ and corresponding magnetic moment $\boldsymbol{\mu}$ interacting with a spatially-dependent weak external magnetic field has the potential energy

$$
\begin{equation*}
V_{m a g}(\mathbf{r})=m_{F} g_{F} \mu_{B}|\mathbf{B}(\mathbf{r})|:=-\boldsymbol{\mu} \cdot \mathbf{B}(\mathbf{r}) . \tag{2.26}
\end{equation*}
$$

Here $m_{F}$ is the projection of the angular momentum onto the axis of the external field, $g_{F}$ is the Landé factor of the state and $\mathbf{B}(\mathbf{r})$ is the magnetic field at point $\mathbf{r}$.

$$
\begin{equation*}
\mu_{B}:=\frac{e \hbar}{2 m_{e} c} \tag{2.27}
\end{equation*}
$$

is the Bohr magneton with $e$ being the electron's charge, $\hbar$ Planck's constant, $m_{e}$ the electron mass and $c$ the speed of light. From (2.26) we see that an atom whose total angular momentum is aligned in the direction of the field ( $m_{F}>0$ ) behaves as a low field seeker. This means that the atom is attracted to the magnetic field minimum. In consequence magnetic fields with a local minimum are possible candidates to create trapping potentials for cold atoms.

[^5]When the atoms at non-zero temperature move around in the trap, their magnetic moment has to follow the local field adiabatically to keep them trapped. This means that the Larmor precession frequency $|\boldsymbol{\mu} \cdot \mathbf{B}| / h$ of the moment $\boldsymbol{\mu}$ has to be much greater than the frequencies associated with the center of mass movement of the atom in the trap. But if the field strength vanishes at the minimum, then the Larmor frequency vanishes as well and the adiabaticity criterion cannot be met any mor $8^{8}$, Therefore only trapping fields can be used that have a strength different from zero at all regions passed by the atoms.

One possibility to create such a field is to combine a quadrupole and a dipole field using a wire configuration as shown in Fig. 2.11(a). Four infinitely long wires arranged around an axis carry currents as indicated in the figure. The currents create a quadrupole field that leads to transverse confinement. Two dipole coils around the axis with currents in the same direction add a bias field along the minimum line of the quadrupole field and give rise to an axial confinement. This field configuration is called Ioffe-Pritchard trap [44].

Let $y$ and $z$ be the directions of radial confinement due to the quadrupole field with gradient $b^{\prime}$ and $x$ be the axis of the dipole field with curvature $b^{\prime \prime}$. Furthermore let $B_{0}$ be the offset due to the dipole field. The magnitude at some point $\mathbf{r}$ is then

$$
\begin{equation*}
B(\mathbf{r})=\sqrt{\left(B_{0}+\frac{b^{\prime \prime} x^{2}}{2}\right)^{2}+\left(b^{\prime 2}-\frac{b^{\prime \prime} B_{0}}{2}\right)\left(y^{2}+z^{2}\right)} . \tag{2.28}
\end{equation*}
$$

Plugging in (2.26) and Taylor developping to second order yields the harmonic potential

$$
\begin{equation*}
V_{\text {mag }}(\mathbf{r})=V_{0, \text { mag }}+\frac{1}{2} m\left(\omega_{x, \text { mag }}^{2} x^{2}+\omega_{\rho, \text { mag }}^{2} \rho^{2}\right) \tag{2.29}
\end{equation*}
$$

with $m$ being the atomic mass, $\rho^{2}=y^{2}+z^{2}$ and $V_{0, \text { mag }}=m_{F} g_{F} \mu_{B} B_{0}$. The oscillation frequencies are

$$
\begin{align*}
& \omega_{\rho, \text { mag }}=\sqrt{\frac{g_{F} m_{F} \mu_{B}}{m}\left(\frac{b^{\prime 2}}{B_{0}}-\frac{b^{\prime \prime}}{2}\right)}  \tag{2.30}\\
& \omega_{x, \text { mag }}=\sqrt{\frac{g_{F} m_{F} \mu_{B}}{m} b^{\prime \prime}} . \tag{2.31}
\end{align*}
$$

For $\omega_{\rho, \text { mag }}>\omega_{x, \text { mag }}$ potential 2.29 gives a cigar shaped harmonic trap as shown in Fig. 2.11(b).

The first experimental demonstration of magnetic trapping for neutral atoms was performed 1985 in the group of Bill Phillips [45].

[^6]

Figure 2.11: Ioffe-Pritchard trap
(a) Possible wire configuration to create the magnetic field (2.28). The wires in blue create the quadrupole field for transverse confinement while the red dipole coils ensure a non-vanishing minimum field strength and axial confinement.
(b) Potential for an atom with its magnetic moment aligned with the field. In this plot $\omega_{\rho, \text { mag }}>\omega_{x, \text { mag }}$. For such frequencies one obtains a cigar shape.

### 2.4 Evaporative Cooling

The laser cooling methods described above all base on microscopic effects acting on the individual atom while interaction between the atoms does not play a role in the cooling itself but at best only limits the temperatures that can be achieved. The situation is completely different for evaporative cooling. There the most energetic particles of a trapped sample are selectively removed causing a reduction in the average energy of the remaining particles. Elastic collisions between the remainders then re-establish thermal equilibrium at a lower temperature than before the removal. Since in practice the atoms are necessarily confined in a trap during this, in addition to reducing the temperature, the process can as well increase the density because a sample with lower energy is also confined to a smaller volume. The achieved gain in phase-space density $D$ can be high enough to reach into the regime of Bose-Einstein condensation. Evaporative cooling is up to date actually the only way BEC of an atomic vapor has been achieved.

Since the atom number decreases during the evaporation process, it is not guaranteed that one reaches below the critical temperature at which the phase transition occurs. Doing the evaporation wrong one might rather end up having no trapped atoms at all any more. Also, there are limits to the final temperature that can be reached. This stems from the fact that there are not only elastic collisions occuring at some rate $\Gamma_{e l}$ but also inelastic collisions happening at a rate $\Gamma_{i e}$. The latter ones, during which not only kinetic energy is exchanged but where the collision partners
also change their internal states, generally lead to heating ${ }^{9}$. Rethermalization has to happen faster than the characteristic timescale of the inelastic processes in order to cool successfully. This means that one has to stay in a regime where the ratio

$$
\begin{equation*}
R:=\frac{\Gamma_{e l}}{\Gamma_{i e}}>N . \tag{2.32}
\end{equation*}
$$

N is the number of elastic collisions necessary for rethermalization. Typically $N \approx 2.7$ [46]. In practice it turns out that one can effectively only go down to a ratio $R$ of about 100 N until undesired effects take over and cooling ceases [47].

A basic result of low energy scattering theory is that the elastic collisional cross section of a particle depends only on the s-wave scattering length $a$. For bosons it reads $1^{10}$

$$
\begin{equation*}
\sigma=8 \pi a^{2} \tag{2.33}
\end{equation*}
$$

and is constant for the weak magnetic fields and low temperatures of our interest. For ${ }^{87} \mathrm{Rb}$ in its $\mathrm{F}=1$ ground state $a=100.5 a_{0}$ [48] with the Bohr radius $a_{0}=$ 0.052917 nm , giving $\sigma=7.10825 \cdot 10^{-12} \mathrm{~cm}^{2}$. Knowing the scattering cross section, the elastic collision rate for atoms at density $n$ with average relative velocity $v_{r e l}$ can be calculated using the relation

$$
\begin{equation*}
\Gamma_{e l}=n \sigma v_{r e l} . \tag{2.34}
\end{equation*}
$$

In the case of very dense clouds typically the inelastic process that limits the cooling is three-body recombination. That is when two atoms go into a bound state while a third one carries away the excess energy. This process is sometimes also called dimerization.

The standard way to remove the hottest atoms from a trap is to slowly reduce the depth and let the highly energetic particles escape. In the case of a magnetic trap this can be done very easily by applying an outcoupling radio-frequency (RF) field at a changing frequency. For one given RF frequency $\omega_{R F}$ all atoms whose kinetic energy is high enough so that they can reach trap regions where their potential energy (2.26) exceeds the value $E_{T}=\hbar \omega_{R F}$ can undergo RF induced spin-flip transitions to different $m_{F}$ states. If the applied RF power is high enough to do this at an appreciable rate then the potential energy of the atoms is effectively limited to $E_{T}$ because for all higher energies the atoms are driven into untrapped states and leave the trap (see Fig. 2.12). Slowly ramping down the applied RF frequency then effectively ramps down the trap depth. This procedure is called forced evaporation.

Another way to see how the trap depth is reduced by the RF field is to use the dressed state approach. That is to treat the field quantum mechanically and

[^7]

Figure 2.12: Evaporation in a magnetic trap
(a) When two atoms in a trap undergo an elastic collistion one can can aquire an energy high enough to leave the trap. The remaining atom then has a lower energy than before. (b) Principle of forced evaporation in a magnetic trap. An atom in a trapped state moves towards a region of high potential energy. At some point $r_{T}$ it becomes resonant with an applied RF field that induces spin-flips into the untrapped state. From there it spin-flips further to an anti-trapped state in which it is expelled from the sample. This can also happen as a coherent process without populating the untrapped state.
considering the system $\{$ Atom + Field $\}$ including the interaction term between them [49]. When calculating the eigenenergies perturbed by the interaction, one finds that the eigenenergies do not actually cross each other as a function of perturbation parameter. They rather repell if the coupling strength is higher than the energy associated with the spontaneous transition rate. This is called avoided crossing and is illustrated in Fig. 2.13. Atoms initially around the trap center in the lowest energy state can move outwards. Following the eigenstate of the dressed system they are adiabatically transferred into untrapped states if their kinetic energy is high enough.

The trapping potential does not need to be purely magnetical or a hybrid between a magnetical and an optical one. With all-optical traps even Cs which is a notoriously difficult candidate [51] is condensable [52]. If no magnetic field is employed for trapping then this additional degree of freedom can be used to tune the scattering behavior using Feshbach resonances [53]. Done the right way, this can suppress undesired collisions.

### 2.5 Optical Tweezer

Similar to the magnetic case considered in the section on the Ioffe-Pritchard trap, particles with permanent electric dipole moment d can be trapped in an external electric field $\mathbf{E}$.

But atomic orbitals have well defined parity and since the dipole operator has odd


Figure 2.13: Evaporation in the dressed state picture
(a) Bare magnetic sublevels $\left|m_{F}\right\rangle$ for an $F=2$ atom in a trap. (b) Dressed eigenstates $\left|m_{F}, n\right\rangle$ of the system $\{$ Atom + Field $\}$ with $n$ photons in the field excluding the interaction. $\nu$ is the average number of photons. (c) Dressed eigenstates including the interaction between the atom and the field. The interaction causes avoided crossings. Figure adapted from 50 .
parity, its expectation value between two identical orbitals vanishes ${ }^{[1]}$. Accordingly neutral atoms do not posses a permanent electric dipole moment in their ground state. Despite this absence, an atom can have a dipole moment induced by the external electric field. With the atomic polarizability $\alpha$, the induced dipole moment $\mathbf{d}=\alpha \mathbf{E}$. The potential at position $\mathbf{r}$ then reads

$$
\begin{equation*}
V_{o p t}(\mathbf{r})=-\frac{\alpha}{2}|\mathbf{E}(\mathbf{r})|^{2} . \tag{2.35}
\end{equation*}
$$

Since the intensity of a laser beam is proportional to the absolute-square of the electric field component, the potential energy of an atom with polarizability $\alpha$ is proportional to the local field intensity $I(\mathbf{r})$. Depending on the sign of $\alpha$, which in turn depends on the atomic species and the laser wavelength in use, the atom can either act as a high-field seeker or as a low-field seeker. If the light is red detuned from all atomic resonances then $\alpha$ is positive and the potential energy is lowest where the field intensity is highest. Atoms can then be trapped at the intensity maximum of a laser beam. Such traps are called optical dipole traps. Since it is possible to move the trapped particles in space by moving the location of the potential minimum, these systems are also called optical tweezers.

In the case of alkalis and far red detuned (much more than the fine structure splitting) laser light at frequency $\omega_{L}$, only the $D_{1^{-}}$and $D_{2}$-line at transition frequencies $\omega_{1}$ and $\omega_{2}$, respectively, give a significant contribution to the atomic polarizability. The optical potential can then be written as [54]

$$
\begin{equation*}
V_{\text {opt }}(\mathbf{r})=-\frac{d^{2}}{3 \hbar \epsilon_{0} c}\left[\frac{\omega_{1}}{\omega_{1}^{2}-\omega_{L}^{2}}+\frac{2 \omega_{2}}{\omega_{2}^{2}-\omega_{L}^{2}}\right] I(\mathbf{r}), \tag{2.36}
\end{equation*}
$$

[^8]

Figure 2.14: Spot size of a Gaussian beam
Within and in vicinity of the Rayleigh range, the spot size varies according to (2.37). Far from focus the beam expands linearly with divergence angle $\Theta=$ $\lambda / \pi w_{0}$. From [55].
where $\epsilon_{0}$ is the vacuum permittivity, $c$ is the speed of light and $d$ is the $S \rightarrow P$ transition dipole matrix element.

