# PHY138Y – Nuclear and Radiation Section

## Supplementary Notes I

## Introductory Nuclear & Atomic Physics

**Contents.**



*About the Supplementary Notes.*
This series of Supplementary Notes (SNI to SNVI) presents all the topics in the Nuclear and Radiation (N&R) section of PHY138. The material in the Appendices is provided for interest or/and to aid your understanding; it is not, however, examinable. The otherwise excellent chapters on Modern Physics in the Knight text are deficient in discussing the application of atomic and nuclear physics to medicine, which is the main thrust of this section. However, in each Supplementary Note, I indicate some of the relevant sections of Knight. This set of notes (SNI) summarizes the minimum background you will need for this N&R section of the course.

*References*.
1. All of the Supplementary Notes owe a debt to those of Professor Kenneth McNeill, who initiated this course many years ago. Dr Pierre Savaria's more recent notes have also been a great help to me, as have been many suggestions from Dr David Harrison.
2. The figures denoted by S&J are copied with permission from the excellent text by Raymond A. Serway and John W. Jewett, Jr, (*Principles of Physics*, 3rd Edition, Brooks/Cole Canada 2002).

*Flash Animations – also accessible from the N&R Web page.*
1. EM Wave from an Accelerating Charge (the generation of electromagnetic waves).
2. The Bohr Model and Line Spectra (photon excitation and emission of an atom).
3. Electromagnetic waves, black body radiation, and much more – a fun site.

# 1. Atomic and Nuclear Physics – a quick review

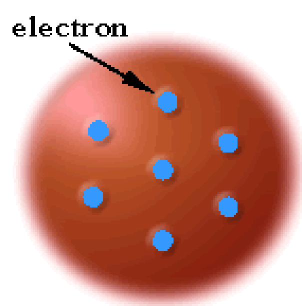
## *1.1 Introduction*

We have become so accustomed to the idea that matter is made of tiny parts – molecules, atoms, electrons, etc., that it is somewhat of a surprise to realize that this fact became accepted only just over a hundred years ago. Indeed it is rumoured that Ludwig Boltzmann, depressed that the rest of the scientific world did not share his belief in the reality of molecules, committed suicide in 1906. Boltzmann had shown that the results of classical thermodynamics could be beautifully explained by the assumption that matter consists of small, indivisible, weakly interacting molecules. His work is summarized in the equation that appears on his gravestone; it relates the entropy (S) of a system to the number of microscopic states (W) that the system can access; $S = k \ln W$, where k is Boltzmann's constant.

Of course, Boltzmann acted precipitously in choosing this 'permanent solution to a temporary problem', since there were many supporters of the theory that matter consisted of indivisible 'atoms' or molecules: Democritus in Greek times, Newton in the 17$^{th}$ century, Dalton and Avogadro in the 19$^{th}$ (see §1.4.4 below). Although some scientists continued to deny reality to objects that could not be observed directly, experimental results were against them; atoms and molecules are much more than merely mathematical fictions. Further, it was becoming clear that the indivisibility of the atom was incorrect; atoms, apparently, consist of other, even smaller objects, whose reality is also incontrovertible.

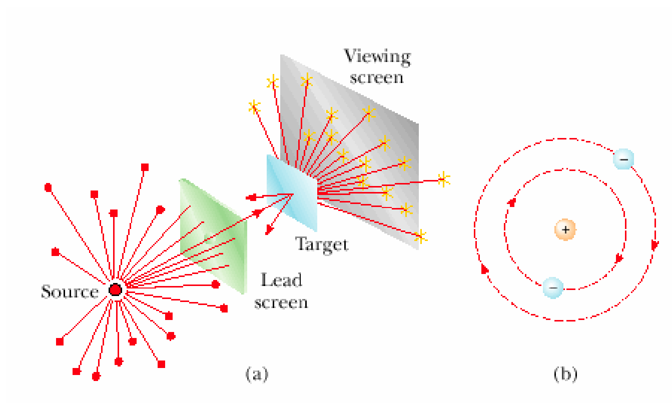
## *1.2 The Discovery of the Nucleus*

In 1897, J.J. Thomson of Cambridge University noticed that particles were emitted from the cathodes of the discharge tubes he was studying. These particles were shown to have a negative charge; we now call them electrons. Since they are emitted from atoms, it seems probable that they are constituents of the atom. Since the atom is neutral, the rest of the atom has to have a positive charge. Accordingly Thomson proposed the 'plum pudding' model, in which the electrons are embedded in a uniform sphere of positive charge, like the plums in a plum pudding. That this model was a poor one was demonstrated by the brilliant experiments of Ernest Rutherford and his collaborators.

