Discrete Transformations: Parity

Parity operation inverts the sign of all spatial coordinates:

Position vector (x, y, z) goes to (-x, -y, -z) (eg $P(\mathbf{r}) = -\mathbf{r}$)

Clearly $P^2 = I$ (so eigenvalues are ±1)

Regular (polar) vectors transform in this way under parity transformation

Regular scalars ($a = \mathbf{b}\cdot\mathbf{c}$) transform like $P(a) = a$ (e.g. they are unaffected)

However, there are other type of scalars and vectors that transform differently:

Axial-vector (also called pseudo-vectors) do not change sign under parity transformation.

e.g. Cross-product of two polar vectors $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, $\mathbf{B}$ (= curl $\mathbf{A}$)

Pseudo-scalar: $\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$ does change sign under a parity transformation

(here, **a**, **b** and **c** are all polar vectors)

Parity conservation

Fundamental particles have intrinsic parity: $P^2 = \mathrm{I}$ (so eigenvalues are ±1)

Quantum Field Theory: the parity of a fermion is opposite to that of its antiparticle
the parity of a boson is the same as its antiparticle

Parity of a composite system is given by the product of the parity of the constituents, with an additional contribution of $(-1)^{\ell}$ according to the orbital angular momentum ℓ

Assign positive parity to the quarks, (thus negative parity to the anti-quarks)

Mesons carry parity $(-1)^{\ell+1}$ Baryons carry parity $(+1)^3 \cdot (-1)^{\ell} = (-1)^{\ell}$

Parity is conserved by both electromagnetic and strong interactions.

Spin-Parity (J^P) Conservation

Note that spin and parity conservation requirements can sometime conflict

In our tutorial last week we looked at the decay $\Delta^{++} \rightarrow p\, \pi^+$

This is spin 3/2 → spin 1/2 + spin 0

Requires either ℓ = 1 or ℓ = 2 in the final state.

Parity conservation will actually pick out the allowed value (since the parity of the two choices will be opposite)

In some cases, the value of orbital angular momentum required for conservation of spin is in conflict with the value required by conservation of parity.

In this case, we say that the decay is forbidden by spin-parity conservation

Parity Violation: the θ-τ puzzle

In the early 1950s there was an "odd" experimental observation: two particles with identical mass, spin, charge, lifetime etc, decayed (weakly) into states of opposite parity:

$$\theta \rightarrow \pi^{+} + \pi^{0} \qquad (P = +1)$$

$$\tau \rightarrow \left\{ \begin{matrix} \pi^{+} + \pi^{0} + \pi^{0} \\ \pi^{+} + \pi^{+} + \pi^{-} \end{matrix} \right\} \qquad (P = -1)$$

Two hypotheses:

1) there are two particles with identical properties except for parity

2) parity is not conserved in weak interaction

That the weak interaction was somehow special had already been established (for instance the lifetime of "strange" particles, non-conservation of strangeness).

Survey of the literature by Lee and Yang showed there was little experimental evidence for parity conservation in weak decays

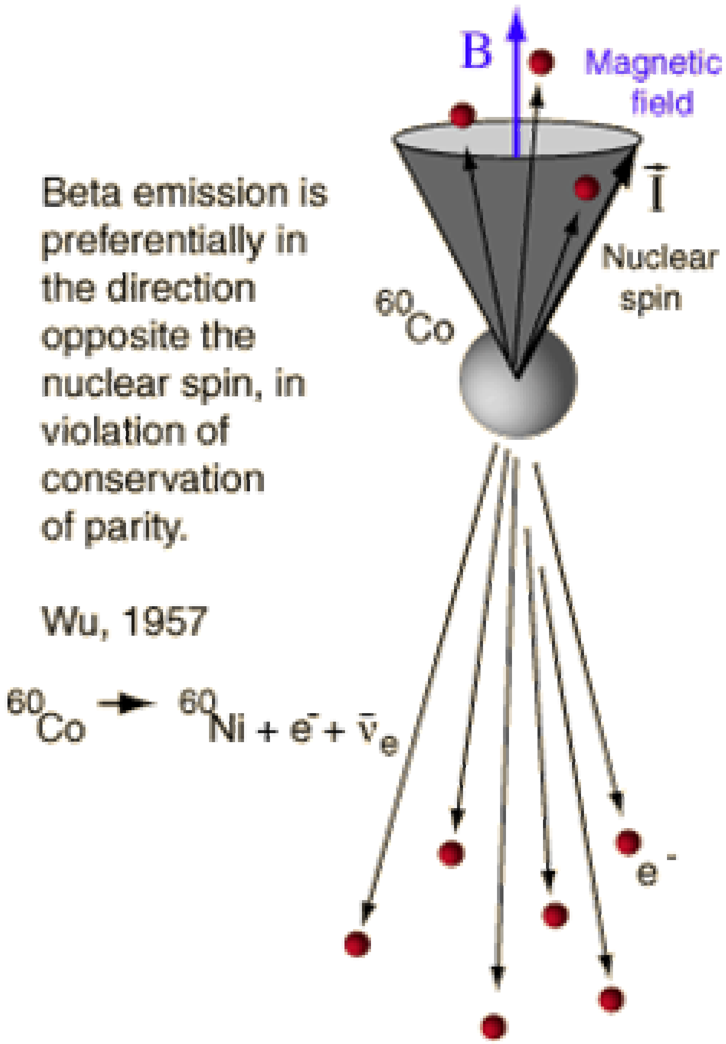
Lee and Yang's historic paper, *Question of Parity Conservation in Weak Interactions*

