

Conservation Laws

All particles will decay to lighter particles unless prevented from doing so by some conservation law.

The timescales for these such decays are

10^{-23} s for the strong interaction

10^{-16} s for the electromagnetic interaction

10^{-13} s – 15 minutes for the weak interaction

Note that some conservation laws are absolute (conservation of energy and momentum for instance) but there are others (parity, which we will get to later) which are conserved by some of the three fundamental interactions but not all.

Note that these conservation laws apply equally well to scattering processes, not simply decays. They are properties of the interactions.

Noether's Theorem

Every symmetry in nature is related to a conservation law and *vice versa*

Invariance under:	leads to
Translations in time	conservation of energy
Translations in space	conservation of momentum
Rotations in space	conservation of angular momentum
Gauge transformations	conservation of charge

One conservation law is well known to us, **conservation of electric charge**

Others that we will discuss today: **conservation of lepton number**

conservation of baryon number

But will mostly focus on **conservation of angular momentum** (spin)

Conservation of Lepton Number

is an empirical fact, based on experiments. It is built into the Standard Model in terms of the allowed interactions (i.e.. allowed vertices) , but it is not a prediction of the theory in the same way that conservation of charge is.

Lepton number defined and conserved separately for electrons, muons and tau leptons (this includes the associated neutrinos)

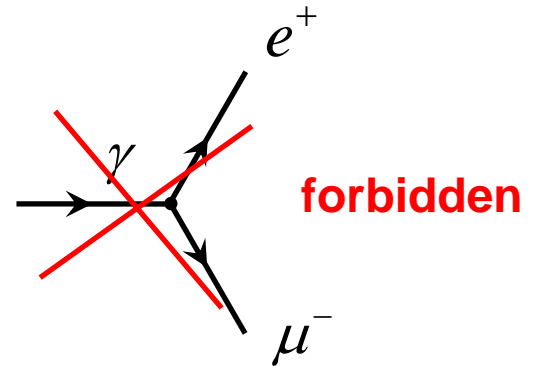
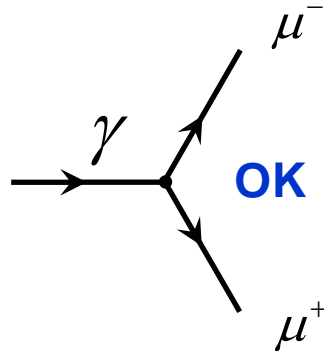
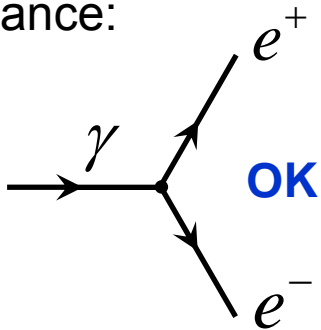
For electrons $L_e = +1$ for electrons and electron neutrinos

-1 for anti-electrons and anti-electron neutrinos

0 for any other particles

This is an additive quantum number (like charge)

