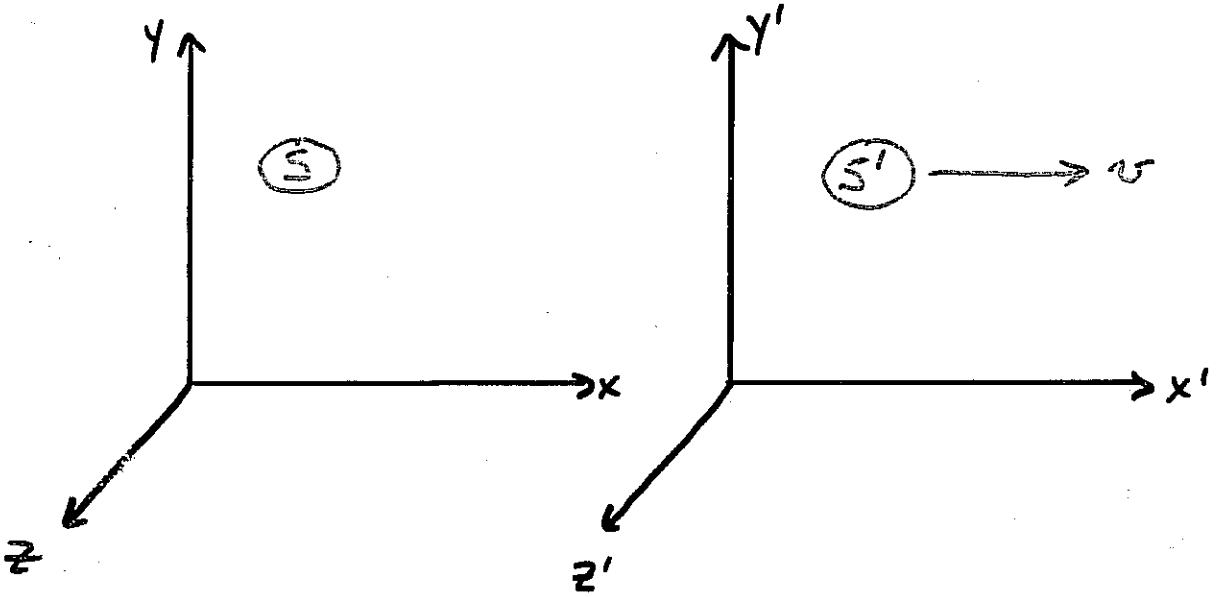
SPECIAL RELATIVITY AND SUBATOMIC PHYSICS

AT HIGH ENERGIES TYPICAL OF NUCLEAR AND PARTICLE PHYSICS, RELATIVISTIC EFFECTS ARE IMPORTANT



$$S \longrightarrow S' \qquad x' = \gamma(x - vt) \qquad y' = y \quad z' = z$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

SPATIAL COORDINATES $\perp \vec{V}$ ARE UNAFFECTED

TIME COORDINATE MIXED WITH SPATIAL COORDINATE $/\!/ \vec{v}$!

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \equiv \text{LORENTZ BOOST FACTOR}$$

$$= \frac{1}{(1-\beta^2)^{1/2}} \qquad \text{WITH} \quad \beta = v/c$$

RELATIVISTIC MOMENTUM $\vec{p} = \gamma m \vec{v}$

DEFINE $p = |\vec{p}|$ $v = |\vec{v}|$ $\Longrightarrow$ $p^2 = \gamma^2 m^2 v^2$

$$v^2 = \frac{p^2}{m^2\gamma^2} = \frac{p^2}{m^2}(1-\beta^2) = \frac{p^2}{m^2}\left(1 - \frac{v^2}{c^2}\right)$$

$$v^2 m^2 c^4 = c^4 p^2 - p^2 v^2 c^2 \qquad v^2\underbrace{(m^2c^4 + p^2c^2)}_{E^2} = c^2 c^2 p^2$$

$$\frac{v^2}{c^2} E^2 = c^2 p^2 \qquad \boxed{\beta = \frac{cp}{E}}$$

RATIO OF A PARTICLES MOMENTUM TO ITS TOTAL RELATIVISTIC ENERGY GIVES A MEASURE OF HOW RELATIVISTIC IT IS.

