Tutorial April 6

Expected odd particle configurations for

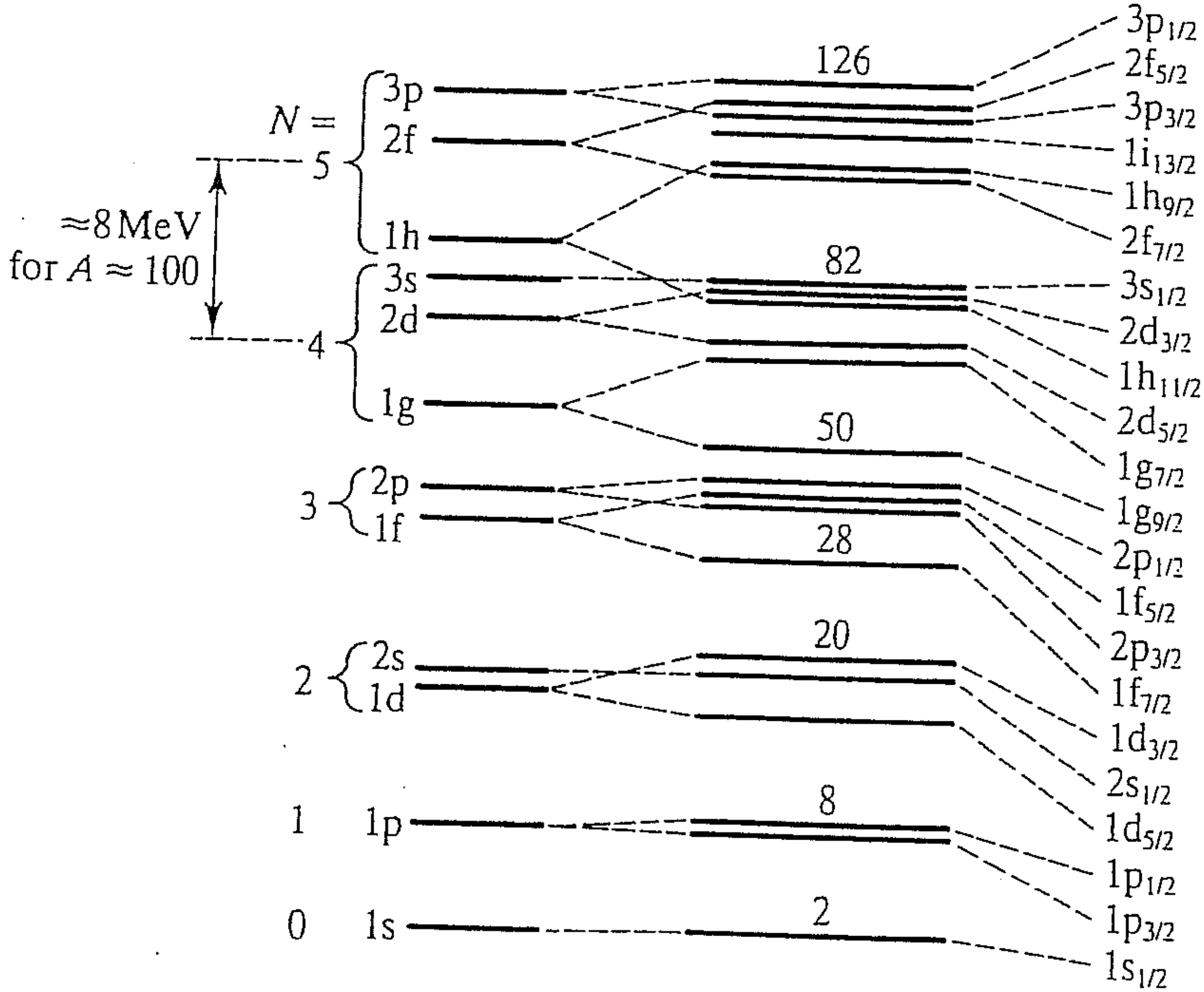
$\boxed{{}^{27}_{13}Al}$ $1d_{5/2}$ (p)

$\boxed{{}^{29}_{14}Si}$ $1s_{1/2}$ (n)

$\boxed{{}^{40}_{19}K}$ $1d_{3/2}$ (p) + $1f_{7/2}$ (n)

$\boxed{{}^{93}_{41}Nb}$ $1g_{9/2}$ (p)

$\boxed{{}^{157}_{64}Gd}$ $1h_{9/2}$ (n)



J^P for

$\boxed{\text{first excited state of } {}^{31}_{14}Si}$ 17n so populating $1d_{3/2}$ state