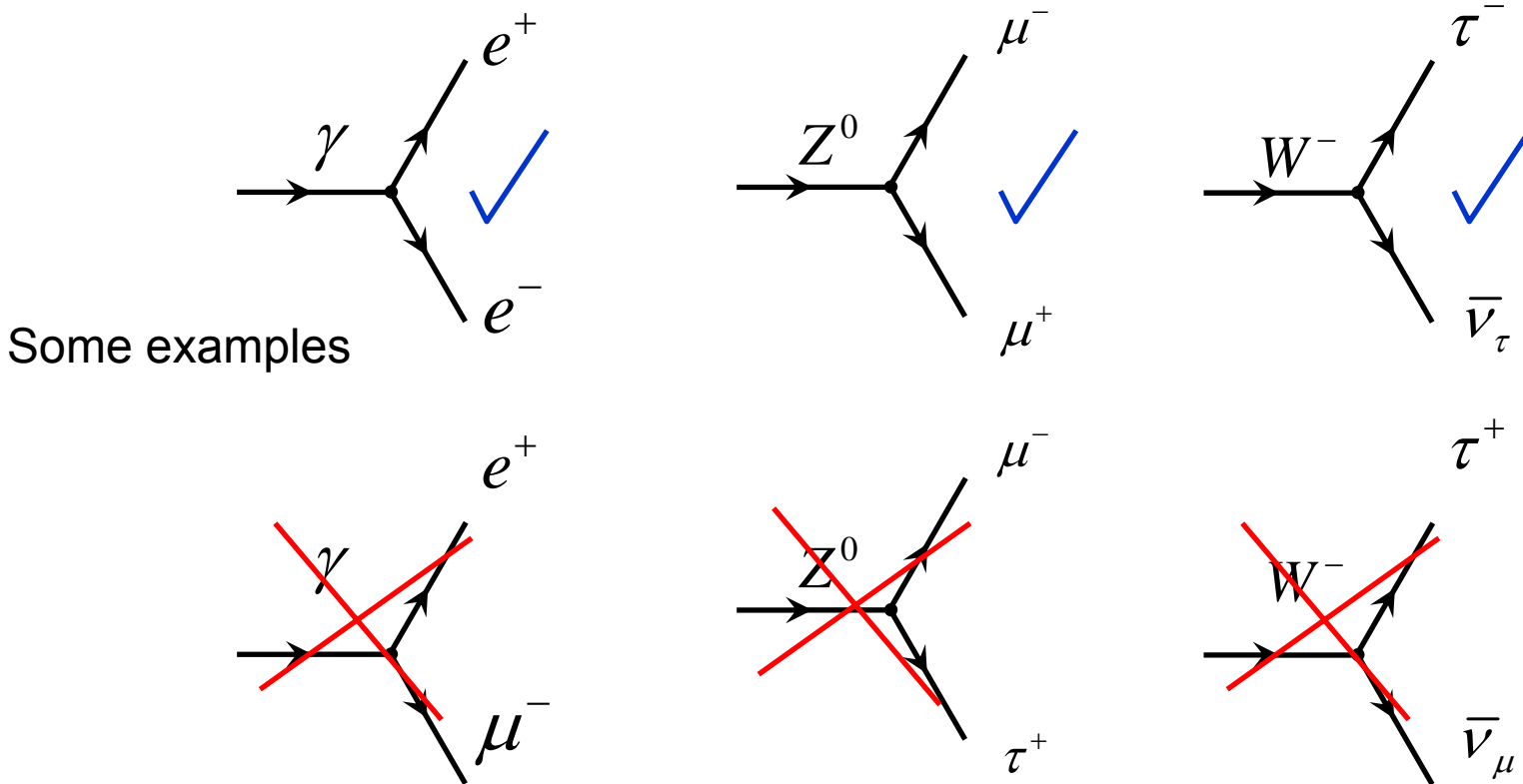
so for instance:



Lepton conservation in EM and Weak Interactions

Strong interaction does not couple to leptons so conservation is trivial there.

Lepton number is an additive quantum number.



In the SM, there are no interactions that mix leptons from different generations.

Conservation of Baryon Number

Proton lifetime many orders of magnitude longer than the age of the Universe

Baryon number conservation originally postulated to explain the fact that the proton is not observed to decay.

In the SM, protons are absolutely stable. They are the lightest baryons, so if baryon number is conserved, they cannot decay....ever.

In (high-energy) extensions to the SM (for instance Grand Unified Theories) protons can decay.

Note that this is effectively conservation of quark number where we assign a baryon number of $+1/3$ to quarks and $-1/3$ to antiquarks.

Baryons then have $B = +1$ and antibaryons have $B = -1$

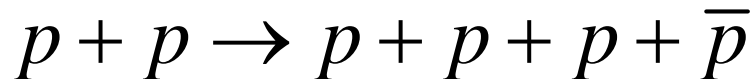
Mesons have baryon number 0.

Discovery of the anti-proton

B is an additive quantum number. It must balance on either side of any process.

There is no equivalent “meson number” that must balance, no such thing as meson number conservation (can always get a quark-antiquark pair from the vacuum if there is enough energy, but cannot, for instance get the three quarks needed for a baryon without also getting three antiquarks).