A Gaussian beam traveling in the $z$-direction has a transverse intensity distribution that depends on the maximum intensity $I_{0}$ and the spot size $w(z)$. In case of a collimated beam the spot size equals the beam waist $w_{0}$ (the smallest spot size along the beam) all along the $z$-axis. If the beam is focussed at some point $z=0$ then the spot size at a distance $z$ from the focal point is given by the Rayleigh formula 55]

$$
\begin{equation*}
w(z)=w_{0}(z) \sqrt{1+z^{2} / z_{0}^{2}} \tag{2.37}
\end{equation*}
$$

Here $z_{0}=\frac{\pi w_{0}^{2}}{\lambda}$ is the Rayleigh range of a beam with wavelength $\lambda$ and beam waist $w_{0}$. The Rayleigh range is the distance at which the spot size is larger than the beam waist by a factor of $\sqrt{2}$ :

$$
\begin{equation*}
w\left(z_{0}\right)=w_{0} \sqrt{2} \tag{2.38}
\end{equation*}
$$

The situation is illustrated in Fig. 2.14
Expressed in the quantities above, the transverse intensity distribution reads

$$
\begin{equation*}
I(\rho, z)=I_{0}\left(\frac{w_{0}}{w(z)}\right)^{2} e^{-2 \frac{\rho^{2}}{w(z)^{2}}} \tag{2.39}
\end{equation*}
$$

where $\rho=\sqrt{x^{2}+y^{2}}$ is the transverse coordinate.
For an atom at the focus of a Gaussian beam, potential (2.36) can be developed around the extremum to

$$
\begin{equation*}
V_{o p t}(\rho, z)=V_{0, o p t}+\frac{1}{2} m\left(\omega_{\rho, o p t}^{2} \rho^{2}+\omega_{z, o p t}^{2} z^{2}\right) \tag{2.40}
\end{equation*}
$$

with $m$ being the mass of the atom and

$$
\begin{equation*}
V_{0, o p t}=-\frac{d^{2}}{3 \hbar \epsilon_{0} c}\left[\frac{\omega_{1}}{\omega_{1}^{2}-\omega_{L}^{2}}+\frac{2 \omega_{2}}{\omega_{2}^{2}-\omega_{L}^{2}}\right] I_{0} . \tag{2.41}
\end{equation*}
$$

The oscillation frequencies are

$$
\begin{align*}
& \omega_{\rho, \text { opt }}=\sqrt{\frac{4 V_{0, o p t}}{m w_{0}^{2}}}  \tag{2.42}\\
& \omega_{z, \text { opt }}=\sqrt{\frac{2 V_{0, o p t}}{m z_{0}^{2}}} . \tag{2.43}
\end{align*}
$$

Optical trapping of particles by radiation pressure was first demonstrated by Art Ashkin at Bell Labs in 1970 [56. The first optical trapping of a sample of neutral atoms was also performed at Bell Labs in the group of Steve Chu in 1986 [57]. Optical tweezers are nowadays also widely used for three-dimensional manipulation of individual particles. An application is for example in-situ control over motion in biological substances.

CHAPTER 3

## Experimental Setup

Our species of choice is rubidium-87. An energy diagram of the levels of interest can be found Appendix C. But we do not only want to create a Bose-Einstein condensate. We rather want to use it as a coherent source of matter-waves, whose de Broglie wavelength stays constant during propagation. To have a coherent source it would be sufficient to only RF-outcouple atoms from a Bose-Einstein condensate which is confined in a magnetic trap. However, the problem in such a setup is that the de Broglie wavelength rapidly decreases as the atoms accelerate under the influence of gravity.

We overcome the problem of gravitational acceleration by outcoupling the atoms into a guide for matter-waves in horizontal geometry. Our concept is to use an optical tweezer as waveguide. To ensure optimum coupling of the matter-waves into the guide, we create the source BEC in a hybrid trap which consists of a magnetic Ioffe-Pritchard trap that is overlapped with the optical tweezer. Atoms that are RFoutcoupled from the BEC and leave the magnetic trap are automatically coupled into the waveguide. The experiment can be run quasi-continuous, which means that atoms are outcoupled continuously until the BEC is depleted and a new condensate needs to be created.

This chapter describes the creation of the BEC in the hybrid trap while the actual guided atom laser will be presented in the following chapter.

### 3.1 Imaging

Before the actual cooling steps of the experiment are described, our means to diagnose the situation of the atomic sample shall be discussed.

After releasing the atoms at the end of an experimental sequence we wait for some time of flight before imaging. Typically this time ranges from a few milliseconds to several tens of milliseconds. The spatial distribution after this time of flight reflects the momentum distribution at the moment of release. This allows us to calculate the temperature at the time of release. We have two different means of imaging at our disposal. Since both methods involve the scattering of photons, our imaging process is always destructive for the cold atomic system.

One method is to illuminate the atomic cloud with light resonant on a cycling transition. The atoms then undergo many cycles of absorption and spontaneous emission. The emitted light is imaged onto a CCD camera which is read out by a computer. With this fluorescence imaging technique we can detect at very low atomic
densities. The actual atom number can be calculated from the known emission rate per atom, the solid angle that is imaged onto the camera, and the camera gain.

The spontaneous emissions are rapidly heating up the atoms, which leads to a spatial diffusion during the imaging process. This reduces the resolution. When exposing the cloud to the pump light for $50 \mu \mathrm{~s}$, we obtain a resolution $15 \mu \mathrm{~m}$. At this exposure time, the fluorescence detection sensitivity is around 0.1 atoms per $\mu \mathrm{m}^{2}$. Fluorescence imaging is not suitable if the cloud is very dense because photons emitted from one atom are then reabsorbed by others and do not reach the camera.

If high spatial resolution is desired or a very dense cloud is imaged then we employ a technique called absorption imaging. There, weak light resonant to an atomic transition is shone directly onto the camera via an imaging system. The presence of atoms in the beam path is then detected because the cloud absorbs photons from the beam. This leaves a shadow on the camera. One image is taken with the cloud and another one is taken shortly after for reference. Thus the intensity distribution of the beam is known with and without atoms present. Knowing the on-resonance scattering cross section we can then calculate the column density along the beam axis. The sensitivity of the absorption imaging system is approximately a few atoms per $\mu \mathrm{m}^{2}$. When using a telescope for imaging, our camera's resolution is $2 \mu \mathrm{~m}$ per pixel. The actual detection resolution, however, is limited to $7 \mu \mathrm{~m}$ by diffraction and abberations of the imaging system.

Our camera detects along the $x$-axis of the experiment(axis of the dipole field in the magnetic trap, see section 3.4). The obtained column density can then be displayed as a pseudocolor image or can be integrated along the $y$-axis (gravity axis) or along the $z$-axis (optical tweezer, see section 3.6). In the latter two cases the resulting linear density can be fitted by an appropriate function to obtain the total atom number, the 1-D velocity distribution and quantities inferred from these. As fit functions we use a Gaussian curve for thermal clouds, a Thomas-Fermi profile for pure BECs and the sum of these two for mixtures of condensed and uncondensed atoms.

The imaging beams are resonant on the $F=2 \rightarrow F^{\prime}=3 D_{2}$-transition. Since the atoms are in the $F=1$ ground state at the time of their release we optically pump them into the $F=2$ ground state before detection.

### 3.2 Vacuum System

Bose-Einstein condensed atoms are very sensitive to collisions with the background gas. Any atom colliding with room temperature particles will immediately be kicked out of the condensate. Therefore BEC experiments need to be performed in ultrahigh vacuum environments. Typical pressures required are in the $10^{-10} \mathrm{mbar}$ (or better) range to allow for condensate lifetimes in the order of some tens of seconds and longer.

On the other hand, it is necessary to create an atomic cloud with a high number of atoms. But to gather enough atoms one either needs rather high pressures in the
$10^{-9}$ mbar and above range or one has to separate the vacuum system into two parts. A higher pressure part is then used to collect atoms and create an atomic beam or successive bunches of atoms that pass to a low pressure part where the atoms are captured and manipulated. We are using such a differential pressure vacuum setup.

A schematic diagram of the system in use is shown in Fig. 3.1. The atomic source is on the left hand side and the ultra-high vacuum part on the right hand side. A chamber connected to an oven that contains rubidium in natural abundance is pumped down to about $10^{-9}$ mbar using a turbo molecular pump followed by a roughing pump. This part is separated from the ultra-high vacuum region by an about 1 m long tube used for the Zeeman slower. The utra-high vacuum part is pumped by a $100 \mathrm{~L} / \mathrm{s}$ ion pump and a titanium sublimation pump. Both pumps are attached to the vacuum system using a four-way cross connected to a six-way cross with viewports at all free flanges. There is an additional four-way cross with a getter pump close to the higher vacuum end of the Zeeman slower.

The location of creation and manipulation of the BEC is inside a quartz cell connected between the tube for the Zeeman slower and the 6 -way cross. This cell is described in more detail in combination with the magneto-optical trap in Sec. 3.3.2.

### 3.3 Laser Cooling Stages

To reach BEC, one has to create an atomic sample at a very high-phase space density, exceeding the value of $2.612 \ldots$ as discussed in section 2.1. This means that the atoms have to be both, very cold and very dense. In order to reach a high value starting from a phase-space density around $10^{-15}$ in our source part, we have to go through several subsequent stages of cooling. The consecutive steps of our sequence are:

- Zeeman slowing, MOT loading
- Dark MOT
- Optical molasses
- Loading of magnetic trap
- First evaporative cooling
- Loading of hybrid trap
- Second evaporation to reach BEC

After the last cooling step we operate the atom laser and image the atomic cloud. All cooling lasers used in the setup are either grating-stabilized or injection-locked. A diagram of the laser frequencies in use is shown in Appendix D. All optical transitions in our experiment are on the $D_{2}$-line. The light generation is described in full detail in previous theses [54, 58].


Figure 3.1: The vacuum system.

(a) Solenoids to create magnetic field. (b) Magnetic field strength along $z$-direction. (c) Atomic transition frequency as function of position (green curve). Different velocity classes are Doppler shiftet into resonance with the counterpropagating light at different locations along the $z$-axis. Fast atoms coming from the left are kept in resonance while being slowed down. $\omega_{L}$ is the laser frequency (dotted) and $\omega_{0}$ is the resonance frequency without magnetic field (dashed). $\omega_{L}$ is red detuned by 133 MHz .

### 3.3.1 Zeeman Slower

The atomic beam coming from the oven has a divergence of about 0.05 rad, determined by the 5 mm aperture and the 10 cm long tube connecting oven and Zeeman slower. The longitudinal velocities of the atoms are around $360 \mathrm{~m} / \mathrm{s}$. The Zeeman slower (see Chapter 2.2.2) decelerates these atoms over a distance of approximately 1 m to have their final velocity at the MOT region within the capture range of the trap (around $20 \mathrm{~m} / \mathrm{s}$ ). The spatially inhomogenous magnetic field along the Zeeman slower is created by two sets of solenoids. The solenoids have different numbers of windings per unit length at different positions (see Fig. 3.2). The created magnetic field starts at 150 G at the beginning of the slower and ends at -70 G close to the MOT. The negative field strength at the end allows us to use slowing light that is far enough red detuned from the cooling transition that the following MOT is not disturbed.

The frequency of the cooling light that we apply is locked to the $F=2 \rightarrow F^{\prime}=$ $2 \times 3$ cross-over transition. It is thus 133 MHz red detuned from the $F=2 \rightarrow F^{\prime}=3$ cooling transition. For an energy level diagram see appendices C and D. The beam has a power of 25 mW and a beam waist of approximately 5 mm . The polarization is circular so that the cooling transition is a closed on ${ }^{1}$. Repumping light is applied

[^9]resonant on the $F=1 \rightarrow F^{\prime}=2$-transition.
We operate the Zeeman slower without transverse cooling. But the spontaneous emission kicks following each absorption causes heating of the atoms in the radial direction. This adds to the divergence that the beam already had when leaving the oven. The result is that not all atoms that passed the Zeeman slower can eventually be trapped in the MOT's finite trapping volume. The effective flux of atoms that are actually trapped is approximately $10^{8}$ atoms per second.

### 3.3.2 Magneto-Optical Trap

The Zeeman slower ends in a quartz cell. Starting with the accumulation of atoms from the slowed beam in a MOT, all the rest of the experiment takes place inside this cell. Our MOT beams are three orthogonal pairs for operation in all three dimensions (see Fig. 3.3). All beams have an intensity of $2 \mathrm{~mW} / \mathrm{cm}^{2}$ and a $1 / e^{2}$ waist of 4 mm . The pairs in each beam have orthogonal circular polarization. The magnetic field is generated by two coils in anti-Helmholtz configuration. The coil wires are microfabricated and attached directly on the cell to minimize their distance and thereby to allow for strong field gradients with moderately high currents applied. The coils have each 6.5 windings and a resistance of $1 \Omega$. Conducting 1 A , the antiHelmholtz coils create a quadrupole field with a gradient of $7.5 \mathrm{G} / \mathrm{cm}$ in the radial (horizontal) direction and $15 \mathrm{G} / \mathrm{cm}$ on the vertical axis. We use extra external coil pairs in Helmholtz configuration to compensate for background magnetic fields. Additional placement coils align the zero-point of the quadrupole field.

Our cooling transition is from $F=2$ to $F^{\prime}=3$. We use a red detuning of $2 \cdot \Gamma=2 \pi \cdot 12 \mathrm{MHz}$. Atoms lost into the $F=1$ ground state by spontaneous emission following off-resonant excitations of the $F^{\prime}=2$ excited state are pumped back by light resonant with the $F=1 \rightarrow F^{\prime}=2$ transition. The repumping light is overlapped with the six cooling beams. In the horizontal plane, the repumping light has an intensity of $100 \mu \mathrm{~W} / \mathrm{cm}^{2}$ in each beam. The vertical repumpers have the intensity $1.5 \mathrm{~mW} / \mathrm{cm}^{2}$.

After loading the MOT for 12 s the Zeeman slower is switched off and the central part of the MOT repumping light is mechanically blocked to create a dark MOT in the center of the cloud. After 50 ms of dark MOT we then have about $10^{9}$ atoms at a temperature of approximately $140 \mu \mathrm{~K}$, in agreement with the Doppler limit $(2.19)$. The density of the cloud is then about $10^{11}$ atoms $/ \mathrm{cm}^{3}$ at phase-space density $D \approx 4 \cdot 10^{-7}$.

### 3.3.3 Optical Molasses

For the optical molasses we use the same beams as for the MOT. The configuration of counterpropagating beams with opposite circular polarization causes polarization gradient cooling to occur.

While switching off the quadrupole magnetic field, the laser power is reduced and the frequency is detuned further to $\delta=-6 \cdot \Gamma=-2 \pi \cdot 36 \mathrm{MHz}$. This reduces multiple


Figure 3.3: MOT beams and coils
The coils for the quadrupole field used in our magneto-optical trap are microfabricated and fixed on the quartz cell. Compensation coils are used to cancel external magnetic fields. The actual position of the magnetic field's zero point can be moved away from the center of the quadrupole coils by adding a bias field with the placement coils.
scattering and allows for high friction forces as explained in section 2.2.4.2,
When the trapping quadrupole field is off, it is crucial that the currents through the compensation coils are adjusted such that the magnetic field at the location of the sample is zero. As discussed in section 2.2.4.2, the presence of a non-vanishing magnetic field in one direction causes polarization gradient cooling to damp the atomic velocity in this direction not to zero but rather to a value proportional to the field strength in the direction. The result is not only a cloud which is displaced after the short but finite time of sub-Doppler cooling, but is also at a higher final temperature than in the case of vanishing field. With well adjusted currents in the compensation coils to cancel all background fields we reach a temperature of about $60 \mu \mathrm{~K}$ after 6 ms of polarization gradient cooling. The number of atoms and the density remain roughly constant during the operation of the optical molasses.