model prediction is thus

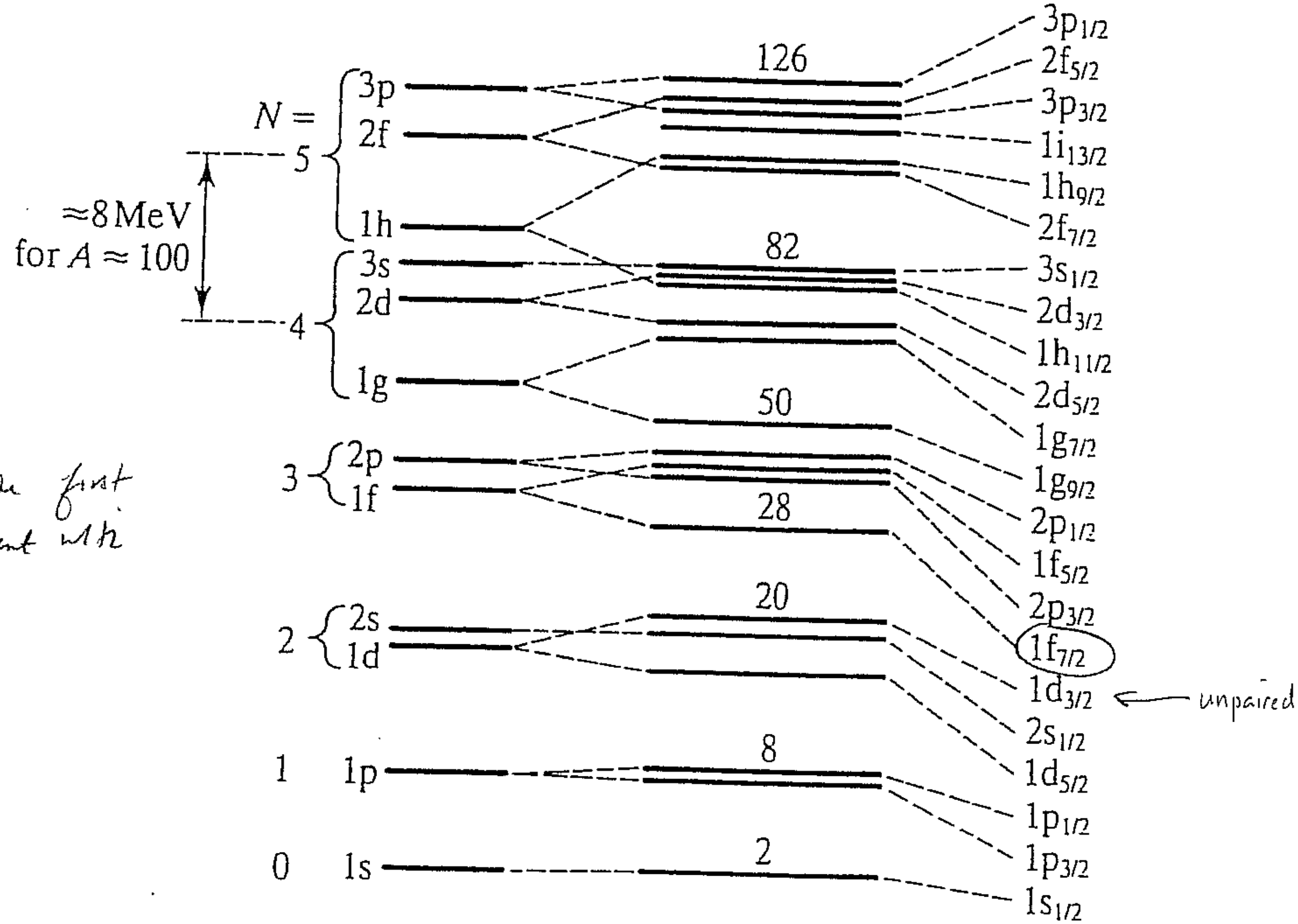
$\frac{7}{2}^-$

but clearly from the energy level diagram promoting a nucleon from the $2s_{1/2}$ state looks like it will cost less energy.

I find the observed J^P for the first excited state is $\frac{1}{2}^+$, consistent with that hypothesis.

One after that is $\frac{5}{2}^+$

(promotion from $1d_{5/2}$?)



Remember. The parity comes from the parity of the unpaired nucleon

It is even for ℓ even (s, d, g ...) and odd for ℓ odd (p, f, ...)

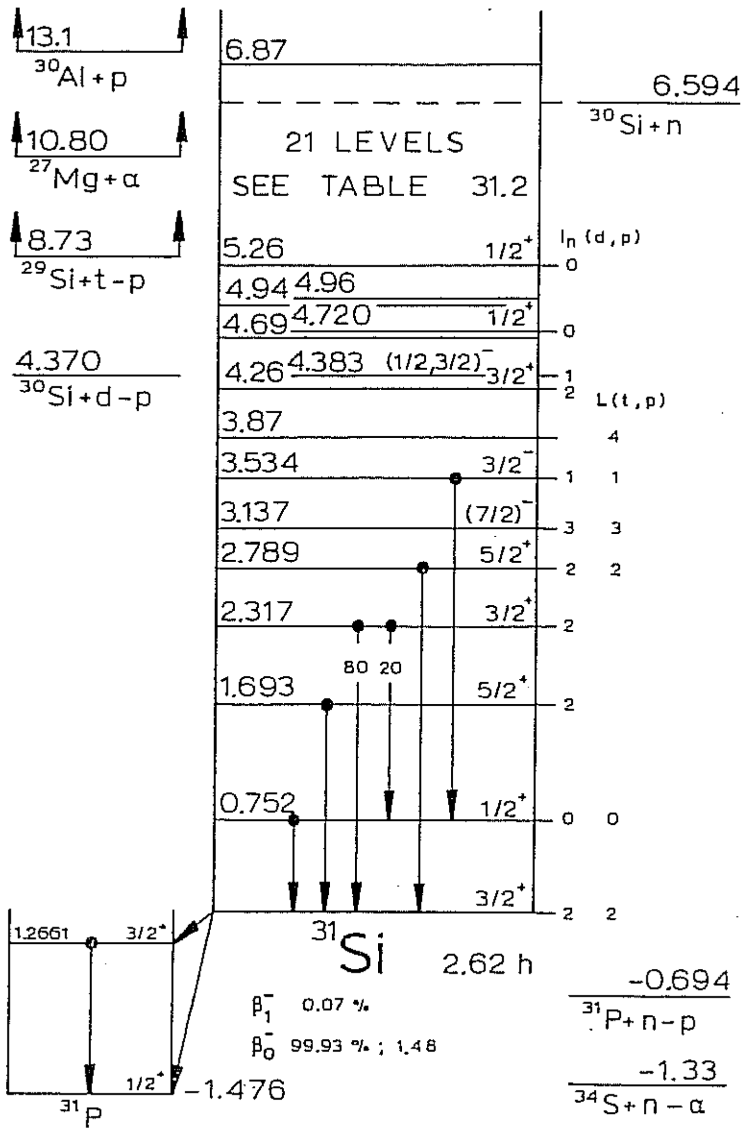


Fig. 31.1. Energy levels of ^{31}Si.

Similarly for ${}^{41}_{19}K$ $\quad$ $\begin{matrix}19\,p \\ 22\,n\end{matrix}$ $\rightarrow$ unpaired p is in $1d_{3/2}$ $\quad$ predict J^P for first excited state of $\frac{7}{2}^-$

