Project Report

# A Hollow-Core Beam



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### Chapter 1

## Introduction

In the so-called "chip" experiment transport phenomena in strongly interacting degenerate Fermi gases of <sup>40</sup>K are studied. The micro-fabricated chip inside the vacuum chamber allows to apply large magnetic fields and field gradients on short time scales [1]. A Feshbach resonance at  $B = 202.1 \,\text{G}$  is utilized to drive the Fermi gas to the strongly interacting regime and to control the interaction strength. To observe the mentioned effects the cloud of atoms is imaged via absorption imaging after turning off all trapping potentials. This time of flight measurement allows to extract the momentum distribution of the Fermi gas.

Compared to an ideal Fermi gas, the system we want to study, a trapped gas has a non-uniform density distribution. This prevents us from observing a sharp step in the momentum distribution according to Fermi-Dirac statistics. It is also harder to compare the experimental results to theory. As first realized in [2] the idea of this project is to transfer the atoms at the outer region of the atomic cloud to a dark state making them invisible for imaging. This way only the center of the cloud with a more uniform density is probed. A so-called hollow-core beam with zero intensity at its center is used for this purpose.

As shown in figure (1.1) the D2 line of <sup>40</sup>K, the transition between the ground state  ${}^{2}S_{1/2}$  and the excited state  ${}^{2}P_{3/2}$  is used. Although operating at high magnetic fields  $B \approx 210 \text{ G}$  at the zero-crossing of the Feshbach resonance the atomic states are labeled by the low-field quantum numbers  $|F, m_F\rangle$  they adiabatically connect to. The cycling transition  $|F = 9/2, m_F = -9/2\rangle \leftrightarrow |F' = 11/2, m'_F = -11/2\rangle$  is used for absorption imaging. Before the final imaging step atoms are transferred



Figure 1.1: Simplified level scheme of the <sup>40</sup>K D2 line. The cycling transition  $|F = 9/2, m_F = -9/2\rangle \leftrightarrow |11/2, -11/2\rangle$  is used to image the atomic cloud. The  $|9/2, -7/2\rangle \leftrightarrow |5/2, -5/2\rangle$  transition is driven by the hollow beam to optically pump atoms at the outer region of the cloud to the  $|7/2, -7/2\rangle$  dark state. The branching ratios of spontaneous emission from  $|5/2, -5/2\rangle$  are .955 and .044 [2].

to the  $|9/2, -7/2\rangle$  state where a pair of hollow-core beams drive the  $|9/2, -7/2\rangle \leftrightarrow$  $|5/2, -5/2\rangle$  transition. This transition is dipole-forbidden at zero magnetic field, but has a finite excitation probability at the desired magnetic field. Spontaneous emission with a branching ratio of 95.5 % transfers the atoms in the  $|7/2, -7/2\rangle$  dark state [2].

The goal of this project is to set up a laser system as described in chapter 2 and to generate a laser beam featuring a hollow core as shown in chapter 3.

## Chapter 2

## Laser Setup

#### 2.1 Laser Frequency



Figure 2.1: Hyperfine energy shift of the ground state ( ${}^{2}S_{1/2}$ , left) and excited state ( ${}^{2}P_{3/2}$ , right) as a function of magnetic field. The highlighted curves correspond to the  $|F = 9/2, m_F = -7/2\rangle$  and  $|F' = 5/2, m'_F = -5/2\rangle$  states, respectively.

