Electromagnetic Absorption in a Disordered Medium near a Photon Mobility Edge

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A frequency regime in which electromagnetic waves in a strongly disordered medium undergo Anderson localization in $d=3$ dimensions is suggested. In the presence of weak dissipation in $d=2+\epsilon$ it is shown that the renormalized energy absorption coefficient increases as the photon frequency ω approaches a mobility edge ω^* from the conducting side as $\alpha \sim (\omega^* - \omega)^{-(d-2)\nu/2}$, $\nu = 1/\epsilon$. This mobility edge occurs at a frequency compatible with the Ioffe-Regel condition.

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The propagation and absorption of electromagnetic waves in a disordered medium is a subject of considerable interest with a variety of applications in physics, chemistry, and engineering. This has largely been studied experimentally in a regime of scattering strengths in which the photon elastic mean free path l is long compared to other relevant lengths such as the electromagnetic wavelength λ and the mean free path l_{inel} for inelastic processes such as absorption. For example, recent experiments on far-infrared absorption from randomly distributed small metal particles[1] have explored the regime where the elastic mean free path is longer than both of these lengths as well as the sample dimension L ($l \gg L \gg l_{\text{inel}} \gg \lambda$). In light scattering from colloidal suspensions,[2] the medium is said to become turbid, on the other hand, when $L \gg l \gg \lambda$. Here, multiple scattering gives rise to diffusive propagation of electromagnetic energy.

In this Letter we discuss the possibility of yet another qualitatively new behavior in a disordered medium in which elastic scattering is sufficiently strong that $l_{\text{inel}} \gg l \sim \lambda$. In the absence of dissipation, the condition $l \sim \lambda$ is precisely that which has been suggested as a criterion for the Anderson localization transition[3,4] of an electron of de Broglie wavelength λ in a disordered solid.[5] It may likewise be shown from first principles[6] that in $d=2+\epsilon$ dimensions, a localization transition for waves in a disordered medium occurs when $(l/\lambda)^{d-1} \sim 1/\epsilon$, $d>2$. Physically, this occurs as a result of strong wave interference from different scattering events and the consequent renormalization of the photon diffusion coefficient to zero. In the presence of a small imaginary part of the dielectric constant, the near vanishing of the diffusivity gives rise to an anomalous rise in the energy absorption per unit length as the photon mobility edge is approached from the conducting side. Unlike the absorption in a tenuous medium ($l \gg L \gg l_{\text{inel}} \gg \lambda$) which is given simply by $2l_{\text{inel}}^{-1}$, the rise in absorption is mediated by the fundamental change in the nature of transport through the disordered medium. The observation of such behavior in a strongly scattering, weakly dissipative medium is suggested as a probe for the existence of a photon mobility edge.

A plane-polarized electromagnetic wave entering a disordered medium loses memory of its initial polarization as a result of scattering on length scales long compared to the mean free path l. Since the phenomena of diffusion and localization occur on precisely such a scale, we will neglect effects due to the vector nature of the photon field and consider for simplicity the scalar wave equation,

$$\nabla^2\phi = [\epsilon(x)/c^2]\partial^2\phi/\partial t^2, \quad c=1, \tag{1a}$$

where

$$\epsilon(x) = 1 + \epsilon_1(x) + i\epsilon_2 \tag{1b}$$

is the complex dielectric constant and hereafter we work in units in which the velocity of light is unity. For the interested reader, the localization transition for a more general vector field has been derived in a previous paper.[7] It is assumed for simplicity that the imaginary part of the dielectric constant ϵ_2 is uniform throughout the medium and that the disorder which enters the real part ϵ_1 is uncorrelated from point to point in space. The latter assumption is valid provided that the photon wavelength is long enough compared to the actual correlation length of the disorder. The effects of correlations which play a role in determining the existence of a mobility edge have been discussed in detail in a previous paper[8] and will be incorporated as needed.

For photon frequencies ω sufficiently far from the mobility edge such that wave-interference effects can be neglected, the local electromagnetic energy density $E(x,t)$ satisfies a diffusion equation. In particular, for a point source of power P in a medium with weak uniform dissipation as described by (1b), the steady-state form of the equation is

$$0=\partial E(x,t)/\partial t$$
$$=D(\omega)\nabla^2 E(x,t)-\tfrac{1}{2}\epsilon_2\omega E(x,t)+P\delta^d(x). \quad (2)$$

Here, $D(\omega)=lc$ is a diffusion coefficient for photons of frequency ω. Such an equation of radiative transfer is analogous to the Boltzmann equation and follows from the addition of wave intensities rather than wave amplitudes.[9] The generalization to other geometries of experimental interest such as a plane wave (collimated beam) impinging on a slab of disordered material follows readily from an appropriate modification of the source term in (2).