but observe $\frac{1}{2}^+$

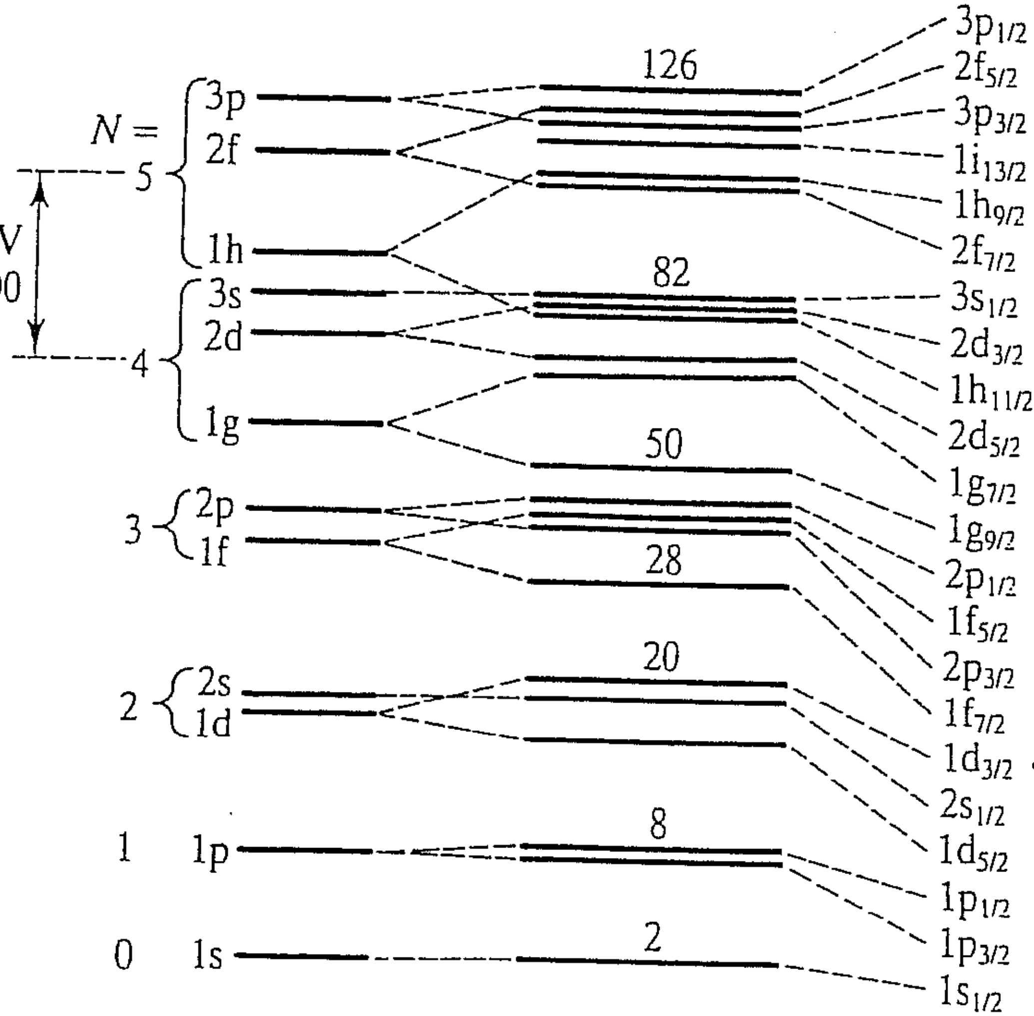
2nd excited state instead

${}^{49}_{21}Sc$ $\quad$ 21 p unpaired in $1f_{7/2}$ so predict excited state $J^P = \frac{3}{2}^-$

but observe $\frac{3}{2}^+$

It is certainly less obvious that this should be the case since the promotion from the $d_{3/2}$ appears to cost more energy. But perhaps pairing energy smaller in lower subshell?

Sequence is $\frac{3}{2}^+, \frac{1}{2}^+, \frac{3}{2}^-$



unpaired ${}^{49}_{21}Sc$

unpaired ${}^{41}_{19}K$

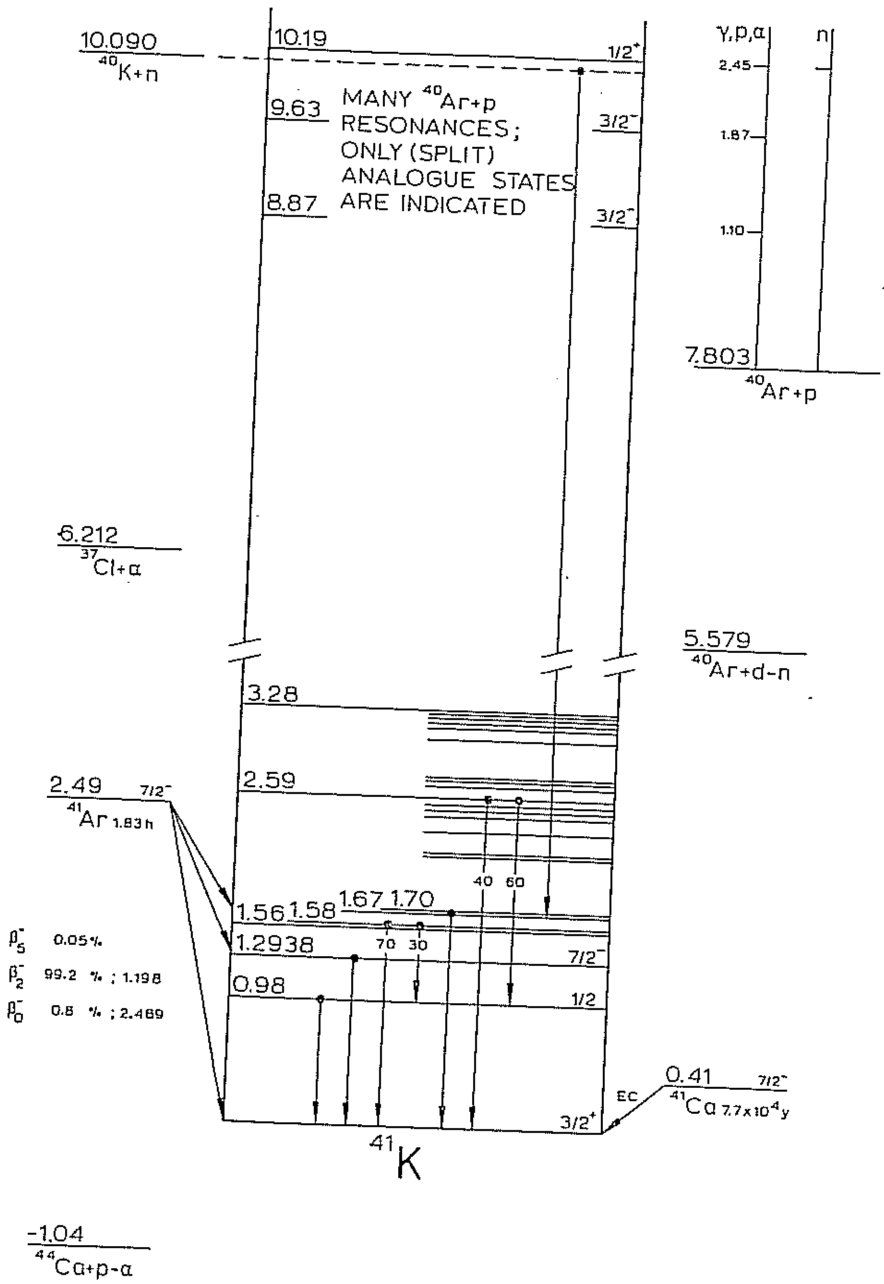


Fig. 41.2. Energy levels of ^{41}K.

