EXPERIMENTS IN NUCLEAR & PARTICLE PHYSICS

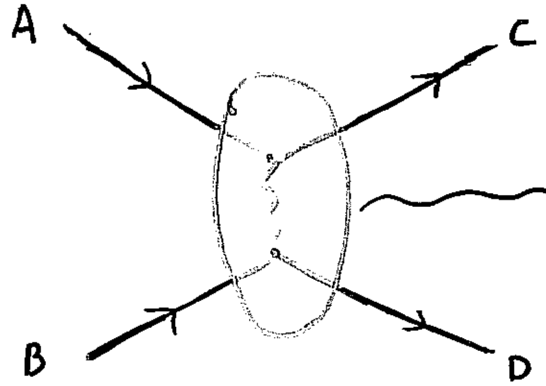
① PROPERTIES OF BOUND STATES

② PARTICLE DECAYS

③ PARTICLE SCATTERING

ALL THREE YIELD INFORMATION ON THE NATURE OF FUNDAMENTAL FORCES.

FOCUS FOR THE TIME BEING ON SCATTERING EXPERIMENTS

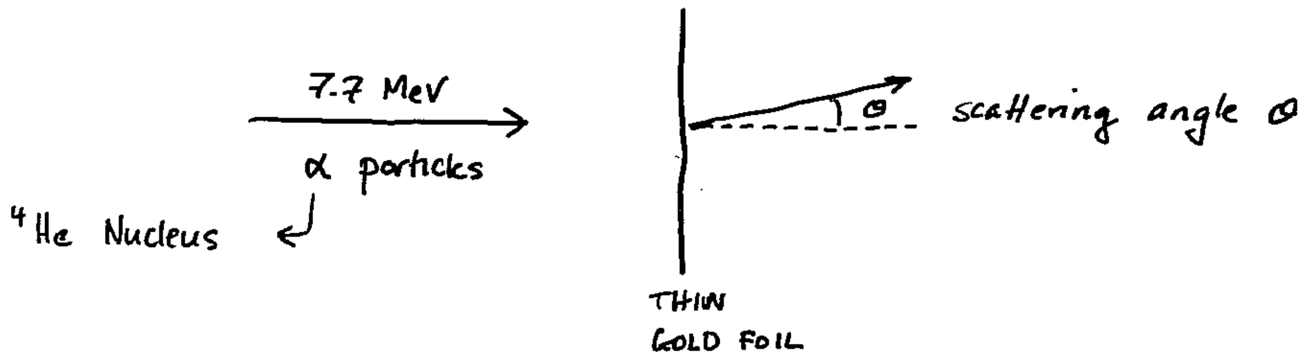


Infer dynamics inside blob from measured properties of ABC&D!

SCATTERING EXPERIMENTS & STRUCTURE OF MATTER

METHODOLOGY FOR INVESTIGATION OF SMALLER & SMALLER DISTANCE SCALES EPITOMISED BY RUTHERFORD SCATTERING EXPERIMENT

PERFORMED BY GEIGER & MARSDEN IN 1909 [UNDER THE SUPERVISION OF RUTHERFORD]



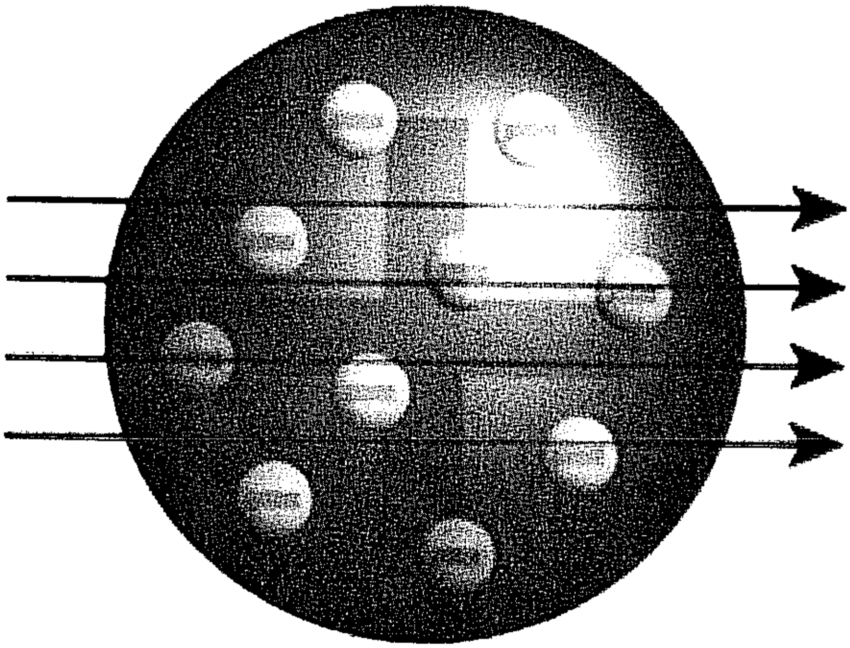
- PROBES THE STRUCTURE OF THE TARGET
- REVEALS THE NATURE OF THE FORCE ACTING BETWEEN BEAM PARTICLE AND TARGET.

