

PHY138Y – Nuclear and Radiation Section

Supplementary Notes II

X-rays - Production, Characteristics, & Use

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References.

1. Mark Oldham, *Radiation Physics and Applications in Therapeutic Medicine*, Physics Education 36, 460-467 (2001): accessible from the N&R Web page.
2. Greg Michael, *X-ray Computed Tomography*, Physics Education, 36, 442-451 (2001): accessible from the N&R Web page.
3. A wonderful site that describes much of this section in an interactive and simple way can be found at <http://www.colorado.edu/physics/2000/index.pl> .
4. Data on the absorption coefficients from the US National Institute of Standards and Technology (NIST); <http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html>
5. A simple introduction to X-rays; <http://www.geocities.com/HotSprings/1368/>

Flash Animations – also accessible from the N&R Web page.

1. [Interaction of a Beam of X-rays](#) (a beam of X-rays passing through matter).
2. [X-ray Interactions with Matter](#) (choose your interaction! PE, Compton, Pair Production).
3. [Pair Production](#) (Pair Production with sound effects!).

2. X-rays - Production, Characteristics, & Use.

2.1 Introduction

In 1895 Wilhelm Roentgen discovered a mysterious penetrating radiation emitted when beams of energetic electrons, traveling in an evacuated glass tube, hit the glass wall. (He was awarded the first Nobel prize in 1901 for this work). We now know that these rays, which Roentgen named X-rays, are electromagnetic waves that have frequencies in the range of about 10^{18} to 10^{21} Hz, corresponding to photon energies in the range of a few keV to several MeV. Gamma rays, emitted from radioactive nuclei are also electromagnetic, with higher energies in the MeV range (§1.6). This set of notes will discuss the physics of the production of X-rays, the interaction of X- and gamma rays with matter, and the application of X-rays to medical diagnosis.

2.2 Production of X-rays

X-rays are produced when high energy electrons strike a metal target. There are two distinct mechanisms of production.

2.2.1 Bremsstrahlung Radiation

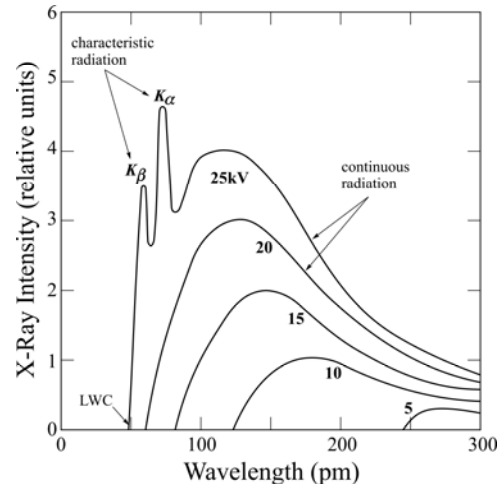
An accelerated charge emits electromagnetic waves (Appendix 1-A). Electrons, shot into a metal target, occasionally pass close enough to a nucleus to be accelerated by the Coulomb force of the positive nucleus, which results in the emission of X-ray radiation. In any one interaction, the electron can lose some or all of its kinetic energy, which is carried off by an X-ray photon. Since the energy of the photon is given by $E_\gamma = hf$ (E_γ – energy, f – frequency, h – Planck's constant), the frequency of the X-rays emitted can vary continuously from a maximum (corresponding to the case when the incoming electron loses all of its energy) to very low values. Since for electromagnetic waves, the wavelength is inversely proportional to the frequency, this *high frequency* cut-off leads to a *low wavelength* cut-off, followed by a continuous range of wavelengths at higher values. The probability of bremsstrahlung radiation increases as the square of the energy of the electron and linearly as the atomic number of the target.

2.2.2 Characteristic X-rays

The electrons entering the metal target also have multiple collisions with the atoms of the target. In many of these collisions, an electron from one of the lower energy levels of the atom is removed. Atomic electrons from higher energy levels of the atom then cascade down to fill the vacancy in the shell, emitting X-radiation that is characteristic of the atoms of the target. The energy of the radiation is given by the difference between the initial and final atomic energy levels ($E_\gamma = hf = E_i - E_f$).

