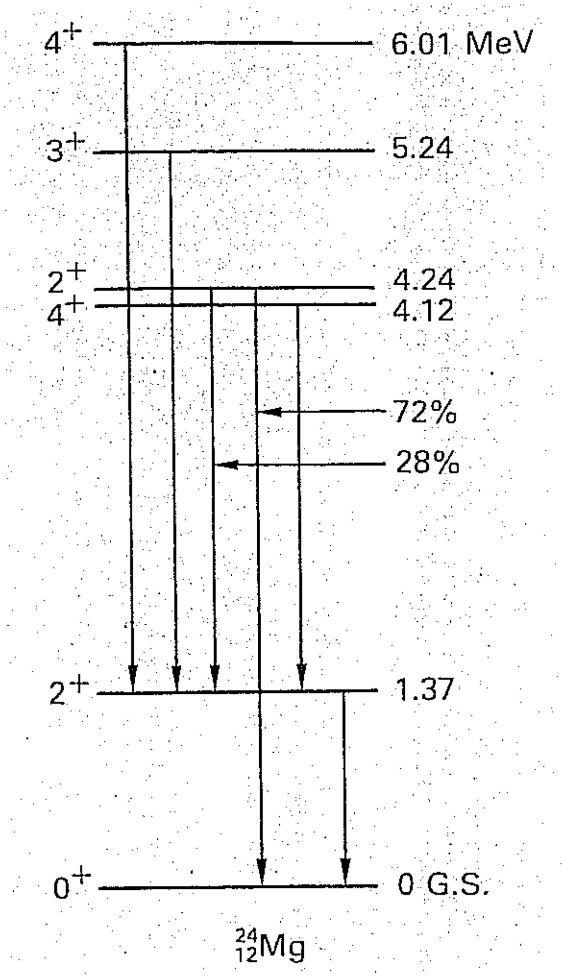
Energy-level diagrams in Nuclear Physics



Energy level diagrams for nuclear states rather similar to the equivalent diagrams from atomic physics.

States usually labeled by their spin-parity J^P

Excited states labeled by their excitation energies.

In the case where multiple decays paths are available, sometime labeled with the relevant branching fractions.

The above plot shows an energy-level diagram for $^{24}_{12}\mathrm{Mg}$ (first 5 excited states)

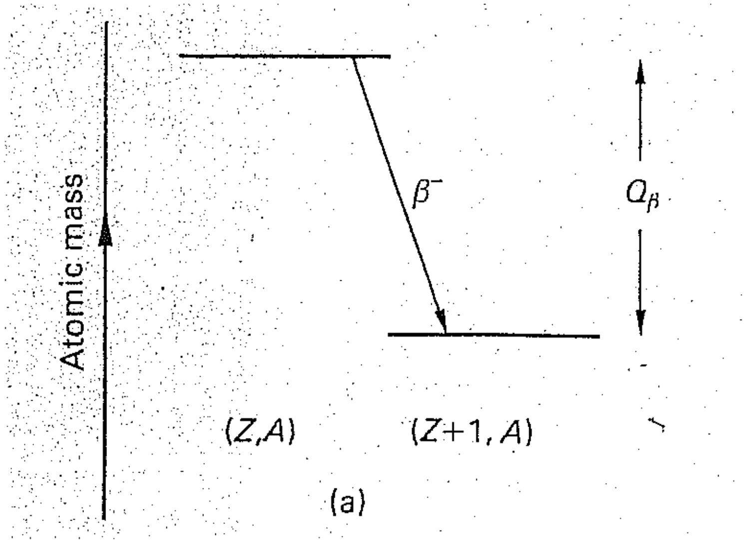
De-excitations from excited states are usually via gamma ray emission

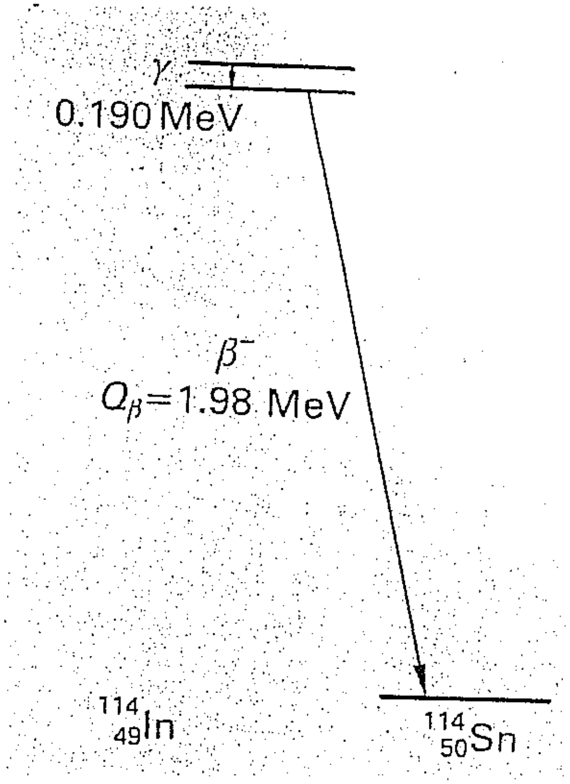
Spin-parity and excitation energies of some light nuclei

Nucleus	Binding energy (MeV)	Binding energy of last nucleon (MeV)	Binding energy per nucleon (MeV)	Spin and parity
$^{2}_{1}H$	2.22	2.2	1.1	1^+
$^{3}_{2}H$	8.48	6.3	2.8	$\frac{1}{2}^+$
$^{4}_{2}He$	28.30	19.8	7.1	0^+
$^{5}_{2}He$	27.34	−1.0	5.5	$\frac{3}{2}^-$
$^{6}_{3}Li$	31.99	4.7	5.3	1^+
$^{7}_{3}Li$	39.25	7.3	5.6	$\frac{3}{2}^-$
$^{8}_{4}Be$	56.50	17.3	7.1	0^+
$^{9}_{4}Be$	58.16	1.7	6.5	$\frac{3}{2}^-$
$^{10}_{5}B$	64.75	6.6	6.5	3^+
$^{11}_{5}B$	76.21	11.5	6.9	$\frac{3}{2}^-$
$^{12}_{6}C$	92.16	16.0	7.7	0^+
$^{13}_{6}C$	97.11	5.0	7.5	$\frac{1}{2}^-$
$^{14}_{7}N$	104.66	7.6	7.5	1^+
$^{15}_{7}N$	115.49	10.8	7.7	$\frac{1}{2}^-$
$^{16}_{8}O$	127.62	12.1	8.0	0^+
$^{17}_{8}O$	131.76	4.1	7.8	$\frac{5}{2}^+$