CONSIDER A 1 MeV ELECTRON ($M_e = 0.5$ MeV/c^2)

total relativistic energy

$$\beta = \frac{pc}{E} \approx \frac{\sqrt{(1)^2 - (0.5)^2}}{1} \approx 0.87$$

ALREADY HIGHLY RELATIVISTIC!

CONSEQUENCES OF SPECIAL RELATIVITY

- RELATIVITY OF SIMULTANEITY
- LORENTZ CONTRACTION ALONG DIRECTION OF MOTION

$$L = L'/\gamma$$

- TIME DIALATION: TIME RUNS MORE SLOWLY IN A MOVING REFERENCE FRAME

$$t = \gamma t'$$

TIME DIALATION IS THE MOST RELEVANT ISSUE IN SUBATOMIC PHYSICS.

EVERY UNSTABLE PARTICLE HAS A LIFETIME THAT IS DEFINED IN THE PARTICLES REST FRAME

→ THIS IS A REFERENCE FRAME THAT ALL OBSERVES CAN EASILY AGREE UPON

FOR A MOVING PARTICLE (IN SOME LAB REFERENCE FRAME)

$$\tau_{LAB} = \gamma \tau$$

DISCUSS THIS FURTHER IN A MOMENT

FOUR VECTORS AND SPECIAL RELATIVITY

DEFINE POSITION-TIME FOUR-VECTOR

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$$

LORENTZ TRANSFORMATION BECOMES

$$x^{0\prime} = \gamma(x^0 - \beta x^1)$$

$$x^{1\prime} = \gamma(x^1 - \beta x^0)$$

$$x^{2\prime} = x^2$$

$$x^{3\prime} = x^3$$

OR $$x^{\mu\prime} = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \qquad \mu = 0, 1, 2, 3$$

can think of these as the elements of some matrix

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

sum over repeated indices is implied

USING EINSTEIN SUMMATION CONVENTION $x^{\mu\prime} = \Lambda^\mu_\nu x^\nu$

LORENTZ INVARIANTS

A CRITICAL CONCEPT IN SUBATOMIC PHYSICS IS THAT OF INVARIANTS: QUANTITIES THAT ARE THE SAME IN ALL REFERENCE FRAMES.

CONSIDER AN ANALOGY USING THREE VECTORS IN THREE DIMENSIONAL SPACE, IN COORDINATE SYSTEMS ROTATED WRT ONE ANOTHER.

VECTOR QUANTITIES SUCH AS POSITION $\vec{x}$, VELOCITY $\vec{v}$ ARE DIFFERENT IN THE TWO FRAMES

SCALAR QUANTITIES SUCH AS $(\text{SPEED})^2 = \vec{v}\cdot\vec{v}$ ARE THE SAME IN THE TWO FRAMES

HOW DO WE CONSTRUCT INVARIANTS IN 4D SPACE-TIME?

$$x^2 = (x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$$

DOESN'T WORK (TRY IT)

BUT CONSIDER $I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$

USE LORENTZ TRASFORMATION TO WRITE THIS IN TERMS OF $x^{\mu\prime}$

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$= (\gamma x^{0\prime} + \gamma\beta x^{1\prime})^2 - (\gamma x^{1\prime} + \gamma\beta x^{0\prime})^2 - (x^{2\prime})^2 - (x^{3\prime})^2$$

$$= (x^{0\prime})^2 - (x^{1\prime})^2 - (x^{2\prime})^2 - (x^{3\prime})^2 \quad \text{[after some algebra]}$$

$$= I \; !$$

THIS QUANTITY IS CALLED LORENTZ INVARIANT. IT IS THE SAME IN ANY INERTIAL REFERENCE FRAME

ANY FOUR COMPONENT OBJECT WHICH TRANSFORMS LIKE x^μ UNDER A LORENTZ TRANSFORMATION IS CALLED A

FOUR VECTOR

GIVEN ANY TWO FOUR VECTORS a, b, THE QUANTITY

$$a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

IS LORENTZ INVARIANT !