The Physical Review 106 vol. 1, (1956)

Freeman Dyson, famed particle physicist later wrote:

A copy of it was sent to me and I read it. I read it twice. I said, `This is very interesting,' or words to that effect. But I had not the imagination to say, `By golly, if this is true it opens up a whole new branch of physics.' And I think other physicists, with very few exceptions, at that time were as unimaginative as I."

Parity Violation: Experiment (Madame Wu, 1956)



Parity Violation

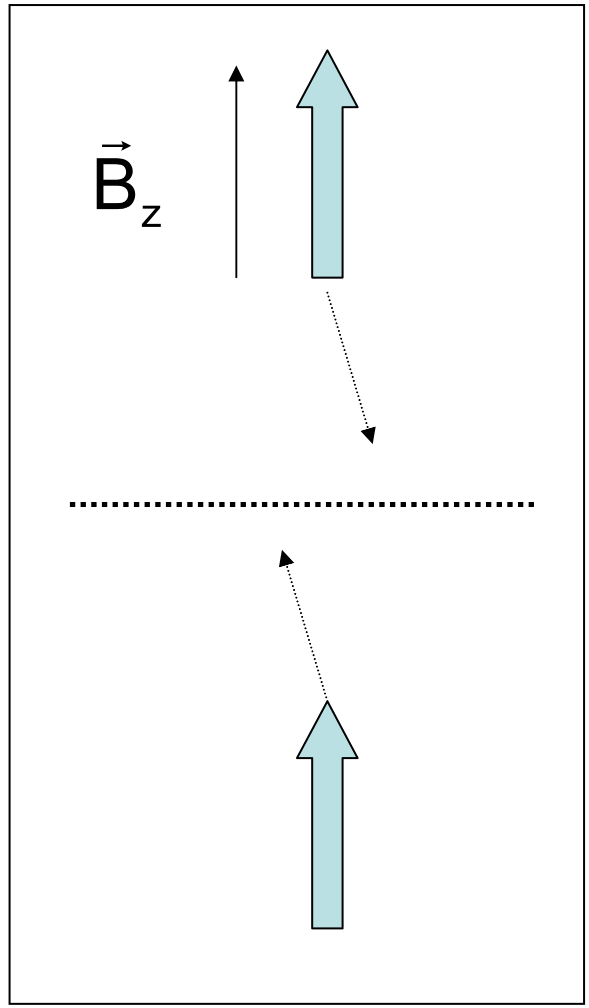
N.B. Spin is an axial vector which does NOT change sign under parity inversion.

Reflect through xy plane (here B and nuclear spin are along z.

$\vec{p}_e \to -\vec{p}_e$

Spin does NOT reflect (it's a pseudovector)

Parity is NOT conserved in weak decays



^{60}Co nuclear spin

$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}_e$

electrons preferentially emitted in direction opposite to nuclear spin

electrons now preferentially emitted in direction of the nuclear spin. This is NOT experimentally observed.

Helicity (Definition)

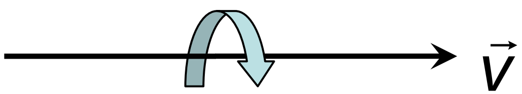
Useful to define a quantity called helicity:

As choice of z-axis for measurement of the spin component, use the axis defined by the particle velocity:

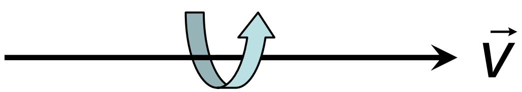
Helicity defined as m_s/s.