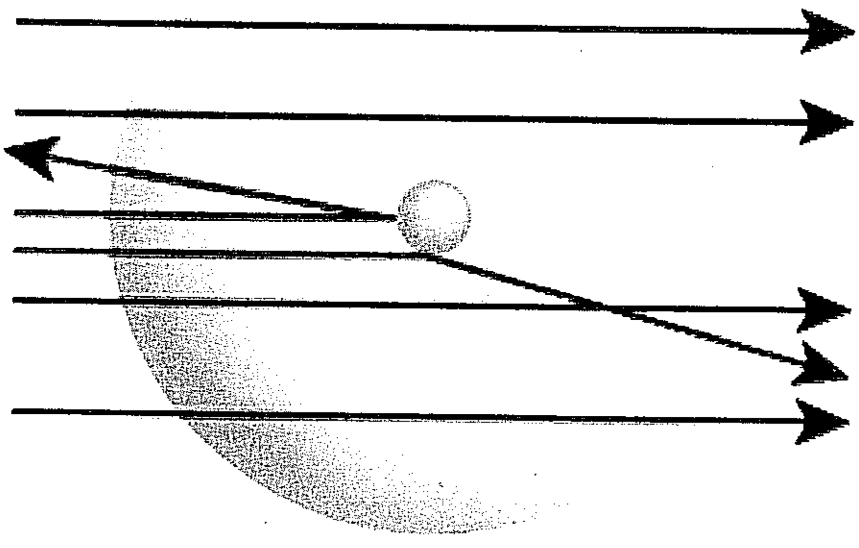
IT WAS KNOWN THAT THE ATOM MUST HAVE POSITIVE CHARGE TO BALANCE THE ELECTRONS [SEEN FOR INSTANCE IN β DECAY].

WHAT WAS NOT KNOWN, WAS HOW THIS POSITIVE CHARGE WAS DISTRIBUTED THROUGH THE ATOMIC VOLUME ($R \simeq 10^{-10}$ m)

THOMSON (DISCOVERER OF ELECTRON) PROPOSED THE PLUM PUDDING MODEL IN WHICH ELECTRONS WERE EMBEDDED IN A UNIFORM DISTRIBUTION OF POSITIVE CHARGE

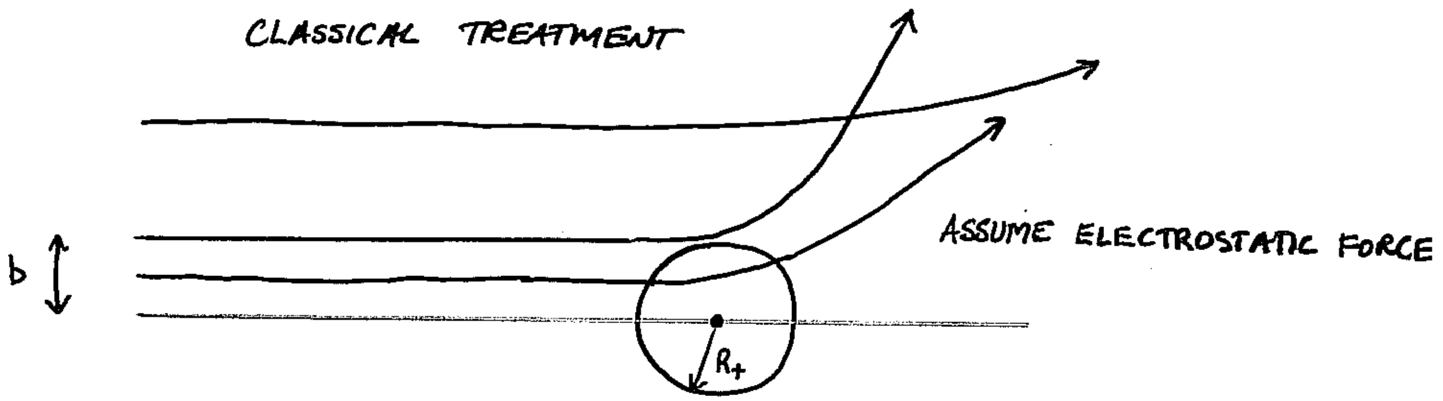
Plum Pudding Atomic Model Vs. Nucleus Model





RUTHERFORD SCATTERING (SIMPLIFIED VIEW)

CLASSICAL TREATMENT



R_+ ≡ radius of +ve charge distribution

b ≡ "impact parameter"

INVESTIGATE SIZE OF POSITIVE CHARGE DISTRIBUTION IN ATOM BY FIRING POSITIVELY CHARGED "PROBE" PARTICLES [α PARTICLES]

IS $R_+ \approx$ ATOMIC SCALE ? ($R_- \sim 10^{-10}$ m)