Electrons in atoms lie in shells (§1.5), each shell containing a few levels of closely similar energies. Each shell corresponds to the “principal quantum number”, n , that takes values 1, 2, 3, The shell with $n=1$ is called the K shell, that with $n=2$ the L shell, that with $n=3$ the M shell, etc. The characteristic X-rays are labeled by the shells where the vacancy occurred; often different Greek suffixes (α , β , etc) are added to identify the shell from which the electron cascades down (α from the shell one above, β from the shell two above, etc.).

X-ray spectra from a tube with a molybdenum target are shown opposite for a variety of tube voltages. The continuous contribution from bremsstrahlung can be seen, which displays the low wavelength cut-off, and, at the highest tube voltage, two spikes caused by the characteristic X-rays. If the manufacturer quotes a single value for the energy of an X-ray machine, the assumption being made is that the full spectrum of X-rays has the same effect as a mono-energetic beam that has **one-third** the value of the maximum energy of the actual spectrum. We will use this approximation to simplify some calculations.

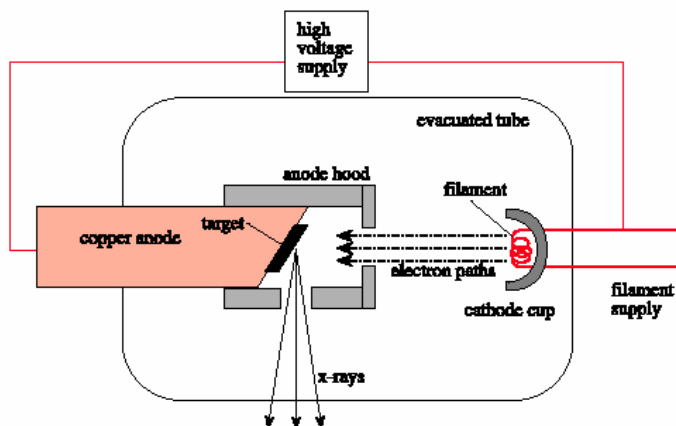


2.2.3 X-ray machines

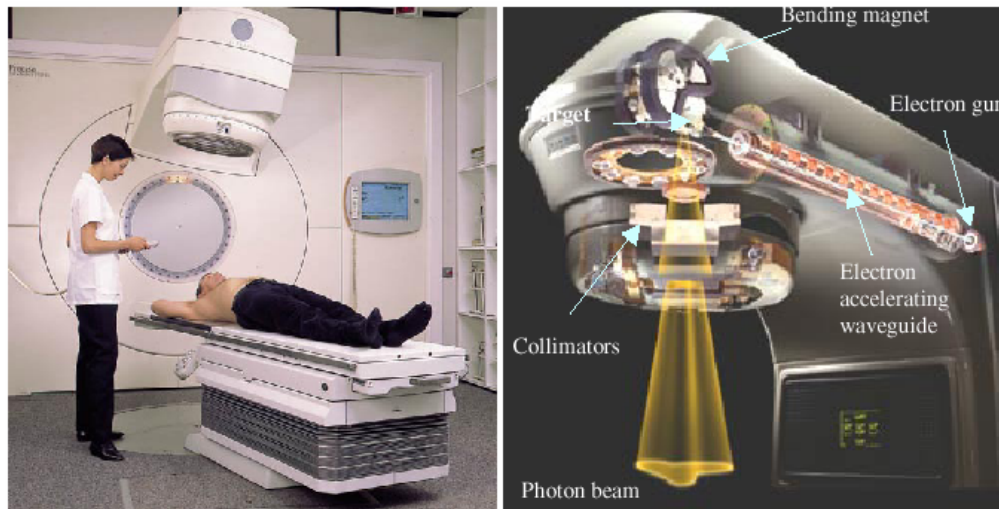
As the electrons are slowed down by multiple collisions with the atoms and nuclei of the target, only a small fraction of their energy (typically less than 1%) is given up as X-rays. Generally, the higher the energy of the electrons, the greater is the fraction of their energy that is converted to X-ray photons. The remainder causes heat generation in the target, and steps must be taken to dissipate this heat.

A schematic of an X-ray tube is shown below. Electrons emitted from a heated filament are accelerated by a high voltage (typically 10 -100 kV). They strike a target, made of a heavy metal – often lead, molybdenum or tungsten – whose high-Z nuclei maximize the electron's acceleration in the Coulomb field. The target is embedded in copper to provide a good conducting path for heat dissipation.