index up

STANDARD CONTRAVARIANT FOUR VECTOR x^{μ}

INTRODUCE COVARIANT FOUR VECTOR $x_{\mu} = g_{\mu\nu} x^{\nu}$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \text{(metric)}$$

$$a^{\mu} = (a^0, \vec{a}) \qquad a_{\mu} = (a^0, -\vec{a}) \qquad a^2 = a_{\mu} a^{\mu} = (a^0)^2 - |\vec{a}|^2$$

$a^2 > 0$ REFERRED TO AS TIMELIKE 4 VECTOR

$a^2 < 0$ " SPACELIKE "

$a^2 = 0$ " LIGHTLIKE "

ANOTHER USEFUL (IMPORTANT) 4 VECTOR IS THE ENERGY-MOMENTUM FOUR-VECTOR

$$p \equiv \left(\frac{E}{c}, \vec{p}\right) \qquad p^2 = \frac{E^2}{c^2} - |\vec{p}|^2 = \frac{|\vec{p}|^2 c^2 + m^2 c^4}{c^2} - |\vec{p}|^2 = m^2 c^4$$

SO p^2 INVARIANT AS REQUIRED

SOMETIMES SEE CALCULATIONS WITH $c = 1$, $p^2 = m^2$. I WILL AVOID THIS

NB THERE ARE OTHER CONVENTIONS FOR DEFINITION OF FOUR-VECTORS AND CORRESPONDING METRIC [SEE PROF. ORR'S NOTES IF YOU ARE INTERESTED] WE WILL STICK TO THE ABOVE DEFINITIONS

DECAYS IN QUANTUM MECHANICS

DECAYS OF PARTICLES (SUCH AS RADIOACTIVE NUCLEI) ARE QUANTUM MECHANICAL PROCESSES THAT ARE INHERENTLY RANDOM

THE DECAY OF A PARTICULAR PARTICLE (OR STATE) IS CHARACTERIZED BY THE MEAN LIFETIME τ WHICH IS DEFINED IN THE PARTICLE'S REST FRAME

THE NUMBER OF PARTICLES DECAYING IN SOME TIME INTERVAL dt IS GIVEN BY

number of undecayed particles at time t

$$dN = -\lambda N(t)\, dt$$

time interval

negative because N is decreasing

decay constant, determines transition rate.

$$dN = -\lambda N(t)\, dt \qquad \frac{dN}{N} = -\lambda\, dt$$

$$\int_{N(0)}^{N(t)} \frac{dN}{N} = -\lambda \int_0^t dt \implies \ln N(t) - \ln N(0) = -\lambda t$$

$$N(t) = N(0)\, e^{-\lambda t}$$

EXPONENTIAL DECAY [SURVIVAL PROBABILITY]

THIS DECAY LAW APPLIES TO ALL UNSTABLE PARTICLES

EG UNSTABLE HEAVY NUCLEI AS WELL AS FUNDAMENTAL PARTICLES LIKE W, Z BOSONS.