Note that even-even nuclei always have ground-state spin-parity of $J^P=0^+$

Decays and Energy Level Diagrams (β-decay)

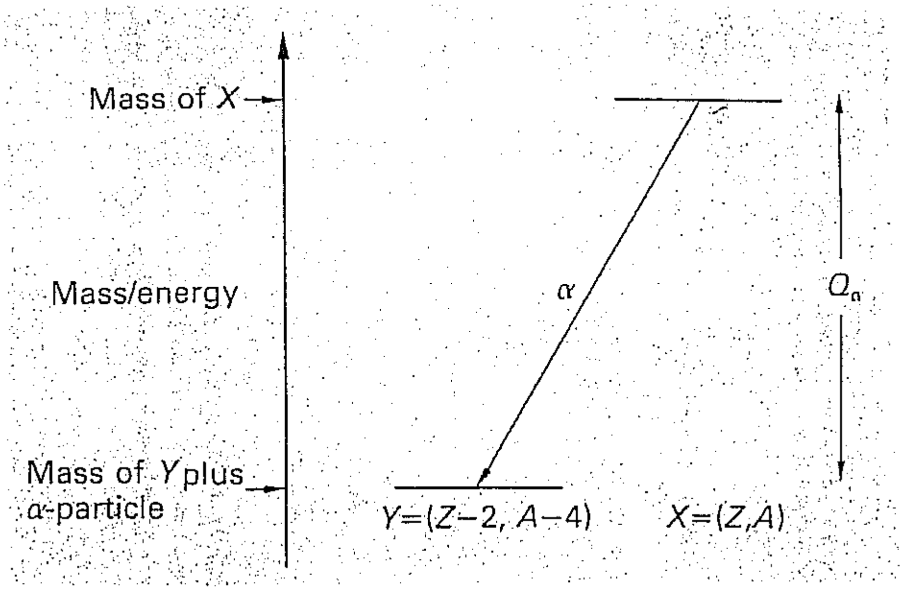


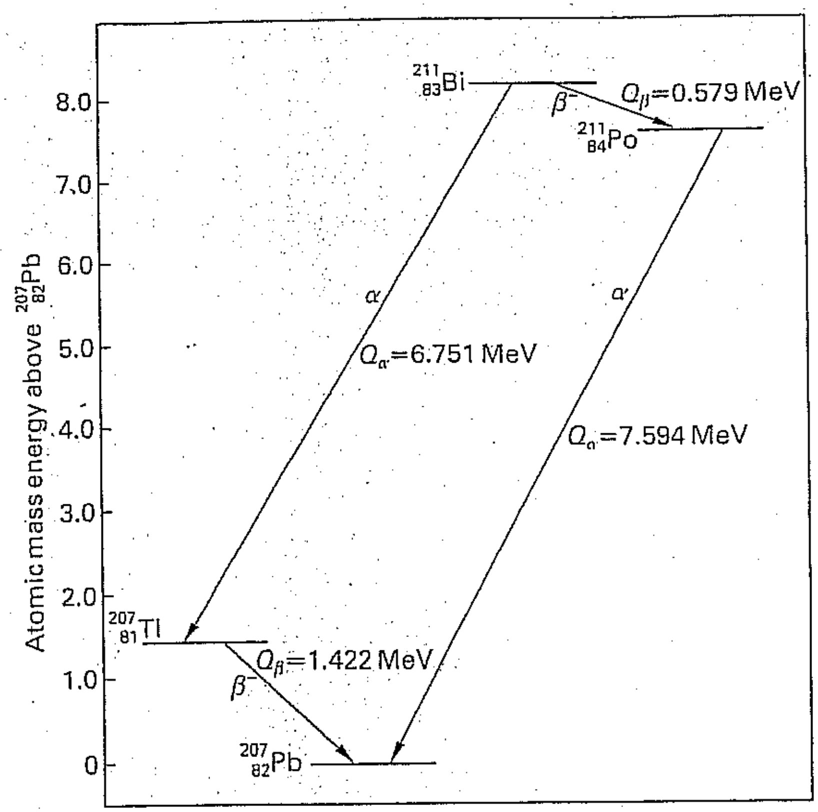


$^{114}_{49}$In produced in excited state. De-excites to ground state and then decays via β decay to $^{114}_{50}$Sn

Q_β = energy release in the decay (available for kinetic energy of final state products)

Decays and Energy Level Diagrams (α-decay)





Using the Liquid-Drop Model for the nucleus, we developed the Semi Empirical Mass Formula (SEMF) which gives a good picture of things like the binding energy per nucleon and the stability of nuclei against α- and β-decay. However, there are numerous nuclear properties that this model simply does not address:

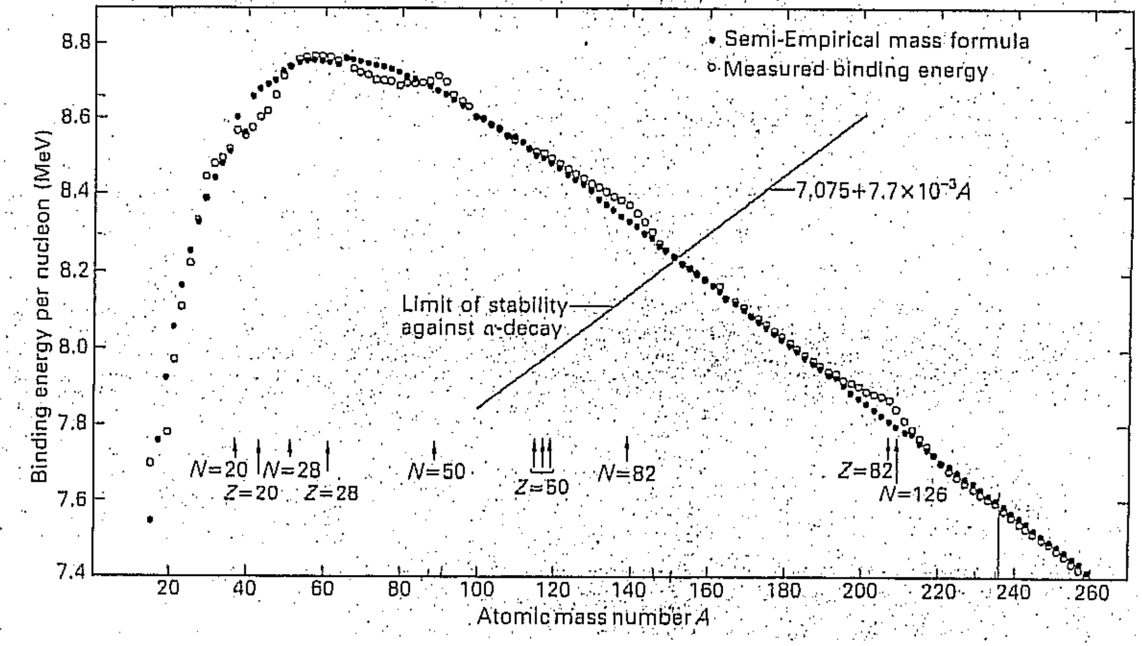
- ground-state spin and parities
- excited-state spin and parities
- existence of magic numbers (deal with this today)
- nuclear Magnetic moments
- nuclear density
- values of the coefficients in the SEMF (except the Coulomb term)