(Ref.1, figure 2)



The low energy X-rays from the tube shown above, typically around several tens of keV, are ideal for diagnostic purposes. However, therapeutic X-rays need to have much higher energies (from around 100 keV for treating superficial skin tumours, for example, up to 25 MeV or so for deeply situated pelvic tumours). For X-rays at these high energies, a linear accelerator is used to accelerate the electrons. A schematic view of a high-energy X-ray machine and a photograph of it in a clinical setting are shown below.



(Ref.1, figure 3)

2.3 The Interaction of Photons with Matter.

When a beam of X-rays or gamma rays passes through matter, it is attenuated (i.e. photons are lost to the beam) by interactions with the atoms and electrons of the matter. The amount of attenuation depends on the energy of the rays and the composition of the matter that allows for contrast between, for example, bone, muscle, and tissue. Some molecular damage is also done to the matter, which will be our concern when we consider the dangers of radiation to biological tissue. Generally, X- and gamma rays produce electrons that, in turn, lose energy by collisions with atomic electrons or nuclei via the Coulomb interaction. Photons have three main methods of producing electrons.

2.3.1 The Photoelectric Effect

For photon energies of less than a few tens of keV or so, their main interaction with matter is via the photoelectric effect. Here the photon is totally absorbed by the atom, giving up all of its energy. The excited atom then de-excites by emitting the electron. (As mentioned in §1.6, it was Einstein's explanation of this effect that demonstrated the particle-like nature of light). If the photon has an energy $e_\gamma = hf$, and the binding energy of the electron to the atom is B_e (the energy required to remove the electron from the atom) then the energy of the ejected electron, $e_e = e_\gamma - B_e$ (since $B_e \sim$ a few eV, we can usually set $B_e = 0$ for the X-ray energies that interest us).

The probability of an X-ray undergoing the photoelectric effect per atom decreases with increasing X-ray energy since if the energy of the photon is too high it will knock the electron it strikes right out of the atom (the interaction is then classified as the Compton effect, see below). The probability of the photoelectric effect increases quickly with the atomic number of the matter being irradiated. This is because a *free* electron cannot absorb a photon and simultaneously conserve energy and momentum. The photon momentum in the photoelectric effect must be absorbed by the whole atom, and larger atoms with higher Z values have more tightly bound electrons close to the nucleus available to perform this function. (The calculation of the exact formula is complicated, requiring advanced quantum mechanics).

2.3.2 The Compton Effect

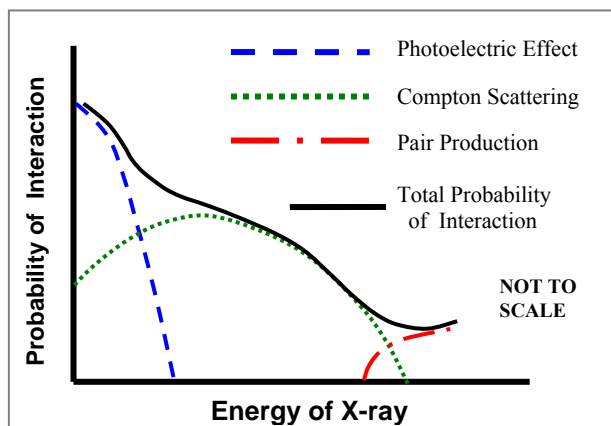
For energies greater than a few tens of keV or so, the effect first explained by Arthur Compton begins to dominate the interaction of photons with matter. Here the photon knocks an electron out of the atom and continues with lower energy on a deflected path; energy and momentum can be simultaneously conserved, and the collision can be considered to be between the photon and a free electron, B_e being negligible at these X-ray energies. The deflected photons can still be in the X-ray range, so shielding must be provided near X-ray machines. The probability of Compton scattering per atom increases with energy from low values (where the photoelectric effect dominates) and then decreases roughly linearly. It increases with the number of electrons available per atom of the scattering material, and is thus approximately proportional to Z .

2.3.3 Pair Production.

In pair production, the equivalence of energy and mass (§1.4.2) is manifested; at high energies, the photon energy converts directly into mass when it passes close to a nucleus, producing an electron-positron pair. The positron, the *anti-particle* of the electron discovered by Anderson in 1932, is a particle that has exactly the same mass as the electron but opposite charge. The energy of the photon has to be at least the sum of the masses of the pair ($1.02 \text{ MeV}/c^2$) for the process to be energetically possible. In practice, the probability of pair production is important in X-ray scattering only above several MeV or so, after which it increases with E and Z .

