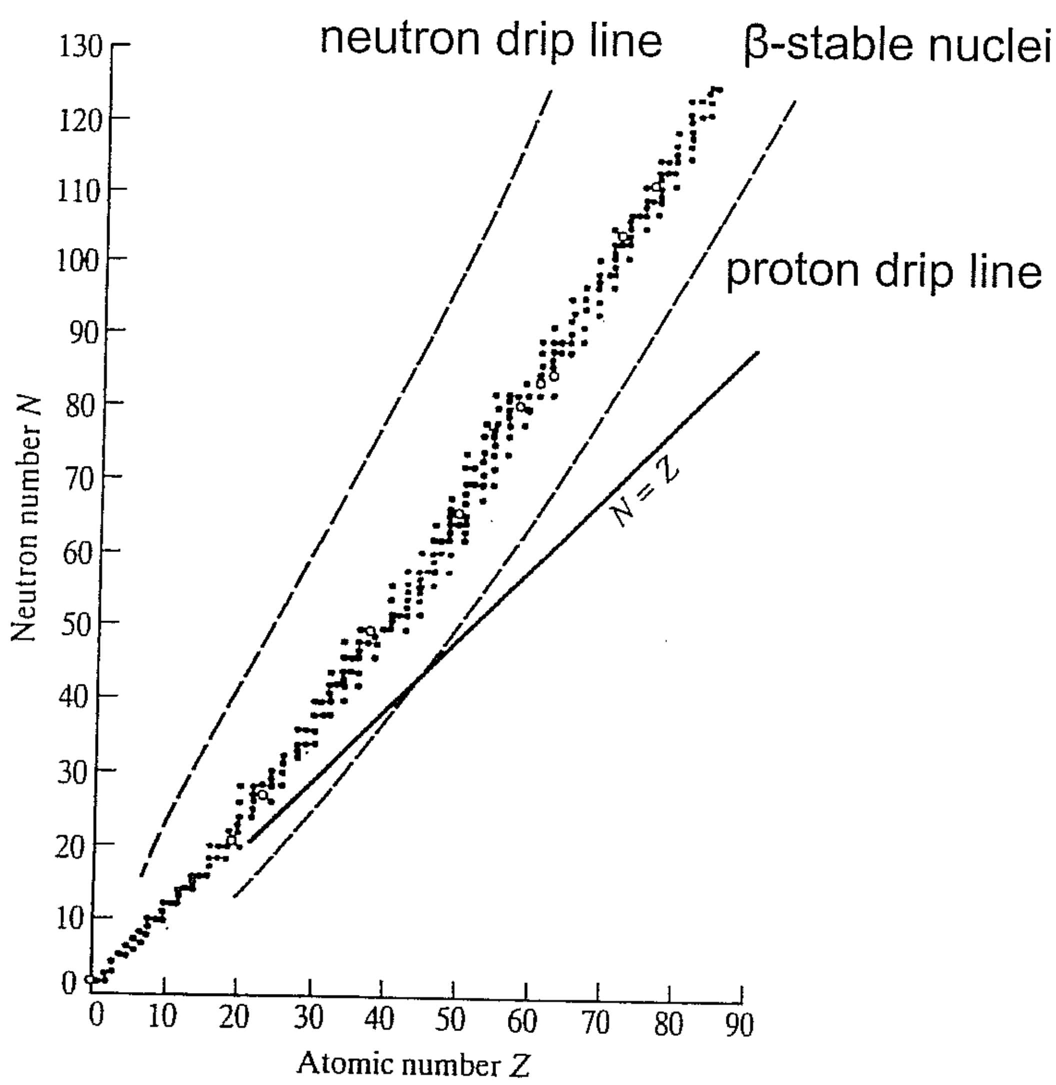
Nuclear Stability



Why are these the only stable nuclei ?

First, some definitions:

Z = atomic number (number of protons in nucleus)

A = atomic mass number (number of nucleons in nucleus)

N = neutron number (A-Z)

Isobars: nuclides with same atomic mass number A

Isotopes: nuclides with same atomic number Z

Isotones: nuclides with same neutron number N

Reminder: Nuclear Decays

Thus far, we have discussed three types of nuclear decay:

α-decay: emission of α-particle (^4He nucleus) [$A \rightarrow A-4$, $Z \rightarrow Z-2$, $N \rightarrow N-2$]

β-decay: e-, e+ emission or e- capture) [changes N,Z keeping A constant]

γ-decay: emission of gamma-ray. Cannot change A, Z, N. De-excitation of excited nuclear state, usually following α- or β-decay.