Particle of spin ½ can therefore have helicity of ±1

Helicity +1 is referred to as Right-handed



Helicity -1 is referred to as Left handed.

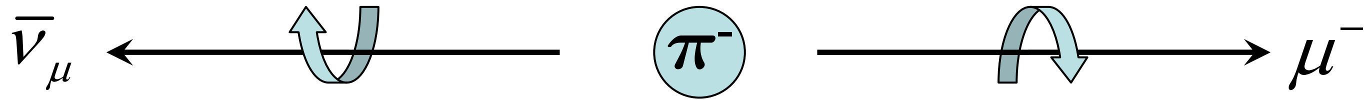


Note that helicity is NOT Lorentz invariant unless the particle is massless

If the particle has mass, can always make Lorentz transformation into an inertial frame with velocity > v, and thus “flip” the helicity.

Neutrino Helicity

Imagine the decay (at rest) of a charged pion $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$



Pion has spin 0, so spins of final state particles must be anti-aligned

Final state particles therefore have the same helicity.

A measurement of the muon helicity therefore gives a measurement of the neutrino helicity.

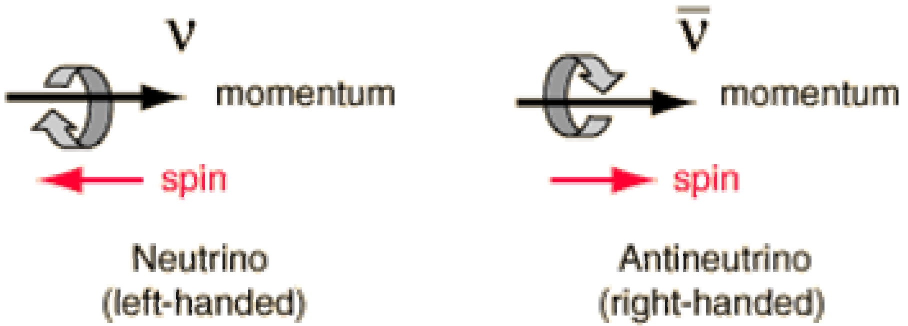
If parity were conserved, in this decay we would expect to see left-handed anti-neutrinos 50% of the time and right-handed anti-neutrinos 50% of the time.

Experimentally <u>ONLY</u> right-handed anti-neutrinos (eg as determined from the muon helicity).

Parity Violation in Weak Decays

<u>All neutrinos are left-handed</u>

<u>All anti-neutrinos are right-handed</u>



This absolute statement is of course not true in the case where neutrinos have mass, which we now know they do.

However, in the rest frame of the pion (as an example) it is still true that the outgoing anti-neutrino is <u>ALWAYS</u> right-handed

We say that parity is maximally violated in weak decays

(eg, there are not simply more left-handed neutrinos than right-handed neutrinos. There are NO right-handed neutrinos at all.)

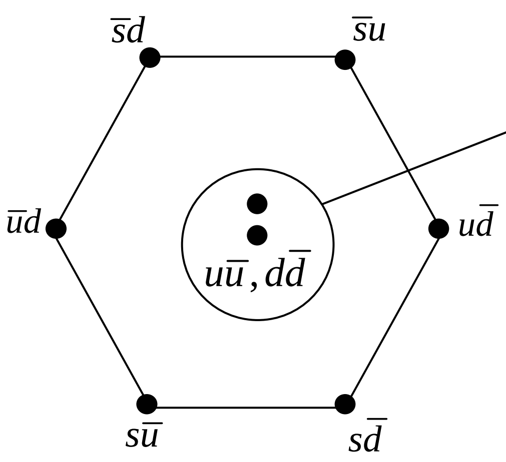
Charge Conjugation

Another symmetry operation: inverts all internal quantum numbers while leaving energy, mass, momentum, spin unchanged.

Internal quantum numbers: lepton number, baryon number, strangeness etc.

Charge conjugation takes a particle into its anti-particle.