INTENSITY OF RADIATION $\equiv$ ACTIVITY

$$I(t) = -\frac{dN(t)}{dt} = \underbrace{N(0)\lambda}_{} e^{-\lambda t}$$

$$I(t) = I(0)\, e^{-\lambda t}$$

$I(0)$ → initial activity or intensity

UNITS OF RADIOACTIVITY: INTERNATIONAL STANDARD IS SI UNITS

1 UNIT OF ACTIVITY = 1 BECQUEREL (Bq)

= 1 DISINTEGRATION / S

ALSO SEE ACTIVITY QUOTED IN CURIES (Ci)

$$\equiv \underbrace{3.7 \times 10^{10} \text{ DISINTEGRATIONS / S}}_{\text{CORRESPONDS TO ACTIVITY OF 1 GRAM OF RADIUM}}$$

UNITS IN SUBATOMIC PHYSICS

QUANTITY	UNIT	SYMBOL	TYPICAL VALUE
TIME	SECOND	s	DISCUSSED LATER ON
LENGTH	METRE	m	10^{-15} m $\equiv$ 1 FERMI (NUCLEAR SIZE)
ENERGY	ELECTRON-VOLT	eV	10^6 eV = 1 MeV 10^9 eV = 1 GeV
MOMENTUM	ELECTRON-VOLT	eV/c	SEE ENERGY
MASS	ELECTRON-VOLT	eV/c^2	PARTICLE MASSES (BELOW)

PARTICLE MASSES (FUNDAMENTAL FERMIONS)

u	300 MeV/c^2	e	0.511 MeV/c^2
d	300 MeV/c^2	μ	106 MeV/c^2
s	500 MeV/c^2	τ	1.784 GeV/c^2
c	1.5 GeV/c^2	ν_e	very small but $\neq 0$
b	4.7 GeV/c^2	ν_μ	very small but $\neq 0$
t	175 GeV/c^2	ν_τ	very small but $\neq 0$

GAUGE BOSONS $M_\gamma = 0$ $M_g = 0$ $M_{W^\pm} = 80.4$ GeV/c^2 $M_Z = 91.2$ GeV/c^2

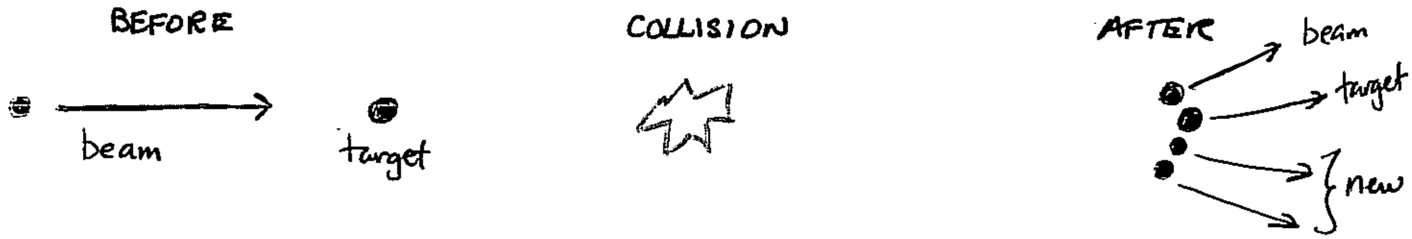
PROTON MASS = 938.2 MeV/c^2 NEUTRON MASS = 939.6 MeV/c^2

HIGGS BOSON MASS   114 GeV/c^2 < M_H < ~1 TeV $\equiv$ 1000 GeV

ENERGY, MOMENTUM & MASS IN SUBATOMIC PHYSICS

TYPICALLY, IN EXPERIMENTS WE SCATTER PARTICLES OFF OF SOME TARGET.

SOME OF THE INITIAL KINETIC ENERGY CAN APPEAR AS MASS IN THE FINAL STATE [AFTER THE COLLISION]



THUS IT IS CONVENIENT TO MEASURE MASSES IN THE SAME UNITS AS ARE USED FOR ENERGY.

rest mass of particle

RECALL

total relativistic energy $\quad E^2 = p^2c^2 + m^2c^4 \qquad p = |\vec{p}|$

momentum $\qquad$ speed of light

FOR A MASSLESS PARTICLE (γ, or approximately, ν)

$$E = pc \qquad p = \frac{E}{c} \implies \text{Units } \frac{[eV]}{c}$$

FOR MASSIVE PARTICLES, CONSIDER THEM IN THEIR REST FRAME

$$E = mc^2 \qquad m = \frac{E}{c^2} \implies \text{Units } [eV]/c^2$$

A PARTICLE WITH A MASS OF 1 MeV/c^2 HAS A TOTAL RELATIVISTIC ENERGY OF 1 MeV WHEN AT REST.

