

Q 4 N&R Problem Set #5 Written Team Problem Set

This homework assignment is due by **5 pm on Friday 30th of March, 2007**. Please submit it in the *Drop Box* for your tutorial.

The Drop Boxes are located in the basement of the Burton Tower of McLennan Physics Labs (MP). On the first floor of MP there is a stairway with a bust of Isaac Newton on the 1st floor; the Drop Boxes are at the bottom of the stairs.

	This page must be the first page of the submitted Problem Set.	
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Solve the problems together in the same team that you have been working with in your tutorials.

	Problem Sets done by a single individual will not be accepted.	
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By filling out and signing the form below, I certify that I took an active role in the solution of all the problems of this problem set.

Name (Please Print)	Signature

Designate one member of your team as the *coordinator*. The coordinator will be responsible for assembling the final copy of your solutions and submitting them in the Drop Box on time.

The coordinator should fill out this part of the form:

Name of Your PHY138 Tutor	
Tutorial Group	
Tutorial Day	
Tutorial Time	
Coordinator's Name	

Show all your reasoning, and work legibly.
The marks given for each question are indicated in brackets.

Note. You may find these problems to be more difficult than ones I am likely to set in Test 4 and the Final Exam. However, they are intended to ensure that you have a good understanding of the topics, and that you discuss them with your team members. If you understand these problems, you will do well in the test and exam. I recommend that every member of your team knows how to solve every one of them!

Readings: Supplementary Notes I to V (to §5.4.2)

1. *Isotopic Dilution.* 3.7×10^4 Bq of ^{47}Ca (half-life 108 hours) was injected into a patient, of mass 60kg, to determine the amount of exchangeable calcium in the body. At 20 hours after injection, the activity of all of the urine collected up to that time was 0.56×10^4 Bq, and a sample of blood taken at that time was found to have 0.90 mg of stable Calcium and 6.0 Bq of ^{47}Ca . Assume that the excretion from the body is exponential and exclusively by urination.

A) What is the mass of the exchangeable Calcium in the patient's body?

You may find the following hints useful:

1. Let $N_{\text{total}}(t)$ be the total number of ^{47}Ca nuclei remaining at time t in the body and in the urine. Write down the equation connecting $N_{\text{total}}(t)$ to $N_{\text{body}}(t)$ and $N_{\text{urine}}(t)$, the number of ^{47}Ca nuclei in the body and in the urine, respectively.
2. Write down the equation that expresses the decay of $N_{\text{total}}(t)$ in terms of the initial number of ^{47}Ca nuclei.
3. Combining these two equations, and multiplying by the nuclear decay constant λ_n , yields an equation which can be solved for the activity in the body at time t (see the footnote in SNV, §5.4.2).
4. The equation relating two ratios (of blood mass to activity) can then be solved for the answer.

(20 marks)

B) Assume that each ^{47}Ca decay deposits approximately 1.1 MeV of beta or gamma energy in the body. Quantitatively evaluate the long-term risk to the patient caused by the injection of this radioisotope. Derive any other formulae you require, based on your calculation of part A above (you may use without derivation any given in the Supplementary Notes). State any assumptions you make, and clearly explain how you arrive at your conclusion.

(20 marks)

[Total 40 Marks]

2. *Biological Uptake.* The formula in SNV concerning the injection of a radioisotope into the body for diagnostic or therapeutic purposes considers the effect on the dose of biological excretion, but assumes that the biological uptake of the radioisotope is instantaneous. In many cases, this is not a realistic assumption, as it often takes a finite time for the injected radioisotope to be absorbed into the target organ. This problem will guide you through some steps that will lead to an expression for the dose delivered to the organ when biological uptake is taken into account.

Suppose a radioisotope of activity $R(0)$ and nuclear decay constant λ_n , is injected into the body at time $t = 0$. Parts A) to D) below lead to an expression for $N_o(t)$, the number of nuclei of the radioisotope in the organ, of mass M , at a time t . Let e_n be the energy deposited in the organ by each nuclear decay. We assume that biological excretion occurs only from the organ under study – i.e. biological excretion from the body as a whole can be neglected.