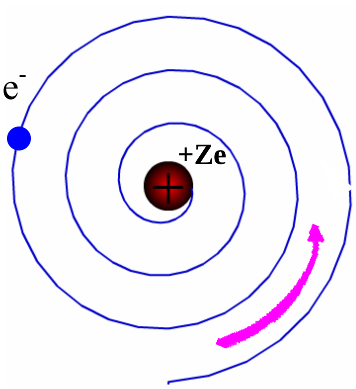


In 1896, the French physicist Antoine Henri Becquerel, noticed that some rocks that he had been studying emitted radiation that could pass through opaque photographic paper. Rutherford – "the father of Nuclear Physics" - identified two of these radiations. He showed that the first, which he called **alpha** radiation, is nothing but Helium atoms with two electrons removed, with, accordingly, a positive charge of plus two. His experiment to probe atoms with these alpha particles showed conclusively that the mass of the atoms is almost totally concentrated in a tiny 'nucleus' at the centre.

The experiments were simple in principle: a beam of alpha particles from a radioactive source were directed at a thin gold leaf. If the Plum Pudding model represented reality, the alpha particles would be expected to suffer only many small deflections as they encounter the spread-out charge of the gold atoms. However, Rutherford, much to his astonishment, noted a significant number of very large deflections, some of them even causing the alpha particles to bounce back from the foil.

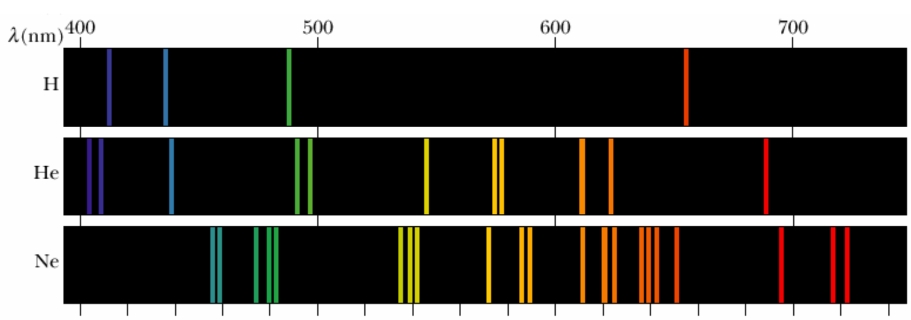



Thus was born the Planetary Model of the atom, in which the electrons circulate around a tiny but very massive nucleus, rather as the planets circulate round the sun (S&J, figures 29.2 and, below 29.3 and 11.18a).




Rutherford's planetary model has an apparently fatal flaw. The accelerating electrons that were supposed to orbit around a central nucleus would, according to classical physics, emit electromagnetic radiation; this loss of energy would cause them to spiral inward to the nucleus till the atom collapsed. That we are here to observe our world lets us know that this does not happen – so what's going on here? (A brief explanation of why accelerating charges emit electromagnetic radiation is given in Appendix 1-A).

There was another difficulty. Atoms do indeed emit electromagnetic radiation, as expected. However, rather than a spectrum that has a continuous range of energies, as classical physics predicts, this radiation appears only at a finite number of discrete energies. (You can study these discrete spectra in the visible region from several atoms in the first year laboratory's eponymous experiment). Both difficulties were 'solved' by Niels Bohr, the Danish genius (see §1.5 below).



## *1.3 The Basics of Nuclear Physics*

We now know that the nuclei of atoms consist of **protons** with positive charge, equal and opposite to that of the electron, and **neutrons** with charge zero; together these are called **nucleons**. The number of protons in a nucleus is denoted by $Z$ and the number of neutrons by $N$. $Z$ is called the **atomic number** of the nucleus since it indicates where on the table of the elements that particular one lies. The **mass number** of the nucleus – the total number of nucleons - is denoted by $A$, which equals $N$+$Z$. Then a particular nucleus **X** is written as ${}^{A}_{Z}\mathbf{X}$ – e.g. ${}^{1}_{1}\mathbf{H}$, ${}^{12}_{6}\mathbf{C}$, ${}^{16}_{8}\mathbf{O}$ etc. Since $Z$ is entirely determined by the chemical symbol, this is often abbreviated by omitting the $Z$ value, - e.g. $^{12}$C, $^{16}$O: pronounced "Carbon-12, Oxygen-16". Nuclei that have the same value of $A$ are called **isobars**.