IF THIS PARTICLE HAS MOMENTUM OF 2 MeV/c THEN IT HAS A TOTAL RELATIVISTIC ENERGY OF

$$E^2 = \underbrace{(2\,\text{MeV}/c)^2 c^2 + (1\,\text{MeV}/c^2)^2 c^4}$$

Note that each term has units of E

$$E^2 = 5\,\text{MeV}^2 \qquad E = \sqrt{5}\,\text{MeV}$$

WHAT IS β FOR THIS PARTICLE? IS IT RELATIVISTIC?

$$\beta = \frac{cp}{E} = \frac{2}{\sqrt{5}} \approx 0.89$$

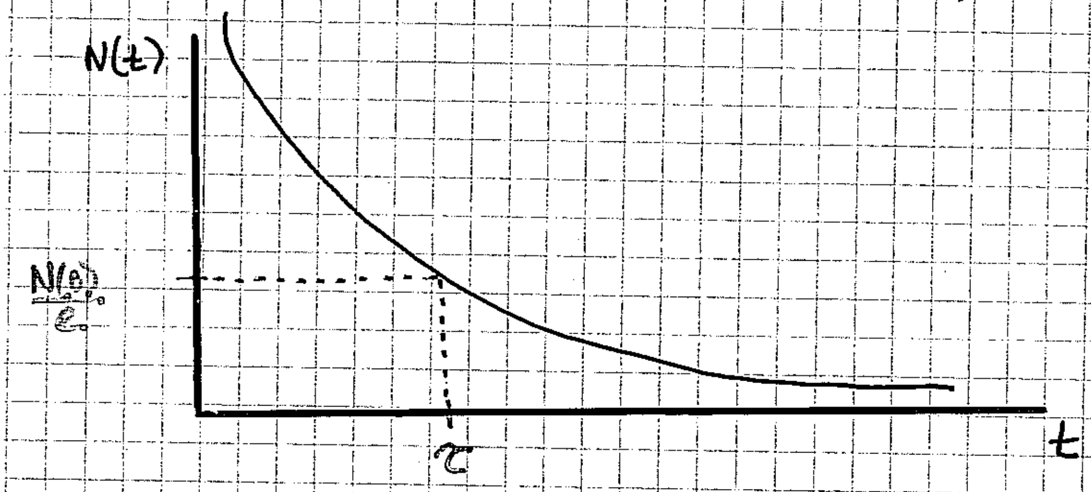
Yes, it is relativistic. Lorentz boost would be

$$\gamma = \frac{1}{\sqrt{1 - 4/25}} = 2.5$$

PARTICLE LIFETIMES

WE HAD $N(t) = N(0)\, e^{-\lambda t}$

DEFINE MEAN LIFETIME AS $\tau = \frac{1}{\lambda}$



IN PARTICLE PHYSICS THIS IS THE USUAL DEFINITION OF THE LIFETIME.