### 3.4 Magnetic Trap

After the optical molasses cooling stage is finished, the atoms are loaded into a IoffePritchard magnetic trap (see Chapter 2.3). For this, the atoms are optically pumped into the $F=1$ ground state by keeping the $F=2 \rightarrow F^{\prime}$ cooling beams on for a short time after the repumping beam is already off. When the magnetic trap is then switched on, $1 / 3$ of the atoms is in the trapped $m_{F}=-1$ sublevel ${ }^{2}$.

Our trapping field is generated by the partially ferromagnetic electromagnet described in [59. Ferromagnetic pieces are excited by coils and create a quadrupole field (see Fig. 3.4). The dipole field is created by a combination of dipole and antidipole coil pairs ${ }^{3}$. Each of these coils consists of 100 windings of copper wire on a water-cooled mount. The dipole coils have a distance of 3 cm from each other and a diameter of 1.5 cm . The anti-dipole coils have a diameter of 7 cm and are at a distance of 10 cm from each other. The two dipole coils have their current running in one direction and the two anti-dipole coils have the current running in the opposite direction. Set up like this the anti-dipole pair partially cancels the bias field of the of the dipole pair but leaves the curvature unaffected. At times when both pairs are on, they are switched in series so that noise in the driving current is also partially cancelled.

A disadvantage of using ferromagnetic materials is that their magnetization show a hysteretis behavior. When the quadrupole excitation coils are switched off, additional coils are turned on to compensate for the hysteresis behavior.

The trap center is aligned with the location of the atomic cloud after the molasses step. At first, the current $I_{Q}$ exciting the quadrupole part is at a rather moderate value of 5 A and the dipole current $I_{D}=15 \mathrm{~A}$. This leads to a large trap into which the atoms are loaded. At this stage the radial confinement is only slightly higher

[^10]

Figure 3.4: Partially ferromagnetic electromagnet
(left) The quadrupole field is created by the ferromagnetic structure (grey) which is excited by copper coils. (right) The dipolar part of the magnetic field is created using pairs of coils in Helmholtz configuration. The inner coils are driven with currents in the opposite direction than the outer (anti-dipole) coils.

Table 3.1: Magnetic trap parameters

| State | $B_{0}[\mathrm{G}]$ | $b^{\prime \prime}\left[\mathrm{G} / \mathrm{cm}^{2}\right]$ | $b^{\prime}[\mathrm{G} / \mathrm{cm}]$ | $\omega_{y, z, \text { mag }}[\mathrm{Hz}]$ | $\omega_{x, \text { mag }}[\mathrm{Hz}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| uncompressed | 54 | 150 | 200 | $2 \pi \cdot 15$ | $2 \pi \cdot 11$ |
| compressed | 7.1 | 82 | 830 | $2 \pi \cdot 280$ | $2 \pi \cdot 8.2$ |

than the axial one. After the transfer, the trap is first compressed transversally by increasing $I_{Q}$ to 60 A . Afterwards the anti-dipole current $I_{A D}$ is ramped up to 15 A , reducing the bias field. The sequence is illustrated in Fig. 3.5.

Changes in the fields and field gradients are made slowly to allow the atoms to adiabatically follow. The criterion of adiabaticity is in this case that the timescales of the changes have to be long compared to the inverse of the trapping frequencies. Our parameters for equation (2.28), describing the magnetic field in the compressed and uncompressed state are summarized in Table 3.1. Also included in the table are the oscillation frequencies $\omega_{x, \text { mag }}$ and $\omega_{y, z, \text { mag }}$ from equations 2.30) and 2.31). The Landé factor of our $\left|F=1, m_{F}=-1\right\rangle$ trapped state is $g_{F}=-1 / 2$. In the compressed state, our trap has a very elongated cigar shape $\left(\omega_{x, \text { mag }}=8.2 \mathrm{~Hz}\right.$ and $\omega_{y, z, \text { mag }}=280$ Hz ).

After loading and compressing the magnetic trap, we have about $1.5 \cdot 10^{8}$ atoms remaining at a temperature around $250 \mu \mathrm{~K}$. The density in the center of the cloud is then about $3 \cdot 10^{11}$ atoms $/ \mathrm{cm}^{3}$. Using (2.34) and (2.2), this corresponds to an elastic collision rate $\Gamma_{e l}=74 / s$ and a phase-space density $D \approx 5 \cdot 10^{-7}$. The remaining seven orders of magnitude by which $D$ has to be increased until BEC sets on are gained by evaporative cooling.


Figure 3.5:

## Electromagnet currents

Time sequence to load magnetic trap. $I_{Q}$ - quadrupole current, $I_{D}$ - dipole current, $I_{A D}$ - antidipole current.

### 3.5 First Evaporation

After the magnetic trap is loaded and compressed, we start our evaporation sequence. It consists of two stages. In the first stage atoms are evaporated in the purely magnetic trap and in the second stage the optical tweezer is turned on and evaporation is finished in the hybrid trap until Bose-Einstein condensation sets on.

As discussed in section 2.4, the depth of the magnetic trapping potential can be reduced by applying an RF field that drives highly energetic particles into untrapped states. We use the MOT coils as antenna to create a field perpendicular to the dipole axis of the Ioffe-Pritchard trap. The power level of the RF field is constant at +15 dBm and the frequency is ramped down from 80 MHz (corresponding to an initial potential depth of $\left.k_{B} \cdot 3.6 \mathrm{mK}\right)$ to a final value of $5.7 \mathrm{MHz}\left(k_{B} \cdot 35 \mu \mathrm{~K}\right)$ during $27 \mathrm{~s} .{ }^{4}$ After this evaporation, about $2 \cdot 10^{6}$ atoms remain in the trap at a temperature of approximately $1 \mu \mathrm{~K}$. The phase-space density is then close to unity. When ramping down further, condensation in this purely magnetic trap would set on around an RF frequency of 4.94 MHz .

### 3.6 Hybrid Trap

After a first evaporation the cloud is kept in the magnetic trap but additionally the optical tweezer is switched on. The optical trapping potential is created by a laser beam propagating horizontally along the $z$-axis (i.e. perpendicular to the long axis of the Ioffe-Pritchard trap) and focussed at the center of the atomic cloud ${ }^{5}$. A sketch of the trapping components is drawn in Fig. 3.6.

[^11]

Figure 3.6: Schematic of the hybrid trap
The field for the magnetic trap is generated by the ferromagnetic electromagnet and dipole coils. The far red detuned laser of the optical tweezer is focussed slightly below the center of the magnetic trap.

As light for the optical tweezer we use the radiation of a Nd:YAG laser at wavelength $\lambda=1024 \mathrm{~nm}$. Our model is an InnoLight Mephisto 2000NE which delivers up to 2 W of output power. The focussing optics are mounted on a remote controllable translation stage. The focal point can thus be moved along the $z$-direction via computer control. The system includes an optical fiber and also diaphragm to filter undesired spatial modes. After these and all other components we have up to 1 W of power available to apply on the atoms. Pointing in the $(x, y)$-plane is stabilized to within $1 \mu \mathrm{~m}$ by a low-frequency (max. 50 Hz ) electronic feedback system.

The beam's transverse intensity profile imaged with a CCD camera is shown in Fig. 3.7(a) for different positions along the propagation axis. Each image shows a region of $250 \times 250 \mu \mathrm{~m}^{2}$. In Fig. 3.7(b) the intensity distribution is plotted along the $x$ - and $y$-axis after integration over the respective other axis. The two curves are Gaussian fits to the measured data points. We use the beam waist as fit parameter and obtain $w_{x}=36.7 \mu \mathrm{~m}$ and $w_{y}=35.7 \mu \mathrm{~m}$. These correspond to a Rayleigh range $z_{0} \approx 4 \mathrm{~mm}$. In Fig. 3.7(c), the maximum intensity as a function of the $z$-position around the focal point is plotted. The asymmetry is due to spherical abberations introduced by the optical system. The typical distances of interest for our experiments are on the order of a few hundred micrometers around focus.

We typically apply 64 mW of optical power to the atoms. This corresponds to a


Figure 3.7: Intensity profile of the optical tweezer
(a) for different $z$-positions around focus and (b) at focus after integration over $y$ (left) and over $x$ (right). (c) shows the maximum intensity as as function of longituninal position.


Figure 3.8:

## Full evaporation sequence

The optical tweezer is loaded during the 1.5 s between the two evaporations. Times are not to scale.
potential depth $V_{0, o p t}=k_{B} \cdot 12 \mu \mathrm{~K}$. The trapping frequencies are then

$$
\begin{align*}
\omega_{x, y, \text { opt }} & =2 \pi \cdot 300 \mathrm{~Hz}  \tag{3.1}\\
\omega_{z, \text { opt }} & =2 \pi \cdot 1.9 \mathrm{~Hz} . \tag{3.2}
\end{align*}
$$

We refer to the trapping of atoms in the combined magnetic and the optical potential as hybrid trap. The effective potential that the atoms experience in it is the sum of both the magnetic and the optical potential. It can be described by effective oscillation frequencies $\omega_{i},(i=x, y, z)$. These frequencies are

$$
\begin{equation*}
\omega_{i}=\sqrt{\omega_{i, \text { mag }}^{2}+\omega_{i, o p t}^{2}} . \tag{3.3}
\end{equation*}
$$

After the first evaporation, the atoms are cold enough to be trapped in the optical tweezer. However, in order to have an effective loading of the tweezer without heating up the atoms the size of the cloud before loading has to match its size in the hybrid trap. Also care has to be taken that the cloud is not compressed too much because then 3-body recombination processes lead to a rapid loss of atoms. Our loading sequence is as follows.

The transverse confinement of the magnetic potential is ramped down to an oscillation frequency $\omega_{y, z, \text { mag }}=130 \mathrm{~Hz}$ over a duration of 500 ms (see Fig. 3.8). The magnetic confinement is then held constant while the optical potential is ramped up to a radial oscillation frequency $\omega_{x, y, o p t}=300 \mathrm{~Hz}$ during another 500 ms . When the tweezer is loaded, the magnetic trap's transverse frequency is lowered further down to $\omega_{y, z, \text { mag }}=35 \mathrm{~Hz}$, again in 500 ms . The effective frequencies in the hybrid trap are then

$$
\begin{align*}
& \omega_{x} \approx \omega_{x, y, o p t}  \tag{3.4}\\
&=2 \pi \cdot 300 \mathrm{~Hz}  \tag{3.5}\\
& \omega_{y} \approx \omega_{x, y, o p t}=2 \pi \cdot 300 \mathrm{~Hz}  \tag{3.6}\\
& \omega_{z} \approx \omega_{y, z, \text { mag }}=2 \pi \cdot 35 \mathrm{~Hz}
\end{align*}
$$

The loading efficienty depends on the strength of confinement in the optical tweezer. For the oscillation frequencies which are used in our experiment, about $25 \%$ of the atoms are transferred into the tweezer. We then have $5 \cdot 10^{6}$ atoms in the hybrid trap. For higher optical powers much greater loading efficiencies can be achieved (see Fig. 3.9) but then the confinement is so strong that trap losses due to 3-body recombination make a successful finishing of the evaporation sequence impossible.


Figure 3.9:

## Loading of hybrid trap

Image of the cloud after holding the atoms in the optical tweezer at higher power ( 145 mW ) for 10 ms . Almost all atoms where kept in the tweezer (top) and only a small number of atoms fell during the 10 ms of holding time (cloud below). Image area is $1 \times 1 \mathrm{~mm}^{2}$.

### 3.7 Second Evaporation

After loading the hybrid trap we finally evaporate for another 3 s until quantum degeneracy is reached. Atoms that are RF-outcoupled from the magnetic trap are still confined in the optical tweezer. But since these atoms are in a magnetically high-field seeking state, they are expelled from the trap center and leave the region of interest.

The optimal final frequency for the evaporation varies from day to day and even within a day due to fluctuations in our magnetic environment. Also the properties of our electromagnet depend slightly on the duty cycle as a result of heating during operation. These changes affect the offset field $B_{0}$.

When the evaporation is finished, some atoms that where outcoupled at the very end still remain in the tweezer close to the cold sample. We wait for $150 \mu$ s to allow them to leave the sample region before either switching off the traps for imaging or operating the guided atom laser.

Fig. 3.10 shows atomic two clouds after 16 ms of free expansion. For the top image the final frequency of the evaporation ramp was above condensation threshold. For the image on the bottom, $\omega_{R F}$ was swept until below threshold. Also shown are the velocity distributions along the $z$-axis. Above the critical temperature, the velocity distributions are thermal and follow a Gaussian curve. When the cloud is well below the critical temperature we have an almost pure BEC. Its velocity distribution is the inverse parabola that reflects the Thomas-Fermi density profile. In between, the atomic distribution has a bimodal structure containing condensed atoms in the center and a thermal cloud surrounding them.

Our threshold of Bose-Einstein condensation is typically at a final RF frequency around 4.94 MHz . Ramping down until a few kHz below threshold we obtain almost pure BECs of approximately $1.5 \cdot 10^{5}$ atoms. Using (2.13) this corresponds to a chemical potential $\mu=h \cdot 3.2 \mathrm{kHz}$. The density of our BECs in the hybrid trap is $n=3 \cdot 10^{13} / \mathrm{cm}^{3}$.

While thermal clouds expand isotropically, the density distribution of a BEC after


$$
\begin{aligned}
\omega_{\mathrm{RF}} & =6.1 \mathrm{MHz} \\
\mathrm{~N} & =2 \times 10^{6} \\
\mathrm{~T} & =1 \mu \mathrm{~K}
\end{aligned}
$$



$$
\begin{aligned}
\omega_{\mathrm{RF}} & =4.932 \mathrm{MHz} \\
\mathrm{~N} & =1.45 \times 10^{5} \\
\mu & =\mathrm{h} \times 3 \mathrm{kHz}
\end{aligned}
$$

Figure 3.10: Clouds at different final RF frequencies
(top) Thermal cloud with Gaussian velocity distribution, (bottom) BEC with Thomas-Fermi profile. Note the different length scales.


Figure 3.11: Expansion of a BEC released from the hybrid trap Images taken after different times of flight. One can clearly see the anisotropy of the expansion due to the interaction. The cloud which is originally elongated in $z$-direction (horizontal) inverses its aspect ratio during the flight.
some time of flight is influenced by the initial distribution at the time of release 60. This is a consequence of the interaction term in the Gross-Pitaevskii equation (2.9) when being in the Thomas-Fermi regime. As the trapping potential is switched off, the atomic mean-field energy is converted into kinetic energy. Fig. 3.11 shows the density distribution at different times after release. One can clearly see the anisotropy in velocity due to the anisotropy of the trapping potential. The cloud is elongated horizontally in a cigar shape when released from the trap. During expansion it assumes the form of a pancake.