A further type is

Spontaneous fission: splitting of nucleus into two smaller nuclei (alpha decay is a special case). *Spontaneous* fission is otherwise relatively unimportant for nuclear stability and will be discussed only briefly. *Induced* fission is important for nuclear reactors)

[For ^{238}U the spontaneous fission rate is about 6 orders of magnitude smaller than the α-decay rate]

Look at stability to β-decay (includes β^- emission, β^+ emission and e^- capture reactions

Consider the case of nuclei with equal mass number A (isobars)

Write an expression for the mass. Note that it is common to use atomic masses rather than nuclear masses since these are generally more precisely known.

$$\mathcal{M}(Z,A) = (A-Z)M_n + ZM_p + Zm_e \qquad \text{[set } c = 1 \text{ here]}$$
$$-a_V A + a_S A^{2/3} + a_C Z^2 / A^{1/3} + a_A (A-2Z)^2 / A + \delta(Z,A)$$

This is just the semi-empirical mass formula that we discussed last time

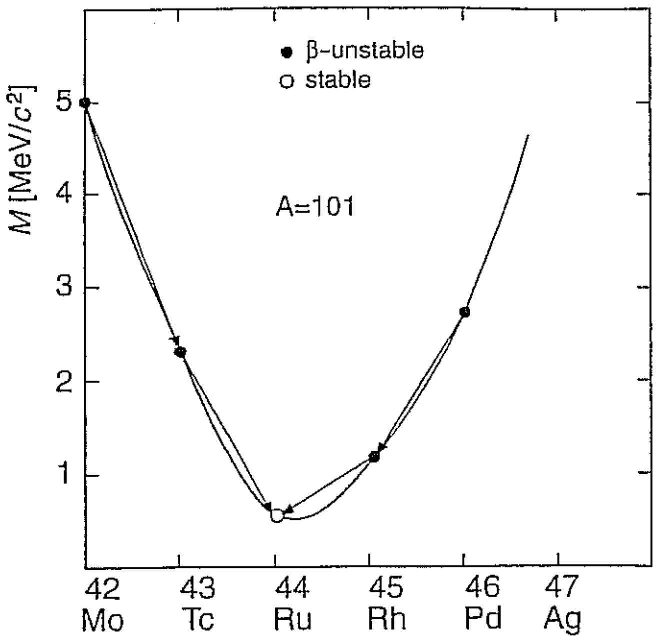
As a function of Z, this describes a parabola:

$$\boxed{\mathcal{M}(Z,A) = \alpha A - \beta Z + \gamma Z^2 + \delta / A^{1/2}}$$

$$\alpha = M_n - a_V + a_S A^{-1/3} + a_A$$
$$\beta = -M_p - m_e + M_n + 4a_A$$
$$\gamma = a_C / A^{1/3} + 4a_A / A$$
$\delta = \pm a_p / A^{1/2}$ (as before).

Recall, this is 0 for odd-A nuclei, +ve for (ee) and –ve for (oo)

First consider odd-A nuclides and look at a set of isobars (here for A = 101)



The figure shows the masses of the various isobars plotted versus Z

The lowest mass state is ${}^{101}_{44}\mathrm{Ru}$ [Ruthenium]

Increasing Z decreases the asymmetry term, but increases the Coulomb term……

Isobars with more neutrons (lower Z) such as ${}^{101}_{42}\mathrm{Mo}$ [Molybdenum] or ${}^{101}_{43}\mathrm{Tc}$ [Technetium] decay via the conversion

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

$$ {}^{101}_{42}\mathrm{Mo} \rightarrow {}^{101}_{43}\mathrm{Tc} + e^{-} + \bar{\nu}_e $$
$$ {}^{101}_{43}\mathrm{Tc} \rightarrow {}^{101}_{44}\mathrm{Ru} + e^{-} + \bar{\nu}_e $$

The kinematic requirement for β-decay to take place is $\mathcal{M}(A,Z) > \mathcal{M}(A,Z+1)$

Note that since these are atomic masses the (free) electron mass is automatically taken care of (we do neglect the neutrino mass however and the e$^-$ binding energy).