I have mentioned the “Magic Numbers” only briefly so, far. Let’s look at these first:

The magic numbers in nuclear physics are 2, 8, 20, 28, 50, 82, 126

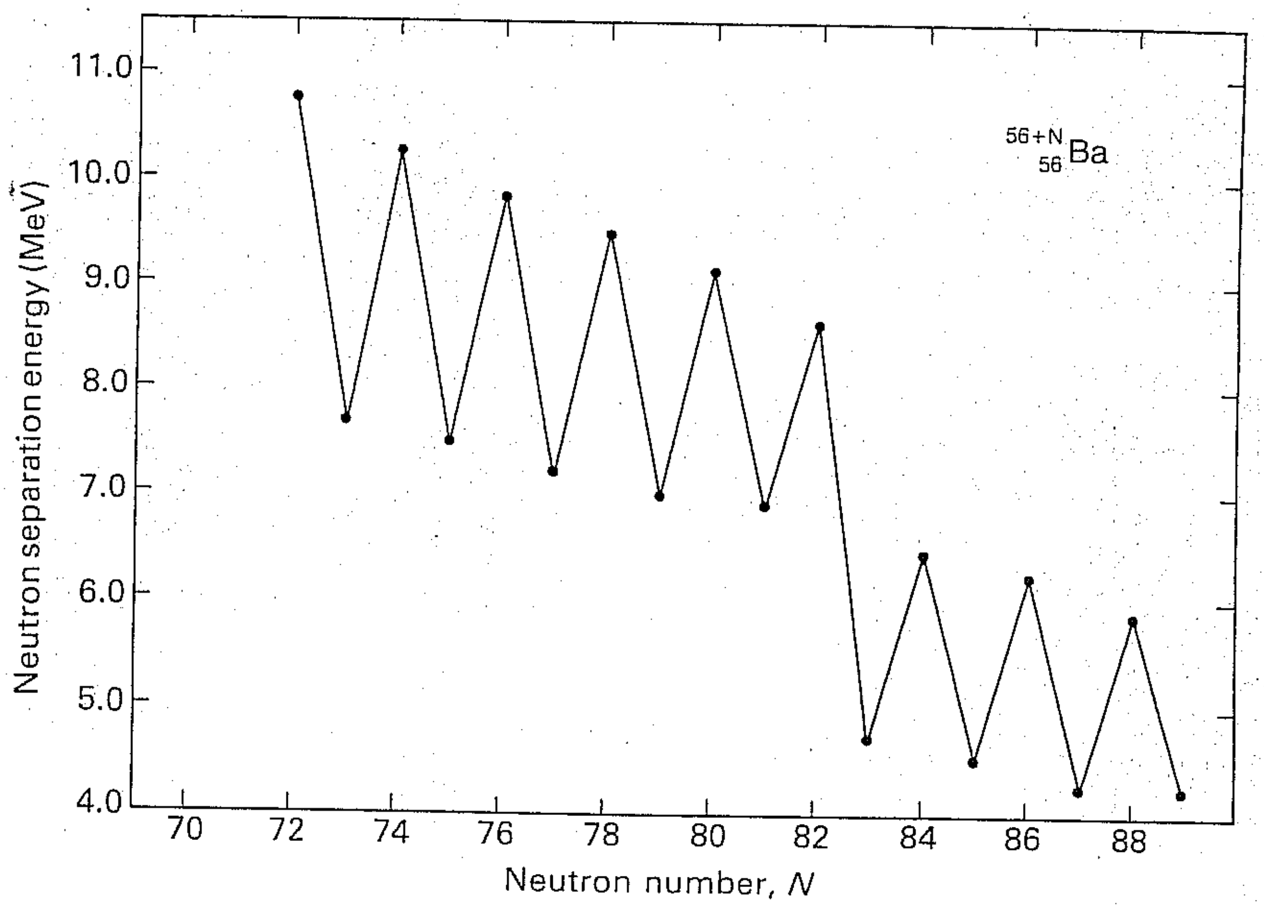
Magic Numbers in Nuclear Physics

Magic numbers are values for the numbers of protons (*Z*) or neutrons (*N*) which result in nuclei that are particularly stable. This manifests itself in a variety of ways. We have already seen (see again below) that in the plot of the binding energy per nucleon, the agreement between the SEMF and the measured values generally agree well for $A > 40$ but do show some deviations associated with particular values of *N* and *Z*:



Note (since we did not mention this before) that this plot is for odd-*A* nuclei. Even-*A* nuclei will of course have curves that are split by the pairing energy term in the SEMF.