CHAPTER 4

## Guided Atom Laser

After obtention of a BEC in the hybrid trap we can then operate the guided atom laser. This is done by continuously outcoupling atoms from the BEC with an RF field. But this time, the RF field is so weak that the atoms are only driven into the untrapped $m_{F}=0$-state and not any further into $m_{F}=+1$. Since the optical potential is independent of the orientation of the atomic total angular momentum, the outcoupled atoms are still confined in the optical tweezer. The tweezer then acts as a guide for the outcoupled matter-waves. Propagation in the tweezer is quasi-onedimensional. Because of our horizontal tweezer geometry the atoms are prevented from falling down under the influence of gravity. Thus their velocity (and with it their de Broglie wavelength) are only determined by the involved optical and magnetical potentials in combination with the mean-field interaction. It will be seen below that this actually allows to choose the de Broglie wavelength initially and then keep it at a constant value.

### 4.1 Outcoupling Process

Like for the evaporation we use the MOT coils as antenna to create the RF field. But this time the applied RF power is much lower, typically around -40 dBm . The outcoupling Rabi frequency $\Omega_{R F}$ is given by

$$
\begin{equation*}
\hbar \Omega_{R F}=\frac{\left|g_{F}\right| \mu_{B} B_{R F}}{\sqrt{2}} \approx 20 \mathrm{~Hz} \tag{4.1}
\end{equation*}
$$

where $B_{R F}$ is the magnitude of the magnetic component of the RF field. $\Omega_{R F}$ is sufficiently low to drive the atoms only from the trapped $m_{F}=-1$ into the untrapped $m_{F}=0$ state but not further into $m_{F}=+1$. During such a process the coherence is preserved, which justifies the expression that the outcoupled particles form a guided atom laser.

RF-outcoupling from the condensate can only happen in a very narrow frequency range which is determined by the chemical potential $\mu$ of the condensate. We take the energy of the $m_{F}=0$-state in the center of the trap excluding interactions as zero and we call $E_{B E C}$ the energy of the BEC in the trap. Outcoupling is only possible if the RF frequency $\omega_{R F}$ is in the range between $E_{B E C} / \hbar$ and $\left(E_{B E C}-\mu\right) / \hbar$ (see Fig. 4.1).

The potential for the outcoupled atoms is essentially flat along the $z$-direction ${ }^{1}$,

[^12]

Figure 4.1: Resonance condition for guided atom laser
Atoms in the BEC experience the sum of the magnetic potential, the optical potential and the mean-field energy (red). Atoms that were outcoupled into the guide only see the mean-field potential $V_{M F}$ (blue) and the transverse optical confinement. The extraction point $z_{e}$ is determined by the RF frequency $\omega_{R F}$.

Only at the location of the remaining condensate, the outcoupled atoms experience the mean-field potential $V_{M F}$. Since the mean-field energy is proportional to the local density, the potential for the outcoupled atoms has the shape of the Thomas-Fermi density profile (see Fig. 4.1).

Classically one can assign an extraction position $z_{e}$ at which the mean-field energy of the condensate is such that

$$
\begin{equation*}
E_{B E C}-\hbar \omega_{R F}=V_{M F}\left(z_{e}\right) \tag{4.2}
\end{equation*}
$$

For a given RF frequency this position is

$$
\begin{equation*}
z_{e}= \pm \sqrt{\frac{2\left(E_{B E C}-\hbar \omega_{R F}\right)}{m \omega_{z, \text { mag }}^{2}}} . \tag{4.3}
\end{equation*}
$$

The outcoupled flux is proportional to the atomic density at the extraction point and assumes its maximum when $z_{e}=0$, i.e., when the atoms are outcoupled at the center of the condensate.

This rises the intuitive expectation that the resonance lineshape as function of $\omega_{R F}$ shows some asymmetry. When $\omega_{R F}$ is too low to spin-flip the atoms into the untrapped state, no atoms are coupled out. If $\omega_{R F}$ is increased, the outcoupling rate should rise quickly and assume a maximum around $z_{e}=0$. Increasing $\omega_{R F}$ further then moves $z_{e}$ towards the edges of the condensate and the flux of outcoupled atoms decreases. Approximately $\mu / \hbar$ above the frequency of maximum flux, the

[^13]

Figure 4.2: Guided atom laser
(a) Schematic of the outcoupling into the guide. (b) Absorption image of the source and the guided atom laser. The size of the displayed region is approximately 1.6 $\mathrm{mm} \times 500 \mu \mathrm{~m}$.
outcoupling should cease. The resonance lineshape will be studied further in section 4.2 in complement with experimental data.

The system is inversion symmetric with respect to the origin. In consequence, atoms are always outcoupled in both halfs of the $z$-axis, with their direction given by the sign of their extraction point. See Fig. 4.2(a) for an illustration. In Fig. 4.2(b) an absorption image of one wing of the guided atom laser is shown.

After an atom is outcoupled, its potential energy in the mean-field is converted into kinetic energy. The resonance condition to create an atom laser with kinetic energy $E$ is

$$
\begin{equation*}
E_{B E C}-\hbar \omega_{R F}=E \tag{4.4}
\end{equation*}
$$

Its de Broglie wavelength is then

$$
\begin{equation*}
\lambda_{d B}=\frac{h}{\sqrt{2 m E}} . \tag{4.5}
\end{equation*}
$$

### 4.2 Resonance Lineshape

The kinetic energy and thus the de Broglie wavelength of the guided atom laser is determined by the applied RF frequency via (4.4). However, not only the de Broglie wavelength of the guided atom laser depends on $\omega_{R F}$ but also the rate at which atoms are actually outcoupled. In the first part of this subsection the theoretical prediction of the outcoupling lineshape will be discussed and in the second part experimental measurements of it will be presented.

### 4.2.1 Theoretical Prediction

A full calculation of the theoretical resonance lineshape can be found in a previous thesis [61]. Here only the outline of the calculation for the coupling rate as function of applied RF frequency shall be given.

The underlying idea is to use Fermi's golden rule [36] which states that for weak and constant perturbation the transition rate from an initial state $i$ to a final state $f$ is given by

$$
\begin{equation*}
R=\frac{2 \pi}{\hbar}\left|V_{i f}\right|^{2} \varrho\left(E_{f}\right) \tag{4.6}
\end{equation*}
$$

where $V_{i f}$ is the perturbation operator's matrix element between the initial and final state and $\varrho\left(E_{f}\right)$ is the density of states with final state energy $E_{f}$. In this case $E_{f}=E$ is the energy of the created atom laser.

The density of states in the outcoupled continuum is the one of free particles 61]

$$
\begin{equation*}
\varrho\left(E_{f}\right)=\frac{1}{2 \pi \hbar} \sqrt{\frac{2 m}{E}} . \tag{4.7}
\end{equation*}
$$

The absolute square of the perturbation matrix element is given by

$$
\begin{equation*}
\left|V_{i f}\right|^{2}=\frac{\hbar \Omega_{R F}^{2}}{2}\left|\int \Psi_{B E C}^{*} \Psi_{G A L} \mathrm{~d}^{3} r\right|^{2}, \tag{4.8}
\end{equation*}
$$

where $\Psi_{B E C}$ and $\Psi_{G A L}$ denote the wavefunction of the condensate and of the guided atom laser, respectively. The wavefunction of the BEC is just the 3-D Thomas-Fermi wavefunction discussed in Chapter 2.1, but the wavefunction of the atom laser is more complicated to calculate. In [61] an analytical solution for the wavefunction is found, but the overlap integral

$$
\begin{equation*}
I=\int \Psi_{B E C}^{*} \Psi_{G A L} \mathrm{~d}^{3} r \tag{4.9}
\end{equation*}
$$

is, however, solved numerically. The solution for $I^{2}$ is plotted in Fig. 4.3 for $\mu=h \cdot 3.2$ kHz.

Knowing $I$, equation (4.6) can be evaluated. Fig. 4.4 shows the resulting outcoupling rate as a function of detuning $\delta \omega_{R F}$ from resonance with extraction at $z_{e}=0$. The Rabi frequency $\Omega_{R F}=20 \mathrm{~Hz}$. Qualitatively the theoretical curve matches very well the intuitive expectations mentioned above. It has a steep sloap on the low frequency side, peaks around $\delta \omega_{R F}=0$ and then decays over a range that is given by the chemical potential.

### 4.2.2 Measurement of the Lineshape

The outcoupling resonance curve has also been measured experimentally. The outcoupling rate was determined by operating the guided atom laser for $\tau=100 \mathrm{~ms}$ and afterwards counting the atoms in the optical tweezer by absorption imaging and


Figure 4.3:

## Numerical solution of $I^{2}$

Solution of the overlap integral (4.9) with $\mu=h \cdot 3.2 \mathrm{kHz}$.


Figure 4.4: Theoretical RF outcoupling resonance curve
Outcoupling rate obtained using Fermi's golden rule and numerically solving (4.9) for $\mu=h \cdot 3.2 \mathrm{kHz}$. The inset is a zoom into the region around maximum rate.


Figure 4.5: Data from 2 April 2007, set D
The red curve is a Gaussian fit to the data with a width of 5.2 kHz and center frequency 0.4 kHz .
integration over the region occupied by the guided atoms. Data points were taken for different detunings $\delta \omega_{R F}=\omega_{R F}-\left(E_{B E C}-\mu\right) / \hbar$. After each measurement a new condensate had to be created which means that the time span between two data points is at least the duration of one experimental cycle (approximately 60 s ).

Fig. 4.5 shows a first set of data. The coupling rate as function of detuning neither clearly shows the expected asymmetric shape nor is its width limited to 3.2 kHz , corresponding to the chemical potential. However, when fitting the data with a Gaussian function one can see that the measured rate rises slightly faster than the symmetric fit function on the low frequency side and decays slightly slower on the positive detuning side. The large width leads to the assumption that the actual curve was convoluted with some noise. The assumption appears quite reasonable considering that the $F=1, m_{F}=-1$ state, in which our condensate is, experiences a Zeeman shift of $0.7 \mathrm{MHz} / \mathrm{G}$. To avoid frequency shifts of the resonance on the order of 1 kHz , the bias field has to be stable to a level of 1.4 mG . Fluctuations of this order can appear in a wide frequency range. High frequencies above the inverse lasing time $\tau^{-1}$ should only broaden the line, but frequencies below can affect the measured outcoupling rate from shot to shot. Very low frequency components below the experimental repetition rate should not influence the outcoupling rate from one


Figure 4.6: Data from 2 April 2007, sets $A, B, C$ Connecting lines are to guide the eye.
shot to the next or broaden the line but can manifest themselves in a constant drift over the time during which data is acquired. In case of the data presented in Fig. 4.5. the overall time span was more than one hour. The different RF frequencies for which the flux rate was measured where not swept linear over the full range during acquisition. The covered frequency range was rather split into small stretches that were covered in a randomly manner. Also the experimental repetition rate was not kept constant. This caused the Ioffe-Pritchard electromagnet to operate at different temperatures, which in turn influenced the offset field $B_{0}$.

In Fig. 4.6, three more data sets are displayed. To avoid some of the effects that might have been present when the data of Fig. 4.5 was acquired, the following precautions where taken:

- The experimental duty cycle was kept constant to ensure that no thermal effects influence the operation of the electromagnet.
- The length of the duty cycle was minimized by reducing the loading time of the MOT and idle times between cycles. The repetition rate was then $1 / 50 \mathrm{~s}$.
- $\omega_{R F}$ was swept linearly within each data set. The duration of data acquisition was $10 \mathrm{~min}($ set A), $15 \mathrm{~min}($ set B) and $29 \mathrm{~min}(\operatorname{set} \mathrm{C})$.


Figure 4.7: Data from 5 May 2007
Lineshape measurement after improving magnetic field environment.

There is clearly some asymmetry present in the data. However, there is still some broadening mechanism acting that causes the frequency range over which atoms can be outcoupled to span much more than the 3.2 kHz of the chemical potential. Also the resonance frequency seems to have shifted by about 1 kHz between each set of data.

The shift in resonance frequency over the long timescale can be very well due to slow changes in the magnetic environment of the laboratory. Using a Gaussmeter we determined that changes on the order of 1 mG over a few hours are quite common in our laboratory.

But the broadening must be attributed to higher frequency components (for example the typical 50 Hz technical noise). Thus several electrical components (controllers for vacuum pumps etc.) were relocated. Furthermore several current supplies for the involved electromagnets where replaced by low-noise Fluke model TTi QL335 ones.

After these modifications the data shown in Fig. 4.7 was aquired. Reduction of the magnetic field noise has dramatically reduced the width of the resonance to below 4 kHz , in coarse agreement with the chemical potential. However, the asymmetry is clearly visible but not as pronounced as theoretically expected (see Fig. 4.4). Supposedly this is due to a remaining broadening caused by the residual fluctuations in the improved setup.


Figure 4.8: Resonance curve before/after improvements of magnetic environment (red) Before and (black) after replacement of current supplies and shielding of other electrical components.

To visualize the improvement, in Fig. 4.8 data set B from Fig. 4.6 (before modifications) and the data from Fig. 4.7 (after) are plotted on the same scale.

It would be interesting to take more datapoints over a wider stretch of frequencies than shown in Fig. 4.7. Unfortunately, at this point technical problems made it temporarily impossible to obtain conditions in which the resonance could be further investigated.

### 4.3 Transverse Mode

The atoms that where outcoupled from the BEC are not necessarily in the transverse ground state of the tweezer. The total kinetic energy of the atoms can be written as the sum of a transverse component and a longitudinal one

$$
\begin{equation*}
E=E_{\perp}+E_{\|} \tag{4.10}
\end{equation*}
$$

with the transverse part being quantized by the oscillation frequency in the tweezer. Due to axial symmetry, the vibrational quantum numbers $n_{x}$ and $n_{y}$ in the $x$ - and $y$-direction are the same,

$$
\begin{equation*}
n_{x}=n_{y}=n . \tag{4.11}
\end{equation*}
$$

The total transverse energy is thus

$$
\begin{equation*}
E_{\perp}=\left(n_{x}+\frac{1}{2}\right) \hbar \omega_{x, y, o p t}+\left(n_{y}+\frac{1}{2}\right) \hbar \omega_{x, y, o p t}=(2 n+1) \hbar \omega_{x, y, o p t}, \tag{4.12}
\end{equation*}
$$

where $n$ is an integer number. In contrast, the tweezer does not much constrain the available values for $E_{\|} .^{2}$ The longitudinal energies that can be assumed form a continuum. Upon release from the condensate, some of the available energy is converted into $E_{\perp}$, the rest becomes kinetic energy in the $z$-direction.