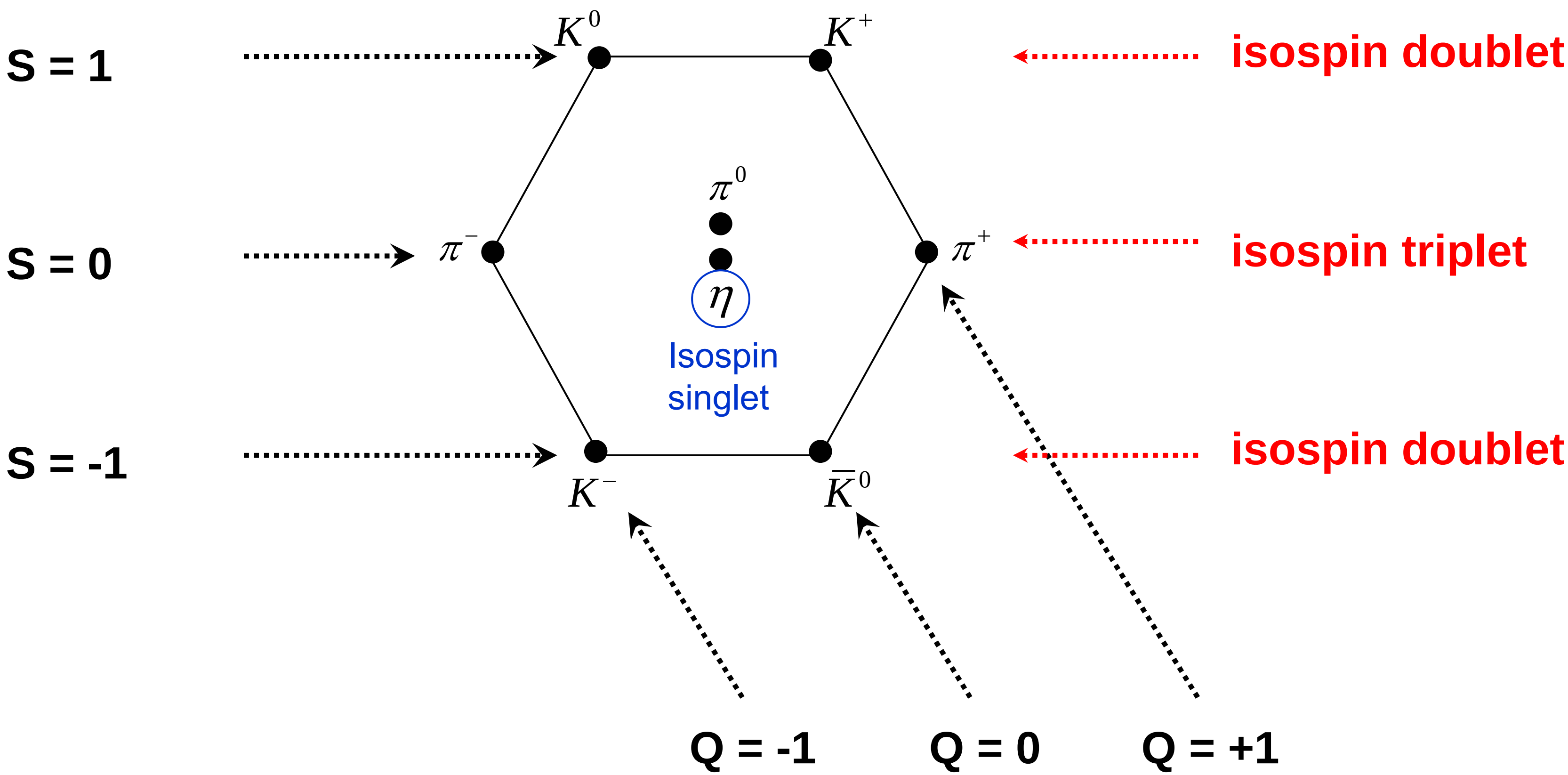
Most particles are not eigenstates of *C* (particle would have to be it's own antiparticle). This is true for photons, and for the central entries in the eightfold way meson octets:



$$\pi^0, \eta, \eta', \rho^0, \phi, \omega, J/\psi$$

Being neutral is necessary but not sufficient: $C|n\rangle \rightarrow |\overline{n}\rangle$

The “Eightfold Way” Meson Octet (u,d,s)



We have not discussed this. We might do so later, but for the moment this is just illustrative. Particles in the centre are combinations of same-flavour $q\bar{q}$ pairs and so are their own antiparticles.

Charge Conjugation

$C^2 = \mathrm{I}$ Eigenvalues are ± 1, so for eigenstates $C\,|\,p\rangle = \pm|\,p\rangle = |\,\overline{p}\rangle$

Electromagnetism is invariant under a change in the sign of all charges

The photon, γ, is the quantum of the EM field, which changes sign under *C.*

Photons "charge conjugation number" is therefore -1

C is a multiplicative quantum number (like parity).