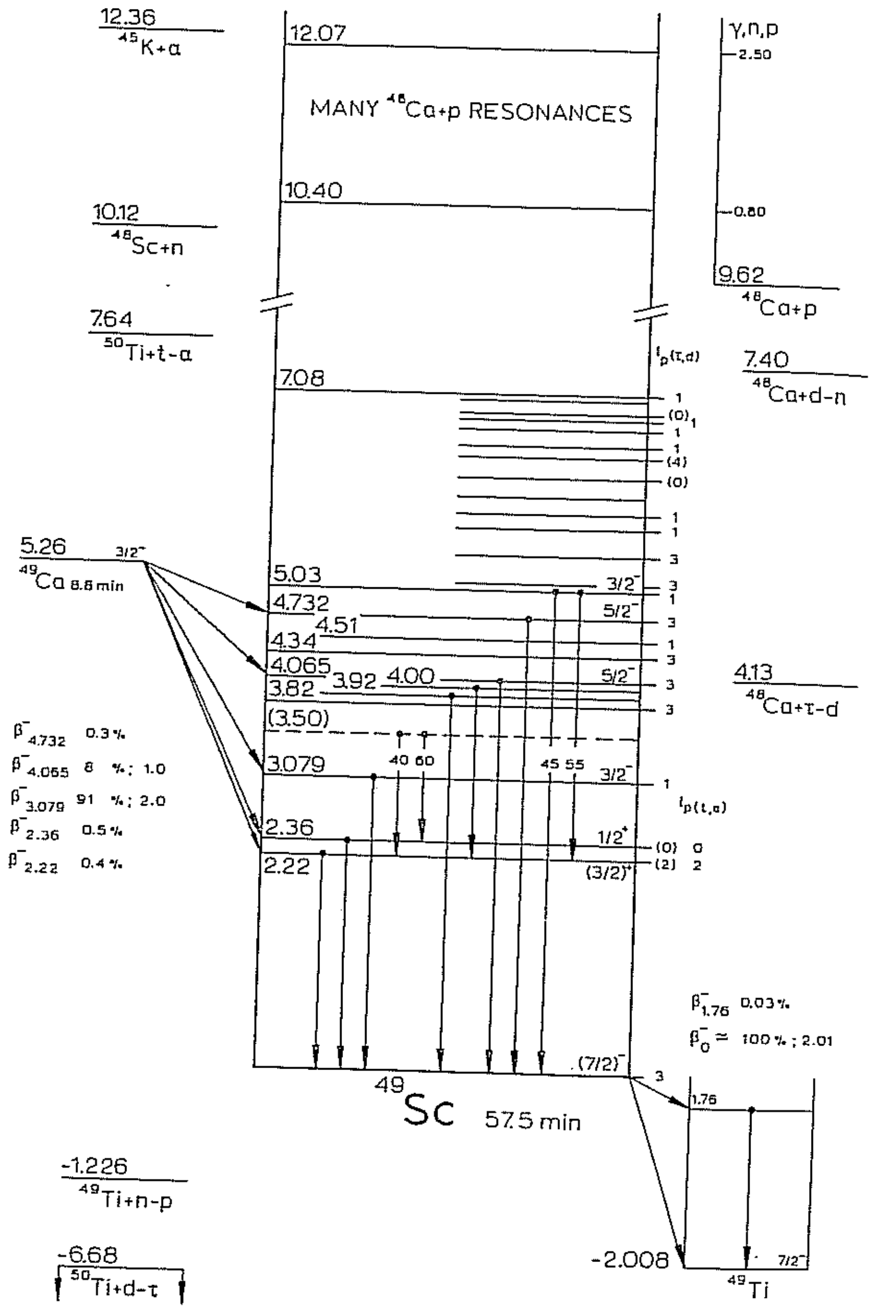


Fig. 49.2. Energy levels of ^{49}Sc.

The low levels of ${}^{39}_{20}Ca$ have J^P values (starting from the ground state) of $\frac{3}{2}^+, \frac{1}{2}^+, \frac{7}{2}^-, \frac{3}{2}^-$
Explain these in terms of the shell model

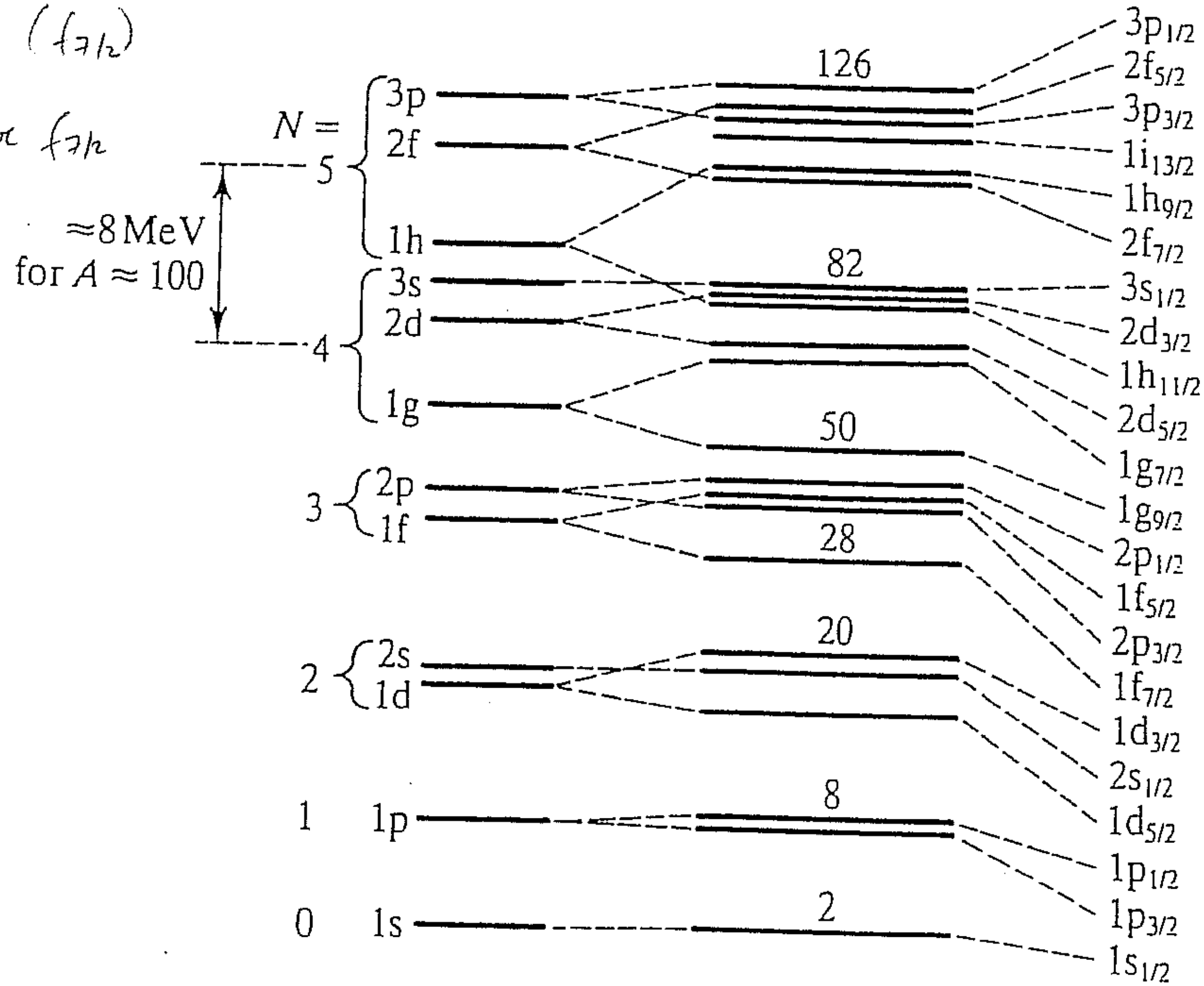
1 unpaired neutron in $1d_{3/2}$ level $(\ell = 2 \;\; j = 3/2) \longrightarrow \frac{3}{2}^+$

first excitation $\frac{1}{2}^+$ is interpreted as promotion of neutron from $2s_{1/2}$

$\frac{7}{2}^-$ excitation is to next level $(f_{7/2})$

$\frac{3}{2}^-$ excitation is to next above $f_{7/2}$

Why no promotion from $1d_{5/2}$?