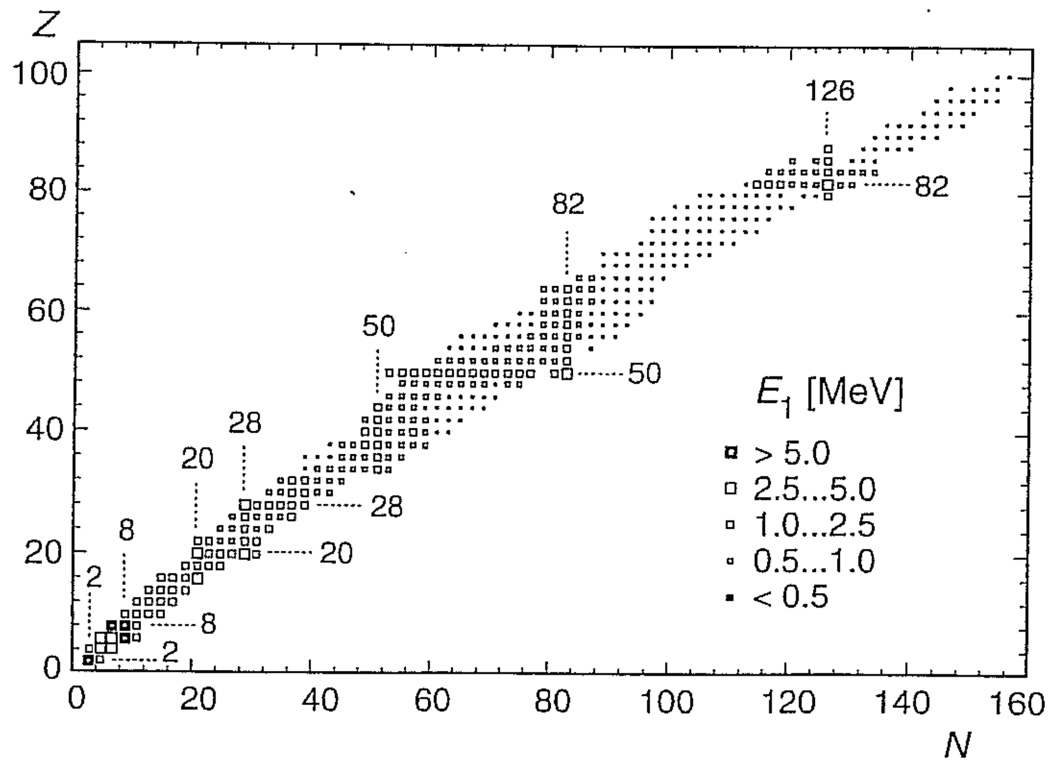
Look for instance at the energy required to remove the last neutron (this is referred to as the neutron separation energy) here for isotopes of ${}_{56}\mathrm{Ba}$



Zig-zag pattern (~ 2 MeV amplitude) is due to differences between odd and even *N*

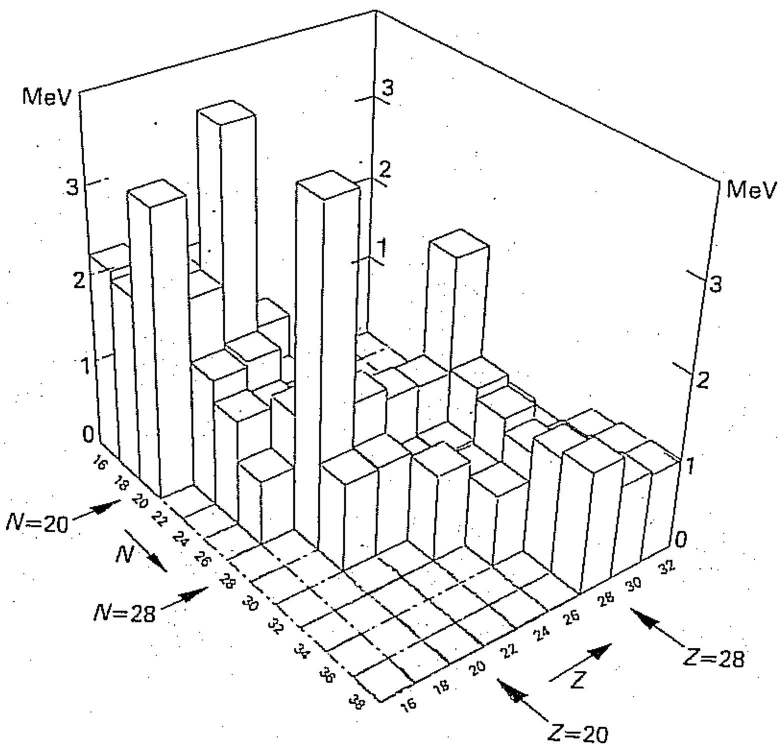
Note that after *N* = 82, the energy drops (additionally) by about 2 MeV

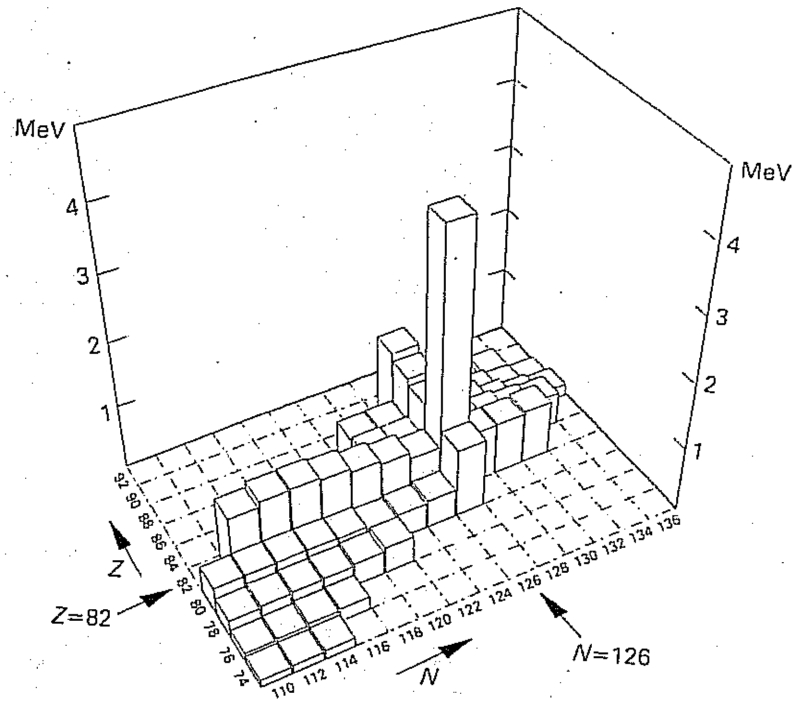
Look at energy of first excited state in the plane of (Z,N)