OR IS $R_+ \ll R_-$? (POSITIVE CHARGE VERY LOCALIZED)

IN PICTURE ABOVE, FOR

$b > R_+$ ELECTROSTATIC REPULSION FALLS OFF WITH INCREASING b

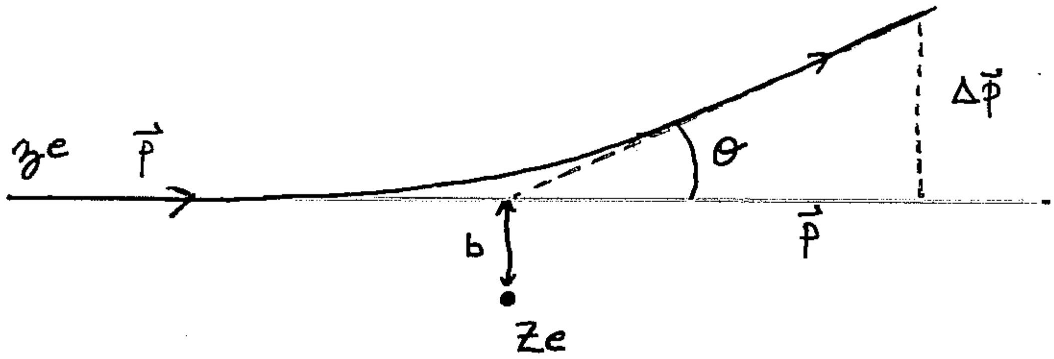
$b < R_+$ α PASSES THROUGH CHARGE DISTRIBUTION, SOME CHARGE REPELS α IN ONE DIRECTION, SOME IN THE OTHER

MAXIMUM DEFLECTION OCCURS FOR $b \sim R_+$

MEASUREMENT OF b FOR MAXIMUM DEFLECTION GIVES MEASURMENT OF $\underbrace{\text{SIZE OF THE POSITIVE CHARGE DISTRIBUTION}}$

$R_+ \sim$ size of the nucleus!

IDEALIZED MODEL FOR SIMPLE CALCULATION



- ASSUME THE θ IS A SMALL ANGLE

ALONG ORIGINAL LINE OF FLIGHT α-PARTICLE IS SLOWED BY THE COULOMB REPULSION. AFTER IT IS SCATTERED IT IS ACCELERATED BY THE SAME AMOUNT.

$\rightarrow E^{\alpha}_{\text{initial}} = E^{\alpha}_{\text{FINAL}}$

[THIS ASSUMES $M_{\text{NUCLEUS}} \gg M_{\alpha}$. ELASTIC SCATTERING]

- APPROXIMATE ALL DEFLECTION AS AN IMPULSE TRANSVERSE TO THE ORIGINAL MOTION, AS α PASSES THE NUCLEUS

FORCE ON THE CHARGE ze AT A DISTANCE b FROM THE NUCLEUS GIVEN BY COULOMB'S LAW

$$\boxed{F = \frac{zZe^2}{b^2}}$$

[NOTE THAT HERE WE ASSUME THAT THE ONLY FORCE IS THE COULOMB FORCE. AT VERY HIGH ENERGIES WE WOULD NEED ALSO TO CONSIDER THE STRONG INTERACTION OR NUCLEAR FORCE]

TO TREAT Δp AS AN IMPULSE DUE TO THE COULOMB FORCE, WE NEED A MODEL FOR HOW LONG THE FORCE ACTS ON THE α.

ASSUME IT ACTS FOR A DISTANCE b ON EITHER SIDE OF THE NUCLEUS, AND IS ALWAYS $\perp$ TO THE ORIGINAL DIRECTION OF MOTION:

TIME OVER WHICH THE FORCE ACTS IS THEN: $\Delta t = \frac{2b}{v}$

↳ velocity

$\frac{\Delta p}{\Delta t} = F$ - (BY DEFINITION!) $\quad \Delta p = F \Delta t$

$$\boxed{\Delta p \approx \frac{Zze^2}{b^2} \frac{2b}{v}}$$

APPROXIMATELY, THE SCATTERING ANGLE θ IS GIVEN BY

$$\sin\theta \approx \theta \approx \frac{\Delta p}{p}$$

$$= \frac{Zze^2}{b^2} \frac{2b}{v} \frac{1}{mv}$$

non-relativistic!

NB $E_\alpha = 7.7$ MeV refers to KINETIC ENERGY! This should be clear since $M_\alpha \approx 4M_p \approx 4$ GeV$/c^2$

So $\beta \approx \frac{7.7}{4000}$ Non-Relativistic approximation valid!

$= \frac{cp}{}$

$$\Longrightarrow \quad \theta \approx \frac{Zze^2}{b} \frac{2}{Mv^2}$$

$$b \approx \frac{Zze^2}{Mv^2\theta} \cdot 2$$

$$= \frac{Zze^2}{E\theta}$$

kinetic energy of α $\frac{1}{2}Mv^2$

"CORRECT" TREATMENT GIVES

$$\frac{Zze^2}{2E} \cot \theta/2$$

$$\cot \theta/2 = \frac{\cos\theta/2}{\sin\theta/2} \approx \frac{2}{\theta}$$

for θ small

GEIGER & MARSDEN OBSERVED MAXIMUM DEFLECTIONS OF ~ 1 RADIAN!