2.3.4 Summary

The graph indicates approximately how the different processes behave as a function of X-ray energy in biological matter (note that the axes are approximately logarithmic). Appendix 2-B gives a more complete discussion.



The interactions described above change the direction or the energy of the original X-ray or gamma ray from the beam; we use the word *attenuation* to describe this process. However the photon energy is not lost, but is transferred to the secondary electrons, photons, and positrons. If these secondaries interact with the atoms of the material being irradiated, thus depositing some or all of their energy in the irradiated matter, *absorption* of the beam is said to have occurred. The electrons, being light, undergo many small Coulomb scatters, giving up their energy to the atoms and other electrons at each collision. The secondary photons, of lower energy than the originals, may continue to interact via the processes described above, and the positrons can encounter an electron and annihilate, producing gamma rays.

These Supplementary Notes, SNII, concentrate on attenuation, since we are most interested here in the transmission of the original beam on to X-ray film or other detectors; the scattered particles contribute to the fuzziness of the picture. We will be more interested in absorption when we come to study the biological effects of radiation in SNV.

We have produced a variety of Flash animations to give you a visual picture of all of these processes; check them out – follow the links from the N&R Web page. You can also study some of these scattering processes in the lab experiment *Gamma Ray Spectra*.

2.4 Units to measure the effect of Radiation on Matter

As X-rays or gamma rays traverse matter, they cause ionization by one or more of the processes discussed in §2.3 above. In turn, the secondary particles can cause damage to the cells through which they pass. We need to define units to measure these effects.

It is convenient to measure the intensity of a beam of X-rays by allowing it to pass through a volume of dry air and collecting and measuring the charge produced. The resultant unit, called the **exposure**, is the **roentgen (R)**, named after the discoverer of X-rays: an exposure of 1R will produce 2.58×10^{-4} C of positive charge in 1 kg of dry air at STP (density = $0.001293 \text{ g.cm}^{-3}$). Thus 1R will produce $(2.58 \times 10^{-4}) / (1.6 \times 10^{-19})$ ion pairs, since each ion carries the electron charge (an ion pair here means a charged ion and the ejected electron). This is one of the oldest units, still in common usage. Note that although the exposure of a beam of X-rays is directly proportional to the *number* of photons in the beam, it is not necessarily proportional to the *energy* of the photons; doubling the energy of each photon will not necessarily double the number of ion pairs, since other factors also influence the ability of photons to ionize atoms.

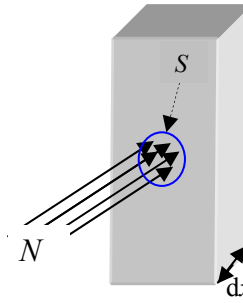
A more useful measure for studying the effects of radiation on matter is the amount of energy deposited by the radiation per unit mass of the material being irradiated. This is called the **absorbed dose** and the SI unit is **Grays (Gy)**, named in honour of Louis Harold Gray (1905-1965). 1 Gray is defined as the radiation dose that deposits 1 Joule of energy in every kilogram of matter. ($1 \text{ Gy} = 1 \text{ J/kg}$). An older unit of absorbed dose, still in use, is the **rad (radiation absorbed dose)**, equal to 0.01 Gy (10 mGy).

Since the production of one ion pair requires an amount of energy that depends on the material ($\sim 1 \text{ eV}$ to $\sim 40 \text{ eV}$; 33.7 eV for air, close to 34 eV for all biological material), the dose is proportional to the exposure, with a constant of proportionality that depends on the material being irradiated. For dry air, we multiply the number of ion pairs produced by one roentgen in 1 kg of dry air by the energy required to produce each ion pair to obtain the energy deposited in 1 kg, or the absorbed dose, D_{air} . The result is $D_{air} = 8.69 \times 10^{-3} X \text{ Gy}$; if X is the exposure in R, D_{air} is in Grays. A useful rule-of-thumb follows; an exposure of 1R yields approximately 1 rad (10 mGy).

In SNIV I will continue this discussion, focusing on the interaction of radiation with biological matter.