The desired laser frequency  $\nu_L$  at  $B = 210 \,\text{G}$  of the hollow beams driving the  ${}^2\text{S}_{1/2} |9/2, -7/2\rangle \leftrightarrow {}^2\text{P}_{3/2} |5/2, -5/2\rangle$  transition in  ${}^{40}\text{K}$  needs to be calculated. As derived in [3] the hyperfine structure Hamiltonian for arbitrary magnetic fields B consists of terms for angular momentum coupling between **I** and **J**, a quadrupole shift for  $I, J \neq 1/2$  and the Zeeman effect.

$$H_{hfs} = \text{I-J coupling} + \text{Quadrupole shift} + \text{Zeeman shift}$$
$$= A_{hfs} \mathbf{I} \cdot \mathbf{J} + B_{hfs} \frac{3(\mathbf{I} \cdot \mathbf{J})^2 + \frac{3}{2}\mathbf{I} \cdot \mathbf{J} - \mathbf{I}^2 \mathbf{J}^2}{2I(2I-1)J(J-1)} + \frac{\mu_B}{\hbar}(g_J m_J + g_I m_I)B \qquad (2.1)$$

A state vector  $|\psi\rangle$  containing each possible state  $|m_J, m_I\rangle$  of the desired manifold is created. It spans a Hamiltonian matrix  $E_{hfs} = \langle \psi | H_{hfs} | \psi \rangle$  which is diagonalized numerically to obtain the hyperfine energy shifts. The results are shown in Figure (2.1). Table (2.2) lists the values at B = 210 G and the obtained transition frequency  $\nu_L$ . The constants used in this calculation are extracted from [4, 5].

#### 2.2 External-Cavity Diode Laser

In 2010 Matthias Scholl created a set of external-cavity diode lasers (ECDLs) stabilized by interference filters [6]. Undergoing continuous development this design has been simplified to reduce costs while maintaining stability and has been successfully tested for wavelengths of 405 and 767 nm [7]. The current revision C by Simon Heun features a front-clamped laser diode aiming for its easier alignment. As shown in figure (2.2, left) the design utilizes an interference filter in contrast to the common grating-stabilized Littrow configurations. Separating laser feedback and the wavelength-selective element leads to increased tunability and reduced sensitivity to misalignment [8].



Figure 2.2: (left) Internal setup of the laser consisting of an anti-reflex coated laser diode, an interference filter and an output coupler in cat's-eye configuration. (right) Output power over diode current measured after the optical isolator (red). A linear fit (blue) determines the lasing threshold current  $I_{TH} = 24$  mA.

The output of an anti-reflex coated laser diode is collimated by the first lens and fed through the interference filter. Two lenses in cat's-eye configuration focus the beam on the output coupler mounted on a piezo ring actuator. The lense after the coupler is used to collimate the beam. The desired wavelength is coarsely adjusted by turning the interference filter, a Fabry-Pérot etalon. Figure (2.2, right) shows the output power after the optical isolator over the current through the laser diode. A linear fit determines the lasing threshold current  $I_{TH} = 24 \text{ mA}$ . This is a simple indicator for the quality of the external cavity's feedback. Table (2.1) shows settings of the temperature, diode current and piezo controller for stable operation at the desired laser frequency  $\nu_L$ .

symbol	value	description
$R_{TH}$	$11.731\mathrm{k}\Omega$	thermistor resistance
$I_{LD}$	$41.81\mathrm{mA}$	laser diode current
$V_{PZ}$	$32\mathrm{V}$	piezo voltage

Table 2.1: Laser settings.

Unfortunately the laser has stability issues after moving the breadboard to the laser table. The breadboard is mounted on 8 inch pedestals leading to increased sensitivity to acoustic and mechanical noise. A correlation between the mode jumps of the laser and a shutter placed nearby was observed. Watching the frequency of the laser on a wavemeter revealed oscillations with an amplitude of  $\approx 1 \text{ GHz}$  and a period on the order of some minutes. With a mode spacing of the laser's external cavity of 1.7 GHz [6] these oscillations lead to frequent mode jumps of the laser. The time scale implies a problem in temperature stabilization of the laser. Turning down the proportional and integral part of the temperature controller's feedback loop decreased the oscillations. This increases the time between mode jumps to roughly 30 min, which is not satisfying. The wavemeter still shows a slow drift in frequency. Further work needs to be done to shield the laser from ambient temperature fluctuations.