The value of $Z$ determines the chemical behaviour of the element. However different nuclei can have the same value of $Z$, yet different values of $N$ (and thus of $A$). Nuclei with the same $Z$ but different $A$ values are called **isotopes**. Thus $^{12}$C, $^{11}$C, and $^{14}$C are isotopes of Carbon. Isotopes are either stable or unstable; the unstable ones emit radiation, as we shall see in SNIII, and are called **radioisotopes**.

Rutherford's experiments indicated the approximate size of the gold nuclei he used as targets. Later experiments showed that the nuclei of all atoms are roughly spherical with a radius that depends on the number of nucleons. This radius is given by $R = r_0 A^{1/3}$ where $r_0 \approx 1.2 \times 10^{-15}$ m. ($10^{-15}$ m is defined as one fermi after the famous Italian physicist; it is also one femtometre. Both have the abbreviation fm). This is an astonishingly small number in comparison to the radius of the atom of about $10^{-10}$ m.

Since nuclei hold together, in spite of the Coulomb repulsion between protons, it is necessary to postulate another force of nature - the so-called strong force – that comes into play only at a distance of less than $10^{-15}$ m or so. This very short-range strong force does not distinguish between neutrons and protons, being attractive between any two nucleons (p-p, p-n, n-n); it is the glue that holds the nucleus together. For very large nuclei, the peripheral protons can lie outside of the range of the strong force and their mutual Coulomb repulsion tends to pull the nucleus apart. However the strong attractive forces between the neutrons tend to moderate this effect. The interplay of these opposing forces sets a limit on the size of stable nuclei.

The mass of a nucleus is less than the sum of the masses of the individual protons and neutrons of which it is composed. This is because energy is needed to pull the nucleons apart; and, as Einstein has shown us (§1.4.2), energy is mass. In a similar way, we would need to apply energy to raise a stone out of a hole in the earth. The difference between the sum of the masses of the nucleons and the mass of the nucleus that they make up is called the **binding energy** of the nucleus.

## 1.4 Notes on Units

### *1.4.1 The Electron Volt*

The SI unit of energy, the joule, is too large a unit to be useful in atomic or nuclear physics. Accordingly we define the **electron volt (eV)**, the energy that an electron would acquire when accelerated through a potential difference of one volt. The electron is now known to have a charge of $e = -1.60 \times 10^{-19}$ Coulombs (first measured by Millikan, whose experiment you can reproduce in the first year laboratory). Thus the electron volt is related to the joule by 1 eV = $1.60 \times 10^{-19}$ joules (can you prove this?).

### *1.4.2 The Most Famous Equation in the World*

For the very small masses encountered in atomic or nuclear physics, the kilogram is not a useful unit (the mass of the electron, for example, is $9.11 \times 10^{-31}$ kg). A more useful unit is defined using the most famous equation in physics, $E = mc^2$, a result of Einstein's theory of Special Relativity (1905). $E$ is the energy, $m$ is the mass, and $c$ is the velocity of light. This equation quantifies the fact that mass and energy are equivalent, so that mass can be expressed in energy terms. For example, if we could somehow completely annihilate an electron, how much energy would be released? Einstein's equation yields:

$$E = (9.11 \cdot 10^{-31}) \cdot (3 \cdot 10^{8})^2 \text{ kg.m}^2\text{.s}^{-2} = 8.199 \cdot 10^{-14} \text{ J}$$
$$= 8.199 \cdot 10^{-14} / 1.6 \cdot 10^{-19} \text{ eV} = 0.511 \text{ MeV.}$$

Thus we can write the electron mass ($m_e = E/c^2$) as 0.511 MeV/$c^2$. **MeV/$c^2$** is a useful unit, since it gives at a glance how much energy a particular mass could yield.