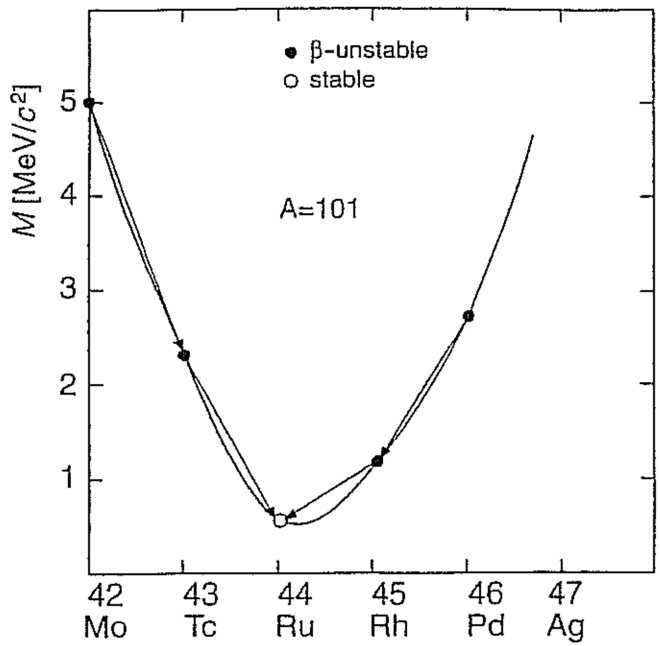
Isobars with a proton excess (relative to ${}^{101}_{44}\mathrm{Ru}$) decay through proton conversion

[i.e. the most stable isobar]

$p \rightarrow n + e^+ + \nu_e$ so for instance:

$${}^{101}_{46}Pd \rightarrow {}^{101}_{45}Rh + e^+ + \nu_e$$
$${}^{101}_{45}Rh \rightarrow {}^{101}_{44}Ru + e^+ + \nu_e$$

The kinematic requirement for β^+ decay is $\mathcal{M}(A,Z) > \mathcal{M}(A,Z-1) + 2m_e$



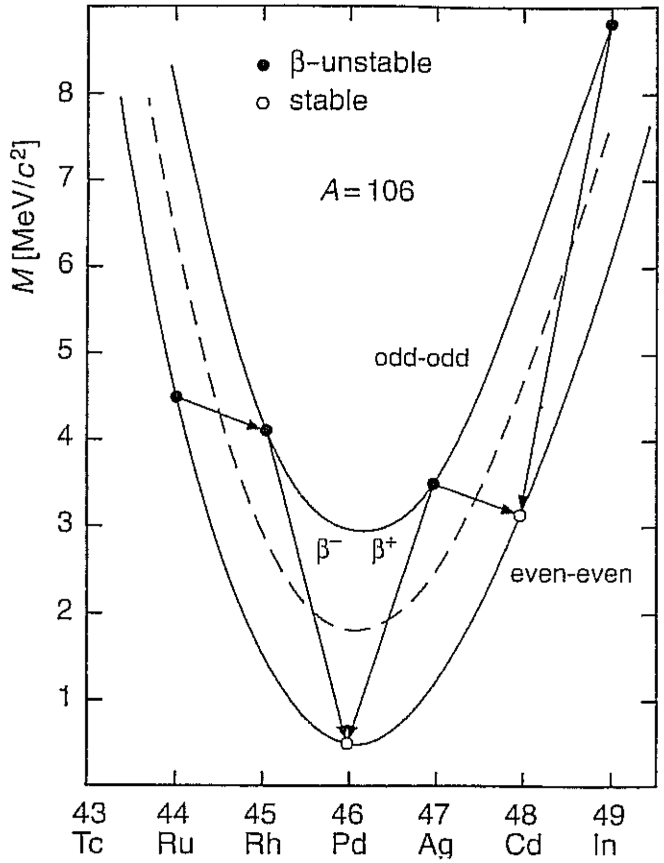
Accounts for the creation of a positron and the existence of an excess electron in the parent atom

[since the atomic mass used for the daughter will not include the mass of the additional electron]

Now consider the situation for even-A nuclei, which has some additional features:

The mass as a function of Z is now split into two parabolas that are separated by twice the pairing energy (which is decreases the binding energy for oo nuclei and increases it for ee nuclei). So one has (here for A = 106 isobars)

[i.e. increases the mass]



Note that in this case it is possible to have more than one stable isobar. The isobars $^{106}_{48}\mathrm{Cd}$ and $^{106}_{46}\mathrm{Pd}$ are on the lower curve with $^{106}_{46}\mathrm{Pd}$ being the more stable.