[THIS IS NOT SMALL ..... BUT]

set $b = R$ (radius of nucleus)

$\theta = 1$ radian

$$R = \frac{Zze^2}{E\theta} = \frac{Zze^2}{E}$$

$Z = 79$, $z = 2$ FOR α ON GOLD TARGET

$E = 7.7$ MeV

$$R = \frac{2 \cdot 79 \cdot e^2}{7.7\text{MeV}}$$

FINE STRUCTURE CONSTANT

$$\alpha = \frac{e^2}{\hbar c} = 1/137$$

$$\Longrightarrow e^2 = \frac{\hbar c}{137} \qquad \hbar c = 197.3 \text{ MeV}\cdot\text{fm}$$

$$e^2 = \frac{197.3}{137}\ \text{MeV}\cdot\text{fm} = 1.44\ \text{MeV}\cdot\text{fm}$$

$$R = \frac{(2)(79)}{7.7} \times 1.44\ \frac{[\text{MeV}]}{[\text{MeV}]}\cdot[\text{M}]\cdot 10^{-15}$$

$$\boxed{R \sim 3 \times 10^{-14}\ \text{m}}$$

RECALL THAT TYPICAL ATOMIC RADIUS IS OF ORDER 10^{-10} m !

OBSERVATION OF THESE LARGE SCATTERING ANGLES DEMONSTRATES THAT THE NUCLEAR RADIUS IS ABOUT 10^4 TIMES SMALLER THAN THE BOHR RADIUS OF AN ATOM.

NEGATIVE CHARGE IN AN ATOM IS RELATIVELY DIFFUSE

POSITIVE CHARGE IS VERY LOCALIZED !

MACROSCOPIC PHENOMENA REVEAL MICROSCOPIC STRUCTURE

eg the scattering angles

THE CALCULATION DONE BY RUTHERFORD WAS SIMPLER EVEN THAN THE ONE OUTLINED ABOVE.

INTERACTIONS IN QUANTUM MECHANICS

DISCUSSION OF RUTHERFORD SCATTERING RELIED ON A CLASSICAL DESCRIPTION OF "FORCE".

WHAT IS MEANT BY "FORCE"? (MICROSCOPICALLY!)

A FORCE IS SOMETHING THAT CHANGES THE MOMENTUM OF A PARTICLE.

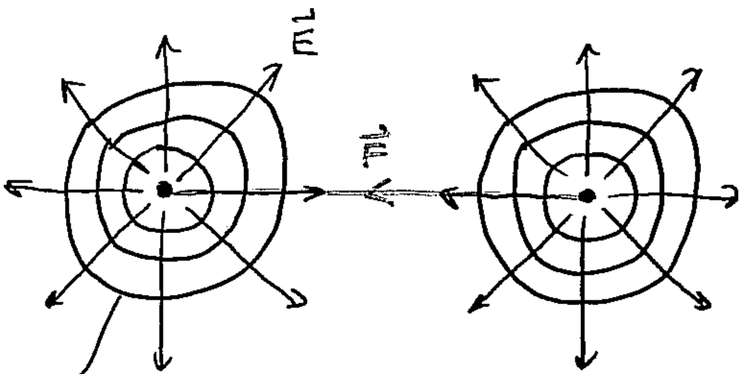
NEWTON

$$\text{FORCE} = \text{MASS} \cdot \text{ACCELERATION}$$
$$= M \frac{\Delta V}{\Delta t} = \frac{\Delta p}{\Delta t}$$

WHAT ONE OBSERVES (MEASURES) IS A MOMENTUM CHANGE

THERE ARE DIFFERENT ABSTRACT MODELS OF WHAT THE FORCE IS THAT "CAUSES" THE MOMENTUM CHANGE

CLASSICALLY WE THINK IN TERMS OF FIELDS AND POTENTIALS



$$\vec{E} = -\vec{\nabla} V$$
$$\vec{F} = q\vec{E}$$

Lines of potential
V

FORCES (INTERACTIONS) BETWEEN FUNDAMENTAL PARTICLES

ARISE DUE TO PARTICLE EXCHANGE!