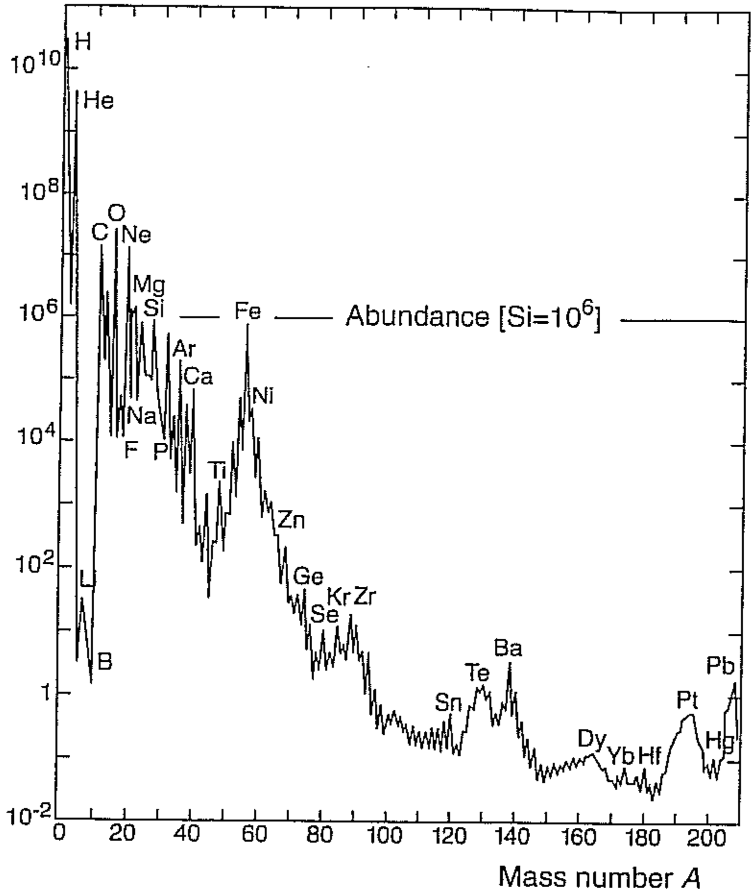
These energies are typically higher for nuclei with either Z or A “magic” and are particularly high for doubly-magic nucei (for instance (Z,A) = (50,82)

Another view:





Nuclear abundances:



peaking at A = 86 -90 due to N = 50

peaking at A = 114-120 due to Z = 50

peaking at A = 138 due to N = 82

peaking at A = 208 due to Z = 82, N = 126

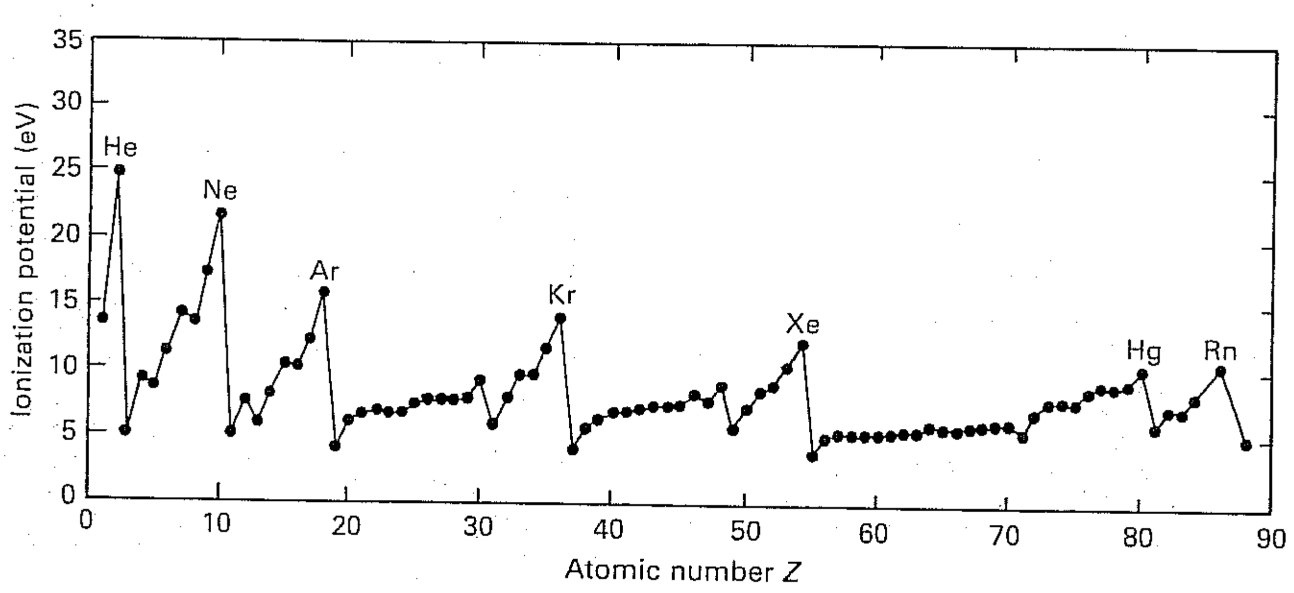
Periodic Table of the Elements

1 H 1.00794																1 H 1.00794	2 He 4.002602
3 Li 6.941	4 Be 9.012182											5 B 10.811	6 C 12.0107	7 N 14.00674	8 O 15.9994	9 F 18.9984032	10 Ne 20.1797
11 Na 22.989770	12 Mg 24.3050											13 Al 26.981538	14 Si 28.0855	15 P 30.973761	16 S 32.066	17 Cl 35.4527	18 Ar 39.948
19 K 39.0983	20 Ca 40.078	21 Sc 44.955910	22 Ti 47.867	23 V 50.9415	24 Cr 51.9961	25 Mn 54.938049	26 Fe 55.845	27 Co 58.933200	28 Ni 58.6934	29 Cu 63.546	30 Zn 65.39	31 Ga 69.723	32 Ge 72.61	33 As 74.92160	34 Se 78.96	35 Br 79.904	36 Kr 83.80
37 Rb 85.4678	38 Sr 87.62	39 Y 88.90585	40 Zr 91.224	41 Nb 92.90638	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.90550	46 Pd 106.42	47 Ag 107.8682	48 Cd 112.411	49 In 114.818	50 Sn 118.710	51 Sb 121.760	52 Te 127.60	53 I 126.90447	54 Xe 131.29
55 Cs 132.90545	56 Ba 137.327	57 La 138.9055	72 Hf 178.49	73 Ta 180.9479	74 W 183.84	75 Re 186.207	76 Os 190.23	77 Ir 192.217	78 Pt 195.078	79 Au 196.96655	80 Hg 200.59	81 Tl 204.3833	82 Pb 207.2	83 Bi 208.98038	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 (269)	111 (272)	112 (277)		114 (289) (287)		116 (289)		118 (293)

58 Ce 140.116	59 Pr 140.90765	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.964	64 Gd 157.25	65 Tb 158.92534	66 Dy 162.50	67 Ho 164.93032	68 Er 167.26	69 Tm 168.93421	70 Yb 173.04	71 Lu 174.967
90 Th 232.0381	91 Pa 231.03588	92 U 238.0289	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)

S.E. Van Bramer, 7/22/99
1995 IUPAC masses and Approved Names from http://www.chem.qmw.ac.uk/iupac/AtWt/
masses for 107-111 from C&EN, March 13, 1995, P 35
112 from http://www.gsi.de/z112e.html
114 from C&EN July 19, 1999
116 and 118 from http://www.lbl.gov/Science-Articles/Archive/elements-116-118.html

“Magic” numbers in atomic physics: reflected in the atomic ionization potential



Arises due to the orbital structure of the atomic electrons, which in turn arises from the solution of Schrödinger’s equation for the Coulomb potential. Increases in the ionization potential occur when the last electron completes the filling of an atomic *s* or *p* shell.