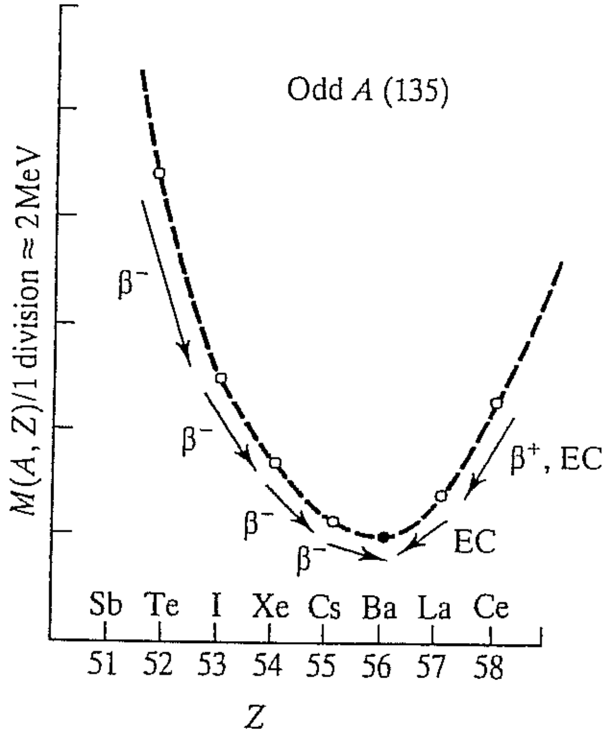
β-decay changes Z only by 1

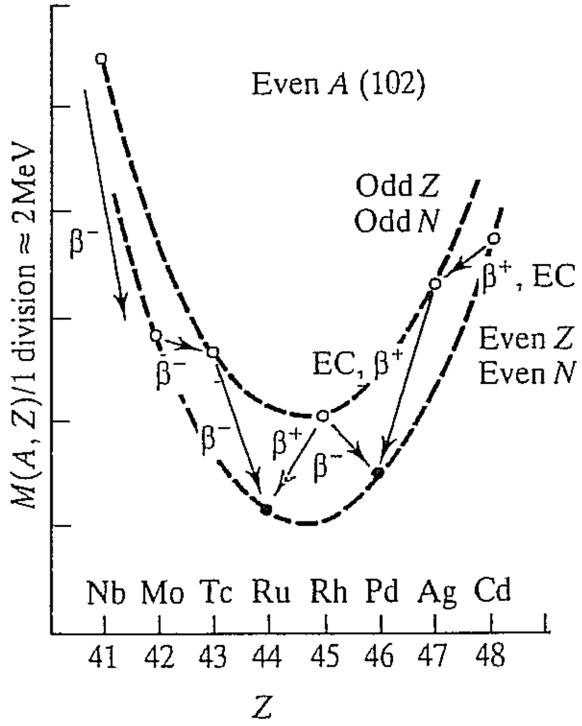
So $^{106}_{48}\mathrm{Cd}$ is stable, because it's two (oo) isobaric neighbors both lie at higher masses.

[Cadmium, Palladium]

$^{106}_{48}\mathrm{Cd} \rightarrow {}^{106}_{46}\mathrm{Pd}$ can proceed only via direct $^{106}_{48}\mathrm{Cd} \rightarrow {}^{106}_{46}\mathrm{Pd} + 2e^{+} + 2\nu_{e}$ decay for which the probability is so low that the state can be considered as stable.

Some more examples





Two conclusions can be drawn:

1) There are no stable oo isobars (this is almost true, there are some light exceptions)

$^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}B$, $^{14}_{7}N$

2) There is often more than one stable isobar for ee nuclei

Reminder: the kinematic requirement for β^+ decay to proceed is

$$\mathcal{M}(A,Z) > \mathcal{M}(A,Z-1) + 2m_e$$

Note that electron capture also contributes to such decays: $p + e^- \rightarrow n + \nu_e$

This reaction occurs mainly in heavy nuclei in which electronic orbits are more compact (there is a finite probability of finding the electron inside the nucleus. This is higher for more compact orbits). Usually the electrons are captured from the innermost shell (the K-shell), after which electrons from higher shells will cascade down resulting in X-ray emission.

This process competes with β^+ decay but has slightly different kinematic requirements:

$$\mathcal{M}(A,Z) > \mathcal{M}(A,Z-1) + \varepsilon$$

where ε is the excitation energy of the atomic shell of the daughter nucleus

Since $\varepsilon < 2m_e$ there will be cases where β^+ decay is forbidden but electron capture can proceed.

Lifetimes of β-unstable nuclei vary between a few ms and 10^{16} years.

There is a strong dependence on the energy E released in the reaction $\tau \propto E^{-5}$