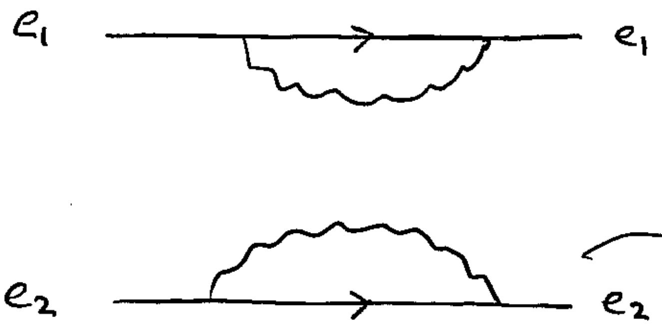
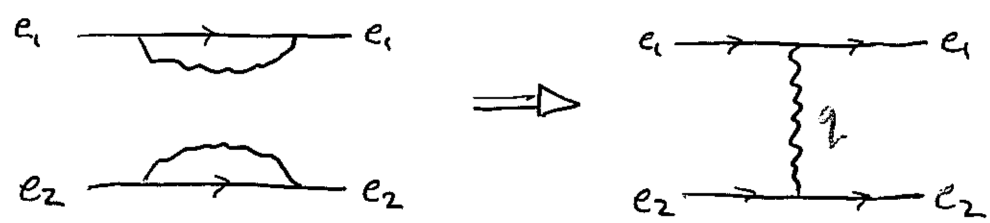


diagram contributes to the mass of the electron

QUANTUM FLUCTUATIONS $\Rightarrow$ PARTICLES ARE CONSTANTLY EMITTING AND REABSORBING OTHER PARTICLES. IN QUANTUM ELECTRODYNAMICS, ELECTRONS ARE CONSTANTLY EMITTING & REABSORBING PHOTONS.

SUCH PARTICLES MAY ALSO BE EXCHANGED BY TWO PARTICLES IF THE ARE CLOSE TOGETHER



q FOUR MOMENTUM TRANSFERRED FROM ONE ELECTRON TO THE OTHER

e.g. ENERGY & MOMENTUM

THE TWO ELECTRONS IN THE INITIAL & FINAL STATE ENSURE THAT ENERGY AND MOMENTUM ARE CONSERVED IN THE INTERACTION.

THEY REMAIN "ON THEIR MASS SHELL"

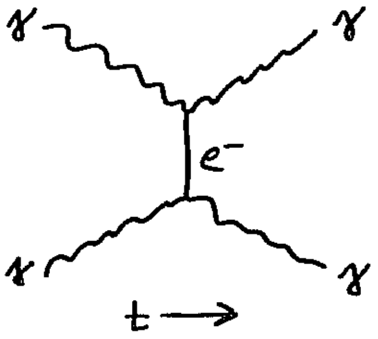
$\hookrightarrow$ meaning $[E^2 = p^2c^2 + m^2c^4]$

DOES NOT APPLY TO VIRTUAL PARTICLES!

THEY ARE "OFF THEIR MASS SHELL"

IN RELATIVISTIC QUANTUM MECHANICS WE THINK OF FORCES AS ARISING FROM THE EXCHANGE OF VIRTUAL PARTICLES.

USUALLY THINK OF THESE AS BEING THE GAUGE BOSONS $(\gamma, W^{\pm}, Z^{0}, g)$ BUT CAN ALSO HAVE (FOR EXAMPLE)



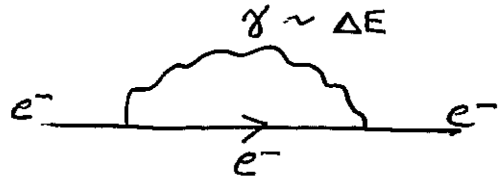
$\gamma\gamma$ scattering via electron exchange.

STILL, WE MOSTLY REFER TO THE GAUGE BOSONS AS THE FORCE CARRIERS

HOW CAN THE EXCHANGE OF A PARTICLE BE A "FORCE"?

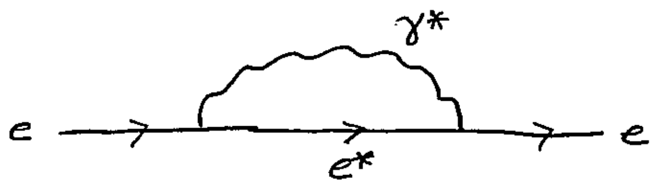
SIMPLE ANSWER IS THAT THE PARTICLE EXCHANGE CAUSES A CHANGE OF MOMENTUM. THAT IS THE <u>DEFINITION</u> OF A FORCE!

LOOK AGAIN AT FREE PARTICLE:



HEISENBERG'S UNCERTAINTY PRINCIPLE $\Rightarrow$ A FREELY PROPOGATING ELECTRON CAN EMIT & REABSORB A PHOTON. REABSORBTION MUST OCCUR WITHIN A TIME

$$\Delta t \lesssim \hbar / \Delta E$$



THE PHOTON CARRIES 4-MOMENTUM [EG ENERGY AND MOMENTUM] AWAY FROM THE ELECTRON

BEFORE THE EMISSION OF THE PHOTON

$$M_e^2 c^4 = E_e^2 - p_e^2 c^2$$

AFTER EMISSION OF THE PHOTON (BUT BEFORE IT IS REABSORBED)

$$M_e^2 c^4 \neq (E_e')^2 - (p_e')^2 c^2 = (m_e^*)^2 c^4$$

FOR A REAL PHOTON $\quad E_\gamma^2 - p_\gamma^2 c^2 = 0 \qquad$ (ie $M_\gamma = 0$)

FOR A $\boxed{\text{VIRTUAL}}$ PHOTON $\quad (E_\gamma')^2 - (p_\gamma')^2 c^2 = (M_{\gamma^*})^2 c^4$

VIRTUAL PARTICLES HAVE UNPHYSICAL MASSES (BUT CAN EXIST FOR ONLY A BRIEF TIME Δt.