### *1.4.3 The Atomic Mass Unit*

Another mass unit used in nuclear physics is the **atomic mass unit** (**amu**, abbreviation **u**), defined as the mass of $\frac{1}{12}^{\text{th}}$ of the mass of the $^{12}$C atom*. $^{12}$C is chosen since it is abundant and extremely stable. Its mass has been measured to be $1.992\,671 \times 10^{-26}$ kg. Thus 1u = $1.660\,559 \cdot 10^{-27}$ kg. In these units the masses of the electron, proton, and neutron are $m_e = 5.485\,799 \cdot 10^{-4}$ u, $m_p =$ 1.007 276 u, and $m_n =$ 1.008 665 u.
Since $c = 2.998 \cdot 10^8$ m/s, a conversion factor to MeV/$c^2$ can be derived via:

$$1\text{u} = [1\text{u}][1.660\,559 \cdot 10^{-27} \text{ kg/u}]\,[2.997\,924 \cdot 10^{8} \text{ m/s}]^2/[1.602\,177 \cdot 10^{-19} \text{ J/eV}]\, c^2$$
$$= 931.49 \text{ MeV/c}^2.$$

### *1.4.4 The Mole and Avogadro's Number*

Here is the modern definition of a mole that we will use. **1 mole of an element is defined as that quantity of the element that contains exactly as many atoms as there are in 12 grams of $^{12}$C.** This number is called **Avogadro's number,** $N_A$; it equals $6.022 \cdot 10^{23}$ g.mole$^{-1}$. Thus one mole of $^{12}$C contains exactly $N_A$ atoms and has a mass of EXACTLY 12g, by definition . This **molar mass** of 12g is just the mass of each $^{12}$C atom in amu multiplied by Avogadro's number, i.e. $12\text{u} \cdot N_A = 12\text{g}$; so we see that $1\text{u} = (1/N_A)\text{g}$. In general the molar mass of an atom is given by the mass of the atom multiplied by $N_A$. Thus the molar mass of a nucleus $^A$X with mass number $A$ is very nearly, but **not exactly**, equal to $A$ grams.

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* This is more properly called the **unified mass unit**. The atomic mass unit was originally based on $^{16}$O. However, many authors continue to use the term 'atomic mass unit' and its abbreviation amu for the newer unit based on $^{12}$C, a practice that I will follow.

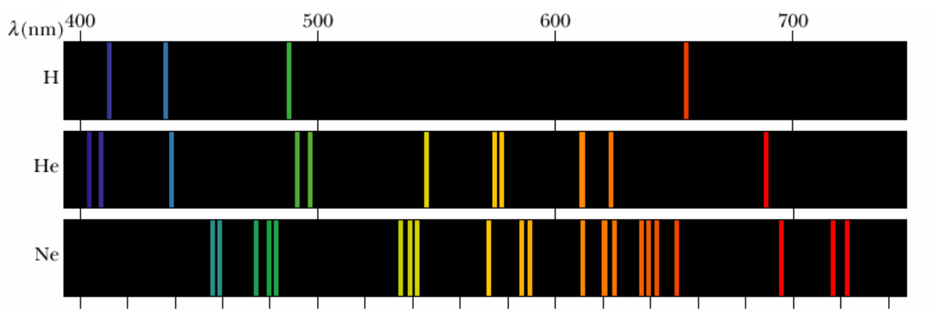
### 1.5 The Bohr Model of the Atom

To address the difficulties inherent in the planetary model of the atom mentioned in §1.2, Bohr arbitrarily stated two postulates.

a. electrons in atoms do not obey classical physics; instead they exist in 'stationary' states (i.e. states that do not radiate electromagnetic energy). The possible energies of these states are discrete, not continuous – i.e. they can occur only with specific values
b. atoms do emit electromagnetic radiation; however they do so by jumping from states of higher energy to states of lower energy.

The first postulate makes no sense from the standpoint of classical physics. However it allowed the further development of the atomic model, though an explanation had to await the full development of quantum mechanics that came years later.

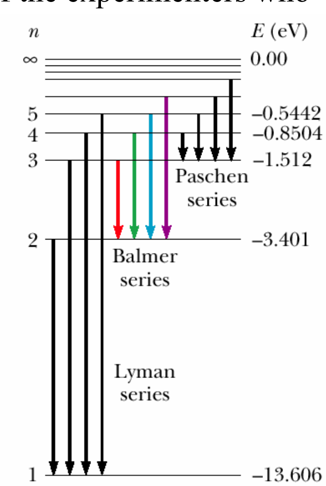
The second postulate explains the observation of discrete atomic spectra, shown below.