#### 2.3 Optics

As illustrated in figure (2.3) the optical setup consists of two parts, the beat note generation and two acousto-optic modulators (AOMs) in double-pass configuration. The latter allow to modulate the laser intensity on the order of tens of MHz with very high extinction ratios of 1 000 000 : 1 as well as fine-tuning the frequency over a range of  $\approx 20$  MHz. Running at 80 MHz both AOMs introduce a frequency shift of +160 MHz. The perpendicularly polarized output of both branches is coupled into a polarization-maintaining optical fiber. After the fiber both polarizations are separated by a polarizing beam splitter with an extinction ratio of  $\approx 300 : 1$  to get



Figure 2.3: Optical setup. The two double-pass AOM configurations (bottom) allow to switch the laser fast and fine-tune the frequency. About 700  $\mu$ W are superimposed with a reference laser on a glass plate and coupled into an optical fiber to detect the beat note signal on a fast photo diode.

two independently switchable laser beams targeting the long and short axis of the atom cloud.

#### 2.4 Beat Note & Offset Lock

To reliably drive atomic transitions with linewidths on the order of a few MHz the laser's frequency needs to be stabilized. In this project the laser with frequency  $\nu_L$  is offset-locked to a reference laser  $\nu_{Ref}$  locked to the D2 line in <sup>39</sup>K at zero magnetic field. Light of both lasers is superimposed on a photodetector to obtain the frequency  $\nu_{Beat}$  of the beat note.

The corresponding electrical field modes of two laser beams with polarization vectors  $\mathbf{E}_i$ , frequencies  $\omega_i$  and relative phase  $\varphi$  can be described as

$$\mathbf{E}(t) = \mathbf{E}_L \cos(\omega_L t + \varphi) + \mathbf{E}_{Ref} \cos(\omega_{Ref} t) .$$
(2.2)

A photodetector detects the intensity I(t) with a finite band with on the order of a few GHz. Therefore all terms oscillating with optical frequencies are negligible:

$$I(t) \propto |\mathbf{E}(\mathbf{t})|^{2}$$

$$= E_{L}^{2} \cos^{2}(\omega_{L}t + \varphi) + E_{Ref}^{2} \cos^{2}(\omega_{Ref}t) + 2\mathbf{E}_{L}\mathbf{E}_{Ref} \cos(\omega_{L}t + \varphi) \cos(\omega_{Ref}t)$$

$$\approx \mathbf{E}_{L}\mathbf{E}_{Ref} \left[\cos\left((\omega_{L} + \omega_{Ref})t + \varphi\right) + \cos\left((\omega_{L} - \omega_{Ref})t + \varphi\right)\right]$$

$$\approx \mathbf{E}_{L}\mathbf{E}_{Ref} \cos\left(\underbrace{(\omega_{L} - \omega_{Ref})t + \varphi}_{\omega_{Beat}}\right) + e^{2}\right)$$

$$(2.3)$$

The scalar product  $\mathbf{E}_L \mathbf{E}_{Ref}$  shows that the intensity is proportional to both electric field amplitudes. Two polarizing beam splitters ensure that both beams have the same polarization to maximize the scalar product's value. About 60  $\mu$ W of light are coupled into the fiber for the photodetector. The obtained beat-note signal is shown in figure (2.5, right). Its width is determined by both laser spectra. Therefore the measured width  $\Delta \nu \approx 1.8$  MHz (FWHM) of the beat note is an upper bound for the laser's spectral width itself. As outlined in Table (2.2) the desired beat note frequency  $\nu_{Beat}$  is 1836.57 MHz.

symbol	frequency [MHz]	description
$\nu_L$	391 016 296.05	$^{40}$ K D2 line
	+ 819.89	$F = 9/2, m_F = -7/2$ at $B = 210 \mathrm{G}$
	+ 581.51	$F' = 5/2, m'_F = -5/2$ at $B = 210 \mathrm{G}$
	- 160.00	double-pass AOM at $80\mathrm{MHz}$
$\nu_{Ref}$	391 016 170.03	$^{39}$ K D2 line
	- 469.15	F = 2 at $B = 0$ G and AOMs
$\nu_{Beat}$	$1 \ 836.57$	$ u_L - \nu_{Ref} $
$ u_{VCO}$	1 811.57	$\nu_{Beat} - 25 \mathrm{MHz}$