2.5 Attenuation of Radiation

If a beam of radiation contains N particles in a cross-sectional area S at right angles to the direction of the beam, the photon **fluence**, Φ , is defined by $\Phi = N/S$ – i.e. the number of particles that cross a unit area at right angles to the beam. (This quantity is sometimes called the flux density). Suppose we have beam of X- or gamma rays, incident on a thin sheet of material with a fluence of $\Phi(0)$; thus $\Phi(0)$ photons arrive at every square metre of the surface. These photons will interact with the atoms of the material (by one or other of the processes discussed in §2.3 above) and will be lost to the beam. This is the process we call **attenuation** of the beam.

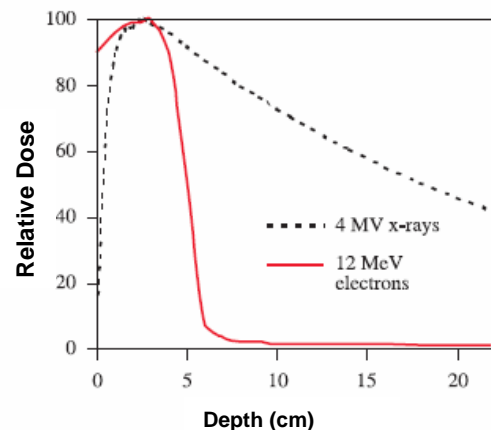


Consider interactions at a depth in the material, x , where the fluence can be written as $\Phi(x)$. The number of photons that interact with the atoms in a small thickness, dx , of the material is obviously proportional to the incident fluence at x , $\Phi(x)$, and to the thickness dx . So the number of particles lost to the beam between a depth x and a depth $(x+dx)$ can be written mathematically as $d\Phi = -\mu\Phi dx$, where the constant of proportionality, μ , is called the **linear attenuation coefficient**. The negative sign in this equation accounts for the fact that the original photons are being lost from the beam – i.e. Φ is being reduced. The solution to this equation is $\Phi(x) = \Phi(0) \exp(-\mu x)$; check by differentiating to recover $d\Phi/dx = -\mu\Phi$. $\Phi(0)$ is, obviously, the fluence at $x = 0$.

The linear attenuation coefficient, μ , depends on the type and energy of radiation and the properties of the material. In particular, μ is proportional to the density of the material (ρ) (see the table below – a ‘proof’ appears in Appendix 2-B), and it is often convenient to define the **mass attenuation coefficient** for a given material m , $(\mu/\rho)_m$.

For dose calculations, both the ‘density’ of the beam, measured by the fluence, and the rate at which the particles arrive are important. If, in a time interval of Δt , the number of particles crossing the unit area is $\Delta\Phi$, we can define the **fluence rate** ϕ , as the number of particles crossing a unit area in a unit time. Obviously the fluence rate will decrease with distance, x , as $\phi(x) = \phi(0) \exp(-\mu x)$ as the beam traverses material.

The penetration of X-rays is much greater than that of electrons, as the figure opposite shows. This is a “depth-dose” curve, which compares the penetrating power in tissue of 4 MV X-rays to that of 12 MeV electrons; the scales have been normalized to make the dose agree at the maxima for each particle. (At these energies, you should be able to judge whether the beams would be used for therapeutic rather than diagnostic purposes). Note that even though the electrons are of much higher energy, they do not penetrate nearly so far into the tissue as do the X-rays.

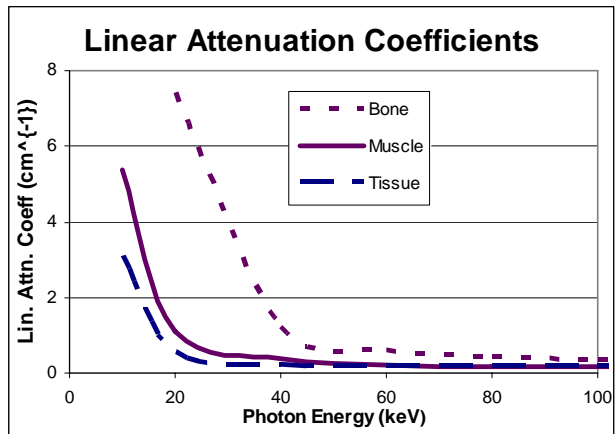


(Ref.1, Figure 6)