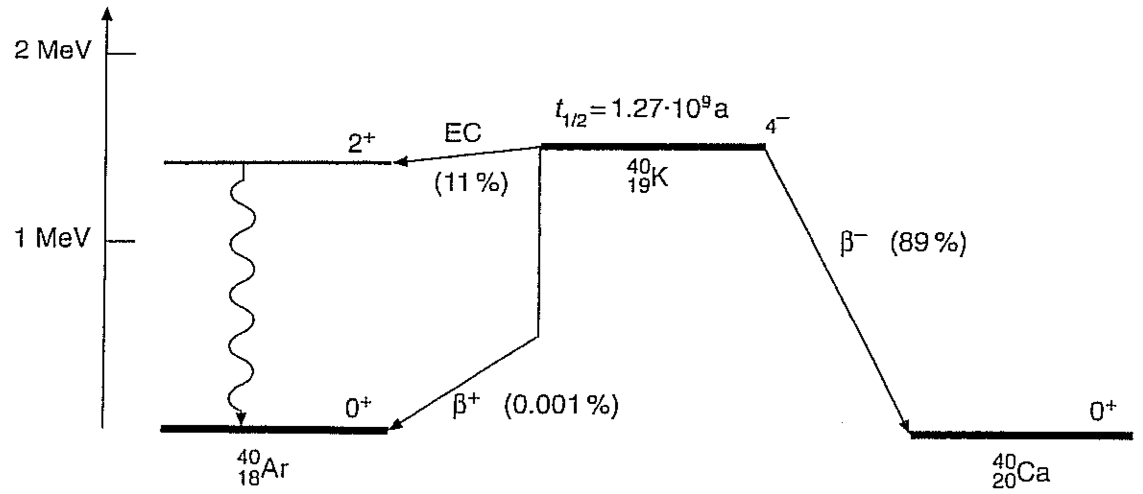
as well as upon the nuclear properties of the mother and daughter nuclei.

A free neutron decays with a lifetime of ~ 896 s, releasing 0.78 MeV of (kinetic) energy

A free proton cannot decay: $p \rightarrow n + e^{+} + \nu_e$ can take place only inside a nucleus when the overall nuclear decay is energetically favoured.

Note that a neutron is absolutely stable in a stable nucleus. In the same way that the decay $p \rightarrow n + e^{+} + \nu_e$ can be facilitated by an energetically favorable nuclear mass difference, the decay of a neutron $n \rightarrow p + e^{-} + \overline{\nu}_e$ bound inside a stable nucleus can be energetically forbidden.

Note that certain nuclei are both β^+ and β^- emitters. One long-lived example is ${}^{40}_{19}K$



Potassium a biologically essential element to both human and other life. It's responsible for signal transmission in the nervous system via exchange of potassium ions.

The fraction of radioactive potassium ${}^{40}_{19}K$ in naturally occurring potassium is about 0.01% and its decay inside the human body accounts for about 16% of our total natural radiation exposure