System consisting of a spin ½ particle and its anti-particle, in a configuration with orbital angular momentum ℓ is an eigenstate of *C* with eigenvalue $(-1)^{\ell+s}$

Invariance under *C*

Strong and electromagnetic interaction are invariant under charge conjugation

Consider the electromagnetic decay of the neutral pion. The π^0 is the lightest meson and so cannot decay strongly. Instead it decays electromagnetically with branching fraction

BR($\pi^0 \rightarrow \gamma\gamma$) = 98.8% and mean lifetime 8×10^{-17} s

C is +1 before and after $(-1)^0$, (-1)(-1) for the photon pair [$(-1)^n$ for system of n photons]

so there is no decay to three photons. Similarly, $\omega \rightarrow \pi^0 + \gamma$ but never $\omega \rightarrow \pi^0 + 2\gamma$.

Other implications of charge conjugation invariance (for example): Consider process

$$p + \overline{p} \rightarrow \pi^+ + \pi^- + \pi^0 \qquad \text{(strong interaction)}$$

Charge conjugation invariance requires the energy distributions of the two charged pions in the final state must be identical. Why is this ?

Charge Conjugation and the Weak Interaction

Charge conjugation is demonstrably NOT a symmetry of the weak interaction.

Consider charge conjugation applied to a neutrino (*C* leaves helicity unchanged)

$$C\left|\nu_L\right\rangle = \left|\bar{\nu}_L\right\rangle \qquad \textbf{No !}$$

We have already stated that all anti-neutrinos are right-handed so this is an unphysical state. So charge conjugation invariance cannot be respected by the weak interaction.

Note though that the combined operations of charge conjugation and parity inversion take a left handed neutrino into a right-handed anti-neutrino

$$CP\left|\nu_L\right\rangle = \left|\bar{\nu}_R\right\rangle \qquad \checkmark$$

(the spin of the neutrino does not transform, but the velocity vector used to define the helicity does).

G Parity (in strong interactions)

Very few particles are eigenstates of the charge conjugation operator *C*

For strong interactions, can extend *C* by combining it with an isospin transformation:

Rotation of 180º about I_2 (R_2) takes I_3 into $-I_3$, for example $R_2\,\pi^+ \rightarrow \pi^-$

Combining *C* and R_2 operations: $CR_2\,\pi^+ \rightarrow \pi^+$

All <u>mesons</u> composed only of u and d quarks and anti-quarks are eigenstates of this operation which we call *G* or *G*-Parity

For particles (u,d mesons) of isospin I, the *G*-parity number is given by $G = (-1)^I C$

Where *C* is the charge conjugation number of the neutral member of the multiplet.

This is a useful symmetry in strong interaction for telling how many pions can be emitted in the final state; an *N* pion system has $G = (-1)^N$

ρ ($I = 1$) can only decay to two pions, ω ($I = 0$) only to three)

Decays of the η

The η meson decays into three different final states (with different branching ratios)

$\eta \to \gamma\gamma$ (39%) $\eta \to 3\pi$ (56%) $\eta \to \pi\pi\gamma$ (5%)

Note that if a particle can decay strongly then strong decays dominate even if decays via other interactions are possible (since the strong decays are much faster).

Lifetime is 7×10^{-19} s (10^{-23} s is typical of the strong interaction and anyway, two of the decays involve photons which indicate electromagnetic decays).

Mass of the η is ~ 549 MeV/c^2, so it has enough mass to decay into 2 or 3 pions.

Why no 2π final state ? Why is the decay to the 3π final state slow ?

First:

What are the other properties of the η ?

It has spin 0, P = -1 and C = +1 (we write $J^{PC} = 0^{+-}$)

It also has isospin I = 0 and thus G = +1 (we write I^G=0^+)

Decays of the η cont'd

$\eta \rightarrow \pi\pi$ is forbidden for both electromagnetic and strong interactions due to spin-parity conservation.

Initial state has negative parity. Two final state particles each have negative parity and thus the final state has parity of $(-1)^{\ell}$. To conserve parity need $\ell = 1$ in the final state (or rather ℓ odd). But there is then no way to conserve angular momentum.

$\eta \rightarrow 3\pi$ The η has $G = +1$. A system of N pions has G-parity $(-1)^N$ so this decay cannot proceed via the strong interaction.

G-parity is not conserved in electromagnetic decays, so the decay can proceed via that interaction.