IN NUCLEAR PHYSICS IT IS MORE COMMON TO USE THE HALF-LIFE

$$\tau_{1/2} = \tau \ln 2$$

↳ time required for half of the particles to disintegrate

THERE IS AN ENORMOUS RANGE OF PARTICLE LIFETIMES

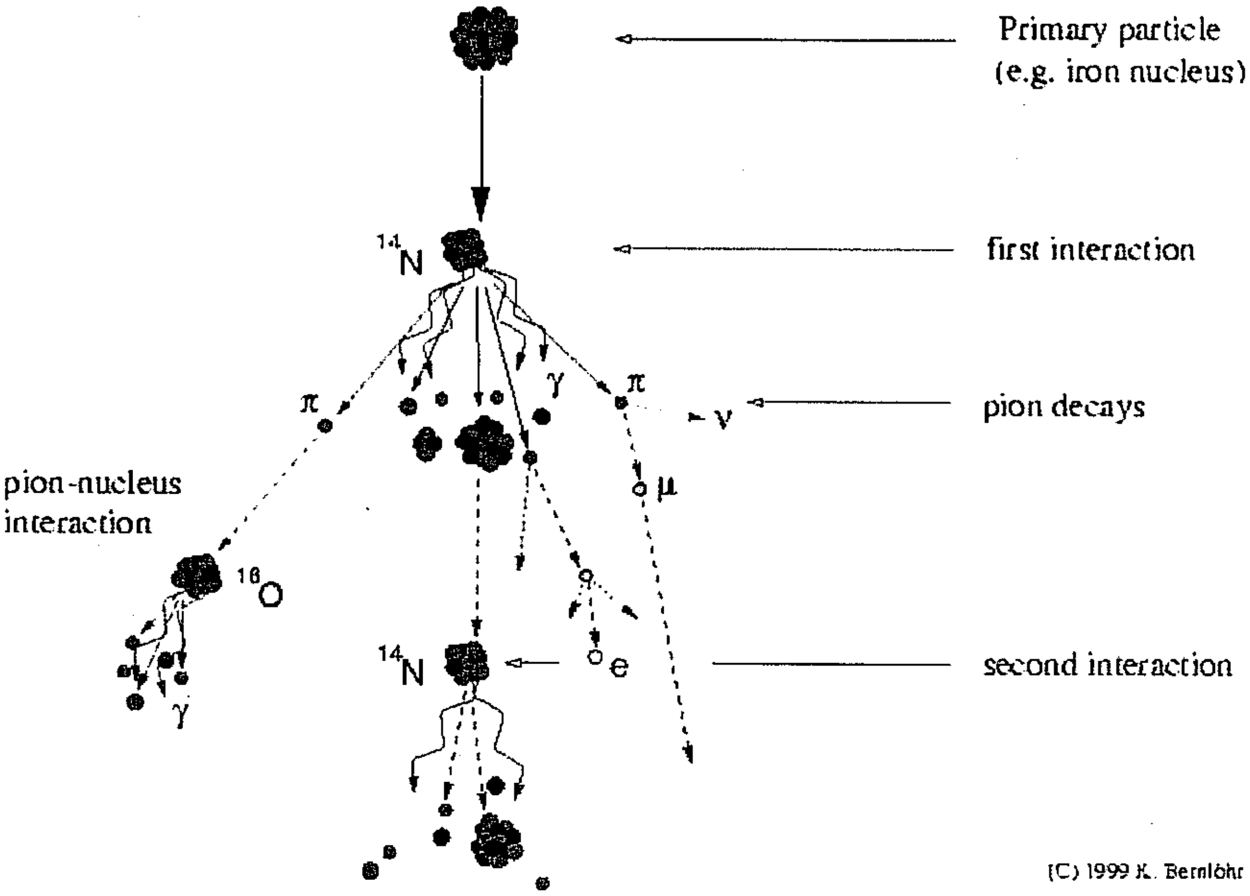
PROTON	$\tau > 10^{31}$ years (>> age of Universe)
$^{238}U \rightarrow {}^{234}U + \alpha$	6.5×10^{9} years
$^{215}Po \longrightarrow {}^{211}Pb + \alpha$	2×10^{3} s
$\mu^+ \longrightarrow e^+ \nu_e \bar{\nu}_\mu$	2.2×10^{-6} s
$\pi^0 \longrightarrow \gamma\gamma$	8.3×10^{-17} s
$\Delta^{++} \longrightarrow p\pi^+$	6×10^{-24} s