Table 2.2: Breakdown of the different transition frequencies and frequency shifts contributing to the beat note frequency  $\nu_{Beat}$ . The desired frequency of the voltage-controlled oscillator  $\nu_{VCO}$  is shifted by 25 MHz due to the locking scheme.

Introduced in [9] the Grimm-type locking scheme offers a simple and inexpensive method to stabilize the beat frequency. The setup here is based upon the work of Daniel Fine for the high-field imaging system [10].

As outlined in figure (2.4) the beat note signal recorded by a *Thorlabs DET025AFC* photo diode is amplified twice by  $+30 \,\mathrm{dB}$  and mixed with the output of a VCO



Figure 2.4: Offset lock schematics. The beat note signal is mixed with the output from a VCO and split into two lines. One of these is  $\Delta l = 2.08 \text{ m}$  longer and therefore introduces a phase delay. Mixing both lines and low-pass filtering gives the error signal.

running at  $\nu_{VCO}$ . The resulting signal is further amplified by +20 dB and lowpass filtered ( $\nu_0 = 35 \text{ MHz}$ ). As for the beat note itself this leads to the difference frequency  $\omega_{Lock}$ .

$$U(t) \propto \cos\left[\left(\underbrace{\omega_{Beat} - \omega_{VCO}}_{\omega_{Lock}}\right) t\right]$$
(2.4)

The low-pass filter surpresses the sum frequency as well as higher-order terms which are due to nonlinearities in the circuit. This signal is then split in two lines. One of these is  $\Delta l = 2.08$  m longer introducing a phase delay. In the used *RG178* cable the propagation velocity is  $\tilde{c} = .694 c$  leading to a time delay of  $\Delta t = \Delta l/\tilde{c}$ . Mixing the two lines results in:

$$U(t) \propto \cos[\omega_{Lock}t + \omega_{Lock}(t + \Delta t)] + \cos[\omega_{Lock}t - \omega_{Lock}(t + \Delta t)]$$
  
=  $\cos[\omega_{Lock}(2t + \Delta t)] + \underbrace{\cos[\omega_{Lock}\Delta t]}_{U_{Error}}$  (2.5)

This signal is low-pass filtered again ( $\nu_0 = 500 \text{ kHz}$ ) to obtain the error signal  $U_{Error}$  corresponding to the right term. For the feedback-loop a zero-crossing of this signal is needed:

$$U_{Error} = \cos[\omega_{Lock}\Delta t] = 0 \quad \Leftrightarrow \quad \frac{\pi}{2} = \omega_{Lock}\Delta t = \omega_{Lock}\frac{\Delta l}{\tilde{c}}$$
(2.6)

This determines the lock frequency to be  $\omega_{Lock} = \omega_{Beat} - \omega_{VCO} = 2\pi \cdot 25 \text{ MHz}$ for the chosen cable and its length. The signal around subsequent zero-crossings is surpressed by the first low-pass filter leaving only the first accessible for a lock. Figure (2.5, left) shows the error signal on an oscilloscope while ramping the piezo voltage. The picture on the right shows the output of the spectrum analyzer with the sharp peak of the VCO and the locked beat-note signal  $\omega_{Lock} = 2\pi \cdot 25 \text{ MHz}$  apart from it.



Figure 2.5: (left) The offset-lock signal on an oscilloscope obtained by ramping the piezo voltage of the laser. The laser is locked to the zero-crossing on the left at  $t \approx -1 \,\mathrm{ms.}$  (right) As shown in the spectrum this results in a constant offset of 25 MHz between VCO and beat note frequency. The horizontal scale is 5 MHz/DIV and the vertical scale is 10 dB/DIV, attenuated by 30 dB.