Schrödinger's equation (central potential) *V(r)*

Generally assume that (stationary-state) wavefunctions can be separated to give

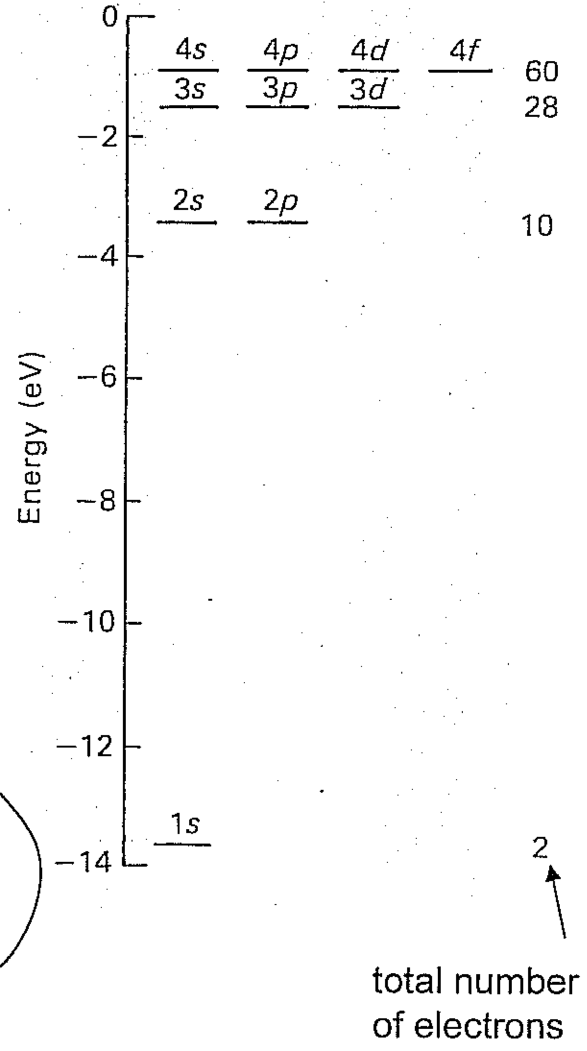
$$\psi(\vec{r}) = R(r)\underbrace{Y_l^m(\theta,\phi)}_{\text{spherical harmonics}}$$

Writing *R(r)* = *U(r)/r*, we obtain

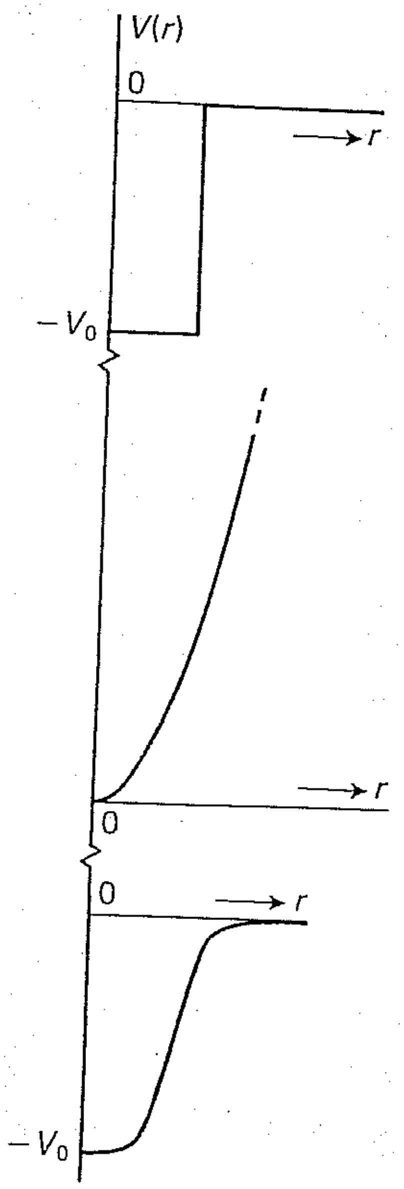
$$-\frac{\hbar^2}{2M}\frac{d^2U(r)}{dr^2} + \left\{V(r) + \frac{l(l+1)\hbar^2}{2Mr^2}\right\}U(r) = EU(r)$$

Angular momentum "barrier" (for *l* > 0) acts like additional potential term, which grows with *l*

Eigenfunctions U_{nl} are defined by *l* and by *n*, which is referred to as the principle quantum number. Energies are E_{nl}



Attempt the same procedure with model of nuclear potential



Square well

$r<R$: $V(r)=-V_0$

$r>R$: $V(r)=0$

For simplicity start by assuming and infinite square well.

Harmonic oscillator

$V(r) = \frac{1}{2}M\omega^2 r^2$

This is also not a great model for the nuclear potential but try it because it is calculable