In the optimum case, all atoms that are coupled out of the BEC end up in the transverse ground state with $E_{\perp}=\hbar \omega_{x, y, o p t}$. They then all propagate along with the same remaining longitudinal energy $E_{\|}$, given by the resonance condition (4.4) and (4.10).

However, the atoms can also occupy higher transverse modes. In particular, considering that the radial Thomas-Fermi length of the condensate at $z=0$ is

$$
\begin{equation*}
R_{T F}=\sqrt{\frac{2 \mu}{m \omega_{x, y, o p t}^{2}}}=3 \mu \mathrm{~m} \tag{4.13}
\end{equation*}
$$

while the Gaussian ground state wavefunction in the tweezer has the characteristic width

$$
\begin{equation*}
l=\sqrt{\frac{\hbar}{m \omega_{x, y, o p t}}}=0.6 \mu \mathrm{~m} . \tag{4.14}
\end{equation*}
$$

[^14]While an outcoupled atom is leaving the condensate region, the transverse part of its wavefunction should adiabatically turn into the eigenstates of the free atom in the optical tweezer. But if the adiabaticity is not provided, it might happen that the atom ends up in higher transverse states of the tweezer.

### 4.3.1 Measurement of the Transverse Energy

We have released the atoms from the tweezer and observed the size of the falling cloud by absorption imaging. The transverse vibrational energy in the tweezer is converted into transverse kinetic energy when the atoms are released (see Fig. 3.11). By measuring the expansion of the cloud, we can determine the velocity and thus calculate the transverse energy. We then know the vibrational quantum number $n$ of (4.12)).

With $x$ being our imaging axis, we can only measure the expansion along $y$. The width $\sigma$ of the cloud after different times of flight was determined by fitting a Gaussian function after integrating the absorption image along the $z$-axis.

After a time of flight $t$, the width of the cloud is given by the initial width in the tweezer $\sigma_{0}$ and the root-mean-square (rms) velocity $\sqrt{\left\langle v_{y}^{2}\right\rangle}$ after release.

$$
\begin{equation*}
\sigma(t)^{2}=\sigma_{0}^{2}+\left\langle v_{y}^{2}\right\rangle t^{2} \tag{4.15}
\end{equation*}
$$

The rms radial velocity $\sqrt{\left\langle v_{\rho}^{2}\right\rangle}$ is related to the rms velocity in the $y$-direction by $\sqrt{\left\langle v_{\rho}^{2}\right\rangle}=\sqrt{2} \cdot \sqrt{\left\langle v_{y}^{2}\right\rangle}$. The transverse kinetic energy is then just

$$
\begin{equation*}
E_{\perp}=\frac{1}{2} m\left\langle v_{\rho}^{2}\right\rangle \tag{4.16}
\end{equation*}
$$

The initial width $\sigma_{0}$ is of the same order as $l$ and below our spatial resolution level. But the width after some time of flight is resolvable. Fig. 4.9 shows the evolution of the width for different $t$. A fit yields

$$
\begin{equation*}
\sqrt{\left\langle v_{y}^{2}\right\rangle}=2.1 \pm 0.1 \mathrm{~mm} / \mathrm{s} \tag{4.17}
\end{equation*}
$$

Thus, the transverse kinetic energy (in terms of the oscillation frequency) is

$$
\begin{equation*}
E_{\perp}=(2\langle n\rangle+1) \hbar \omega_{x, y, o p t}=(5.4 \pm 0.8) \hbar \omega_{x, y, o p t} . \tag{4.18}
\end{equation*}
$$

The average vibrational quantum number $\langle n\rangle=2.2 \pm 0.4$, which means that the atom laser is not propagating in the ground state of the optical tweezer. One reason for this excitation could be vibrations of the optical tweezer itself, but when stabilizing the pointing of the beam with a servo system, the situation was not improved. Supposedly the reason why the atoms are not in the transverse ground state is unadiabaticity in the outcoupling. The expanding clouds did not feature any spatial patterns that could be associated with occupation of only one mode. Atoms are probably present in various transverse states.


Figure 4.9:
Transverse mode of atom laser

Measured width of the cloud after time of flight $t$. The offset on the vertical axis is due to the limited resolution.

Figure 4.10:

## Acceleration of atom laser

The length of the atom laser versus lasing time $\tau$. The red line is the fit of a second order polynomial.

### 4.4 Longitudinal Energy

One of the key aspects of our experiment is that the de Broglie wavelength of the atoms propagating in the optical tweezer is constant. To test this property we have measured the distance traveled by the outcoupled atoms after different propagation times. Fig. 4.10 shows the measured length of the cloud in the tweezer as a function of outcoupling time. The optical tweezer was exactly focussed onto the BEC.

A fit with a polynomial of second order gave an acceleration of $0.07 \pm 0.06 \mathrm{~mm} / \mathrm{s}^{2}$. The typical velocities of the atoms in the laser are between around $2 \mathrm{~mm} / \mathrm{s}$ and 5 $\mathrm{mm} / \mathrm{s}$, depending on the choice of outcoupling frequency. For our short experimental times (typically around 100 ms ), the acceleration is negligible. The de Broglie wavelength is between $0.9 \mu \mathrm{~m}(5 \mathrm{~mm} / \mathrm{s})$ and $2.1 \mu \mathrm{~m}(2 \mathrm{~mm} / \mathrm{s})$.

Care has to be taken that the focus of the optical tweezer is well aligned onto the BEC. Since the atoms are high-field seeking, they are otherwise accelerated towards the focus. On the other hand, this can be used to apply a well defined acceleration.


Figure 4.11: Top view of components to create the optical barrier

After intentionally translating the focus by 1 mm along the $z$-axis, the acceleration was measured to be already $0.36 \pm 0.04 \mathrm{~mm} / \mathrm{s}^{2}$. More detailed and also more accurate studies of the longitudinal energy will be made in the future by probing the transmission through barriers of well defined height.

### 4.5 Optical Barrier

To allow for further characterization of the guided atom laser and as a preparation for future studies, we have also implemented an optical barrier for the atoms in the guide.

The barrier is formed by light far blue detuned from the $D_{1}$ and $D_{2}$ transitions. Opposite to the case of the red detuned light of the optical tweezer, the atoms then act as low-field seekers: their potential energy increases when they enter the region that is illuminated by the light and is maximized at the point where the intensity is highest.

The light for the repulsive potential is created by the radiation of a free running diode laser that emits at 405 nm . We use a Melles Griot model 561 CS 325. Its maximum output power is 30 mW . The beam is focussed into the vacuum chamber using a cylindrical telescope (see Fig. 4.11). The numerical aperture of the optical system is $N A=0.15$. This value is limited due to geometrical constrains imposed by the axial electromagnets of the Ioffe-Pritchard trap.

The beam propagates along the $x$-direction of the experiment (magnetic dipole axis). It is focussed onto the optical tweezer $80 \mu \mathrm{~m}$ next to the BEC (see Fig. 4.12).

With the prospect to study quantum transport phenomena in the future, it is necessary to have potentials that change on a length scale which is shorter than or comparable to the de Broglie wavelength of the atom laser. To provide this for a wide range of kinetic energies, the structure should be made as small as possible. But when focussing light of wavelength $\lambda$ with an optical system of numerical aperture NA, the minimum obtainable spot size is limited by diffraction to

$$
\begin{equation*}
w_{\min }=\frac{\lambda}{2 N A} . \tag{4.19}
\end{equation*}
$$

For our values $(\lambda=405 \mathrm{~nm}$ and $N A=0.15), w_{\text {min }}=1.35 \mu \mathrm{~m}$.


Figure 4.12: Position of the optical barrier along the tweezer


Figure 4.13:

## Size of the optical barrier

Intensity distribution of the optical barrier at focus, measured using a CCD camera.

A measurement of the intensity distribution in the $(y, z)$-plane at focus is shown in Fig. 4.13. The $1 / e^{2}$ beam waist in the $z$-direction (optical tweezer axis) is $w_{z, \text { blue }}=$ $1.3 \mu \mathrm{~m}$, in agreement with the theoretical limit. The beam waist in the $y$-direction is $w_{y, \text { blue }}=28 \mu \mathrm{~m}$.

Given the beam waists and the power $P$, the maximum intensity is

$$
\begin{equation*}
I_{0}=\frac{2 P}{\pi w_{y, b l u e} w_{z, b l u e}} \tag{4.20}
\end{equation*}
$$

The optical potential at maximum power of 30 mW is then (see Chapter 2.5) $V_{\text {blue, max }}=h \cdot 300 \mathrm{kHz}$. The transitions giving the main contribution to the potential are the $D_{1}\left(5^{2} S_{1 / 2} \rightarrow 5^{2} P_{1 / 2}\right)$ and $D_{2}\left(5^{2} S_{1 / 2} \rightarrow 5^{2} P_{3 / 2}\right)$-line at 795 nm and 780 nm , respectively. A smaller contribution (around $10 \%$ ) stems from the $5^{2} S_{1 / 2} \rightarrow 6^{2} P_{1 / 2}$ and $5^{2} S_{1 / 2} \rightarrow 6^{2} P_{3 / 2}$ transitions at 422 nm and 420 nm , respectively.

By adding a $\pi$-phase plate in half of the beam path (see Fig. 4.14), we can also create a double bump potential. The measured intensity distribution of the light which then forms the two barriers is shown in Fig. 4.15. The intensity maxima are $3 \mu \mathrm{~m}$ apart from each other.


Figure 4.14: Optical setup to create double bump potential
A phase plate is added inside the beam to create a $\pi$ phase shift between two sides of the beam. Destructive interference then causes an intensity minimum in the center of the beam.


Figure 4.15:
Size of the double bump structure
Intensity distribution of the double structure, measured using a CCD camera. The distance of the two peaks is $3 \mu \mathrm{~m}$.

### 4.6 Interference between Two Guided Atom Lasers?

Two atom lasers can be simultaneously extracted from the condensate when two different RF frequencies are applied. Observing an interference pattern between them would be an important proof that the outcoupled atoms are actually phase coherent.

We have amplitude modulated the outcoupling RF by modulation frequencies $\omega_{\text {mod }}$. According to (B.1) this corresponds to applying a waveform with two frequency components at $\omega_{R F} \pm \omega_{\text {mod }}$. Two atom lasers should then be outcoupled with their difference in energy given by twice the modulation frequency. Both lasers are extracted from different points $z_{e}$, given by 4.3. If spatial coherence over the distance of the two points exists, interference fringes should be seen when the two lasers overlap during propagation. In the case of free falling atom lasers this method has been used to demonstrate the coherence [20].

Unfortunately, it was not possible for us to observe interference fringes between two guided atom lasers for any values of $\omega_{R F}$ and $\omega_{m o d}$. There are different reasons that might have prevented us from seeing them.

- One possible reason is that multiple transverse modes were occupied and the fringes where washed out because not only two waves but rather many waves at different energies with different phase interfered ${ }^{3}$.
- The linewidth of the atom laser might have been too broad. Fluctuations in the magnetic field lead to outcoupling of atoms with different kinetic energies. The energy difference between the two atom lasers is fixed by the difference of outcoupling frequencies and is not affected by changes in the background field. Their linewidth, however, is affected. The fringe contrast vanishes if the linewidth of each atom laser is comparable to their energy difference.
- It might also be possible that the two different atom lasers do not propagate in the same transverse mode. The transverse part of the atomic wavefunction is given by eigenfunctions of the harmonic oscillator potential of the optical tweezer. Since the eigenstates of a harmonic oscillator are orthogonal, two lasers in different transverse modes to not interfere.
- Another possibility why no interference was observed is that there might have been no phase coherence between different outcoupled atoms. With other words, it could be that in our case the outcoupling process into the guide does not preserve the fixed phase relation between the different atoms. Unadiabaticity of the outcoupling (in particular at high flux rates) might cause excitations in the source BEC. The atoms which leave the magnetic trap might then experience different mean-field potentials (i.e. due to density fluctuations associated with the excitations). Such an effect might also be responsible for

[^15]the broadening of the resonance curve compared to the theoretical prediction, which was made under the assumption of perfect adiabaticity.

Creating a beat note between two or more atom lasers still remains an important goal for the future work on the experiment.

## CHAPTER 5

## New Cooling Lasers

The experiment will be moving into a new laboratory shortly after the writing of this thesis. The new setup will use the same laser cooling sequence as presented in Chapter 3, but the laser light will be generated by an improved diode laser system. The new system is described in this chapter.

The required light frequencies for the experiment are as follows (see also Chapter 3 and Appendix D):

- Cooling light for MOT and optical molasses with detuning 12 MHz and 36 MHz , respectively, to the red from the $F=2 \rightarrow F^{\prime}=3$ transition. The frequency adjustment has to happen without any change in beam pointing.
- Zeeman slowing beam on the $F=2 \rightarrow F^{\prime}=2 \times 3$ cross-over.
- Repumping light resonant on the $F=1 \rightarrow F^{\prime}=2$-transition.
- Probe beam resonant on the $F=2 \rightarrow F^{\prime}=3$-transition.
- Blow-away beam resonant on the $F=2 \rightarrow F^{\prime}=2$-transition.

All the transitions mentioned above are on the $D_{2}$-line.
A schematic diagram of the core part of the new laser system is shown in Fig. 5.1. One master laser is locked to the $F=2 \rightarrow F^{\prime}=2 \times 3$ cross-over transition using saturated absorption spectroscopy in a rubidium vapor cell. A slave laser is phase-locked to the master laser at a variable frequency offset. The light of the slave laser is amplified by a tapered amplifier. Parts of the beams from the two lasers are frequency-shifted by acousto optical modulators. At a later point also a repumping laser will be locked to the master laser.

### 5.1 Diode Lasers

The external cavity diode lasers used in the new setup were originally designed at the Observatoire de Paris [62, 63]. They use an interference filter inside a long FabryPérot cavity for a rough wavelength selection while the long external cavity itself effectuates the fine selection.

In our implementation (see Fig. 5.2), the long cavity has a length $L=10 \mathrm{~cm}$ and is formed by the laser diode at one end and a lens focussing on a planar outcoupling mirror at the other end. It has a free spectral range $\nu=c / 2 L=1.5 \mathrm{GHz}$. The so


Figure 5.1: Optical arrangement for cooling lasers.
AP - anamorphic prisms, OI - optical isolator, PBS - polarizing beam splitter, BS - 50:50 beam splitter, AOM - acousto-optical modulator, TA - tapered amplifier system.