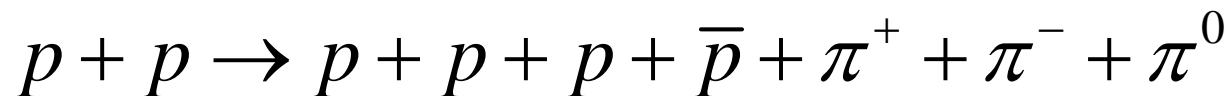
Antiprotons first produced experimentally in proton proton collisions:



B = 2 before 1 + 1 \rightarrow 1 + 1 + 1 + (-1) and 2 after

This is the minimal possible final state containing an anti-proton (in pp collisions)

Could also for instance have



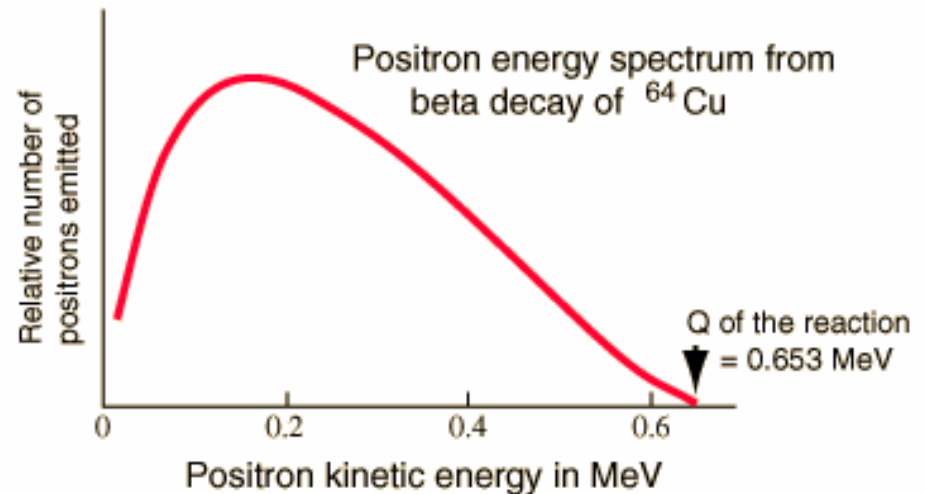
e.g. no mesons in the initial state but some number in the final state (here 3)

Radioactive Beta Decay

$A \rightarrow B + e$ Conservation of energy and momentum leads to

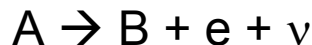
$$E_e = \left(\frac{m_A^2 - m_B^2 + m_e^2}{2m_A} \right) \quad \text{BUT}$$