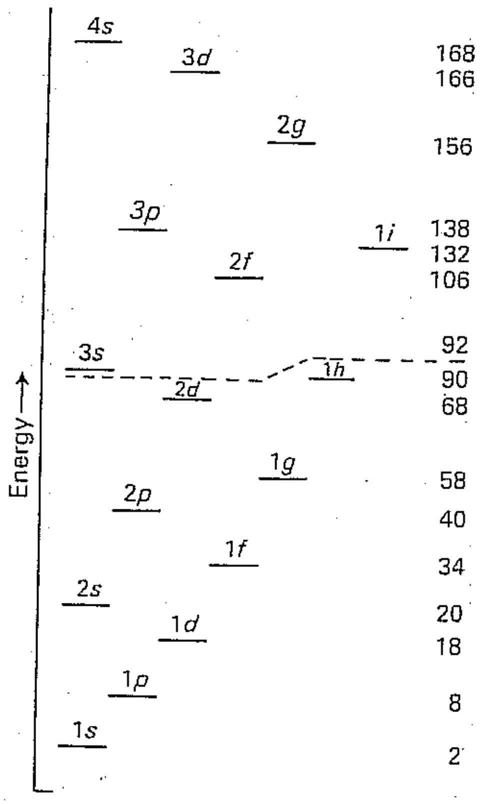
Saxon–Woods

$$V(r)= \frac{-V_0}{1+\exp\left(\frac{r-R}{d}\right)}$$

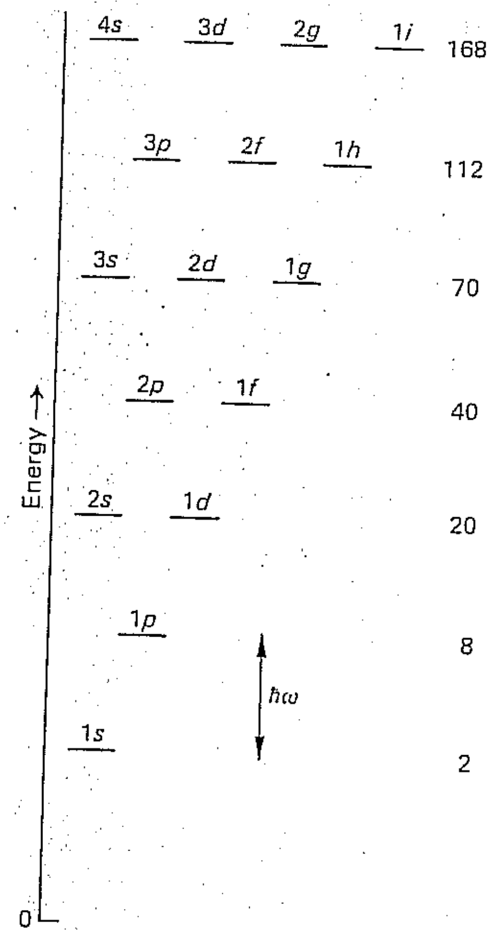
Really expect something more like this, which reflects what we learned from scattering experiments, about distributions of charge and matter in the nucleus. This is more difficult to deal with.

Solve Schrödinger equation for infinite square well and for harmonic oscillator and look at accumulated occupancy of energy levels, to try to reproduce the observed values of the magic numbers: 2, 8, 20, 28, 50, 82, 126

infinite square well



harmonic oscillator

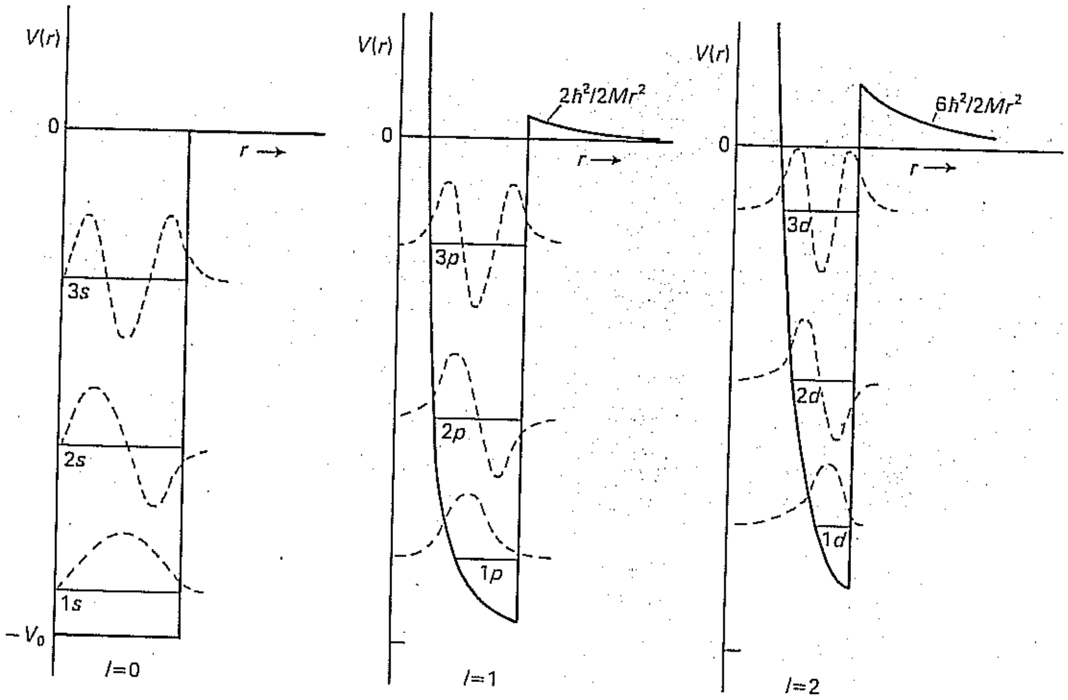


Can reproduce the magic numbers 2, 8 and 20, but not the higher values

Missing effects that are more important at higher energies ?

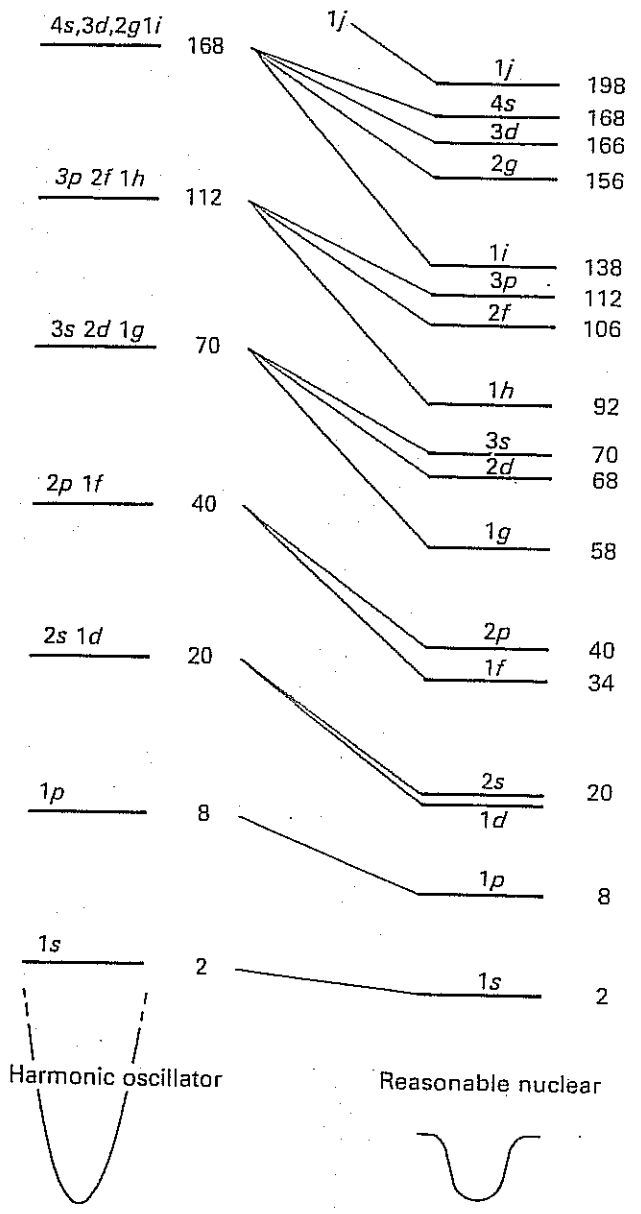
Attempt to be more realistic: truncate the well so that it is not of infinite height (since we know that the nuclear force saturates).