Figure 5.2: Schematic of the cavity components.
LD - laser diode, FP - Fabry-Pérot interference filter, BS - outcoupling beam splitter, COL1 - collimation lens, COL2 - output collimator
called cat's eye arrangement with a lens focussing on a mirror assures that collimated light which passes through the lens will be reflected straight back even if the mirror is slightly misaligned. The outcoupling mirror has a dielectric coating for $30 \%$ reflection on the inner side of the cavity and is anti-reflection coated on the outer surface.

Since the output of the diode is strongly divergent, there is a first collimation lens with focal length $f_{1}=4.55 \mathrm{~mm}$ mounted close to the diode. The light that is coupled out of the cavity was focussed by the cat's eye lens ( $f_{2}=18.4 \mathrm{~mm}$ ) and is collimated again by a another lens with focal length $f_{3}=11 \mathrm{~mm}$ to form a telescope.

The outcoupler is mounted on a piezo actuator. Its position along the beam axis can be changed by applying a voltage to the actuator. The change of cavity length resulting from this movement tunes the resonance frequency for the fine selection of the output wavelength.

The interference filter is also of Fabry-Pérot type. We are using a $6 \times 6 \times 1 \mathrm{~mm}^{3}$ fused silica plate with narrowband high-reflection coatings on both sides, custom manufactured by Research Electro Optics. The coatings have a center wavelength of 780 nm and a spectral width of 0.2 nm . Given the thickness of 1 mm , the free spectral range $\nu_{\text {filter }}=150 \mathrm{GHz}$.

For the laser diode itself we use the type GH0781JA2C from Sharp. This model is specified to have a free running center wavelength of 784 nm when operated at $25^{\circ} \mathrm{C}$. The diode is mounted inside a small temperature-stabilized copper piece. The copper part itself is mounted to one end of the long cavity block using thermally isolating screws and washers. The holding block for all optical components is also temperature-stabilized. This extra temperature-control is necessary because heating of the mounting block during operation would cause its length to change and consequently the distance between the outcoupler and the diode as well. This would result in a change of output frequency. Actually, a drift of output frequency as large as 75 MHz per minute is observed during the first hour after switch-on when the additional temperature servo is deactivated.

The diodes are driven using either homemade laser diode drivers or the current
supply unit of a Thorlabs ITC502 controller. The piezos holding the outcouplers are driven by model SVR 500-3 piezo drivers from Piezomechanik GmbH. The tuning coefficient of the external cavity is $c_{P Z T} \approx 8.5 \mathrm{MHz} / \mathrm{V}$ and the coefficient for the injection current is $c_{\text {Current }} \approx-100 \mathrm{MHz} / \mathrm{mA}$. Signals applied to the piezo are limited to a maximum frequency of about 1 kHz due to mechanical resonance.

The laser diodes are specified for operation currents $I_{o p} \approx 140 \mathrm{~mA}$. To spare the diodes and extend their lifetime we inject only around 100 mA . We then get approximately 35 mW output power at the desired wavelength. The beam has the elliptic profile typical for diode lasers owing to the rectangular shape of the gain region on the output facet. Anamorphic prism pairs outside the laser housing are used to compress the beam in one dimension, resulting in a more circular shape.

### 5.2 Saturated Absorption Spectroscopy

All our cooling and detection lasers are required to output light with certain well defined frequencies around the rubidium- $87 D_{2}$ line. To ensure this, they are stabilized to rubidium resonances in vapor cells.

When determining the resonance frequency of an atomic transition, one desires to resolve the transition down to its natural linewidth. But when an atomic vapor sample is illuminated with monochromatic laser light, the Doppler shift causes each individual atom $i$ of the sample to experience the light at a different frequency

$$
\begin{equation*}
\omega_{i}=\omega_{L}-\mathbf{k} \cdot \mathbf{v}_{i} \tag{5.1}
\end{equation*}
$$

where $\omega_{L}$ is the laser frequency in the laboratory frame and $\mathbf{v}_{i}$ is the velocity of the atom. With $c$ being the speed of light, the wavevector $\mathbf{k}$ has the magnitude $k=|\mathbf{k}|=\omega_{L} / c$ and points in the direction of the beam.

Rubidium- 87 atoms at temperature $T=300 \mathrm{~K}$ have a Gaussian velocity distribution with a root-mean-square (rms) velocity $v_{r m s}=\sqrt{3 k_{B} T / m}=294 \mathrm{~m} / \mathrm{s}\left(k_{B}\right.$ is the Bolzmann constant and $m$ is the atomic mass). The resulting inhomogeneous broadening of a resonance at frequency $\omega_{0}$ in the atomic sample causes its spectrum to have a Doppler width $\Delta \omega_{\text {Doppler }}=\omega_{0} \cdot v_{r m s} / c$. For the $\mathrm{D}_{2}$-transitions of rubidium $\Delta \omega_{\text {Doppler }}=2 \pi \cdot 376 \mathrm{MHz}$ and is much larger than the natural linewidth $\Gamma=2 \pi \cdot 6$ MHz . The Doppler width is also larger than the energy spacing between different hyperfine levels in the $\mathrm{D}_{2}$ excited state manifold, therefore not allowing to resolve them individually.

The basic idea of saturated absorption spectroscopy is to saturate a transition with a pump beam from one direction and to probe the transition with a beam from the opposite direction. This enables one to obtain resolutions only limited by the natural linewidth because of cancelling Doppler shifts due to the opposite beam directions. This will become clear in the following discussion of the shape of the absorption signal.


Figure 5.3: Hole burning in the velocity distribution
Hole burning effect for saturation parameters $s=0.5$ (red), $s=1$ (green) and $s=5$ (blue).

### 5.2.1 Lineshape

For a moving two-level atom interacting with a laser field close to a single-photon resonance, the steady-state probability of it being in its excited state is [25]

$$
\begin{equation*}
\rho_{2}(v)=\frac{1}{2} \cdot \frac{s}{1+s+4\left(\frac{\omega_{L}-\omega_{0}-k v}{\Gamma}\right)^{2}} . \tag{5.2}
\end{equation*}
$$

Here $v$ is the velocity of the atom along the beam direction. $s=2 \frac{\Omega^{2}}{\Gamma^{2}}$ is called the the saturation parameter with $\Omega$ being the Rabi frequency.

With the laser light far off-resonance, the number of the ground state atoms as function of velocity has a Gaussian distribution

$$
\begin{equation*}
n(v)=\frac{N \sqrt{3}}{v_{r m s} \sqrt{\pi}} e^{-\left(\frac{v \sqrt{3}}{v_{r m s}}\right)^{2}} . \tag{5.3}
\end{equation*}
$$

Here, $N$ is the total number of atoms. When a pump beam is now tuned close to a transition frequency then the light becomes resonant for atoms in a certain velocity class around $v_{0}=\left(\omega_{L}-\omega_{0}\right) / k$ due to the Doppler shift. The beam then pumps a hole in the ground state velocity distribution as illustrated in Fig. 5.3.

A counterpropagating probe beam at the same frequency as the pump also burns a hole in the velocity distribution but not centered around velocity $v_{0}$ but rather at exactly the opposite velocity $-v_{0}$. When the frequency moves closer to resonance the two holes in the velocity distribution approach each other until they exactly overlap at $v=0$ when the beams are exactly resonant. The resulting dip at zero velocity is called the Lamb dip.

For low probe beam intensities the occurence of this Lamb dip for light on resonance can be detected by measuring the beam's reduced absorption using a photo detector.

In a system with one ground state and multiple excited states, absorption dips not only occur exactly on an atomic resonance but also at so called cross-over resonances in the center between two atomic resonances. To see this, consider a system with one ground state and two excited states $a$ and $b$. Let $\omega_{a}<\omega_{b}$ be the respective transition frequencies. The laser frequency in the rest frame of atoms moving with velocity $v$ towards the probe beam is $\omega_{p r o b e}^{\prime}=\omega_{L}+k v$ for the probe and $\omega_{p u m p}^{\prime}=\omega_{L}-k v$ for the pump light. If $\omega_{p r o b e}^{\prime}$ is close to $\omega_{b}$ then probe light gets absorbed by the atoms. But if $\omega_{p u m p}^{\prime}$ is for the same atoms close to $\omega_{a}$ then the ground state population is reduced due to the hole burning of the strong pump beam and the absorption of the probe beam is consequently reduced as well. This effect leads to a dip for the probe absorption when the laser frequency equals the cross-over resonance frequency $\left(\omega_{a}+\omega_{b}\right) / 2$.

### 5.2.2 Electronically Extracting the Derivative Signal

The signal of the photodetector $S\left(\omega_{L}\right)$ can be Taylor expanded in the vicinity of the resonance of interest:

$$
\begin{equation*}
S\left(\omega_{L}\right)=S\left(\omega_{0}\right)+\left.\left(\omega-\omega_{0}\right) \frac{\mathrm{d}}{\mathrm{~d} \omega} S\left(\omega_{L}\right)\right|_{\omega_{0}}+\left.\frac{1}{2}\left(\omega-\omega_{0}\right)^{2} \frac{\mathrm{~d}^{2}}{\mathrm{~d} \omega^{2}} S\left(\omega_{L}\right)\right|_{\omega_{0}}+\ldots \tag{5.4}
\end{equation*}
$$

Modulating the laser frequency around resonance with a modulation depth $\alpha$ and modulation frequency $\omega_{\text {mod }}$, i.e.

$$
\begin{equation*}
\omega(t)=\omega_{0}+\alpha \sin \omega_{m o d} t \tag{5.5}
\end{equation*}
$$

gives in (5.4)

$$
\begin{equation*}
S\left(\omega_{L}\right)=S\left(\omega_{0}\right)+\left.\alpha \sin \omega_{m o d} t \frac{\mathrm{~d}}{\mathrm{~d} \omega} S\left(\omega_{L}\right)\right|_{\omega_{0}}+O\left(\alpha^{2}\right) \tag{5.6}
\end{equation*}
$$

For small modulation depth we drop higher order terms in $\alpha$, since $\alpha^{2} \ll 1$. The remaining signal $S\left(\omega_{L}\right)$ can now be demodulated to extract the derivative term: Mixing with a local oscillator at frequency $\omega_{\text {mod }}$ at phase $\phi$ gives the signal

$$
\begin{align*}
E^{\prime}=S\left(\omega_{L}\right) \cdot \sin \left(\omega_{\text {mod }} t+\phi\right) & \approx S\left(\omega_{0}\right) \sin \left(\omega_{\text {mod }} t+\phi\right)+\left.\alpha \frac{\mathrm{d}}{\mathrm{~d} \omega} S\left(\omega_{L}\right)\right|_{\omega_{0}} \sin \omega_{\text {mod }} t \sin \left(\omega_{\text {mod }} t+\phi\right) \\
& =S\left(\omega_{0}\right) \sin \left(\omega_{\text {mod }} t+\phi\right)+\left.\alpha \frac{\mathrm{d}}{\mathrm{~d} \omega} S\left(\omega_{L}\right)\right|_{\omega_{0}} \frac{1}{2}\left[\cos \phi-\cos \left(2 \omega_{\text {mod }} t+\phi\right)\right] \tag{5.7}
\end{align*}
$$

where we have used (B.1). Low pass filtering the mixer output $E^{\prime}$ with a cutoff frequency below $\omega_{\text {mod }}$ yields the error signal

$$
\begin{equation*}
E=\left.\frac{\alpha}{2} \cos \phi \frac{\mathrm{~d}}{\mathrm{~d} \omega} S\left(\omega_{L}\right)\right|_{\omega_{0}} . \tag{5.8}
\end{equation*}
$$

This error signal is proportional to the slope of the Doppler free absorption signal around resonance with a zero-crossing when the laser is exactly on resonance. The


Figure 5.4: Line pulling effect
Two individual resonances (red and green) and their sum (blue) are shown. In the sum of the two absorption components the position of the bottoms of the dips are shifted due to the sloping background caused by the other resonance.
amplitude of the error signal depends on the relative phase $\phi$ between the signal from the photo detector and the local oscillator. This phase can be adjusted electronically to maximize $E$.

Unfortunately most investigated systems (such as rubidium in our case) do not only have one resonance line but rather several ones. If there are mechanisms like Doppler broadening that lead to an overlap of multiple resonances, then an effect called line pulling occurs. It can be understood as follows: Consider a resonance at some frequency $\omega_{0}$. Let another resonance be at frequency $\omega_{1}$, having a width $\Gamma_{1} \gtrsim\left|\omega_{0}-\omega_{1}\right| \|^{\mid}$Then the Doppler free absorption signal of the resonance at $\omega_{0}$ has a superimposed sloping background due to the neighboring resonance. The position of the absorption dip and therefore the zero-crossing of the derivative of the signal is consequently shifted. See Fig. 5.4 for an illustration.

Practically there are different ways to handle the problem of line pulling. One way is to simply add an electronic offset to have the derivative of the resonance of interest be symmetric around zero. Alternatively one can increase the modulation depth to also include higher order terms of the Taylor expansion (5.6). Then one can demodulate at the third harmonic of the modulation frequency to extract the third instead of the first derivative of the absorption signal. This signal has smaller amplitude than the first derivative but does not feature the constant offset.

Another way is to modulate the pump and the probe beam at different frequencies and to lock-in detect at the sum and the difference of the modulation frequencies. In these intermodulation signals the saturation dip is separated from the sloping background [64].

An approach allowing for higher modulation frequencies and thereby allowing

[^16]

Figure 5.5: Setup used to identify the Rubidium resonances.
The rubidium cell can be placed in either position 1 for single photon spectroscopy or in position 2 for Doppler free spectroscopy. The quarter-wave plate is adjusted to change the incoming linear polarization into a circular one. OI - optical isolator, PBS - polarizing beam splitter, PD - photo detector.
a higher locking bandwidth is to modulate only the pump beam with a frequency around the natural linewidth and to lock-in detect the absorption signal of the probe beam. The output in this case contains information about the phase response of the atomic system with respect to the pump frequency and is not affected by the line pulling effect. This technique is called modulation transfer spectroscopy [65].

### 5.2.3 Practical Implementation

The first step to lock the lasers to a specific frequency is to rotate the interference filter while observing the output frequency with a wavelength meter. Turning the angle between the filter and the optical axis changes its effective length and thereby the frequencies that are transmitted. Once the filter is set to a position where the system lases around the desired frequency, a finer tuning can be done by changing the injection current into the diode until the output is within 1 pm of the desired mode. After setting the current to support lasing close to target frequency one can perform the finest tuning by changing the voltage that is applied to the piezo in order to scan the long cavity.