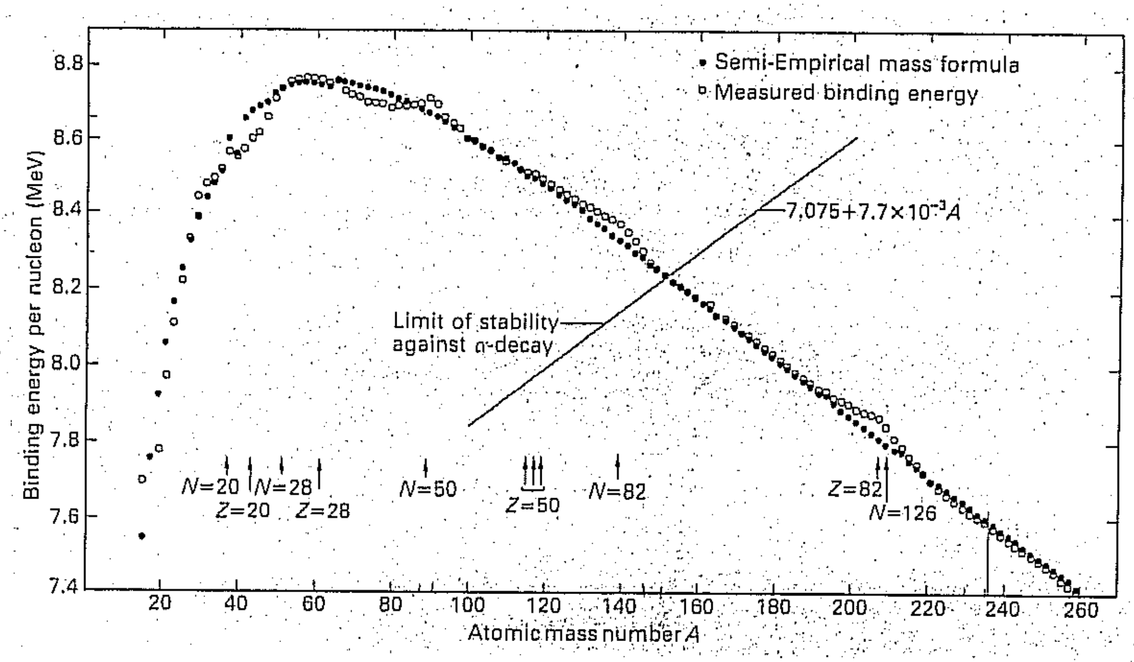
Periodic Table of the Elements

1 H 1.00794																1 H 1.00794	2 He 4.002602
3 Li 6.941	4 Be 9.012182											5 B 10.811	6 C 12.0107	7 N 14.00674	8 O 15.9994	9 F 18.9984032	10 Ne 20.1797
11 Na 22.989770	12 Mg 24.3050											13 Al 26.981538	14 Si 28.0855	15 P 30.973761	16 S 32.066	17 Cl 35.4527	18 Ar 39.948
19 K 39.0983	20 Ca 40.078	21 Sc 44.955910	22 Ti 47.867	23 V 50.9415	24 Cr 51.9961	25 Mn 54.938049	26 Fe 55.845	27 Co 58.933200	28 Ni 58.6934	29 Cu 63.546	30 Zn 65.39	31 Ga 69.723	32 Ge 72.61	33 As 74.92160	34 Se 78.96	35 Br 79.904	36 Kr 83.80
37 Rb 85.4678	38 Sr 87.62	39 Y 88.90585	40 Zr 91.224	41 Nb 92.90638	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.90550	46 Pd 106.42	47 Ag 107.8682	48 Cd 112.411	49 In 114.818	50 Sn 118.710	51 Sb 121.760	52 Te 127.60	53 I 126.90447	54 Xe 131.29
55 Cs 132.90545	56 Ba 137.327	57 La 138.9055	72 Hf 178.49	73 Ta 180.9479	74 W 183.84	75 Re 186.207	76 Os 190.23	77 Ir 192.217	78 Pt 195.078	79 Au 196.96655	80 Hg 200.59	81 Tl 204.3833	82 Pb 207.2	83 Bi 208.98038	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 (269)	111 (272)	112 (277)		114 (289) (287)		116 (289)		118 (293)

58 Ce 140.116	59 Pr 140.90765	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.964	64 Gd 157.25	65 Tb 158.92534	66 Dy 162.50	67 Ho 164.93032	68 Er 167.26	69 Tm 168.93421	70 Yb 173.04	71 Lu 174.967
90 Th 232.0381	91 Pa 231.03588	92 U 238.0289	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)

S.E. Van Bramer, 7/22/99
1995 IUPAC masses and Approved Names from http://www.chem.qmw.ac.uk/iupac/AtWt/
masses for 107-111 from C&EN, March 13, 1995, P 35
112 from http://www.gsi.de/z112e.html
114 from C&EN July 19, 1999
116 and 118 from http://www.lbl.gov/Science-Articles/Archive/elements-116-118.html

Next consider α-decay: we saw last time that in medium and heavy nuclei, protons and neutrons are bound inside the nucleus by about 8 MeV. So an individual nucleon cannot generally escape from the nucleus (except for very neutron or proton rich nuclei, beyond the so-called drip lines). However, in some cases it can be energetically favorable for a bound system of nucleons to be emitted (since the binding energy of the system increases the available energy for the process).