### Chapter 3

### Hollow-Core Beam

#### 3.1 Spiral Phase Plates

A hollow-core beam is a laser beam featuring zero intensity at the optical axis. [11] gives an in-depth overview of the different types of hollow-core beams and the different methods to generate them, i. e. using a pair of axicons or a forked diffraction grating. With the recent advancements in litography spiral phase plates (SPPs) became commercially available. A SPP is the most straightforward way of generating an optical vortex and therefore a hollow-core beam. It is a glass plate coated with a substrate with optical thickness increasing proportional to the azimuth angle  $\phi$ . Therefore it imprints a phase factor  $e^{il\phi}$  onto an incident beam. The integer l is called the "charge" of the resulting approximation to a Laguerre-Gaussian beam LG<sub>l0</sub>, a type of hollow-core beam. The SPP VPP-1b (see figure 3.6) used here features charge l = 1 vortices for various wavelengths  $\Lambda$ . Although there is a mismatch between the wavelength  $\lambda = 767$  nm of the laser and the design wavelength  $\Lambda = 735$  nm of the vortex in use the obtained hollow-core beam is well-defined. For the following calculations this mismatch is taken into account by defining an effective charge  $l = \lambda/\Lambda \approx 1.04$ .

#### 3.2 Propagation Dynamics

The electric field amplitude of an incident  $\text{TEM}_{00}$  Gaussian beam with wave number  $k = \frac{2\pi}{\lambda}$ , waist  $w_0$  at z = 0 and Rayleigh range  $z_0 = \frac{\pi w_0^2}{\lambda}$  at position  $(r_1, \phi_1, z_1)$  is

$$E_G(r_1,\phi_1,z_1) = E_0 \frac{w_0}{w(z_1)} \exp\left[-\frac{r_1^2}{w(z_1)^2}\right] \exp\left[-ikz_1 - i\frac{kr_1^2}{2R(z_1)} + i\zeta(z_1)\right]$$

The usual abbreviations for the width  $w(z_1) = w_0 \sqrt{1 + (z_1/z_0)^2}$ , the radius of curvature  $R(z_1) = z_1[1 + (z_1/z_0)^2]$  and the Gouy phase  $\zeta(z_1) = \arctan(z_1/z_0)$  are used. A SPP placed at this position (see figure 3.1) results in an electric field

$$E(r_1, \phi_1, z_1) = E_G(r_1, \phi_1, z_1) e^{il\phi}$$
(3.1)

breaking the radial symmetry of the problem.

The propagation of such a transformed Gaussian beam can be calculated numerically via the Fresnel or Fraunhofer diffraction integrals. A less cumbersome way is the application of the Collins-Huygens integral [12], a paraxial variation of the Fresnel integral in terms of the ABCD matrix formalism. This yields an analytical description of the electric field  $E(r, \phi, z)$  of a Gaussian beam modified by a SPP and an ABCD lens system:

$$E(r,\phi,z) = -\frac{i}{\lambda B} e^{ikz} \int_0^\infty \mathrm{d}r_1 \, r_1 \int_0^{2\pi} \mathrm{d}\phi_1 \, E(r_1,\phi_1,z_1) \times \exp\left[\frac{ik}{2B} \left(Ar_1^2 + Dr^2\right)\right] \exp\left[-\frac{ikrr_1}{B}\cos(\phi - \phi_1)\right]$$
(3.2)

As shown in [13] substituting the incident electric field from equation (3.1) and



Figure 3.1: An incident Gaussian beam with waist  $w_0$  at z = 0 is altered by a spiral phase plate at  $z = z_1$  followed by propagation through an ABCD lens system.