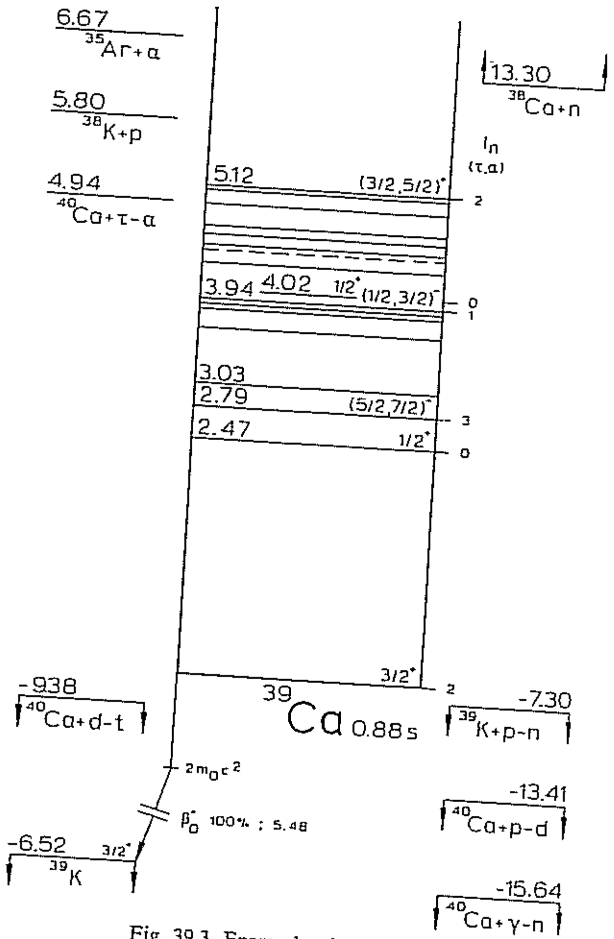


Fig. 39.3. Energy levels of ^{39}Ca.

4.b

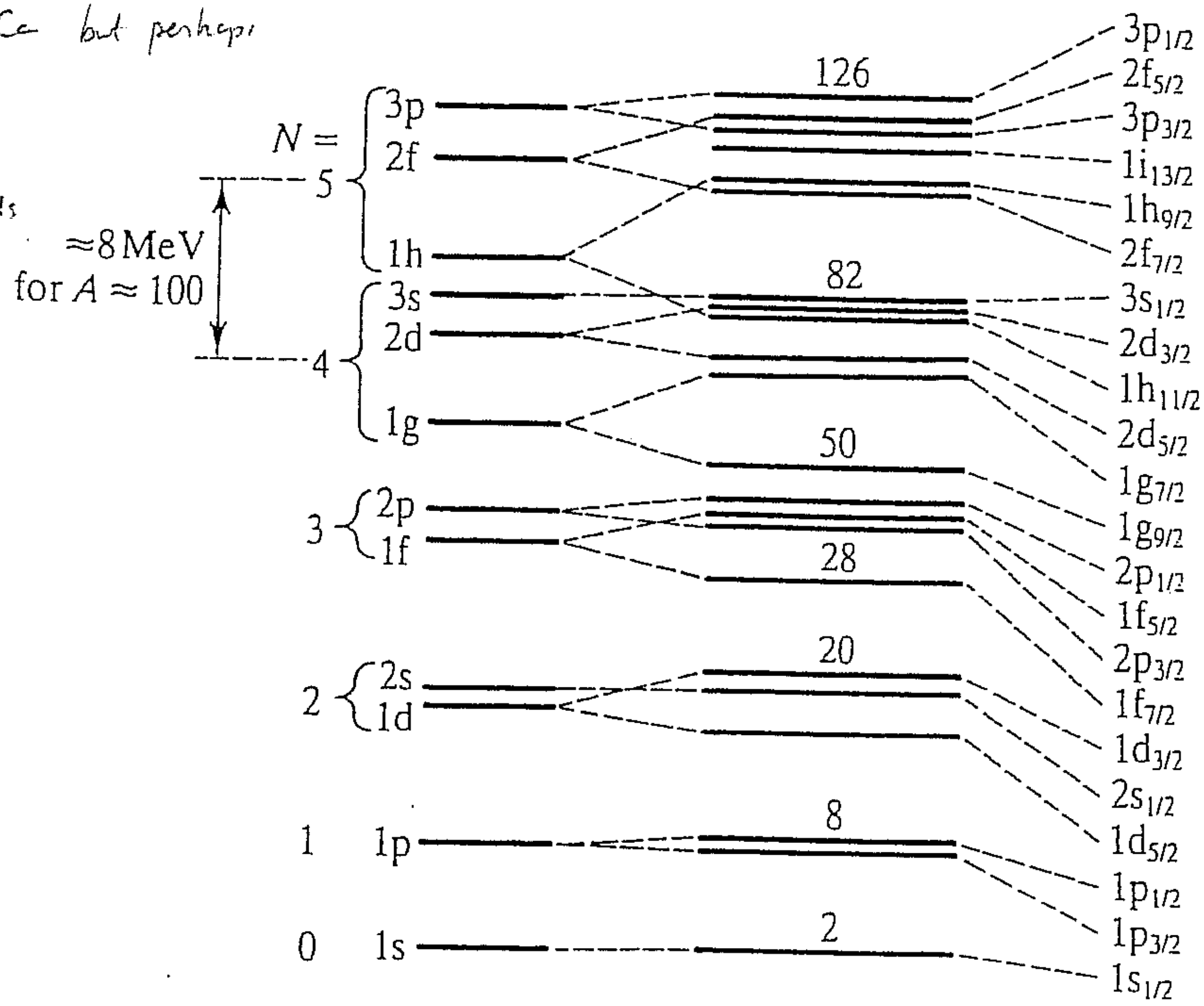
$^{43}_{20}Ca \rightarrow$ unpaired neutron in $1f_{7/2}$ shell just above $N = 20$ (large jump)

$\hookrightarrow$ ground state $J^P = \frac{7}{2}^-$

expect excited states $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{1}{2}^-$ $\rightarrow$ see

draw the same conclusions for $^{41}_{20}Ca$ but perhaps expect better agreement since there is a single unpaired neutron and two magic filled shells