i.e. E_e fixed for fixed M_A , M_B

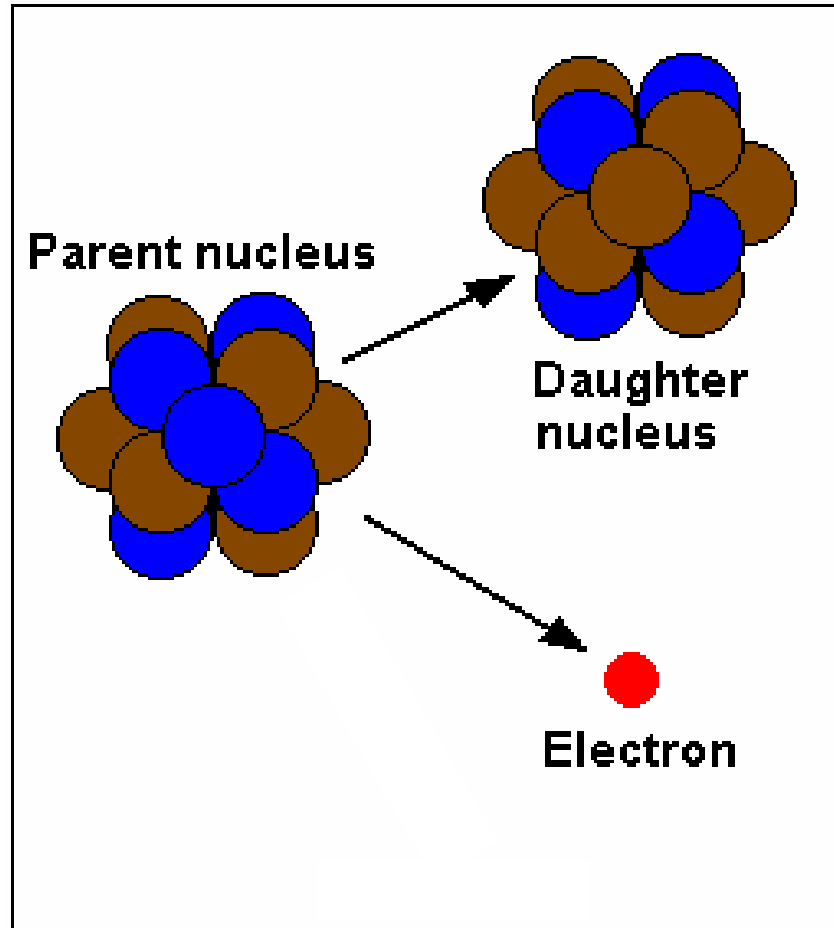


Fermi was ready to abandon conservation of energy !!!!! (one of his few mistakes)

Pauli suggested the existence of a third (undetectable) particle in this decay, which carries off some fraction of the energy. This is now called the neutrino.



Beta Decay Revisited



Beta decay



**"I have done a
I have postulated a particle
terrible thing
that cannot be detected."**

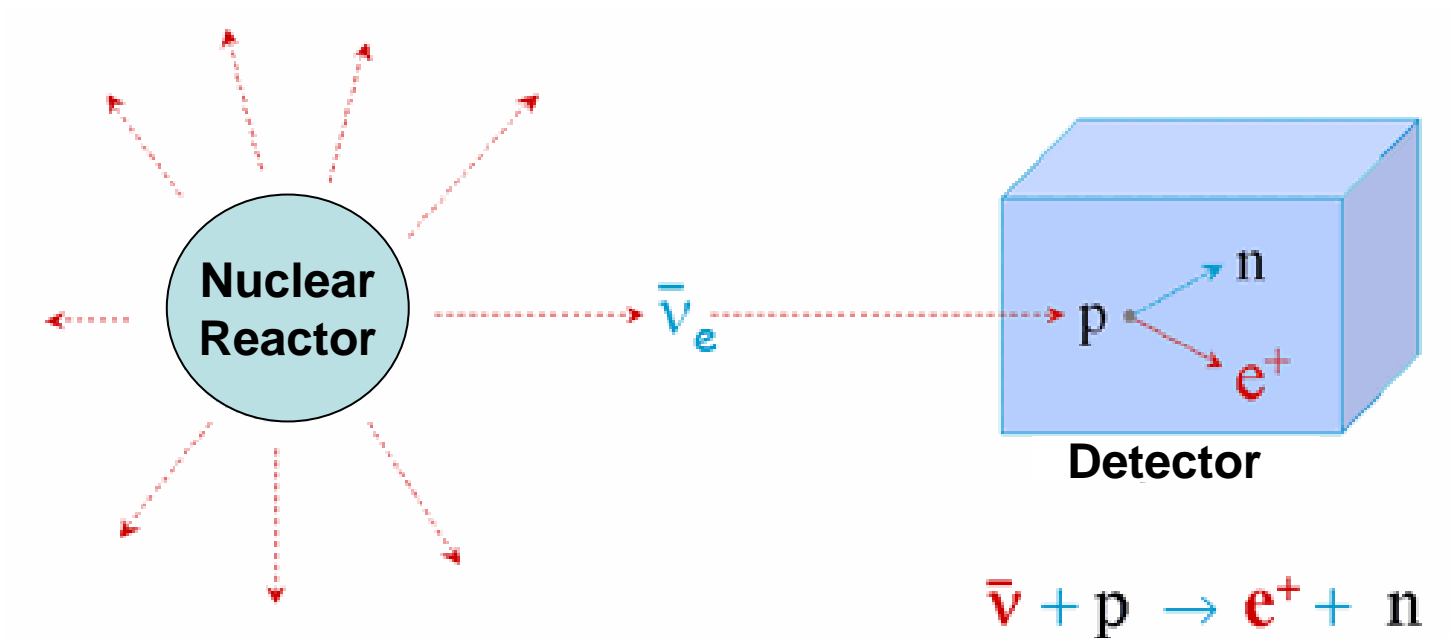
Wolfgang Pauli, Austrian physicist,
after concocting the neutrino to
help balance an equation.

Pauli was wrong too. We can detect
neutrinos. It's just not that easy.....