introducing abbreviations

$$E_{00}(r, z_1) = E_0 \frac{w_0}{w(z_1)} \exp\left[\frac{ikr^2D}{2B}\right] \exp[-ikz_1 + i\zeta(z_1)]$$
(3.3)

$$\frac{1}{R_c^2(z_1)} = \left[\frac{1}{w^2(z_1)} + \frac{ik}{2R(z_1)} - \frac{iAk}{2B}\right]$$
(3.4)

$$\frac{1}{r_c} = \frac{kr}{B} \tag{3.5}$$

leads to the closed-form solution

$$E(r,\phi,z) = 2\pi^{3/2} (-i)^{|l|+1} \frac{E_{00}(r,z_1) R_c^3}{8r_c \lambda B} \times e^{ikz} e^{il\phi} e^{-\frac{R_c^2}{8r_c^2}} \left[ I_{\frac{1}{2}|l|-\frac{1}{2}} \left(\frac{R_c^2}{8r_c^2}\right) - I_{\frac{1}{2}|l|+\frac{1}{2}} \left(\frac{R_c^2}{8r_c^2}\right) \right] .$$
(3.6)

 $I_n(x)$  denotes the modified Bessel functions of the first kind. The angle dependence  $e^{il\phi}$  corresponds to a helical propagation of the wave-front. This effect can be used to transfer orbital angular momentum  $\hbar l$  to an ensemble of atoms, e. g. to rotate a Bose-Einstein condensate in order to create vortices. In this experiment we are only interested in the doughnut-shaped intensity distribution:

$$I(r,z) \propto |E(r,\phi,z)|^{2} = \frac{\pi^{3}}{2^{4}\lambda^{2}} \left| \frac{E_{00}(r,z_{1})R_{c}^{3}}{r_{c}B} e^{-\frac{R_{c}^{2}}{8r_{c}^{2}}} \right|^{2} \left| I_{\frac{1}{2}|l|-\frac{1}{2}} \left( \frac{R_{c}^{2}}{8r_{c}^{2}} \right) - I_{\frac{1}{2}|l|+\frac{1}{2}} \left( \frac{R_{c}^{2}}{8r_{c}^{2}} \right) \right|^{2} .$$
(3.7)

Surprisingly the rotational symmetry along the z axis is restored. This equation is used to obtain the gray theoretical curves in figures (3.2 & 3.3). Unlike a Gaussian beam the shape of the intensity distribution is not maintained along the optical axis. Therefore it is not straightforward to define a width of the beam's hollow core. Instead the first maximum at  $r = r_{max}$  is calculated numerically and used to describe the width of the hollow core. In order to compare this result to images of the beam the rescaled intensity

$$\hat{I}(r,z) = \frac{I(r,z)}{I(r=r_{max},z)} \in [0,1]$$
(3.8)

is introduced.

The free parameters of this calculation are shown below along with the values used for these figures.

symbol	value	description
$w_0$	$1.07\mathrm{mm}$	waist size of incident Gaussian beam
$z_1$	$-5\mathrm{m}$	distance of waist to SPP
$\lambda$	$767\mathrm{nm}$	wavelength
$\Lambda$	$735\mathrm{nm}$	design wavelength of the SPP
$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$	see below	ray transfer matrix
-	$\left(\begin{array}{cc}1&z\\0&1\end{array}\right)$	M in figure $(3.2, \text{ top})$
	$\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/0.3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.05 \\ 0 & 1 \end{pmatrix}$	M in figure $(3.2, bottom)$
	$\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/0.3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2.0 \\ 0 & 1 \end{pmatrix}$	M in figure $(3.3)$

Table 3.1: Overview of the free parameters of equation (3.7) along with the values used for the curves in figures (3.2 & 3.3).

#### 3.3 Data



Figure 3.2: Propagation of the hollow-core beam in free space (top) and after a lens with focal length f = 300 mm (bottom). The images shown are taken with a CCD camera at distance z after the SPP and the lens, respectively. The calculated intensity distribution (gray) is shown along with the data obtained by a cut through the center of the image (orange).  $r_{max}$  denotes the position of the first maximum.