but see $\frac{3}{2}^-$ $\frac{3}{2}^+$ $\frac{3}{2}^-$



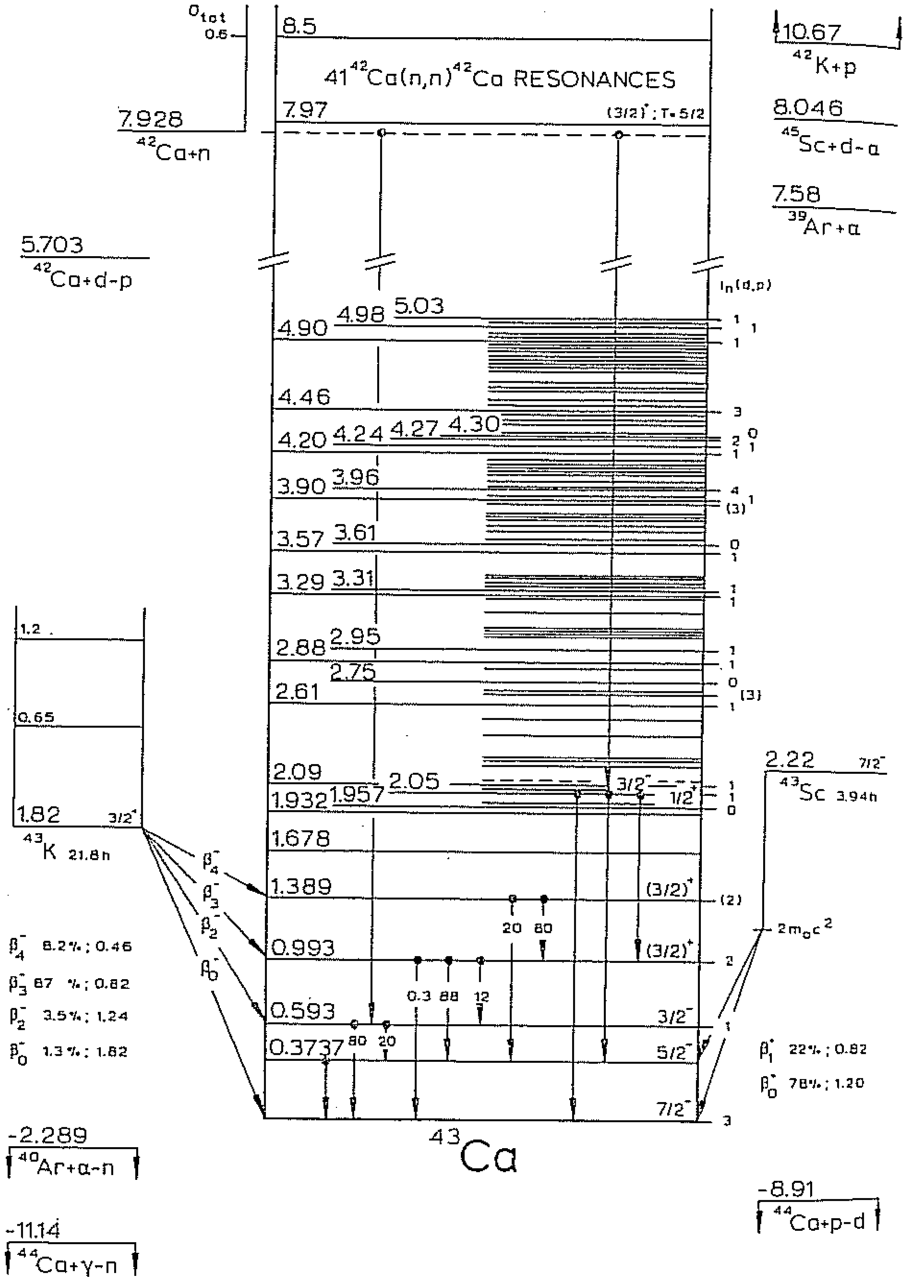


Fig. 43.1. Energy levels of ^{43}Ca.

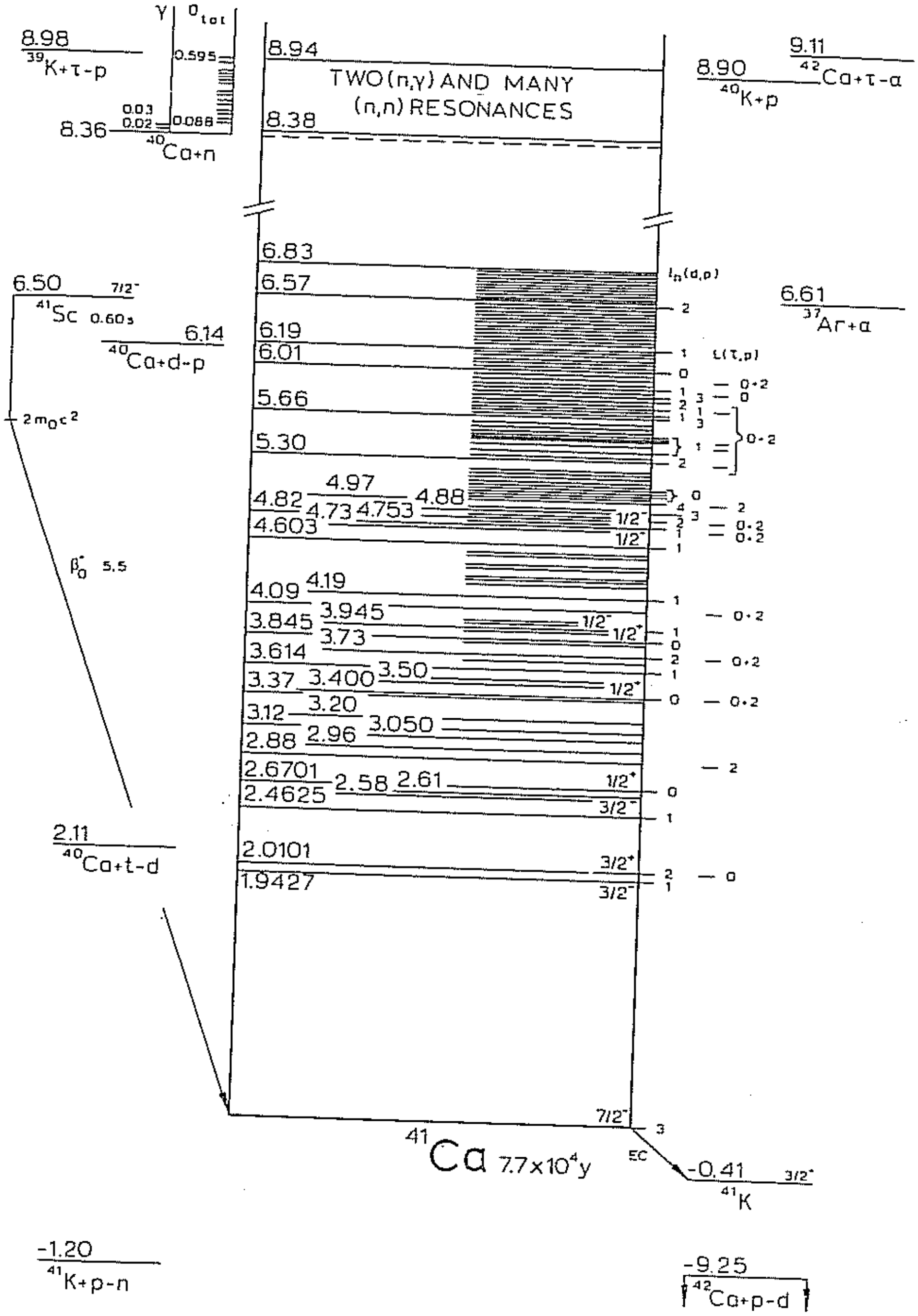


Fig. 41.3. Energy levels of ^{41}Ca; for γ decay see fig. 41.4.