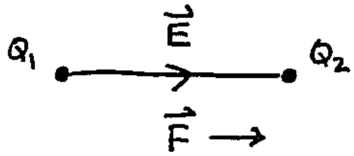
THESE PARTICLES (HERE γ^*, e^*) ARE SAID TO BE

"OFF THEIR MASS SHELL" OR SIMPLY "OFF SHELL"

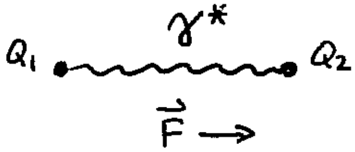
Eg Consider $e^+e^- \to e^+e^-$ in CM Frame $\qquad \longrightarrow \gamma^* \longleftarrow$

energy of virtual photon is all in the form of MASS!

FORCES & INTERACTIONS: CLASSICAL/QUANTUM ANALOGY



Classical Field



Exchange of Virtual Quantum

SINCE $\Delta E \Delta t \lesssim \hbar$ PROPOGATION TIME OF VIRTUAL PARTICLE IS:

$$t \approx \Delta t \lesssim \frac{\hbar}{mc^2}$$

$\sim \Delta E$

→ mass of virtual particle

ASSUME VIRTUAL PARTICLE PROPOGATES AT ~ SPEED OF LIGHT

RANGE OF FORCE $R \approx c\Delta t \lesssim \frac{\hbar}{mc}$

FOR $M \rightarrow 0$ $\quad R \rightarrow \infty$ $\quad$ [COULOMB FORCE $M_\gamma = 0$]

$M \rightarrow$ LARGE $\quad R \rightarrow$ SMALL [FORCE WILL ACT OVER LIMITED RANGE]

CLASSICALLY: $\vec{F}_2 = \vec{E}_1(\vec{r})\,Q_2 = \dfrac{Q_1 Q_2}{r^2}\,\hat{r}$

QUANTUM MECHANICALLY: FORCE IS DUE TO ABSORPTION OF γ WITH MOMENTUM q.

$$\Delta p \, \Delta x \gtrsim \hbar$$
$$\Rightarrow \quad \Delta q \, \Delta r \gtrsim \hbar$$

TRANSFER OF THIS MOMENTUM TAKES A TIME $\Delta t = r/c$

$$\frac{dq}{dt} \sim \frac{\hbar}{r} \cdot \frac{c}{r} = \frac{\hbar c}{r^2} \equiv \text{FORCE}$$

THIS ARGUMENTS YIELDS THE CORRECT $\frac{1}{r^2}$ BEHAVIOUR

NUMBER OF γ EMITTED/ABSORBED $\propto Q_1 Q_2$

$$\Rightarrow \quad F \sim \frac{Q_1 Q_2}{r^2}$$

TWO DIFFERENT ABSTRACT CONCEPTS $\Rightarrow$ SAME FORCE LAW

QUANTUM FIELD	CLASSICAL FIELD
VIRTUAL PARTICLE EXCHANGE	ACTION AT A DISTANCE

FEYNMAN DIAGRAMS

ARE GRAPHICAL REPRESENTATIONS OF QUANTUM MECHANICAL AMPLITUDES

PROBABILITY OF SOME QM PROCESS $\propto$ $[\text{AMPLITUDE}]^2$

IF A PROCESS CAN PROCEED IN TWO DIFFERENT WAYS

$$\text{PROB} \propto |A_1 + A_2|^2 = (A_1 + A_2)(A_1 + A_2)^*$$

A_1, A_2 ARE TYPICALLY COMPLEX

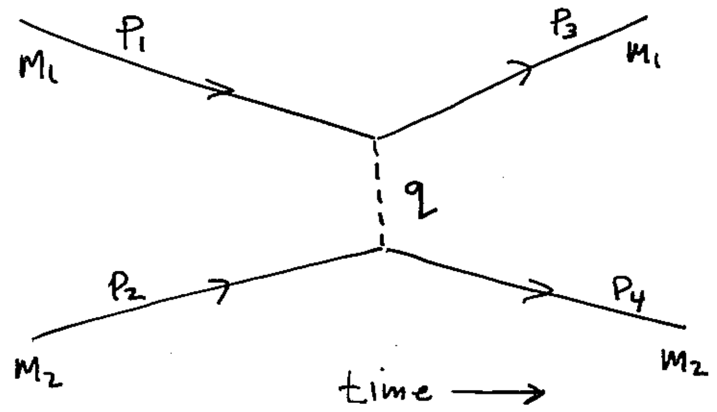
$\Longrightarrow$ INTERFERENCE EFFECTS

(FAMOUS 2 SLIT EXPERIMENT)

[SEE FEYNMAN'S THEORY OF FUNDAMENTAL PROCESSES OR QED]

ASSOCIATED WITH EACH FUNDAMENTAL INTERACTION (QED, QCD, ELECTROWEAK) IS A SET OF RULES, THE FEYNMAN RULES OF THE THEORY, STATING HOW TO CONVERT A DIAGRAM INTO A QUANTUM MECHANICAL AMPLITUDE.