2.6 Diagnostic Use of X-rays

The use of X-rays for diagnostic purposes depends on the fact that the attenuation coefficient is different for bone, muscle, soft tissue and tumours. When X-rays pass through heterogeneous material, e.g. the human body, the reduction in the fluence will depend on the variety of attenuation coefficients corresponding to the different types of material encountered. If one type of material of thickness x_i has a linear attenuation coefficient μ_i , the simple equation above is replaced by $\Phi = \Phi(0) \exp(-\sum \mu_i x_i)$. Representative values of mass attenuation coefficients for some biological materials of interest are given in the table below (Ref.4): the graph shows the corresponding behaviour of the linear attenuation coefficients with energy.

| Mass Attenuation Coefficients ($\mu/\rho)_m$ (cm^2g^{-1}) | | | |
|---|-------------|-------------|-------------|
| Energy (keV) | Tissue | Muscle | Bone |
| 10 | 3.268 | 5.356 | 28.51 |
| 20 | 0.568 | 0.821 | 4.000 |
| 40 | 0.239 | 0.269 | 0.666 |
| 60 | 0.197 | 0.205 | 0.315 |
| 80 | 0.180 | 0.182 | 0.223 |
| 100 | 0.169 | 0.169 | 0.186 |
| 200 | 0.137 | 0.136 | 0.131 |
| 500 | 0.097 | 0.096 | 0.090 |
| 1000 | 0.071 | 0.070 | 0.066 |
| 2000 | 0.049 | 0.049 | 0.046 |
| 5000 | 0.030 | 0.030 | 0.029 |
| 10000 | 0.021 | 0.022 | 0.023 |
| 20000 | 0.017 | 0.018 | 0.021 |
| Density (g/cm^3) | 0.95 | 1.00 | 1.85 |



Remember that a typical spectrum of diagnostic X rays covers a band of energies from ten to a hundred keV or so, while therapeutic X-rays have energies in the several MeV range, up to a maximum of around 25 MeV.

Two features are immediately obvious; the linear attenuation coefficients increase rapidly with *increasing* density of the material and with *decreasing* energies. Thus as X-rays pass into matter, the lower energy rays are preferentially attenuated and there is a 'hardening' of the beam. This feature is used to screen out the very low energy ('soft') X-rays that are not sufficiently penetrating to reach the X-ray film, but deposit their unwanted energy in the patient. A thin foil of Aluminum is placed just after the target in diagnostic X-ray tubes in order to 'filter' out some of this unwanted portion of the emitted spectrum.

The linear attenuation coefficient for air is very much smaller ($\sim 10^{-3}$) than biological tissue or metals, thanks to the much lower density of air. Therefore, useful attenuation of X-rays can be achieved in air only by increasing the distance from the source (giving an $1/r^2$ reduction) or, most commonly, by inserting a thickness of heavy metal as shielding.

2.6.1 X-Ray Images

X-rays are a form of electromagnetic radiation like light; they are of higher energy, however, and can penetrate the body to form an image on film. Structures will appear as shades of gray on the photographic negative, depending on their density (bone appears white, corresponding to maximum attenuation, and air appears black). X-rays can provide information about obstructions, tumors, and other diseases. The first picture below shows a PA chest X-ray (so-called because the X-rays direction is from the Posterior (back) to the Anterior (front) of the body). The bones show up well, and some of the major arteries and blood vessels can be distinguished. Chest X-rays are usually PA, often supplemented by lateral views.

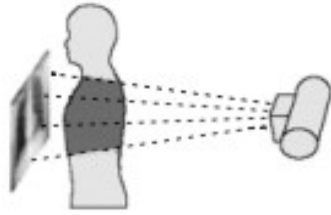


(This picture and the two below are from Ref. 5: www.geocities.com/HotSprings/1368/)

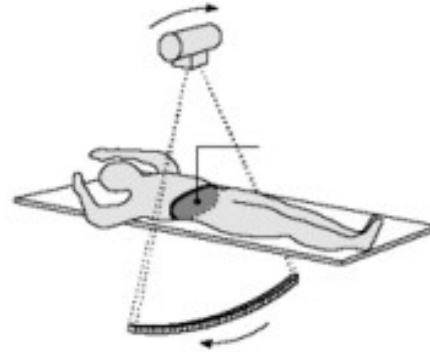


X-ray pictures of the abdomen are shown above. The lighter parts of the image correspond to greater attenuation of the X-ray beam. In the left one, the bones show up well but the soft tissues – that give a contrast of only several percent - are not well distinguished. Thus dyes or other more attenuating substances are often inserted or ingested to show up the soft tissue. The picture on the right was taken after the patient had been given a “barium meal”; a previously ingested laxative has cleared out any other solid material. The stomach and lower bowel show up well. Increasingly, film is being replaced by digital pictures viewed directly on a computer.