For comparison with this theoretical prediction the intensity distribution of the hollow beam is recorded with a CCD camera, an inexpensive *Logitech* webcam. The images shown in figures (3.2 & 3.3) are taken by subtracting the background and averaging over 30 such frames. A cut through the center shown in orange gives the intensity distribution I(r, z) below at a certain distance z after the last optical element. Both the theoretical curve (gray) and the data (orange) are rescaled to  $\hat{I}(r = r_{max}, z) = 1$  and agree well visually. Most of the deviation compared to the

theoretical curve is due to the CCD camera. Its intensity scaling is prone to be nonlinear and the images show diagonal fringes due to the sensor's coating being in the same range as the incident wavelength.

While figure (3.2, top) shows the beam propagating in free space with z measured directly after the SPP, figure (3.2, bottom) shows the beam after an achromatic lens doublet with a focal length of f = 300 mm placed d = 50 mm behind the SPP. Figure (3.3, left) shows the same lens placed d = 2.0 m behind the SPP. Surprisingly a second focal spot appears at z = 351 mm after the lens. With the first maximum at  $r_{max} = 26.9 \,\mu\text{m}$  this focal spot features an even narrower hollow core compared to the conventional focal spot of the lens with  $r_{max} = 56.8 \,\mu\text{m}$ .

Figure (3.3, right) shows the position of the maximum over the distance to the lens placed d = 1, 2, 3, 4, 5 m after the SPP. This reveals a slight shift by few mm of the conventional focal spot due to a non-perfect collimation of the incident Gaussian beam ( $z_1 = -5$  m). A perfectly collimated beam would have a waist placed far away from the SPP, therefore  $z_1 \to \pm \infty$ . But more importantly the second, narrower focal spot moves towards the first for increasing d.



Figure 3.3: (left) Images of the first and second focal spot at z = 300 mm and z = 351 mm after a f = 300 mm. (right) Position of the maximum  $r_{max}$  over distance z after the lens placed d = 1, 2, 3, 4, 5 m behind the SPP. For increasing d the second focal spot moves towards the first.

For the optical pumping process it is crucial that the center of the hollow-core beam has zero intensity in order to preserve the density of atoms at the center of the cloud. Therefore the ratio of power  $P(r_H)$  through several pinholes with radii  $r_H$  over the full power of the beam  $P_0$  is measured. A higher power beam  $(P_0 = 117 \text{ mW}, \lambda = 855 \text{ nm}, \Delta \lambda = 10 \text{ nm})$  is chosen to not be limited by the power meter's resolution. The normalized intensity function  $\hat{I}(r) \in [0, 1]$  gets an offset  $\Delta I$  as a fit parameter for the through-hole intensity:

$$\frac{P(r_H)}{P_0} = \frac{\int_0^{r_H} dr \, r \, \left[\hat{I}(r) + \Delta I\right]}{\int_0^\infty dr \, r \, \hat{I}(r)}$$
(3.9)

The resulting power fractions are shown in figure (3.4, left) without (black) and with an offset of  $\Delta I = 10^{-3}$  (red) acquired through a fit.

#### 3.4 Optical Pumping Process



Figure 3.4: (left) Measured and theoretical power fraction through a pinhole with radius  $r_H$  as described by equation (3.9). Compared to the black line, the red line introduces an offset  $\Delta I = 10^{-3}$  to the intensity through the hole. (right) Model of the optical pumping process with a hollow-core beam. The inhomogeneous Gaussian density distribution  $(1/e^2 \text{ radius of } 5\,\mu\text{m}, \text{dashed})$  is altered by the hollow-core beam with offsets  $\Delta I = 0$  (black) and  $\Delta I = 10^{-3}$  (red). The laser power is  $P = 1\,\mu\text{W}$ with a pump time of  $\tau = 100\,\mu\text{s}$ .