LIFETIMES (DECAY RATES) ARE LARGELY DETERMINED BY THE STRENGTH OF THE UNDERLYING FUNDAMENTAL INTERACTION [AS WELL AS KINEMATICAL CONSIDERATIONS

TYPICAL TIMESCALES

WEAK INTERACTION	10^{-13} s to 15 minutes
ELECTROMAGNETIC INTERACTION	10^{-16} s
STRONG INTERACTION	10^{-23} s

Development of cosmic-ray air showers



SPECIAL RELATIVITY AND PARTICLE LIFETIMES

REMEMBER: THE LIFETIME τ IS DEFINED IN THE PARTICLE'S REST FRAME

IN A REFERENCE FRAME IN WHICH THE PARTICLE IS IN MOTION THE APPARENT LIFETIME IS INCREASE BY A FACTOR OF γ [TIME DIALATION]

<u>EXAMPLE</u> COSMIC RAYS IMPINGING ON THE UPPER ATMOSPHERE CREATE MANY SHORT-LIVED SECONDARY PARTICLES WITH VELOCITIES DIRECTED TOWARDS THE EARTH.

CONSIDER MUONS PRODUCED AT AN ALTITUDE OF 8000 m TRAVELLING TOWARDS THE GROUND AT A SPEED CLOSE TO THE SPEED OF LIGHT. $\beta = 0.998$

DO THEY LIVE LONG ENOUGH TO BE DETECTED BY GROUND BASED DETECTORS?

CLASSICALLY THE ANSWER IS NO. $\tau(\mu) = 2.2 \times 10^{-6}$ s

$$\text{distance} = \text{velocity} \times \text{time} = [2.2\times10^{-6}\,\text{s}][0.998][3\times10^{8}\,\text{m/s}]$$

$$= 660\,\text{m}$$

RELATIVISTICALLY THE SITUATION IS RATHER DIFFERENT

DISTANCE = velocity × time   time = $\gamma\tau$

$$\gamma = \frac{1}{\sqrt{1-(0.998)^2}} = 15.8$$

DISTANCE = $15.8\ (2.2\times10^{-6}\,s)\ (0.998)\ (3\times10^{8}\,m/s) = 10{,}400\ m$

SO MOST MUONS MAKE IT TO THE GROUND

SENSITIVE NUCLEAR AND PARTICLE PHYSICS EXPERIMENTS SOMETIMES NEED TO BE LOCATED DEEP UNDERGROUND TO BE SHIELDED FROM COSMIC RAYS WHICH CAUSE THEM PROBLEMS

THIS IS PARTICULARLY TRUE FOR NEUTRINO DETECTORS SUCH AS

THE SUDBURY NEUTRINO OBSERVATORY [SNO]

NATURAL RADIOACTIVITY [RADIOACTIVE DECAY]

He nucleus 2p+2n

α - DECAY : EMISSION OF AN α PARTICLE IN NUCLEAR DECAY

$$^{238}U \rightarrow {}^{234}U + \alpha$$

mono energetic
[cons. of momentum, energy]

NUCLEAR NOTATION $^{A}_{Z}X$

A → number of nucleons (p+n)

X → elemental symbol

Z → number of protons in nucleus

NUCLEI WITH THE SAME Z BUT DIFFERENT A ARE CALLED

ISOTOPES

β- DECAY : EMISSION OF AN ENERGETIC ELECTRON [FROM $n \rightarrow p + e^- + \bar{\nu}_e$]

$Z \rightarrow Z+1$
A FIXED

$$^{40}_{19}K \rightarrow {}^{40}_{20}Ca + e^- + \bar{\nu}_e$$

continuous energy spectrum
from 0 to some maximum.

γ- DECAY: EMISSION OF GAMMA RAY (γ) IN DE-EXCITATION OF EXCITED NUCLEAR STATE

* denotes excited state.

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$