Both the steady-state diffusion equation (2) and its generalization to include the effects of wave interference may be obtained from first principles from the Green's function of the associated wave equation (1a):

$$[\nabla^2+\omega_+^2\epsilon(x)]G(x,0;\omega_+)=\delta^d(x),$$
$$\omega_\pm=\omega\pm i\eta. \quad (3)$$

The ensemble-averaged energy density $E(x,t)$ is determined by the averaged two-particle Green's function $\langle|G(x,0;\omega_+)|^2\rangle_{\text{ensemble}}$ which may be conveniently evaluated by use of a replica functional integral representation.[10,11] Saddle-point evaluation of the functional integrals leads to a nonlinear σ model[6] for which the dimensionless coupling constants are the conductance,

$$\Sigma(\omega)=g^{-1}=\tfrac{1}{2}\pi\rho(\omega)D(\omega)l^{d-2}, \quad (4a)$$

and a replica-symmetry-breaking field,

$$h=\tfrac{1}{4}\pi\epsilon_2\omega\rho(\omega)l^d, \quad (4b)$$

associated with dissipation. Here all lengths are measured in units of the photon elastic mean free path l and $\rho(\omega)$ is the photon density of states. The bare propagator of the nonlinear σ model takes the form

$$\int d^dx\, e^{iq\cdot x}\langle|G(x,0;\omega_+)|^2\rangle_{\text{ensemble}} \propto (h+g^{-1}q^2)^{-1}, \quad (5)$$

in accord with the form suggested by the steady-state diffusion equation (2). The effects of wave interference may now be incorporated systematically by momentum-shell integration of the nonlinear σ model, leading to renormalization-group flows at the one-loop level for the conductance and dissipation in $d=2+\epsilon$ of the form[12]

$$\frac{dg}{d\ln L}=-\epsilon g+\frac{1}{4}\frac{S_d}{(2\pi)^d}\frac{g^2}{1+hg}, \quad (6a)$$

$$\frac{dh}{d\ln L}=dh. \quad (6b)$$

Here, S_d is the surface area of the d-dimensional unit sphere. The form of the propagator (5) suggests that an incident plane wave of intensity I_0 decays with distance x traversed in the disordered medium as

$$I=I_0e^{-\alpha x}, \quad \alpha=(hg)^{1/2}/l. \quad (7)$$

Linearization of the recursion relations about the Anderson-Wegner fixed point $(g^*,h^*)=(4\epsilon(2\pi)^d/S_d,0)$ yields the homogeneity relation

$$\alpha(\Delta g,h)=b^{-1}\alpha(b^\epsilon\Delta g,b^dh), \quad (8a)$$

where

$$\Delta g=g-g^*\propto\omega-\omega^*, \quad (8b)$$

describing the renormalization of the absorption coefficient due to strong wave interference near the photon mobility edge ω^*. With the choice $b=(\Delta g)^{-1/\epsilon}$, it follows that the absorption increases with frequency as

$$\alpha(\omega)\sim h^{1/2}(\omega^*-\omega)^{-(d-2)\nu/2}, \quad \nu=1/\epsilon, \quad (9)$$

from the conducting side provided that $\Delta g>h^{(d-2)/d}$. Here, the assumption of weak dissipation,

$$l_{\text{inel}}=[\omega\,\text{Im}(1+i\epsilon_2)^{1/2}]^{-1}\gg l, \quad (10)$$

has been made since inelastic effects act as a long-distance cutoff of the renormalization of the photon diffusivity and accordingly a cutoff to the divergence (9) of the absorption. In particular, since at the fixed point g is independent of length scale, it follows from (4a) that the residual diffusivity at the mobility edge ω^* scales with l_{inel} as

$$D(\omega^*)\sim 1/(g^*l_{\text{inel}}^{d-2}). \quad (11a)$$

With use of (10), (7), and the definitions (4a) and (4b), it follows that the absorption peak

$$\alpha(\omega^*)\propto[\epsilon_2/D(\omega^*)]^{1/2}\sim\epsilon_2^{(3-d)/2} \quad (11b)$$

persists for arbitrarily small but nonzero ϵ_2 in three dimensions. With use of the free-photon density of states