Neutrino Observation: Cowan and Reines 1958

So called “**Appearance Experiment**” $\bar{\nu}_e p \rightarrow e^+ n$

Anti-neutrinos from beta-decay in a nuclear reactor



We still do appearance experiments. This is how the tau neutrino was first directly observed in 2001 at Fermilab (DONUT experiment)

Addition of angular momenta

To evaluate the effects of angular momentum conservation on particle decays, we first need to understand how to add angular momentum vectors in quantum mechanics.

Brief Review

Orbital angular momentum \vec{L} is quantized in integer units of \hbar

In QM one cannot simultaneously specify more than one component of \vec{L}

The best we can do is to specify the magnitude and one component (L_z)

$$L^2 = \vec{L} \cdot \vec{L} = \ell(\ell + 1)\hbar^2 \quad \ell = 0, 1, 2, \dots \quad L_z = m_\ell \hbar \quad m_\ell = -\ell, -\ell + 1, \dots, 0, 1, 2, \dots, \ell - 1, \ell$$

(2ℓ + 1 values)

Spin angular momentum \vec{S} is quantized in half-integer units of \hbar

Same rules apply.....

$$S^2 = s(s + 1)\hbar^2 \quad s = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \quad S_z = m_s \hbar \quad m_s = -s, -s + 1, \dots, 0, 1, 2, \dots, s - 1, s$$

(2s + 1 values)

Conservation of angular momentum in particle interactions

Look again at the problem of β decay, prior to neutrino hypothesis

$$\begin{array}{cccc} & & n & \rightarrow & p & + & e \\ \text{spin} & & \frac{1}{2} & & \frac{1}{2} & & \frac{1}{2} \end{array}$$

As you probably know (but as we will see) there is no way to combine two spin $\frac{1}{2}$ particles to produce a total angular momentum (spin) of $\frac{1}{2}$.

Could also have predicted the existence of the neutrino based on conservation of angular momentum

If neutrino is spin $\frac{1}{2}$ then the decay can conserve angular momentum

Intrinsic Spin vs Total Angular Momentum

For fundamental particles, the spin is an intrinsic property of the particle. We sometimes denote this as \vec{S} and sometimes as \vec{J} , which is the conventional symbol for total angular momentum $\vec{J}=\vec{L}+\vec{S}$

Fundamental fermions are all spin $\frac{1}{2}$

Force carrying particles (gauge bosons) are spin 1

For hadrons (which are composite) we will always discuss the total angular momentum \vec{J} which comes from the addition of the spin angular momentum \vec{S} which comes from the spin of the quark spins (for the appropriate orientation, and any relative orbital angular momentum \vec{L} between them

$$\vec{J}=\vec{L}+\vec{S}$$

Mesons are always bosons (integer spin)

Baryons are always fermions (half-integer spin)

Addition of Angular Momentum (Again)

Assume that we add two angular momenta and we want to know the possible values of the total angular momentum

$$\vec{J} = \vec{J}_1 + \vec{J}_2$$

First of all, please note that we do not need to make special reference to whether the quantities we are adding are intrinsic spin, orbital angular momenta or total angular momenta or some combination thereof.

The z components add, so that $J_z = J_{1z} + J_{2z}$

The magnitudes do not add, instead, a measurement of J^2 can yield and value

$$J^2 = j(j+1)\hbar^2 \quad \text{For any } j \text{ running from } j = |j_1 - j_2| \quad \text{to} \quad j = |j_1 + j_2|$$

(in integer steps)

With what probabilities ????

Clebsch-Gordan Coefficients

$$|j_1 m_1\rangle |j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{j_1+j_2} C_{m m_1 m_2}^{j j_1 j_2} |j m\rangle \quad m = m_1 + m_2$$

Two systems with spin j_1 and j_2 and z components m_1 and m_2 can be combined to give a system which (quantum-mechanically) is a linear combination of states having spin j from $|j_1-j_2|$ to j_1+j_2 (in integer steps) each having a z component of $m = m_1 + m_2$.

So if we make a measurement of the total angular momentum of a state made up of two spin states as defined above, the square of the Clebsch-Gordan coefficient

$$C_{m m_1 m_2}^{j j_1 j_2}$$

represents the probability of obtaining a measurement of $j(j+1)\hbar^2$

Tables of Clebsch-Gordan Coefficients

$\mathbf{j}_1 \times \mathbf{j}_2$

Table for combining spin j_1 with spin j_2

$$|j_1 m_1\rangle |j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{j_1+j_2} C_{m m_1 m_2}^{j j_1 j_2} |j m\rangle \quad m = m_1 + m_2$$

		J	J	...
		m	m	...
m_1	m_2	$C_{m m_1 m_2}^{j j_1 j_2}$		
m_1	m_2			
.	.			
.	.			
.	.			