MOMENTUM TRANSFER & SCATTERING ANGLES



$$P_i = (E_i, \vec{p}_i)$$

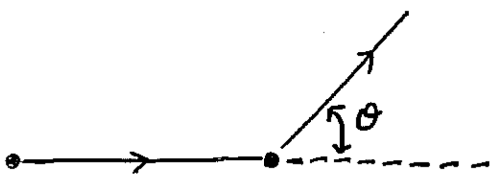
$$(\Delta p)^2 = q^2 = (p_3 - p_1)^2 = \underbrace{p_3^2}_{m_1^2c^2} + \underbrace{p_1^2}_{m_1^2c^2} - 2\, p_1 \cdot p_3$$

$$= m_1^2c^2 + m_1^2c^2 - 2\left(\frac{E_1}{c}, \vec{p}_1\right)\cdot\left(\frac{E_3}{c}, \vec{p}_3\right)$$

$$= m_1^2c^2 + m_1^2c^2 - \frac{2E_1E_3}{c^2} + 2\,\vec{p}_1 \cdot \vec{p}_3$$

$$= m_1^2c^2 + m_1^2c^2 - \frac{2E_1E_3}{c^2} + 2|\vec{p}_1||\vec{p}_3|\cos\theta$$

IN LAB FRAME (BEAM ON TARGET)



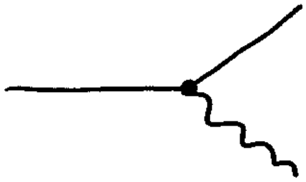
$E_1 \gg m_1c^2 \qquad |\vec{p}_1|^2 = E^2 - m^2c^2 \approx E^2$

$$(\Delta p)^2 = q^2 = -2E_1E_3(1-\cos\theta)$$

↳ momentum transfer given by lab frame kinematics

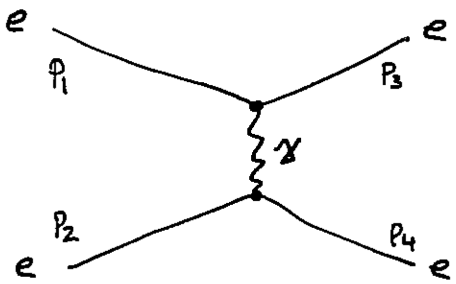
QUANTUM ELECTRODYNAMICS (AS AN EXAMPLE)

SIMPLEST THEORY: ONLY ONE FUNDAMENTAL INTERACTION VERTEX:



$ee\gamma$ COUPLING, STRENGTH $\sqrt{\alpha}$

LOWEST ORDER PROCESS (AS AN EXAMPLE) $e^- e^- \to e^- e^-$



incoming electron

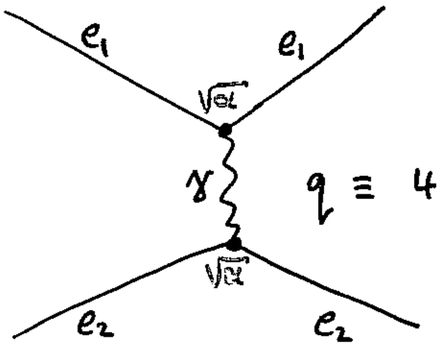
AMPLITUDE $\sim$ $\bar{\psi}_3 \psi_1 \; \dfrac{1}{q^2} \; \bar{\psi}_4 \psi_2$

outgoing electron

$q^2 \to (p_3 - p_1)^2$ $\qquad \dfrac{1}{q^2} \equiv$ propogator

FEYNMAN RULES FOR QED DICTATE THE FACTORS WRITTEN DOWN FOR

- EXTERNAL LINES (INCOMING & OUTGOING PARTICLES)
- VERTICES
- INTERNAL LINES (PROPOGATOR)



$q \equiv$ 4 MOMENTUM TRANSFER

PROBABILITY OF SCATTERING THROUGH ANGLE θ $\quad P(\theta) = P(q^2)$

$$= \left| A(q^2) \right|^2 \qquad q^2 \text{ related to } \theta$$

$\hookrightarrow$ QM scattering amplitude

LARGE ANGLE SCATTERING REQUIRES HIGH q^2

$$\text{QED FEYNMAN RULES} \Rightarrow A(ee \to ee) \sim \frac{\overbrace{\sqrt{\alpha}\sqrt{\alpha}}^{\text{vertex factors}}}{\underbrace{q^2}_{\text{propagator}}}$$

$$\sim \frac{e^2}{q^2} \qquad \text{SO} \quad P \propto |A|^2 \sim e^4/q^4$$

SCATTERING DEPENDS ON STRENGTH OF INTERACTION $\sqrt{\alpha}$

SCATTERING THROUGH LARGE ANGLE REQUIRES HIGH q^2

REQUIRES VERY VIRTUAL PHOTON (FAR OFF MASS SHELL)