$$\rho(\omega)=[Sd/(2\pi)^d]\omega^{d-1}, \quad (12)$$

it follows from (4a) $[D(\omega)=l]$ and the fixed-point condition that the mobility edge in $d=3$ occurs at a frequency ω^* such that

$$(\omega^* l)^2 \simeq 1/(2\pi) \quad (d=3). \tag{13}$$

As a guide to the amount of disorder required to achieve this condition, we consider the scattering from a collection of highly conducting metallic or insulating dielectric spheres immersed in an otherwise uniform background. For wavelengths long compared to the sphere radius, a, the photon is Rayleigh scattered with a cross section $\sigma \sim (\omega/c)^4 a^6$, and for arbitrarily high sphere density n_s (as geometrically allowed), the mean free path $l \sim 1/(n_s \sigma)$ is long compared to the wavelength and the states are extended. In the opposite limit of wavelengths short compared to the sphere size, the cross section approaches the value $\sigma = 2\pi a^2$ given by geometrical optics.[2] Here again, the wavelength is short compared to the mean free path for arbitrarily close-packed spheres and the waves propagate classically. However, there exists an intermediate regime $\lambda \sim a$, for which the mean free path may become comparable to λ as the interparticle spacing is reduced to the smallest allowed values. This is summarized schematically in Fig. 1 for disorder characterized by a correlation length a. The behavior of the mean free path may alternatively be derived by considering a Poisson distribution of spheres and regarding the scattering as arising from fluctuations in their local number density.[6]

The range of wavelengths for which $l \sim \lambda$ corresponds to a region where weak localization may occur and which separates the high- and low-frequency regimes of extended states. In the presence of weak dissipation, such a region should be signaled by an anomalous rise in absorption of the form (9). The example of spheres serves only as an illustration. The accessibility of the localization regime depends more generally on the preparation of a disordered medium which scatters waves sufficiently strongly that the Ioffe-Regel condition[5] (13) is satisfied and for which the dissipation can be made sufficiently weak. Another possibility might be a medium consisting of a dense tangle of coated metal wire. Systems occurring in nature such as rain clouds or colloidal suspensions, which although capable of sustaining diffusive scattering,[9] in general do not achieve a condition analogous to the close packing of dielectric spheres with the onset of precipitation.

In summary, it has been demonstrated that a signature of an electromagnetic mobility edge in a disordered medium is the anomalous rise in energy absorption due to localization fluctuations in the photon diffusivity as the critical frequency ω^* is approached. An illustration with randomly distributed spheres suggests that this may be observed for a small range of photon wavelengths comparable to the correlation length of a sufficiently strongly disordered medium. The analysis presented is general and may be applicable to other systems such as phonons in a disordered solid. The electromagnetic case, however, has the advantage that the speed of light is sufficiently fast compared to the possible motion of scatterers even at moderate temperatures so as to present a nearly static disordered medium on the time scale required for scattering and interference associated with localization.

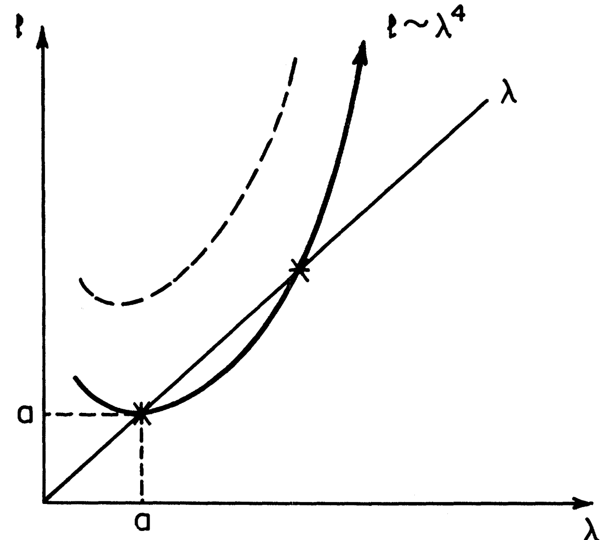

FIG. 1. Behavior of the elastic mean free path as a function of wavelength. In the long-wavelength Rayleigh-scattering limit $l \sim \lambda^4$. In the short-wavelength limit, $l \gtrsim a$, the correlation length. For a strongly scattering disordered medium (solid curve) there may exist a range of wavelengths for which $2\pi l/\lambda \simeq 1$, exhibiting weak localization. This would not occur in a dilute impurity limit (dashed curve).

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