2.6.2 Computer Axial Tomography (CAT, or CT Scans)



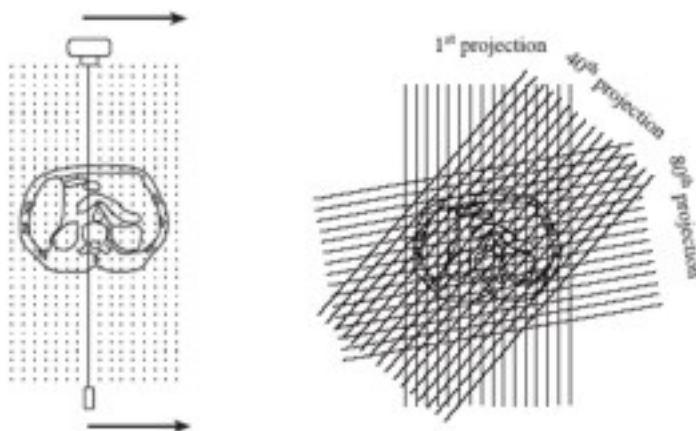
Standard PA Chest X-ray



X-ray Tomography

(Ref.2, figures 1 & 2)

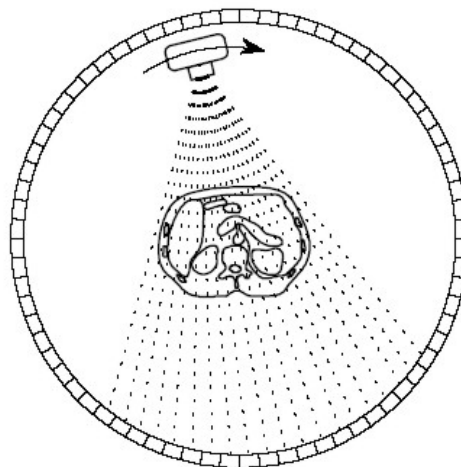
‘Tomography’ means the graphical representation of slices; the ‘axial’ refers to the plane at right angles to the central axis of the body (to be correct, it should be called ‘trans-axial’). In a standard X-ray exposure, the relative number of X-rays removed from a narrow beam measures only the total attenuation, but gives no information about the different materials in the path; the images of all the structures in the path of the X-rays fall on top of each other in the photographic image, as you can see in the diagrams above. To obtain information about the different structures, Godfrey Hounsfield in 1972 pointed out the advantage of taking many exposures at a variety of different angles. (Hounsfield and Cormack shared the 1979 Nobel Prize in Physiology or Medicine.) These can then be combined using sophisticated computer programmes to create a ‘picture’ of 2-dimensional slices of the exposed volume. To make reconstruction computationally possible the photographic film is replaced by many small solid state detectors.



(Ref. 2, figure 7)

The way this works can perhaps be imagined from the diagrams opposite. A much better understanding can be obtained from the dramatic and interactive Web site of Ref. 3: <http://www.colorado.edu/physics/2000/index.pl> (Click on *Einstein's Legacy* and then on *CAT Scan*).

Modern CT scanners are now much more sophisticated. Shown opposite is a “4th generation” scanner that involves a moving X-ray tube and a circular array of detectors that surrounds the patient. In more recent types of scanner, both the X-ray tube and the detector array spin around the patient.



(Ref.2, figure 13)

Here is a CT scan of the abdomen. The liver, the spinal column, and the kidneys can all be clearly seen.



(Ref.2, figure 2)

When the different 2-dimensional images are combined, a sophisticated computer calculation can then provide what amounts to a 3-D reconstruction. (Similar computational techniques are used in Magnetic Resonance Imaging, studied in SNVI).

Read an excellent article on an advanced CT scanner, just purchased for the Toronto General at the Globe and Mail site (<http://www.theglobeandmail.com/> ; search the archives for “Images reveal life’s secrets, one slice at a time” – 26th November, 2007). The article by Greg Michael (Ref. 2) has interesting details on the detectors used in CT scans, some outstanding images, and discussions of methods of computation, image display, and other applications that we will not have time to study in depth.