ground state magnetic moments for $^{7}_{3}Li$, $^{39}_{19}K$, $^{45}_{21}Sc$

$\mu = g_j j$

	$^{7}_{3}Li$	$^{39}_{19}K$	$^{45}_{21}Sc$
J^P	$\frac{3}{2}^-$	$\frac{3}{2}^+$	$\frac{7}{2}^-$
unpaired	p	p	p
	$p_{3/2}$	$d_{3/2}$	$f_{7/2}$
$\ell =$	1	2	3

$$g_j = \overset{\nearrow 5.5856}{g_s} \frac{j(j+1) - \ell(\ell+1) + s(s+1)}{2j(j+1)} + \underset{\swarrow 1}{g_\ell} \frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)}$$

for $^{39}_{19}K$ which has $j = \ell - 1/2$

$$g_j = g_s \left(\frac{\frac{3}{2}(\frac{5}{2}) - 6 + 3/4}{3(5/2)} \right) + g_\ell \left(\frac{\frac{3}{2}(\frac{5}{2}) + 6 - 3/4}{3(5/2)} \right)$$

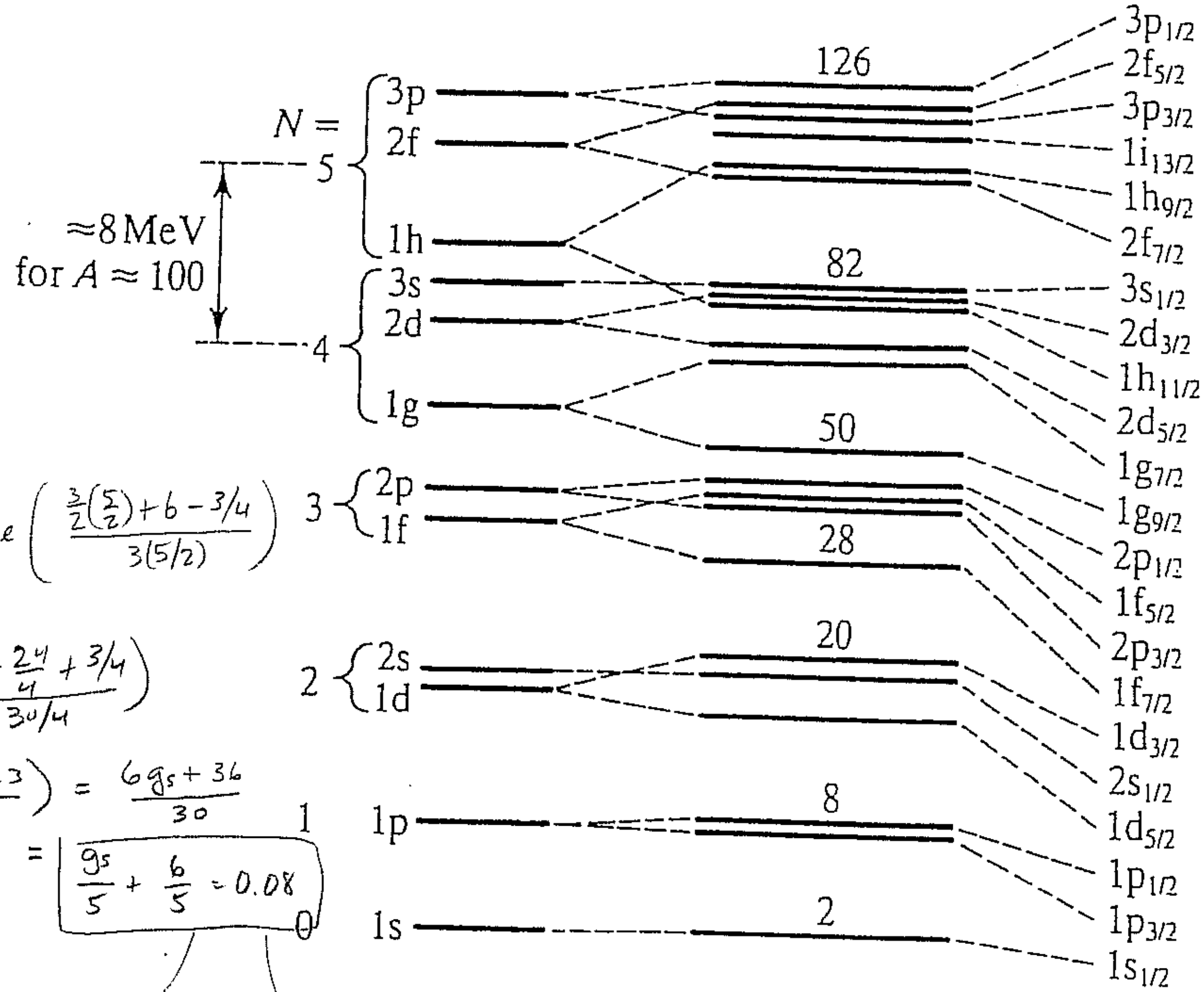
$$= g_s \frac{\frac{15}{4} - \frac{24}{4} + \frac{3}{4}}{30/4} + g_\ell \left(\frac{\frac{15}{4} + \frac{24}{4} + 3/4}{30/4} \right)$$

$$= g_s \left(\frac{15 - 24 + 3}{30} \right) + \left(\frac{15 + 24 - 3}{30} \right) = \frac{6 g_s + 36}{30}$$

$$= \boxed{\frac{g_s}{5} + \frac{6}{5} = 0.08}$$

measure 0.391

$\Rightarrow$ $^{7}_{3}Li$ $\quad$ $^{39}_{15}K$ $\quad$ $^{45}_{21}Sc$



~~for a nucleus with only one unpaired~~

More generally, this expression simplifies to

$$g_j = \left(g_\ell \pm \frac{g_s - g_\ell}{2\ell+1} \right) \quad \begin{matrix} + : j = \ell + 1/2 \\ - : j = \ell - 1/2 \end{matrix}$$

for $j = \ell + 1/2 \Rightarrow g_j = g_\ell\, \ell + \frac{g_s}{2}$

this simplifies the other two

$$g_j\left({}^{7}_{3}\mathrm{Li}\right) = g_\ell + \frac{g_s}{2} = 3.79 \qquad \text{(measured } 3.2564\text{)}$$

$$g_j\left({}^{45}_{21}\mathrm{Sc}\right) = 3g_\ell + \frac{g_s}{2} = 5.79 \qquad \text{(measure } 4.756\text{)}$$

ıtes predicted by the shell model. the model, e.g.