At this point a temporary spectroscopy setup was used to identify the different atomic resonances. A schematic of this setup is shown in Fig. 5.5. The atomic sample employed for the spectroscopy is a gas of rubidium in natural abundance inside a glass cell at room temperature. The natural abundance of rubidium consists of about $72 \%$ ${ }^{85} \mathrm{Rb}$ and $28 \%{ }^{87} \mathrm{Rb}$. The sample cell can be placed in either position 1 or position 2 of Figure 5.5 to observe either Doppler broadened single photon resonances or Dopplerinsensitive saturated absorption lines. An optical isolator was inserted into the beam path to avoid unwanted feedback due to back reflection from optical elements.

Figure 5.6 shows a rubidium spectrum that was obtained using this setup. The


Figure 5.6: Part of the rubidium natural abundance $D_{2}$ spectrum. Lines were taken with the same laser settings. The rubidium cell was placed in position 1 of Fig. 5.5 for single-pass and position 2 for double-pass.
frequencies are with respect to the frequency of the cross-over resonance that was later used for locking. The Doppler free resonances can be clearly seen without having to attenuate the beam before its second pass through the cell.

The wide feature at higher frequencies can be identified as the ${ }^{85} \mathrm{RbF}=3 \rightarrow \mathrm{~F}$ ' transitions of the $\mathrm{D}_{2}$-line. The excited state hyperfine levels are too close to each other to be resolved in the single-pass configuration. The resonance at lower frequencies is due to the broadened ${ }^{87} \mathrm{RbF}=2 \rightarrow \mathrm{~F}^{\prime}$ transitions.

Using the Doppler free configuration one can clearly see three narrow resonances of both ${ }^{85} \mathrm{Rb}$ and ${ }^{87} \mathrm{Rb}$. The latter three can be identified in order of increasing frequency as:

- cross-over between the $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=1$ and $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=3$ transitions
- cross-over between the $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=2$ and $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=3$ transitions
- $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=3$ transition.

The central and strongest resonance is the one to which the laser was eventually locked.

To extract the derivative of the signal, the frequencies were modulated and the derivative lock-in detected at the output of the photo detector as discussed in Section 5.2.2. In order to avoid noise on the laser, the modulation was not done on the light itself but rather on the atomic resonance frequencies using the Zeeman effect. A coil of length $L=5 \mathrm{~cm}$ with $N=100$ turns was wound around the 2.5 cm diameter cell


Figure 5.7: Electronic setup for saturated absorption spectroscopy with modulation. PS - phase shifter, MX - mixer, LP - low-pass filter, Int - integrator, HV - high voltage amplifier, PI - proportional - integral servo, PZT - piezo to actuate mirror, MOD - modulation input of current supply. The integrator and the PI-servo have input stages with an adjustable offset.
and supplied with an AC current of amplitude $55 \mathrm{~mA}_{p p}$ oscillating at $f_{\text {mod }}=40 \mathrm{kHz}$. The magnitude B of the created magnetic field is given by

$$
\begin{equation*}
B=\mu_{0} \frac{N}{L} \tag{5.9}
\end{equation*}
$$

where $\mu_{0}$ is the vacuum permeability. The peak-to-peak amplitude of the magnetic field in the cell is thus $B_{p p}=0.69 \mathrm{G}$. The created magnetic field along the beam axis causes a Zeeman shift of $0.70 \mathrm{MHz} / \mathrm{G}$ and $0.93 \mathrm{MHz} / \mathrm{G}$ for the ground and excited state, respectivly. The differential shift causes a modulation in the photo detector signal similar to the case when the laser frequency itself is modulated. In order to always maintain a well defined quantization axis the AC current is offset by a DC value of 30 mA to avoid zero-crossings of the magnetic field. The resulting frequency shift is corrected by adding an offset to the final error signal as well. A schematic diagram of the electronics used to generate the error signal is depicted in Fig. 5.7.

Fig. 5.8 shows the demodulated signal. After identification of the different signal components, a more compact setup for permanent use was built. A schematic diagram of it is depicted in Fig. 5.9.

The photo detector used in the permanent setup is a homemade high gain one utilizing a BPW34 photo diode followed by an OP 27 high precision operational amplifier. Using this system peak-to-peak error signals of $1.4 \mathrm{~V}_{p p}$ with a signal-tonoise ratio of $140: 1$ are obtained using only 0.6 mW of optical power.

The error signal after the low-pass filter is integrated with an about 1 ms time constant and fed to the high voltage amplifier driving the piezo of the long laser


Figure 5.8: Rubidium $F=2 \rightarrow F^{\prime}$ lock error signals.
$\mathrm{F}^{\prime}=\mathrm{AxB}$ denotes a cross-over resonance between $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=\mathrm{A}$ and $\mathrm{F}=2 \rightarrow \mathrm{~F}^{\prime}=\mathrm{B}$. The slightly sloping background is due to the line-pulling effect.


Figure 5.9: Saturated absorption spectroscopy schematic.
PBS - polarizing beam splitter, PD - photo detector, MIR - $0^{\circ}$ mirror. The rubidium cell is surrounded by a solenoid to create the modulated magnetic field.


Figure 5.10: Double-pass acousto optical modulator.
The incoming light is linearly polarized. The quarter wave plate is adjusted to change the polarization to circular on the first pass. After the returning pass through the waveplate the polarization is linear again, but orthogonal to the incident one. This way the outgoing beam is separated from the input by the polarizing beam splitter. The distance between the AOM and the lens as well as between the mirror and the lens equal the focal length.
cavity. Another path of the error signal is input to a proportional-integral servo whose output is then fed to the current modulation input of the laser diode current controller.

### 5.3 Phase-lock

The slave laser that is used to inject the tapered amplifier needs to have different frequencies for the magneto-optical trapping and for the optical molasses stage.

One way to change a laser beam's frequency is to use and acousto-optical modulator (AOM). An AOM deflects the beam by an angle proportional to its RF drive frequency $\omega_{R F}$ and detunes the deflected beam by $\omega_{R F}$ with respect to the incident frequency. However, pointing changes of the beams are undesired, so using an AOM in a single-pass configuration is not advisable here because in this case the output moves unavoidably when switching from MOT to molasses. A popular method to circumvent the problem is to use the AOM in a double-pass configuration where the beam is first shifted, then backreflected and shifted again (see Fig. 5.10). Exactly counterpropagating backreflection for different deflection angles can be assured by placing a lens at its focal distance between the AOM and the retroreflector to have a cat's eye configuration. The incident and the returning beam are then overlapped but can be separated using a polarizing beam splitter if the light's polarization is changed during the double-passing by using a quarter wave plate (again, see Fig. 5.10).

The shifted beam can be derived from the master laser and then injected into the slave laser [66, 67] to lock it to the shifted frequency. This is done in the old atom laser setup presented in Chapter 3 (see also [58, 54].) However, there are some problems that arise when using this technique.

- First, the system is sensitive to alignment errors. Slight misalignments of the
cat's eye can cause pointing changes of the output beam when the driving frequency of the AOM is varied.
- Second, the AOM's defletion efficiency is sensitive to the angle between the beam and the acoustic waves in the modulator. If the Bragg condition is not well met, the deflection efficiency is reduced. Since the correct incident angle depends on the drive frequency, best efficiency for the first pass can only be obtained for one frequency.
- Third, AOMs use a piezo transducer to drive their crystal. An LC circuit (the piezo acts as a capacity) is used as resonator to have maximum power buildup. Such a circuit is resonant at only one frequency. By designing a circuit with a lower Q factor one can have a wider useful frequency range but only at the price of loosing peak power buildup and consequently loosing maximum deflection efficiency.

Furthermore, even a perfect double pass AOM system always has a limited efficiency, thus always costing some optical power.

A better approach is used in the new setup. Beam samples of the master and the slave laser are beat on a photo detector and the frequency difference is locked. The most accurate way to do this is by phase-locking the beat signal to a stable reference oscillator. The frequency of the reference oscillator can then be changed in order to tune the slave laser frequency without any changes in output pointing or optical power.

The AC part of the beat signal on the photo detector can be written as

$$
\begin{equation*}
S_{R F}=A_{R F} \sin \left(\left(\omega_{\text {Master }}-\omega_{\text {Slave }}\right) t+\phi\right) \tag{5.10}
\end{equation*}
$$

where $A_{R F}$ is the signal amplitude, $\omega_{\text {Master }}$ and $\omega_{\text {Slave }}$ are the laser frequencies and $\phi$ is their relative phase. For our purpose the beat frequency $\omega_{\text {Master }}-\omega_{\text {Slave }}:=\omega_{R F}$ is in the radio frequency region. We mix $S_{R F}$ with a reference signal

$$
\begin{equation*}
S_{L O}=A_{L O} \sin \left(\omega_{L O} t+\phi_{L O}\right) . \tag{5.11}
\end{equation*}
$$

The mixer multiplies the two input signals and produces the output (using B.1)

$$
\begin{equation*}
O_{m i x}=\frac{g}{2} A_{R F} A_{L O}\left\{\cos \left[\left(\omega_{R F}-\omega_{L O}\right) t+\phi-\phi_{L O}\right]-\cos \left[\left(\omega_{R F}+\omega_{L O}\right) t+\phi+\phi_{L O}\right]\right\}, \tag{5.12}
\end{equation*}
$$

with a gain factor $g$ below 1 for our passive Mini Circuits model ZAD-1+ mixer. $O_{m i x}$ has two components. One is oscillating at the sum of the two input frequencies, the other component oscillates at the difference. Filtering out the high frequency part and leak-through from the inputs, one obtains the signal

$$
\begin{equation*}
E=\frac{g}{2} A_{R F} A_{L O} \cos \left[\left(\omega_{R F}-\omega_{L O}\right) t+\phi-\phi_{L O}\right] . \tag{5.13}
\end{equation*}
$$

$E$ is suitable to be used as error signal to phase-lock the two lasers. When the beat frequency and the local oscillator frequency are equal, it changes linearly if the


Figure 5.11: Electronic diagram of the phase-locked loop.
The error signal after the double-balanced mixer (DBM) is fed back to the laser via three paths for different frequency ranges. High frequencies above 100 kHz are fed directly to the diode head via a bias tee. Frequencies up to 100 kHz are fed back by modulating the DC current output of the diode laser controller. Very slow components up to 1 kHz are fed to the piezo by a proportional-integral servo followed by a high voltage amplifier. LP - Low-pass filter.
relative phase between the lasers $\phi$ deviates from the value $\phi_{L O}-\pi / 2$. By locking $\phi$, $\omega_{R F}$ is automatically locked to $\omega_{L O}$, too.

In order to obtain lock it is necessary to have a higher servo bandwidth than the free running laser linewidth, in our case several hundred kiloherz. Feedback is done via three paths covering different frequency ranges with individual gains and correction ranges:

- The fastest path uses a Mini Circuits ZFBT-6GW-FT bias tee to directly feed the error signal via proportional feedback to the laser diode. The RF input of the bias tee operates a frequencies above 100 kHz .
- An intermediate speed path for the range between 1 kHz and 100 kHz is fed proportionally to the modulation input of the laser diode controller.
- Frequency components below 1 kHz are send to the piezo after a proportionalintegral servo followed by a high voltage amplifier. This stage has sufficient correction range to lock away large long-term drifts that cannot be covered by the current modulation stages.

A diagram of the electronics is depicted in Fig. 5.11.
The input signal $S_{R F}$ has a peak-to-peak amplitude around 2 V , allowing to adjust the proportional gains by simply adding attenuators instead of using active components.


Figure 5.12: Beat spectrum.
Displayed is the noise spectral density of the error signal in $\mathrm{dBc} / \mathrm{Hz}$, measured inloop. Horizontal span is $250 \mathrm{kHz} /$ div, center frequency is 80.1 MHz , the resolution bandwidth is 300 Hz .

The in-lock beat signal between the two lasers is shown in Fig. 5.12. One can clearly see the delta-like peak at the lock frequency. The rise in phase noise around 1 MHz away from carrier frequency indicates the end of the locking bandwidth. In the figure the phase-noise is already displayed in $\mathrm{dBc} / \mathrm{Hz}$. The noise pedestal is at a level of approximately $-65 \mathrm{dBc} / \mathrm{Hz} .^{2}$

### 5.4 Master Oscillator Power Amplifier System

The power of the slave laser is amplified by injecting the beam into a tapered semiconductor amplifier chip. In this cofiguration the tapered amplifier is used as a master oscillator power amplifier (MOPA) system. We are using a mechanical design identical to the one described in [68]. The gain chip is a model EYP-TPA-0780-01000-3006-CMT03-0000 provided by eagleyard Photonics.

The mount for the chip is temperature-stabilized with a water-cooled heat sink. Aspherical lenses with effective focal length $f=8 \mathrm{~mm}$ focus the input light on the chip and collimate the output on the other side. The mounts for the chip and for the lenses are precision machined to ensure accurate transverse and tilt alignment. The distance of the lenses is adjusted by screwing the mounts closer to or and further

[^17]away from the chip using fine threads.
Our slave laser delivers 35 mW of power to inject into the MOPA. When driven with a current of 2.5 A , our amplifier chips are able to provide more than 600 mW of output power at the injected frequency, even at much lower input levels [68].

### 5.5 Discussion

The new cooling laser system has three significant advantages compared to the old one that was used in the experiment described in chapters 3 and 4 :

- First, the tapered amplifier provides high optical power. The old cooling system was severely limited by insufficient laser power. In the new experiment it will be possible to use larger MOT beams at same intensity to have an increased capture volume. The case is similar for the Zeeman slowing beam. It will also be possible to implement an additional transverse cooling of the atomic beam inside the Zeeman slower by creating a two-dimensional optical molasses in the region where currently the getter pump is located. Reducing the atomic beam divergence by transverse cooling in combination with a larger MOT capture volume will increase the loading rate of the MOT. The higher loading rate will allow to accumulate a higher number of atoms and will lead to a shorter duration of the MOT stage, reducing the total time of each experimental cycle.
- Second, the use of a phase-lock to change the frequency of the cooling beam between the MOT and the optical molasses ensures stable pointing of the beams at all times with short switching times.
- Third, avoiding to have many master-slave combinations of lasers, the new setup has much less components than the old one. This will significantly improve the reliability and should boost it to a satisfactory level.

The new lasers are intended to be used for Zeeman slowing, magneto-optical trapping and polarization gradient cooling. For all these techniques the laser frequencies do not need to be locked to the desired values extremely precisely. The accuracies reached with saturated absorption spectroscopy as employed in this setup are sufficient. Also, it is not crucial to phase-lock the slave laser to the master with low residual noise. Actually it already suffices to have the average frequency of the slave follow the frequency which is given by the master laser and the local oscillator ("frequency-lock").