(S&J, figure 11.18a, and, below, figure 1.22).

Since there are only specific energy states available to electrons in an atom, the energies of the emitted radiation, acquired by jumps between these states, must also have only specific values. To calculate these energies, Bohr used quasi-classical ideas that gave a surprisingly good explanation of the experimentally observed spectral lines of Hydrogen.

The energy level diagram for Hydrogen is shown; the names of the experimenters who first studied the different series are indicated. The possible states are labeled by the 'principal quantum number', n, where n is an integer taking all positive values from one to infinity. The energy of each state is then given by ($-C/n^2$), where $C$ is a constant for the atom under consideration (equal to 13.6 eV for Hydrogen). The explanation of the minus sign is as follows. In order to remove an electron from an atom, positive energy must be applied (think of lifting a stone from the bottom of a well). For instance if exactly $+13.6/3^2$ eV were applied to an electron in the third energy level of Hydrogen, the electron would just escape from the atom, with no remaining kinetic energy. If less than this amount of energy were applied, the electron could not escape; if more than this amount were applied, the electron could escape with positive kinetic energy (equal to the difference between the energy applied and $+13.6/3^2$ eV).



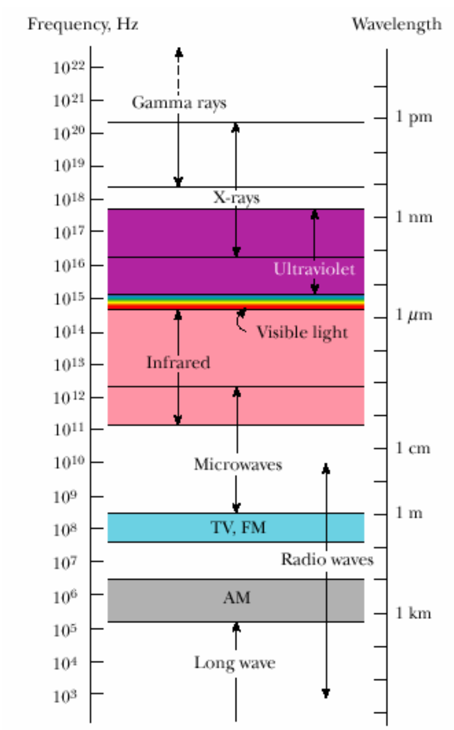
The energy of the radiation emitted by a transition from the $n^{th}$ to the $m^{th}$ energy levels is given by $[-C(1/n^2 - 1/m^2)]$. The minimum energy required to remove an electron from an atom is the **ionization energy**; the ionization energy of Hydrogen is thus +13.6 eV.

There were many more or less successful attempts to extend Bohr's simple picture to multi-electron atoms, but a consistent and accurate theory had to await the development of Quantum Mechanics and the discovery of electron 'spin' (discussed in SNVI). The electrons in multi-electron atoms lie in shells, each shell containing a few levels of closely similar energies that depend in a complicated way on the electron spins. Each shell corresponds to the "principal quantum number", n, that can take the values 1, 2, 3, …. It turns out that nuclei also have discrete energy levels, as we shall see in SNIII.

## *1.6 The Photon*

Whether light is particle-like or wave-like has been a topic of controversy for over 400 years since Newton, Hooke, and Huygens, among others, debated it in the 17th century. Thomas Young's experiments (which you can reproduce in the lab) in 1801 seemed to settle the question; light was undoubtedly wavelike in nature. Its colour is determined by its wavelength, $\lambda$, which is related to its frequency, $f$, by the equation $c = \lambda f$, where $c$ is the speed of light in vacuum. However, in 1905, Einstein's explanation of the **photoelectric effect** demonstrated, equally conclusively, that a ray of light consisted of a stream of particles that we now call photons!

Finally, quantum mechanics resolved the paradox by accepting it. Nothing in our everyday experience allows us to form a visualization of an object that sometimes looks like a wave (spreads out, exhibits interference), and sometimes like a particle (located at a point in space, doesn't exhibit interference). That concept, and many others that apparently fly in the face of common sense, is one we must get used to if we want to understand the world of the very small, where quantum mechanics holds sway.