IT IS MORE DIFFICULT TO CREATE PARTICLES FAR OFF MASS SHELL

IN MODEL OF RUTHERFORD SCATTERING WE HAD

$$(\Delta p)^2 \sim q^2 \sim p^2\theta^2$$

SO WE WOULD PREDICT $\quad P(\theta) \sim \dfrac{e^4}{p^4\theta^4}$

SO EXPECT SCATTERING PROBABILITY AS A FUNCTION OF θ TO FALL RAPIDLY

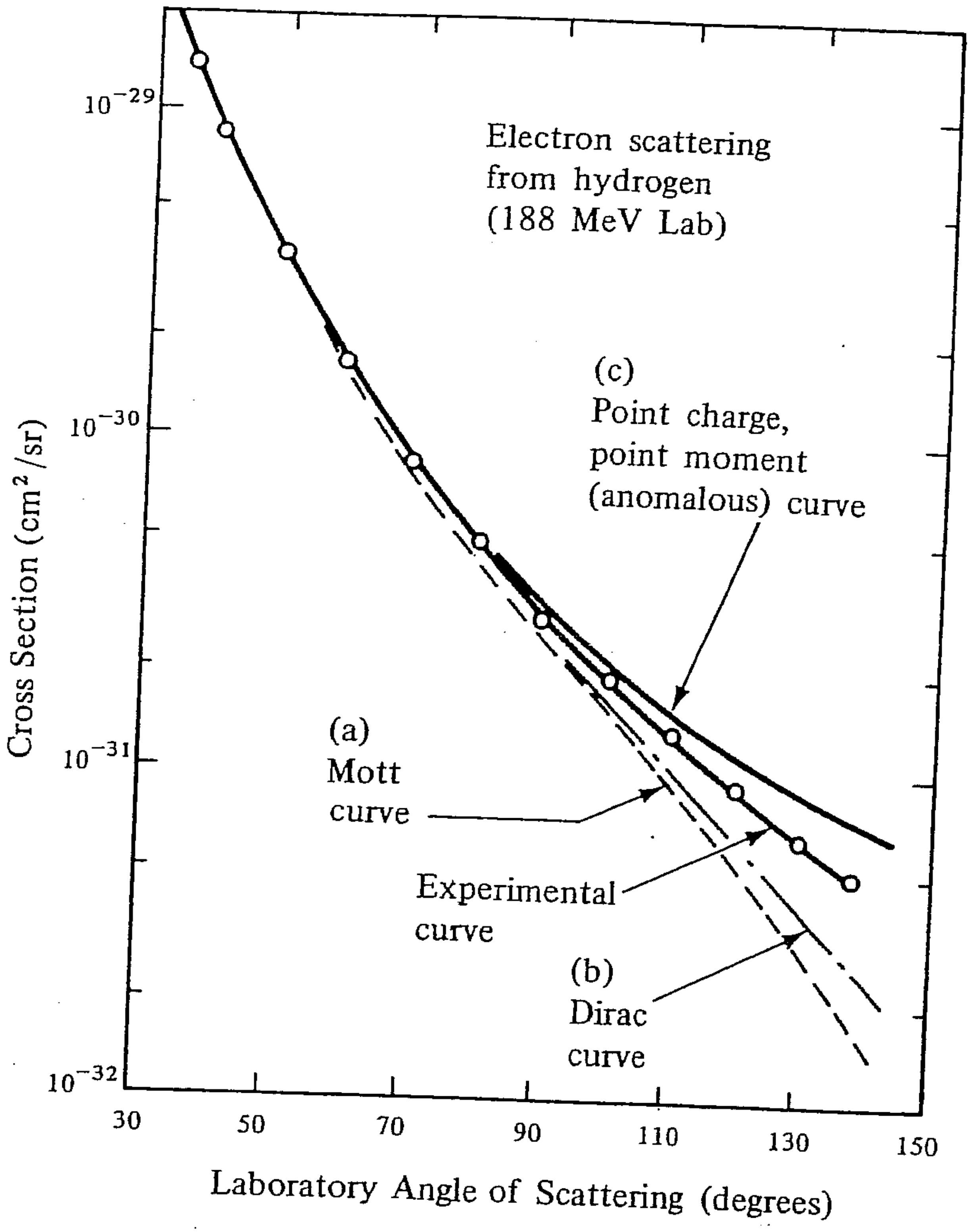


Fig. 6.13. Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1; 0), Dirac (1;1), anomalous (1; 2.79).