Consider the infinite square well. Making the depth finite means that the wave-functions do not need to be zero at the boundaries. They can leak (tunnel) out past the effective end of the potential, which reduces their “curvature” and lowers the energy, relative to the infinite square well solutions.



e.g. including the angular momentum barrier

The second and third plots show the effective finite square-well potential for l = 1, 2. Note that the nucleons get moved closer and closer to the surface, as l increases.



Truncating the potential (e.g. making the potential of finite height) lowers the energy levels.

Energy levels with different *l* are split: increasing the orbital angular momentum moves the nucleon closer to the surface where the potential is higher: lose the degeneracies that are present for the harmonic oscillator.

Note that the magic numbers still do not appear.

Potential missing something ? So far we have ignored the fact that the nucleons have spin.

Add a contribution from the interaction between the nuclear spins and their orbital angular momentum.

We have not discussed this, but such a term is necessary in the nucleon-nucleon potential in order to explain even the properties of the deuteron, which is the simplest compound nucleus.

Here, spin and orbit refer to the attributes of a single nucleon moving in the assumed nuclear potential. This term is also found in atomic physics and is necessary for explanation of the ionization potentials we looked at earlier.

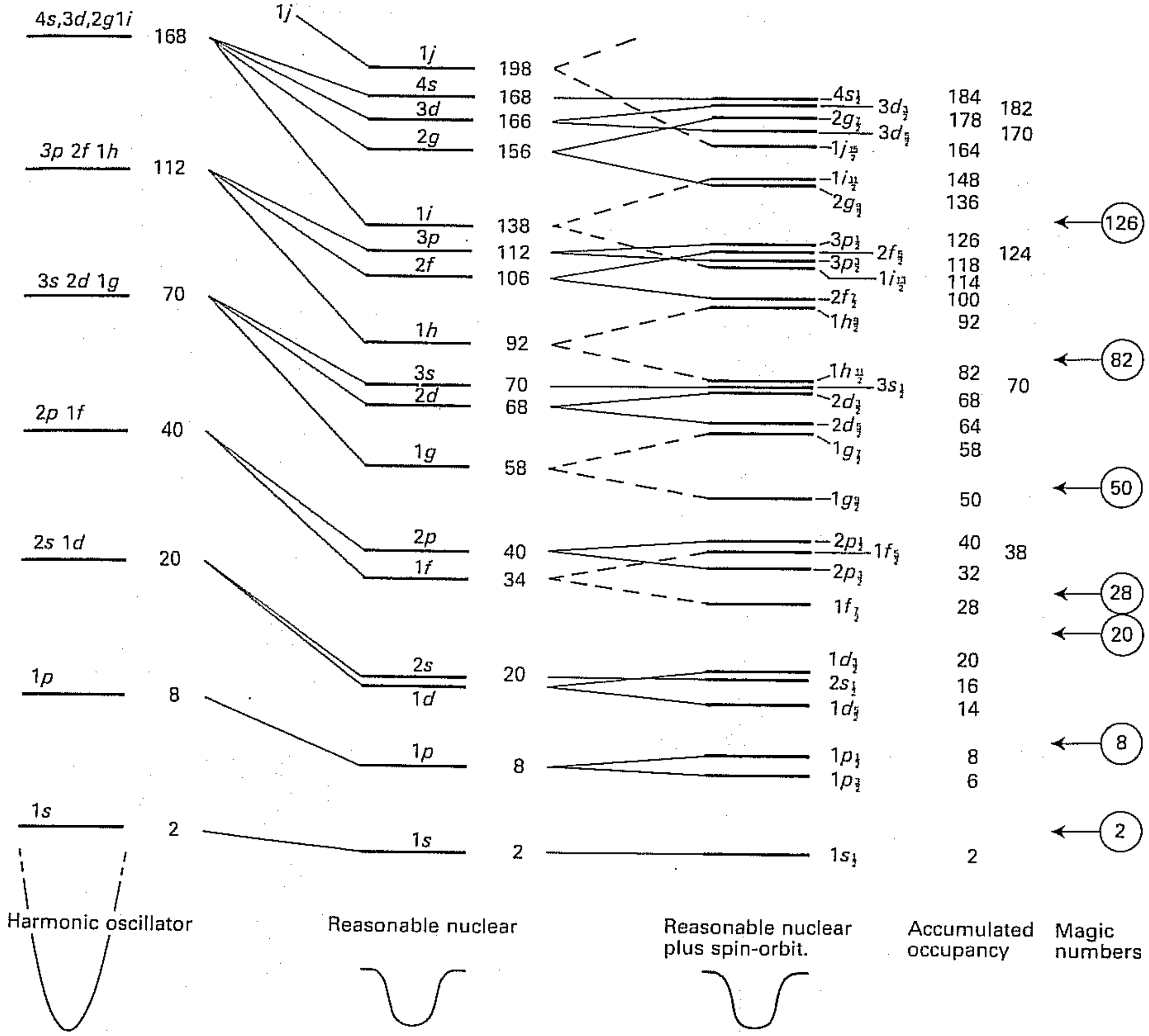
So we modify the potential: $V(r) \rightarrow V(r) + W(r)\vec{L}\cdot\vec{S}$ some function of *r*

For a given value of the orbital angular momentum *l* the nucleon total angular momentum *j* is *l* + ½ or *l* – ½. The spin-orbit term in the potential is different for these two possibilities: the eigenvalues of $\vec{L}\cdot\vec{S}$ are:

$$\frac{1}{2}\left[j(j+1)-l(l+1)-s(s+1)\right]\hbar^2 = \frac{1}{2}l\hbar^2 \qquad (j=l+\frac{1}{2})$$

$$= -\frac{1}{2}l(l+1)\hbar^2 \quad (j=l-\frac{1}{2})$$

Check effect on “shell” structure of nuclear energy levels



Reproduces the observed values of the magic numbers