The situation is different, however, when the lasers are to be used for Raman pulses, e.g., as coherent beam splitters for atom interferometry [69]. In this case actual phase-lock between the counterpropagating Raman lasers is necessary.

When the lasers are to be used for coherent beam splitting, then it will also be necessary to characterize the low frequency noise. If, for example, beam splitting pulses are applied for $\tau=100 \mu$ s then noise has to be suppressed up to frequencies around $1 / \tau=10 \mathrm{kHz}$ [5]. Higher frequency components average out over the pulse
duration. The low-frequency noise can be measured by mixing the beat signal down with a stable reference oscillator at the same frequency as the beat. The low-pass filtered output of the mixer then directly contains the noise in vicinity of the carrier frequency.

For really accurate tests of the laser stability it will also be necessary to measure the beat signal with a separate photo detector out of the servo loop. In-loop measurements tend to underestimate the actual noise because they do not reveal errors of the detection system itself. When operated with the tapered amplifier then the beam sample of the slave laser should be taken after amplification to ensure that no noise is introduced by the MOPA.

There are still some remaining insuffiencies in the design of the new extended cavity diode lasers. The main one is the way how the laser is directly screwed onto the optical table. When bending the optical table, the baseplate bends as well and the laser cavity with it. But this changes its length, affecting the resonance frequency. Since these movements happen slower than the frequency limit of the feedback system, the effects are removed by the servo when the lasers are in lock. It should be mentioned, however, that the free running output frequency can change by as much as several tens of MHz when just pushing on the table by hand.

Eventually the saturated absorption spectroscopy should be abandoned and replaced by a technique that allows for higher modulation frequencies and has less and smaller systematic errors. A good candidate would be modulation transfer spectroscopy 65]. The locking bandwidth achievable with this method can by far exceed the acoustic frequency domain. This would further increase the lock stability.

In the old setup the repumping laser on the $F=1 \rightarrow F^{\prime}=2 D_{2}$-transition has its own spectroscopy system. But since the $F=1 \rightarrow F^{\prime}$-transitions all have rather low oscillator strength, the lock of the repumper does not work particularly well. In the new setup the repumper should also be phase- or frequency-locked to the master laser using the same methods as described in section 5.3.

CHAPTER 6

## Conclusion

I have presented an improved setup for a quasi-continuous guided atom laser. Quasicontiunous means that the flux out of the BEC is continuous until the source of $1.5 \cdot 10^{5}$ atoms is depleted. The injection efficiency of atoms into the guide is $100 \%$ since the BEC is directly created in its trapping potential. Due to the horizontal geometry of the guide, the outcoupled atoms are not accelerated by gravity. The acceleration of the atoms during propagation is adjustable and can be close to zero. As demonstrated in Chapter 4.4, the de Broglie wavelength is constant over a long propagation distance ( 1 mm ). The actual value of the de Broglie wavelength is controlled by the applied outcoupling RF frequency. Being transversally confined, the atoms in the guide propagate quasi-one-dimensional. The flux of atoms leaving the condensate is adjustable by varying the power of the RF field. Ability to continuously change the flux allows to tune between the regimes of high and low linear atomic densities.

We have measured the outcoupling rate as function of detuning from resonance. An asymmetry of the lineshape has been observed. After several technical improvements, the resonance width was reduced to the order of the chemical potential of the source condensate.

In Chapter 4.3 the transverse mode of the guided atom laser has been studied. It was found that the atoms were not propagating in the ground state of the radially confining potential of the guide. The mean value of their transverse quantum number was found to be $\langle n\rangle=2.2 \pm 0.4$. There is no evidence that all atoms in the guide actually occupy the same transverse mode. It is likely that atoms are present in various transverse modes, each also having different longitudinal energies.

## Near future

Finally, the cooling laser system for the next generation of the experiment has been presented. A more reliable setup in combination with modifications of the vacuum apparatus to allow for better optical access will make it possible to use the atom laser as tool for other studies.

But first several further investigations of the properties of the guided atom laser are required. With the improved magnetic field configuration, the outcoupling resonance curve has to be investigated in more detail. Also a theoretical study of the coupling rate including the possible population of higher transverse modes is necessary. This is of particular interest with respect to selection of the de Broglie wavelength. If the outcoupling rate is to be kept fixed when changing the RF frequency (and thus the wavelength), the resonance curve has to be well known so the RF power can be
adjusted accordingly.
It will also be necessary to improve the transverse mode of the atoms in the guide. A configuration has to be found in which all atoms occupy the same mode, ideally the lowest one. Having the optical barrier at our disposal, the longitudinal energy of the guided atom laser can now be studied more thoroughly.

Eventually it will be necessary to prove the coherence properties of the outcoupled atoms by creating an interference pattern between two or more guided atom lasers.

In the new experiment some technical improvements of the apparatus will be made. The availability of more optical power in the new cooling lasers allows to also transversally cool the beam in the Zeeman slower, using high-intensity optical molasses [70]. Sisyphus cooling similar to the effect described in Chapter 2.2.4.1 will then reduce the transverse spread of the atomic beam. As discussed in Chapter 5.5 this, combined with larger MOT beams will increase the number of available atoms before evaporation. A shorter MOT loading along with an optimized evaporation sequence will reduce the duration of the experimental cycles.

The vacuum chamber in the new experiment will feature an additional area to which the BEC can be transported with the optical tweezer [71. Good optical access (high numercal aperture) to that area will then allow to create optical potentials with smaller sizes than the current barrier. With the barrier size small compared to the de Broglie wavelength, additional quantum effects can then be studied. It will then also be possible to prepare a new sample in the MOT cell while manipulating the previous sample in the new science chamber. The new atomic sample might be used to replenish the BEC in the science chamber [72].

## Applications

Once the different technical problems of the setup are under control, a quasicontinuous, adjustable flux of coherent matter-waves will be available with controllable de Broglie wavelength. The setup can then be used as a source of matter-waves to study other systems.

The constant wavelength, in combination with the fact that the atoms are in a guided geometry, renders our setup as a useful tool to study quantum transport phenomena. The adjustable flux can be very useful, since, by adjusting the flux, the linear density can be varied and one has the ability to continuously tune between the non-interacting and the interacting regime. A system similar to our optical barrier could be used to create an attractive potential of variable height. In combination with the adjustable interaction strength of our guided atoms, this could be used to study the problem of saturating quantum reflection at attractive potentials[73].

With the double bump potential which we have implemented, it will also be possible to create the matter-wave analogon to an optical Fabry-Pérot cavity. In such a system the interaction between atoms can quickly cause nonlinear behavior [10.

Various problems can be studied with our quasi-1-dimensional system that are different or not present in higher dimensionalities. For example Anderson localization could be easier to observe in one dimension than in 2-D or 3-D[74].

Thus, it is worth fixing the remaining imperfections of the experiment and to
characterize it well in order to pave the way for future work. After these steps, the guided atom laser can be converted from a subject of study in itself into a new tool for research on other systems.

## APPENDIX A

## Light Polarization

Light can have linear, circular or a superposition of the two (called elliptical) polarization. In a cartesian coordinate system with orthonormal unit vectors $\hat{x}, \hat{y}, \hat{z}$ one can define the polarization vectors

$$
\begin{align*}
\sigma^{+} & :=\frac{1}{\sqrt{2}}(\hat{x}+i \hat{y})  \tag{A.1}\\
\sigma^{-} & :=\frac{1}{\sqrt{2}}(\hat{x}-i \hat{y}) \tag{A.2}
\end{align*}
$$

for light propagating in the $\hat{z}$-direction. These vectors describe circular polarized light since the $\hat{y}$-components are $90^{\circ}$ advanced or retarded with respect to the $\hat{x}$-component at same amplitude.

In the context of optical pumping of atoms we call light $\sigma^{+}-\left[\sigma^{-}\right]$-polarized when its wavevector $\mathbf{k}$ is parallel to the quantization axis $\hat{z}$ and the spin of the photons is in [against] the direction of the quantization axis. The electric field component is then described by (A.1) [(|A.2)]. We refer to $\pi$-polarized light as light whose electric field component $\mathbf{E}$ is parallel to the quantization axis. Its magnetic field component $\mathbf{B}$ is then perpendicular to it since $\mathbf{B} \propto \mathbf{k} \times \mathbf{E}$.


Figure A.1:

## Light polarization

$\sigma^{-}$-polarized light propagating in the $\hat{z}$ direction. Due to the phase shift between the $\hat{x}$ - and $\hat{y}$-component, the sum rotates around the propagation axis.

## APPENDIX B

## Some Useful Formulas for Signal Modulation and Demodulation

The following simple but very useful formulas for the multiplication of signals at different frequencies are used on various occasions throughout this text.
Use $\cos x=\frac{1}{2}\left(e^{i x}+e^{-i x}\right)$ and $\sin x=\frac{1}{2 i}\left(e^{i x}-e^{-i x}\right)$. Then

$$
\begin{align*}
\sin x \cdot \sin y & =-\frac{1}{4}\left(e^{i x}-e^{-i x}\right)\left(e^{i y}-e^{-i y}\right) \\
& =-\frac{1}{4}\left(e^{i(x+y)}-e^{-i(x+y)}-e^{i(x-y)}+e^{-i(x-y)}\right) \\
& =\frac{1}{2} \cos (x-y)-\frac{1}{2} \cos (x+y) \tag{B.1}
\end{align*}
$$

Similarly

$$
\begin{align*}
\cos x \cdot \cos y & =\frac{1}{2} \cos (x-y)+\frac{1}{2} \cos (x+y)  \tag{B.2}\\
\sin x \cdot \cos y & =\frac{1}{2} \sin (x-y)+\frac{1}{2} \sin (x+y) \tag{B.3}
\end{align*}
$$

## APPENDIX C

## Rubidium-87 Levels



Figure C.1: Rubidium $87 D_{2}$ Line Energy Diagram adapted from [75].

## APPENDIX D

## Laser Frequency Generation



Figure D.1: Laser frequency generation
All lasers used for cooling and imaging are diode lasers. Master lasers are grating-stabilized and locked to rubidium resonances using saturated absorption spectroscopy. Slave lasers are injected by beams derived from master lasers.

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## APPENDIX E

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## Everywhere

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[^0]:    ${ }^{1}$ The ratio $E_{i n t} / E_{k i n}$ can be estimated as follows: The ground state wavefunction of a harmonic oscillator with frequency $\bar{\omega}$ is a Gaussian function of width $l=\sqrt{\hbar / m \bar{\omega}}$. Accordingly the density is of the order $N l^{-3}$. The ratio $E_{\text {int }} / E_{k i n} \approx g N^{2} l^{-3} / N \hbar \bar{\omega} \propto N a / l$.

[^1]:    ${ }^{2}$ The emission direction is not completely random due the anisotropy of the dipole radiation pattern. But since the pattern is symmetrical, the transferred momentum averages out over many emission cycles.
    ${ }^{3}$ When considering single atoms then the phrase "population of a state" refers to the probability of the atom to be in this state.

[^2]:    ${ }^{4}$ For definition of light polarizations see Appendix A

[^3]:    ${ }^{5}$ Due to the $\Delta F=0, \pm 1$ selection rule only transitions to $F^{\prime}=0,1,2$ could be driven. But these are all far out of resonance.

[^4]:    ${ }^{6}$ For $\pi$-polarized pumping the transition rate for $|1, \pm 1\rangle \rightarrow|1,0\rangle$ scales as $3 \cdot 3=9$ (Fig. 2.8) while the rate for $|1,0\rangle \rightarrow|1, \pm 1\rangle$ scales as $4 \cdot 1=4$. Thus the steady state population is $4 / 17,9 / 17$, $4 / 17$ for the states $|1,-1\rangle,|1,0\rangle,|1,1\rangle$, respectively.

[^5]:    ${ }^{7}$ e.g. cesium hyperfine clock transition measurements and the cesium photon recoil measurement [5].

[^6]:    ${ }^{8}$ Another way to think about this is to realize that in regions of very weak magnetic field the quantization axis is not well defined and Majorana flips into untrapped or antitrapped states can occur.

[^7]:    ${ }^{9}$ To understand the connection to heating consider the case of a magnetic trap. Collision rates are largest at the bottom where the density is highest. But there the atoms have energies lower than average. If now one of these atoms spin-flips into an $m_{F} \geq 0$ state then it gets lost from the sample leaving the other atoms at higher average energy.
    ${ }^{10}$ For a very recent derivation in the context of cold atoms see the lecture notes of C. CohenTannoudji at the workshop on quantum gases April 23 to July 20, 2007 held at Institut Poincaré, Paris. Lecture notes are available online at http://www.phys.ens.fr/\%7Ecastin/cohen1.pdf.

[^8]:    ${ }^{11}$ The expectation value is then given by the integral of an anti-symmetric function over a symmetric interval.

[^9]:    ${ }^{1}$ The atoms cycle between $\left|F=2, m_{F}= \pm 2\right\rangle$ and $\left|F=3, m_{F}= \pm 3\right\rangle$

[^10]:    ${ }^{2}$ Optical pumping to $m_{F}=-1$ with $\sigma^{-}$-polarized light proved not to be very efficient because of the cloud's high density causing multiple scattering events to occur. Since the scattered light has random polarization, it disturbs the optical pumping.
    ${ }^{3}$ Both coil pairs are in Helmholtz configuration. "Anti" here refers to the fact that one coil pair creates a field in the opposite direction than the other one

[^11]:    ${ }^{4}$ Strictly speaking the RF sweep is not one long linear ramp but rather consists of many smaller sections with different slopes and durations. The segments are optimized experimentally by looking at the atom number, elastic collision rate, etc. after each step.
    ${ }^{5}$ For this, the tweezer is aligned to have the focal point slightly below the center of the magnetic trap to account for the gravity sag.

[^12]:    ${ }^{1}$ Strictly speaking there is still the quadratic Zeeman effect and the longitudinal trapping due to

[^13]:    the tweezer. But these two effects have opposite sign and similar magnitude, so they approximately cancel each other.

[^14]:    ${ }^{2}$ The lowest possible value for $E_{\perp}$ is $\hbar \omega_{x, y, o p t}$, so the maximum available longitudinal energy is not exactly $E$ but rather $E-\hbar \omega_{x, y, o p t}$.

[^15]:    ${ }^{3}$ It these waves still had a fixed phase relation, a beat analogous to the output of a mode-locked optical laser should be seen.

[^16]:    ${ }^{1}$ This width includes inhomogeneous broadening.

[^17]:    ${ }^{2}$ Note that reference level in this picture is 20 dB above carrier.