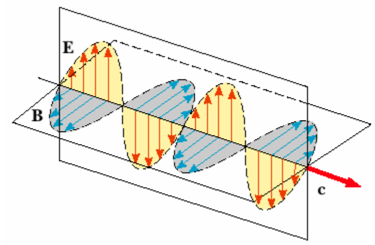

Electromagnetic waves cover a huge range of frequency; they include visible light (a tiny portion), radio waves (first discovered by Hertz), X- and gamma-rays (S&J, figure 24.13).

For most of our discussion of medical applications, it will be most useful to consider the electromagnetic radiation that will interest us – X-rays or gamma rays – to be particles, each with an energy E that is related to the frequency of the wave, $f$, by $E = hf$, where $h$ is called **Planck's constant**; see Appendix 1-A). You will notice the 'schizophrenic' nature of the last equation, which has a particle property (energy) related to a wavelike property (frequency). Such is the world we inhabit.

### *Appendix 1-A. Electromagnetic Radiation*

Faraday's law shows that a changing magnetic field causes electrons to move in a conducting medium. We interpret this to mean that a changing magnetic field generates an electric field that causes the electrons to move. But moving electrons constitute an electric current, and, as Oersted showed in 1819, an electric current generates a magnetic field. Since the electrons have their own electric field, a current of electrons produces a moving electric field. To summarize, a uniformly moving magnetic field generates a constant electric field; a uniformly moving electric field generates a constant magnetic field.

Now consider what happens when we generate an *oscillating* electric field, for instance by waving an electric charge in air. The generated magnetic field also oscillates; and this oscillating magnetic field will, in turn, generate another oscillating electric field, with its own associated oscillating magnetic field, and so on! What happens to this apparently self-generating system? The electric and magnetic fields, inextricably joined, dissociate from the accelerating charge and speed off as *electromagnetic waves*.




(S&J figure 24.6).

Check out the excellent Electromagnetic Wave animations at [http://www.upscale.utoronto.ca/PVB/Harrison/Flash/EM/EMWave/EMWave.html](http://www.upscale.utoronto.ca/PVB/Harrison/Flash/EM/EMWave/EMWave.html) or [http://www.colorado.edu/physics/phet/web-pages/simulations-base.html](http://www.colorado.edu/physics/phet/web-pages/simulations-base.html) (click on 'Light and Radiation' and choose the animation entitled 'Radios, Waves, and Electromagnetic Fields').

Here is a condensed (and inaccurate!) mathematical sketch to make this process plausible:
Faraday's Law yields $E \infty \mathrm{d}B/\mathrm{d}t$.
Oersted's observation, as represented by the Biot-Savart Law yields

$$B \infty I \text{ (current)} \infty \mathrm{d}E/\mathrm{d}t.$$

$$\text{Thus}\quad \mathrm{d}^2E/\mathrm{d}t^2 = \mathrm{d}(\mathrm{d}E/\mathrm{d}t)/\mathrm{d}t \infty \mathrm{d}B/\mathrm{d}t \infty E.$$

$$\text{and, similarly, } \mathrm{d}^2B/\mathrm{d}t^2 = \mathrm{d}(\mathrm{d}B/\mathrm{d}t)/\mathrm{d}t \infty \mathrm{d}E/\mathrm{d}t \infty B.$$

The solution of these equations yields oscillatory behaviour (sines and cosines) for $E$ and $B$. (Remember that the second differential coefficient of a sine or a cosine gives back the sine or the cosine respectively; e.g. $\mathrm{d}^2(\sin X)/\mathrm{d}t^2 = -\sin X$; don't worry about the sign!).

The correct mathematical treatment leads to four equations, called Maxwell's equations after James Clerk Maxwell. They brilliantly summarize all of electricity and magnetism. Maxwell showed that the speed of these waves in vacuum was given by a very simple formula which includes only the permittivity and permeability of free space. This is the speed of light in vacuum, $c \approx 3 \times 10^8$ m.s$^{-1}$.

Maxwell's work seemed to confirm the belief that light was a wave, and demonstrated that it was the $E$ and $B$ fields that 'waved'. In 1905, Albert Einstein had a 'magical' year, producing no less than three papers, each one of which changed our understanding of the world. The most well-known of these papers ('On the Electromagnetic Dynamics of Moving Bodies') addressed some of the difficulties that arose from this new understanding of the nature of electromagnetic radiation; thus was born the theory of Special Relativity. The second paper began to suggest that this classical view of electromagnetic waves was not the whole story (see Appendix 1-B, below).