Appendix 2-A. Some Useful Physical Quantities.

| Material | Density (g cm ⁻³) | Effective Atomic Number, \bar{Z} | No. of Electrons per gm. (ZN_A/A) (x 10 ⁻²³ g ⁻¹) |
|----------------|-------------------------------|------------------------------------|--|
| Air | 0.001293 | 7.6 | 3.01 |
| Water | 1.00 | 7.4 | 3.34 |
| Muscle | 1.00 | 7.4 | 3.36 |
| Adipose Tissue | 0.95 | 5.9 | 3.48 |
| Bone | 1.85 | 13.8 | 3.00 |
| Lead | 11.7 | 82 | 2.38 |

Note that bone is about twice the density of tissue or muscle, and that the electron density (can you show that this is just ZN_A/A ?) is approximately constant across the periodic table.

Appendix 2-B. Photon Scattering and Attenuation Coefficients

The probability of scattering depends on the **cross-section for scattering** of the atoms that compose the irradiated material. It is easiest to think of this cross-section as the area that the atom presents to the incident beam for the particular type of scattering under consideration; I will denote it by σ (pronounced ‘sigma’). Consider a very thin slice of the material, of thickness dx , on to which a beam of photons of cross-sectional area S impinges. The probability of scattering, P , is then equal to the total area of the cross-sections of the n atoms presented to the beam, divided by the total area of the beam. (The choice of a very thin slice of material ensures that no atom or electron is hidden behind others.) Thus $P = n\sigma/S$. Now n , the number of scattering centres encountered by the beam in the area S , is given by the number of atoms per mole, N_A , multiplied by the number of moles in the volume of the slice being irradiated, equal to $\rho S dx/A$, where ρ is the density of the material and A is its mass number.

$$\text{Thus } P = (N_A)(\rho S dx/A) (\sigma / S) = (N_A/A)\sigma\rho dx.$$

P is equal to the fraction of photons in the beam that are scattered, dN/N . (Since $\Phi = N/S$ – see §2.5 – we have also that $dN/N = d\Phi/\Phi = P$). Inspection of the definition of the linear attenuation coefficient in §2.5 yields $\mu = (N_A/A)\sigma\rho$, and the mass attenuation coefficient, $\mu/\rho = (N_A/A)\sigma$.

Now we are in a position to consider the factors that determine how an X-ray beam is attenuated by different physical processes.

The Photoelectric Effect. Quantum mechanical calculations show that the probability of interaction per atom is approximately proportional to $Z^4 E_\gamma^{-3}$, where Z is the atomic number of the atom, and E_γ is the energy of the X-ray. Thus at low X-ray energies, where the photoelectric effect dominates, we have, approximately, $\sigma \propto Z^4 E_\gamma^{-3}$.

Thus $\mu = (N_A/A)\sigma\rho \propto (N_A/A)Z^4 E_\gamma^{-3}\rho = (ZN_A/A)Z^3 E_\gamma^{-3}\rho$. Now (ZN_A/A) is approximately constant across the periodic table, $\mu \propto Z^3\rho$ for a given energy. The beam attenuation for a given energy is thus given by

$$\Phi(x) = \Phi(0) \exp(-\mu x) \propto \Phi(0) \exp(-Z^3 \rho x).$$

Compton Scattering. At higher energies, we have to consider Compton Scattering; here the scattering probability per atom depends approximately on the number of electrons available. Thus σ is directly proportional to Z , and, since Z/A is approximately constant across the periodic table, the mass attenuation coefficient, μ/ρ , is approximately equal for bone, muscle and tissue at energies where the Compton effect dominates (you can check this prediction in the table in §2.6). Similarly, at a given energy, the beam attenuation is

$$\Phi(x) = \Phi(0) \exp(-\mu x) \propto \Phi(0) \exp(-\rho x).$$

The greater the density, the more attenuation a given thickness of material at a given energy will produce. This effect will be greatly enhanced at low energies, where the photoelectric effect dominates, due to the Z^3 factor. Thus the best contrast between bone and tissue occurs at lower X-ray energies.

Appendix 2-C. Backup Reading in Knight

The table lists some of the relevant sections in Knight.

| Section in SNII | Corresponding Sections in Knight |
|------------------------|---|
| 2.1 | 24.2 (p.760) |
| 2.3.1 | 38.1 (p.1220,1221) |
| 2.3.3 | 36.10 (p.1186) |
| 2.4 | 42.7 (p.1373,1374) |

7 December 2007