With the rescaled intensity distribution  $\hat{I}(r)$  according to equation (3.8) at hand we are able to model the optical pumping process as described in [2]. As outlined in figure (2.1) the hollow-core beams drive the transition  $|F = 9/2, m_F - 7/2\rangle \leftrightarrow$  $|5/2, -5/2\rangle$  with a branching ratio of  $\eta = .044$  to pump atoms in the outer region of the cloud to a dark state. After transfer to the  $|9/2, -9/2\rangle$  state the density distribution of the remaining atoms can be imaged. Therefore the position-dependent probability p(r) of an atom remaining in the  $|9/2, -7/2\rangle$  state is of interest. In other words such an atom scatters zero photons with the probability p(r). Assuming a two-level system this probability is given by

$$p(r) = \exp[-\gamma(r) \sigma \tau]$$
(3.10)

with photon flux  $\gamma(r) = I(r)\frac{\lambda}{hc}$ , resonant absorption cross section  $\sigma = \frac{3}{2\pi}\lambda^2\eta$  and pump time  $\tau$ . The photon flux is proportional to the derived intensity distribution

$$I(r) = I_0 \left[ \hat{I}(r, z = f) + \Delta I \right] = \frac{P}{2\pi \int_0^\infty \mathrm{d}r \, r \, \hat{I}(r, z = f)} \left[ \hat{I}(r, z = f) + \Delta I \right] \quad (3.11)$$

for a beam with optical power P at the focal spot.

For the sake of simplicity we assume a thermal cloud of cold atoms in a harmonic trap as a target for the hollow-core beams. The inhomogeneous density distribution

$$n_{Inh}(r) = e^{-2r^2/w_c^2} \tag{3.12}$$

is well described by a Gaussian with width  $w_c$ . Shining in the hollow-core beams alters this density distribution leading to a more homogeneous one:

$$n_{Hom}(r) = p(r) n_{Inh}(r)$$
 (3.13)

This behavior is shown in figure (3.4, right) for a cloud width of  $w_c = 5 \,\mu\text{m}$  and an incident beam with power  $P = 1 \,\mu\text{W}$  turned on for  $\tau = 100 \,\mu\text{s}$ . As can be seen there the altered density distribution (black) is narrower. Introducing the offset  $\Delta I = 10^{-3}$  as shown in red leads to a loss of atoms ( $\approx 20 \,\%$ ) in the center of the cloud.

Finally, this calculation demonstrates the general idea of this project and theoretically shows its feasibility with the chosen components.

#### **3.5** Setup in the Experiment

Figure (3.5) shows the proposed setup of the hollow-core beams in the experiment. After the outcoupler the beam is altered by the SPP and split into two parts with



Figure 3.5: Proposed setup of the hollow-core beams in the experiment.

perpendicular polarization. Each beam goes through a lens that focuses the beam on the atom cloud and is then combined on a 50/50 beam splitter. The high field imaging beams drive a  $\sigma^-$  transition ensured by a final quarter-wave plate not shown in the picture. Unfortunately the hollow-core beams are supposed to be  $\sigma^+$  polarized. This can be solved by slightly rotating the wave plate leading to a decreased imaging efficiency.



#### Fact Sheet – Vortex Phase Plate

### Vortex Phase Plate

#### VPP-1b

Properties		
Master	u9182E	
Pattern dimension	~50 x 50	
Vortex aperture	10 x 10mm	
Charge	1	
Material	Polymer replicated on glass substrate	
Transmission spectrum	350-2000nm	
Temperature range	-150°C to 120°C	
Defects	None	



Wavelength Map for $2\pi$ Phase Depth						
339	365	372	375	398		
407	418	432	463	542		
548	539		552	582		
683	671	708	735	794		
858	953	969	1193	1450		
∧ *Pattern side up						



Figure 3.6: Data sheet of the used spiral phase plate VPP-1b manufactured by RPC Photonics. For the final setup the vortex with design wavelength  $\Lambda = 735$  nm is used.

### Chapter 4

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