### *Appendix 1-B. The Quantum World*

Many books have been written about the development of quantum mechanics, and controversy around its extraordinary results still appears in the scientific journals. The following ridiculously brief summary is the least that a well-educated student should know. I assume that you have had at least a brief introduction to quantum physics at school or in one of your other university courses. For those of you who become fascinated by this most fascinating subject, you might consider taking PHY100H – *The Magic of Physics,* or PHY201H – *Concepts of Physic*s.

When any material is heated, it emits light; the visible part of the emitted spectrum is red at lower temperatures, becoming yellow to white as the temperature increases. Given the knowledge about the origin of electromagnetic radiation, the theoretical calculation of the observed spectra, emitted from the oscillating charges of the heated material, should have been easy. In fact, calculations based on classically correct assumptions failed spectacularly. In 1900 Max Planck realized success by assuming that the light emitted from the oscillating charged particles, is emitted in lumps, or packets, rather than in the continuous manner that Maxwell's equations predict. The energy of these lumps is proportional to the frequency of the light, and the constant of proportionality was, appropriately, called Planck's constant, denoted by $h$. Initially, this seemed to be an intriguing mathematical trick, with no real physical significance.

However, five years later, the second of the Einstein's 1905 papers showed that Maxwell's 'classical' view of electromagnetic waves could not explain some of the details of the phenomenon of the photoelectric effect (to be discussed briefly in §2.3.1). This effect – in which light knocks electrons out of a metal - can be explained only if light is also made of lumps, which we now call photons. The energy ($E$) of these photons is related to the frequency of the light ($f$) by the formula $E = hf$, where $h$ is Planck's constant.

Planck received the Nobel Prize for his work in 1918; Einstein, for his, in 1921.

The animation at http://www.colorado.edu/physics/phet/web-pages/simulations-base.html gives more details about the 'black body' spectrum that inspired Planck's work (click on 'Heat and Thermo' and choose 'Blackbody Spectrum').

Astonishingly, it turns out that particles (electrons, protons, neutrons, etc.), under certain circumstances, behave as waves! This extraordinary feature of nature was first proposed by de Broglie in 1923, and experimentally observed by the Scotsman G.P. Thomson, among others. The latter was the son of J.J. Thomson, which caused a wag to remark that J.J. proved that the electron was a particle and his son proved that it was a wave! However further discussion will take us too far afield.

### *Appendix 1-C. Backup Reading in Knight*

The text by Knight has some excellent sections on the material contained in this set of Supplementary Notes, SNI. The detail is often more than we require, but you may find that even a cursory reading will greatly increase your understanding, particularly where you find my explanations hard to understand or too abbreviated. With luck, you may find yourself transported by the fascination of modern physics and the insights it brings to this amazing universe we inhabit.

The table below lists the main sections in Knight that correspond to the material in these Supplementary Notes I; these sections are not examinable.

| **Section in SNI** | **Corresponding Sections in Knight** |
|---|---|
| 1.1 | 37.1 (p.1196) |
| 1.2 | 37.4 (pp.1200, 1201, 1203); 37.6 (pp.1205-1210) |
| 1.3 | 37.7 (pp.1210,1211); 42.1 (pp.1352-1356); 42.2 (pp. 1356-1359); 42.3 (pp.1359,1360) |
| 1.4.1 | 37.6 (p.1208) |
| 1.4.2 | 14.2 (p.1354,1355); 36.10 (p.1186) |
| 1.4.3 | 16.2 (p.448) |
| 1.4.4 | 16.2 (pp.488-490) |
| 1.5 | 24.1 (pp.758-760); 37.8 (p.1213,1214); 38.5 (p.1234-1238); 38.7 (p.1243-1245) |
| 1.6 | 38.2 (p.1224,1225);38.3 (p.1228); |
| Appendix 1-A | 33.1 (pp.1041-1043); 33.6 (pp.1061,1062); 34.5 (p.1102,1104); 34.6 (p.1104) |
| Appendix 1-B | Chapters 39, 40 |

*25 April 2007*