+)

·)

1d)^3, parity even (+)

ıultiples of the fundamental spin to be the spin of both proton and en together with magnetic dipole by optical hyperfine spectroscopy riments using both liquids and ns of atoms and molecules. The utrons to pair off suggests, quite that *even-N–even-Z nuclei have* lly, it also follows in a model- an odd-A nucleus is equal to the $j = l + s$ of the odd nucleon, all le to zero resultant. The spins of ınge abruptly at the magic num- ld-proton nuclei in Table 7.1. a nucleon number just below a le excited states with a spin value of the ground state, e.g. $I_e = \frac{9}{2}$ eason for isomerism (Sect. 9.2.2)

ɔdd-mass nuclei which are well lel can be predicted by a simple ı of the Landé g-factor in atomic ɔy the equation

$$_I\mu_N I \qquad (7.10)$$

ıtum number, μ_N is the nuclear ;yromagnetic ratio given by

$$- g_s)\frac{l(l+1)-\frac{3}{4}}{j(j+1)}\Bigg] \qquad (7.11)$$

ɜ the orbital and intrinsic spin ucleon concerned (see Appendix ɔtal angular momentum quantum :he nuclear spin quantum number

ı knowledge of I, the l-value and ıte can be predicted. In general, ıuantized, change when spin val- he same periodicity, illustrated in

TABLE 7.1 Properties of selected odd-proton nuclei

l_j is the state of motion of the odd proton. For the deformed nuclei l is not a good quantum number. The symbol I^π gives ground-state spin and parity.

Z	A	*Atom*	l_j	I^π	μ_I/μ_N	Q_I/b	*Predicted configuration of protons* (Fig. 7.4)				
							$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$		
1	1	H	$s_{1/2}$	$\frac{1}{2}^+$	2·793	—	1				
3	7	Li	$p_{3/2}$	$\frac{3}{2}^-$	3·256	−0·04	2	1			
7	15	N	$p_{1/2}$	$\frac{1}{2}^-$	−0·283	—	2	4	1		
8											
							$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$		
9	19	F	$s_{1/2}$	$\frac{1}{2}^+$	2·629	—	1				
11	23	Na	$(d_{5/2})^3$	$\frac{3}{2}^+$	2·218	0·14	3				
19	39	K	$d_{3/2}$	$\frac{3}{2}^+$	0·391	0·055	6	2	3		
20											
							$1f_{7/2}$				
21	45	Sc	$f_{7/2}$	$\frac{7}{2}^-$	4·756	−0·22	1				
25	55	Mn	$(f_{7/2})^3$	$\frac{5}{2}^-$	3·444	0·4	5				
27	59	Co	$f_{7/2}$	$\frac{7}{2}^-$	4·62	0·4	7				
28											
							$2p_{3/2}$	$1f_{5/2}$	$2p_{1/2}$	$1g_{9/2}$	
29	63	Cu	$p_{3/2}$	$\frac{3}{2}^-$	2·223	−0·18	1				
31	69	Ga	$p_{3/2}$	$\frac{3}{2}^-$	2·016	0·19	3				
35	79	Br	$p_{3/2}$	$\frac{3}{2}^-$	2·106	0·31	4	3			
47	107	Ag	$p_{1/2}$	$\frac{1}{2}^-$	−0·114	—	4	6	1		
49	113	In	$g_{9/2}$	$\frac{9}{2}^+$	5·523	0·82	4	6	2	9	
50											
							$1g_{7/2}$	$2d_{5/2}$	$1h_{11/2}$	$2d_{3/2}$	$3s_{1/2}$
51	121	Sb	$d_{5/2}$	$\frac{5}{2}^+$	3·359	−0·29	1				
51	123	Sb	$g_{7/2}$	$\frac{7}{2}^+$	2·547	−0·37	1				
63	151	Eu	$d_{5/2}$	$\frac{5}{2}^+$	3·464	1·1	8	5			
67	165	Ho	—	$\frac{7}{2}^-$	4·12	3·0					
71	175	Lu	—	$\frac{7}{2}^+$	2·23	5·6	Deformed nuclei				
73	181	Ta	—	$\frac{7}{2}^+$	2·36	4·2					
75	185	Re	$d_{5/2}$	$\frac{5}{2}^+$	3·172	2·7	8	6	11		
79	197	Au	$d_{3/2}$	$\frac{3}{2}^+$	0·145	0·58	8	6	12	3	
81	203	Tl	$s_{1/2}$	$\frac{1}{2}^+$	1·612	—	8	6	12	4	1
82											
							$2f_{7/2}$	$1h_{9/2}$	$3p_{3/2}$	$2f_{5/2}$	
83	209	Bi	$h_{9/2}$	$\frac{9}{2}^-$	4·080	−0·35	1				

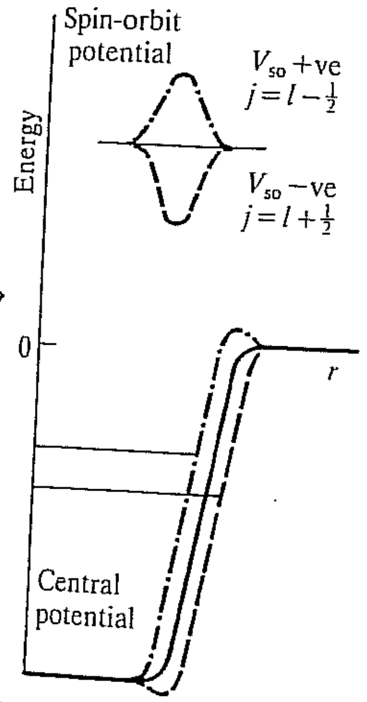
Notes: [1] For ^{19}F, ^{79}Br, ^{121}Sb, ^{185}Re and ^{209}Bi deviations from the level order of Fig. 7.4 are seen.

For ^{23}Na and ^{55}Mn the observed spin must be explained by coupling of equivalent nucleons or by deformation (Ch. 8).

4.11 Taking the nuclear density in the shell model to have the radial dependence given by the (Woods–Saxon) expression

$$\rho = \frac{\rho_0}{1 + \exp(r - R)/a}$$

obtain an expression for the spin–orbit potential V_{so}. Using the results of example 4.5, verify qualitatively the conclusions illustrated in figure 4.4.



we wrote $W(r)$ in class

$$V_{SO}(r) = V_{LS}(r)\, \vec{L} \cdot \vec{S}$$

For constant density expect that the spin-orbit effects cancel (so in the interior of the nucleus) [see Frauenfelder & Henley § 17.3

$$\langle \vec{L} \cdot \vec{S} \rangle = \frac{1}{2}\left[j(j+1) - \ell(\ell+1) + s(s+1) \right] \hbar^2$$

So expect that the effect is important only where the density is non-uniform, eg near the surface, (where the direction of the gradient of the density defines a direction in space, relative to which the orbital angular momentum of a single nucleon can be defined)

$$V_{LS}(r)\, \vec{L} \cdot \vec{S} \sim \frac{1}{r}\left(\frac{\partial \rho}{\partial r}\right) \vec{L} \cdot \vec{S} = -\frac{1}{r} \frac{\rho_0}{\left[1 + e^{(r-R)/a}\right]^2} \frac{e^{(r-R)/a}}{a} \vec{L} \cdot \vec{S}$$

$$\langle \vec{L} \cdot \vec{S} \rangle = \frac{1}{2} \ell \hbar^2 \quad \text{for } j = \ell + 1/2$$

$$-\frac{1}{2} \ell(\ell+1) \hbar^2 \quad \text{for } j = \ell - 1/2$$

So potential (including spin-orbit effect) is as shown above; deeper (more attractive) for the case $j = \ell + 1/2$.