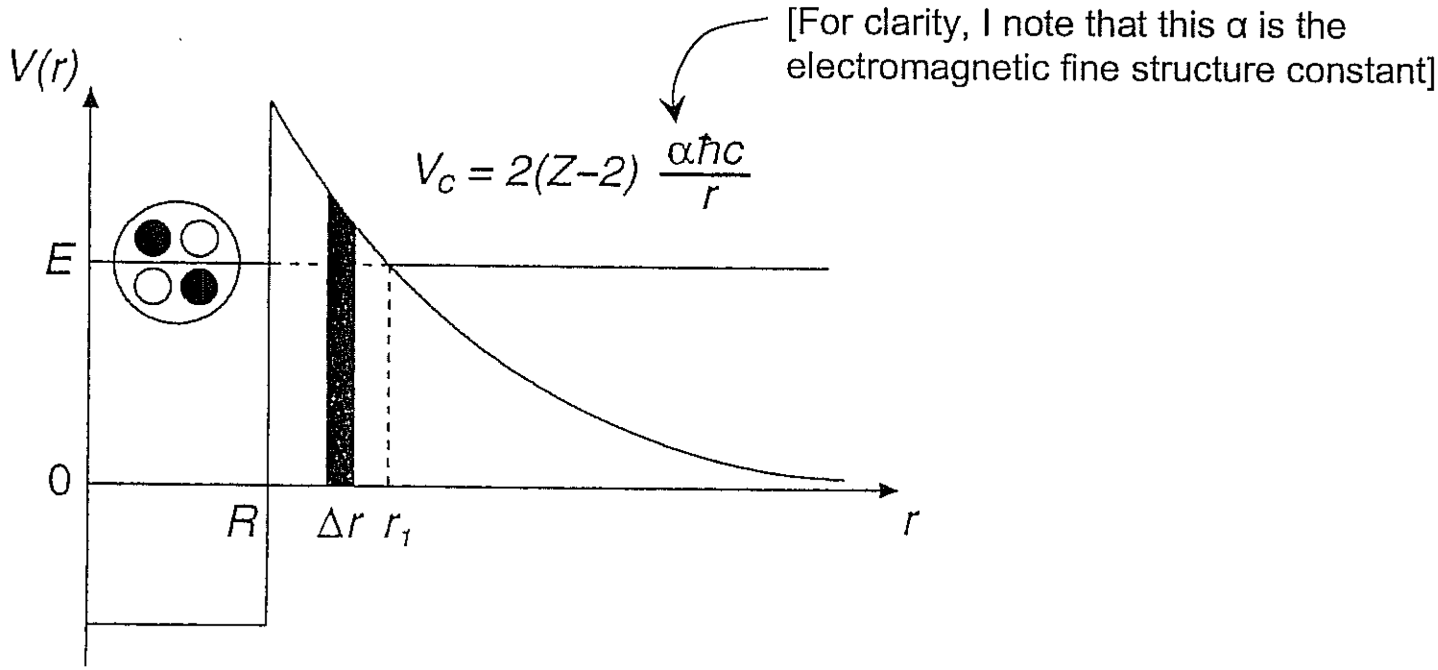


The probability for this to happen decreases rapidly with the number of nucleons in the emitted bound state. The most significant decay is that of a ^4He nucleus which has a very large binding energy relative to systems of 2 or 3 nucleons (remember, the deuteron is very weakly bound). These so-called α-particles are bound with about 7MeV/nucleon.

Now consider α-decay (an α-particle is a ^{4}He nucleus $A=4$, $Z=2$)

α-decay can be viewed as a quantum-mechanical tunneling effect:

This is illustrated in the figure below:



This shows the potential energy of an α-particle as a function of its distance from the centre of the nucleus. Beyond the nuclear force range (which saturates, as we have seen) the particle feels only the Coulomb potential V_C which increases closer to the nucleus. Within the range of the nuclear force, the strongly attractive nuclear potential prevails. The total energy of the α particle (in the case where α-decay is allowed) must be positive. This is the energy that is emitted in the decay.

Measured range of lifetimes for α-decay varies from 10ns to 10^{17} years

Tunneling calculation: break potential barrier up into thin "walls"

Probability to tunnel through one of these is $T \approx e^{-2\kappa\Delta r}$ $\kappa = \sqrt{2m\,|E-V|}\,/\,\hbar$

barrier thickness

E is the energy of the α-particle and V is the height of the potential barrier

The full transmission can be shown to be given by the expression $T \approx e^{-2G}$

for the Gamow factor

$$G = \frac{1}{\hbar}\int_R^{r_1} \sqrt{2m\,|E-V|}\,dr \approx \frac{2\pi(Z-2)\alpha_{em}}{\beta}$$

β is the velocity (v/c) of the outgoing α-particle; other quantities are as defined in the diagram

Probability / unit time for emission of an α-particle is proportional to

- $w(\alpha)$ the probability of finding an α-particle inside the nucleus
- the number of collisions of the α-particle with the barrier $\propto v/2R$
- the transmission probability T.

Thus we have a transition rate of $$\lambda = w(\alpha)\frac{\mathrm{v}}{2R}e^{-2G}$$

Observed, wide variation in lifetimes mostly due to the Gamow factor in the exponent

$$G \propto \frac{Z}{\beta} \propto \frac{Z}{\sqrt{E}}$$ so small differences in the energy of the α-particle have a large effect on the lifetime.

Most α-emitters are heavier than lead. For lighter nuclei ($A \leq \sim 140$) α-decay is still possible, but the energy release is so small that the lifetimes are usually so long that decays are not observed.

Energy conditions for α-decay to take place:

The energy release Q_α in the decay process is

$$Q_\alpha = [\mathcal{M}(Z,A) - \mathcal{M}(Z-2,A-4) - \mathcal{M}(2,4)]c^2$$

or, in terms of the binding energies

$$Q_\alpha = B(Z-2,A-4) + B(2,4) - B(Z,A)$$

so the decay may proceed if

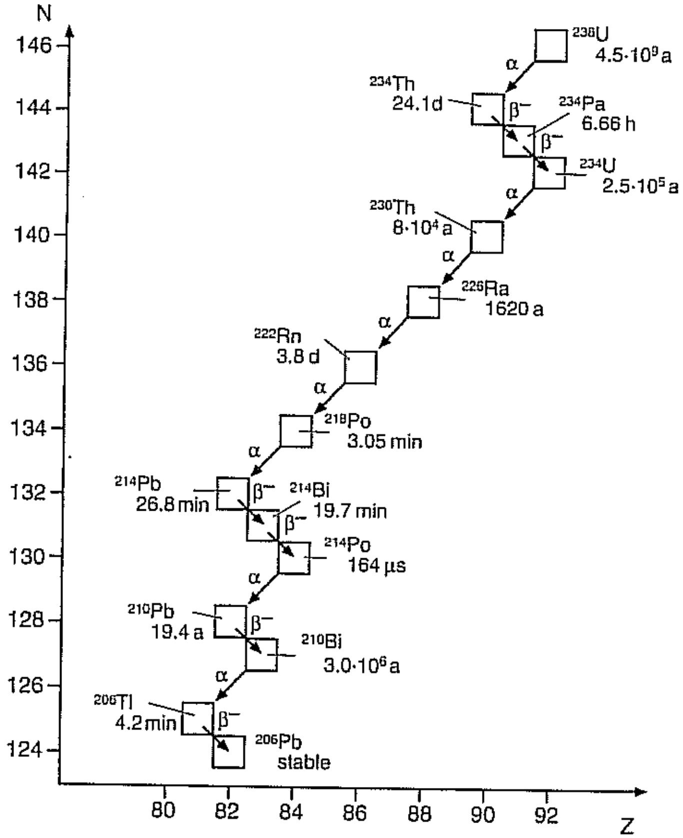
[-7.7×10^{-3} MeV]

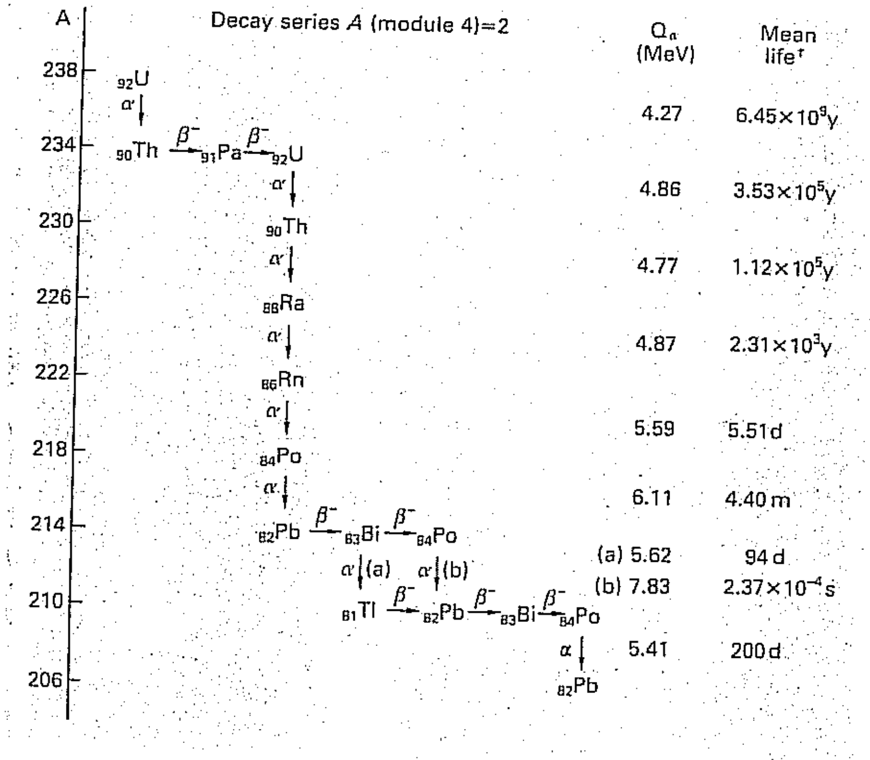
$$B(2,4) > B(Z,A) - B(Z-2,A-4) \approx 4\frac{dB}{dA} = 4\left[A\frac{d(B/A)}{dA} + \frac{B}{A}\right]$$

^4He binding energy is 28.3 *MeV*: 28.3 = 4(*B/A*-7.7x10-3*A*) plotted on earlier figure

Radioactive decay sequence for ^{238}U (two views)

Q_α = energy released





Each α-decay increases *N/Z* until *β*-decay takes over. Speed of descent depends on lifetimes of intermediate states. Create secular equilibrium…..still have natural abundances of short-lived states since they are in equilibrium with production from these decay chains.

Note that the small differences in the released energies for various decays in the sequence results in very large differences in the lifetimes.

Note that this decay sequence is also responsible for a large fraction of the natural radiation dose that people are exposed to.

Uranium compounds are common in granite, and it's radioactive daughters can be found in the stone walls of buildings.

This is particularly true of Radon (^{222}Rn) which can escape from the walls and be inhaled.

The α-decay of ^{222}Rn is typically responsible for about 40% of the average natural human radiation exposure.