Combine 2 spin $\frac{1}{2}$ particles

$\frac{1}{2} \times \frac{1}{2}$

		1			
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
	+ 1/2		1/2	1/2	1
	- 1/2		1/2	- 1/2	- 1
			- 1/2	- 1/2	1

$$\left| \begin{array}{c} 1 \\ 2 \end{array} \right\rangle \left| \begin{array}{c} 1 \\ 2 \end{array} \right\rangle = \left| \begin{array}{c} 1 \\ 1 \end{array} \right\rangle$$

Combine 2 spin $\frac{1}{2}$ particles

$\frac{1}{2} \times \frac{1}{2}$

			1		
			+ 1	1	0
+ 1/2	+ 1/2		1	0	0
	+ 1/2	- 1/2		1/2	1/2
	- 1/2	+ 1/2		1/2	- 1/2
		- 1/2		- 1/2	- 1
					1

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} \left| 10 \right\rangle + \frac{1}{\sqrt{2}} \left| 00 \right\rangle$$

Combine 2 spin $\frac{1}{2}$ particles

$\frac{1}{2} \times \frac{1}{2}$

			1		
			+ 1	1	0
+ 1/2	+ 1/2	1	0	0	
	+ 1/2	- 1/2	1/2	1/2	1
	- 1/2	+ 1/2	1/2	- 1/2	- 1
		- 1/2	- 1/2	- 1/2	1

$$\left| \frac{1}{2} \begin{array}{c} 1 \\ -1 \\ 2 \end{array} \right\rangle \left| \frac{1}{2} \begin{array}{c} 1 \\ 2 \end{array} \right\rangle = \frac{1}{\sqrt{2}} \left| \begin{array}{c} 1 \\ 0 \end{array} \right\rangle - \frac{1}{\sqrt{2}} \left| \begin{array}{c} 0 \\ 0 \end{array} \right\rangle$$

Combine 2 spin $\frac{1}{2}$ particles

$\frac{1}{2} \times \frac{1}{2}$

			1		
			+ 1	1	0
+ 1/2	+ 1/2		1	0	0
	+ 1/2	- 1/2		1/2	1/2
	- 1/2	+ 1/2		1/2	- 1/2
				- 1/2	- 1/2
					1
					- 1
					1

$$\left| \frac{1}{2} \begin{array}{c} 1 \\ -1 \\ 2 \end{array} \right\rangle \left| \frac{1}{2} \begin{array}{c} 1 \\ -1 \\ 2 \end{array} \right\rangle = \left| \begin{array}{cc} 1 & -1 \end{array} \right\rangle$$

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle = |11\rangle$$

		1			
	+1	1	0		
+1/2	+1/2	1	0	0	
	+1/2	-1/2	1/2	1/2	1
	-1/2	+1/2	1/2	-1/2	-1
		-1/2	-1/2	1	

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} |10\rangle + \frac{1}{\sqrt{2}} |00\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} |10\rangle - \frac{1}{\sqrt{2}} |00\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |1-1\rangle$$

Spin triplet (symmetric under exchange of particles 1,2)

$$|11\rangle = \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$|10\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle + \frac{1}{\sqrt{2}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$|1-1\rangle = \left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

Spin singlet (anti-symmetric under exchange of particles 1,2)

$$|00\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle - \frac{1}{\sqrt{2}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

Spin triplet

Symmetric under $1 \leftrightarrow 2$

$$|11\rangle = \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$|10\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle + \frac{1}{\sqrt{2}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$|1-1\rangle = \left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

$$|00\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle - \frac{1}{\sqrt{2}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

Spin singlet

Anti-symmetric
under $1 \leftrightarrow 2$

			1		
		+ 1	1	0	
+ 1/2	+ 1/2	1	0	0	
	+ 1/2	- 1/2	1/2	1/2	1
	- 1/2	+ 1/2	1/2	- 1/2	- 1
			- 1/2	- 1/2	1