- A) Write down the number of radioactive nuclei, $N(t)$, in the injection, in terms of the given quantities. (2 marks)
- B) Write down an expression for $N_o(t)$ when nuclear decay and biological excretion are taken into account, but biological uptake is instantaneous (i.e. $N_o(0) = N(0)$). Let $\lambda_{eff} (= \lambda_n + \lambda_{be})$ be the effective decay constant. (4 marks)
- C) Assume that the biological uptake is exponential, governed by a biological uptake constant, λ_{bu} . If there is no nuclear decay or biological excretion, so that the sole influence on $N_o(t)$ is the biological uptake, guess at a reasonable equation of proportionality for $N_o(t)$ in this case. (Hint: this expression should increase with time to a maximum value). By considering the behaviour of $N_o(t)$ as $t \rightarrow \infty$ (i.e. when only biological uptake is operative), determine the constant of proportionality. (7 marks)
- D) Now combine these two equations to obtain an equation for $N_o(t)$ when nuclear decay, biological excretion and biological uptake are taken into account (Hint: their effect is multiplicative!). Your equation should contain $N(0)$, λ_{eff} , and λ_{bu} . (5 marks)
- E) Let e_n be the energy deposited in the organ by each nuclear decay and let the activity of the radioisotope at time t be $R(t)$. Write down an equation for the dose rate [$dD(t)/dt = \dot{D}_m(t)$] at this time t in terms of $R(t)$, e_n , and M ; then express $R(t)$ in terms of $N_o(t)$. (Hint: read the note at the bottom of p6 of SNV). (3 marks)
- F) This last equation can now be integrated from $t = 0$ to $t = \infty$ to yield the expression for the committed dose to the organ in the presence of nuclear decay, biological excretion, and biological uptake. (Hint: $\int_0^{\infty} \exp(-ax)dx = 1/a$). (6 marks)
- G) Check that your expression makes sense in the limiting cases of $\lambda_{bu} = 0$, and $\lambda_{bu} = \infty$. (5 marks)
- H) Finally, sketch a graph of dose rate against time. (3 marks)

[Total 35 Marks]

3. On the 1st of November 2006, an ex-KGB agent called Alexander Litvinenko, who had sought political asylum in the UK, was taken to hospital. 22 days later he was dead. Before he died he accused Russian president Vladimir Putin of having engineered his death. Exhaustive tests on Litvinenko established that he had been poisoned by a radioactive isotope, Polonium-210. The Guardian newspaper states that the amount of polonium-210 found in the Russian's body would have cost as much as £20m (~ \$50 million) to acquire.

^{210}Po was discovered by Marie and Pierre Curie in 1898, who named it after Marie's native Poland. It occurs naturally in uranium ores, being one of the decay products of uranium, and is made artificially at nuclear reactors. It also appears in tobacco plants which have been fertilized by calcium phosphate; one cigarette contains around 0.04 pCi and the US Surgeon General has claimed that this radioactivity, rather than the tar, accounts for 90% of all smoking related cancers. ^{210}Po is five times more deadly when inhaled than ingested.

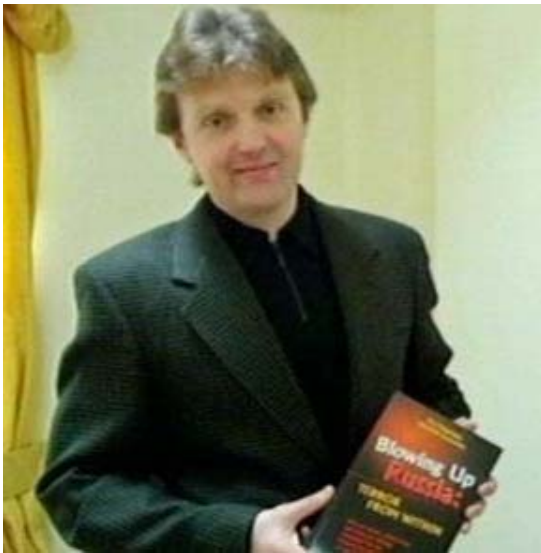
^{210}Po decays with a half-life of 138 days by emitting 5.41 MeV alpha particles, which are easily stopped by skin, or by a few centimeters of air. However, when ingested, the soluble Polonium circulates to every cell and tissue in the body, where the alpha particles deposit all of their energy. The half-life for biological excretion is about 50 days.

Comment on the accuracy of **all** of the numbers in the following quotes of the time:

- A) The average American receives about $\frac{1}{4}$ Gy in a lifetime from cosmic rays, the soil, and medical exposure (*USA Today*) (6 marks)
- B) ^{210}Po is about five thousand times as radioactive per gram as Radium (*Globe & Mail*) (6 marks)
- C) 1 mCi of Polonium would weigh only 0.2 millionths of a gram (*Globe and Mail*) (2 marks)
- D) 3 mCi of ^{210}Po would be sufficient to kill (*Health Physics Society*) (5 marks)
- E) a fatal 4-Sv dose can be caused by ingesting 8 MBq (200 microcurie), about 50 nanograms (ng) (*Wikipedia*) (6 marks)

No calculation is required for part A, just a little research – but remember to quote your sources. A complete answer to parts B) through E) will show all of the working you need to verify or refute the numbers.

[Total 25 Marks]



Mr Litvinenko with his book, which is highly critical of Mr Putin.



Mr Litvinenko in hospital, a few days before his death.