PHY357 Assignment 1, Due Feb 2, 2006

The first three questions have to do with decays. As discussed in class, the same decay law applies both to the decays of fundamental particles and to the decays of radioactive nuclei. However, in a discussion of the decay of radioactive nuclei, there can be interesting equilibrium effects, as discussed in questions 2 and 3. The remaining problems are on simple relativsitic kinematics (problem 4) and and on drawing Feynman diagrams (problems 5 and 6).

1. Assume that some particle can decay into two different final states. There are characteristic decay rates asscociated with each of the decays, λ_1 and λ_2.

   (a) Derive an expression for the lifetime τ of this particle, in terms of the two decay constants.

   (b) A particle that can decay into N different final states is said to have N *decay modes*. The *Branching Ratio* or *Branching Fraction* for the i^{th} decay mode is defined as the ratio of the number of decays into the i^{th} final state to the total number of decays. For the scenario described above write an expression for the branching ratio into the first of the two final states.

2. Consider the situation where some target material is exposed to a source of neutrons (which for instance can be produced from a nuclear reactor). Certain materials can absorb a neutron and become *activated*, that is, subject to subsequent radioactive decay. For instance:

$$^{23}_{11}\mathrm{Na} + n \rightarrow^{24}_{11}\mathrm{Na} \rightarrow^{23}_{12}\mathrm{Mg} + e^- + \bar{\nu}_e$$

   In a normal sample (*e.g.* many atoms) the depletion of the target source is negligible (there are always enough $^{23}_{11}$Na atoms for the neutrons to interact with) so the rate of production of $^{24}_{11}$Na can be treated as constant (assuming the reactor is operating at constant power so that the neutron rate onto the $^{23}_{11}$Na is constant).

   (a) As soon as the $^{24}_{11}$Na is produced it is subject to radioactive β-decay with a decay rate λ. Assuming a constant production rate p of $^{24}_{11}$Na, write down and solve the differential equation for the number of $^{24}_{11}$Na atoms at time t, assuming that there are no $^{24}_{11}$Na atoms present at $t = 0$. What happens as t become very large ?

   (b) A sample of gold (Au) is exposed to a neutron beam of constant intensity such that 10^{10} neutrons/s are absorbed in the reaction

$$^{179}_{79}\mathrm{Au} + n \rightarrow^{198}_{79}\mathrm{Au} + \gamma$$

   $^{198}_{79}$Au undergoes β-decay to $^{198}_{80}$Au with a mean lifetime of $\tau = 3.89$ days. How many atoms of $^{198}_{79}$Au will be present after six days of irradiation ? What is the equililibrium number of $^{198}_{79}$Au atoms ?

   Note that the technique of radio-carbon dating is based on a similar principle. It is utilizes the radiactive decay of $^{14}_{6}$C which is produced by the interaction of cosmic rays with our atmosphere and which decays with a half-life of 5730 years. There is thus some equilibrium concentration of $^{14}_{6}$C in atmospheric CO_2 which is the source of biological carbon. Any living organism will carry this equilibrium concentration of $^{14}_{6}C$ until death, afterwhich the $^{14}_{6}$C is no longer replenished and the fraction of $^{14}_{6}$C relative to $^{12}_{6}$C decreases with the characteristic liftime. This fraction can me measured either via decays (which is not so efficient) or by mass spectroscopy.

3. Now consider a similar case in which we again produce a radioactive state which subsequently decays with some characteristic lifetime. However, in this case let us assume this state is produced in the radioactive decay of some parent nucleus. Assume the decay rate of the parent is λ_1 and that of the daughter is λ_2. The situation is similar to that in problem 2, except now the production rate of the daughter state is not constant. For this scenario:

   (a) Write down and solve the differential equation for the number of daughter nuclei as a function of time. Describe the situation in the limit where:

      i. $\lambda_1 >> \lambda_2$

      ii. $\lambda_2 >> \lambda_1$

      iii. $\lambda_1 \sim \lambda_2$

   (b) Consider the decay sequence

$$^{210}_{83}\mathrm{Bi} \rightarrow ^{210}_{84}\mathrm{Po} + e^- + \bar{\nu}_e$$

$$^{210}_{84}\mathrm{Po} \rightarrow ^{206}_{82}\mathrm{Pb} + \alpha$$

   If the lifetimes associated with these two decays are 7.2 days and 200 days respectively, at what point in time is the α-particle emission maximal ?

4. A pion traveling at speed v decays into a muon and a muon anti-neutrino, $\pi^- \rightarrow \mu^-\bar{\nu}_\mu$. If the neutrino emerges at 90° to the original pion direction, show that the muon comes off at an angle given by $\tan\theta = (1 - \mathrm{m}_\mu^2/\mathrm{m}_\pi^2)/(2\beta\gamma^2)$.

5. As a simple introduction to Feynman diagrams, consider a theory in which there are only three particles, A, B and C, and there is one fundamental interaction vertex that couples the three particles together. Assume that these particles are their own anti-particles (then we don't need to worry about the arrows on the diagrams). Assume, as well, that the mass, M_A, of particle A exceeds the sum of the masses of particles B and C ($M_A > M_B + M_C$). Then the lowest order process in the theory is the decay $A \rightarrow B + C$.

   (a) Draw the leading order and all of the next-to-leading order diagrams contributing to this process.

   (b) Draw the leading-order diagrams for the scattering process $AB \rightarrow AB$

   (c) Draw the leading-order diagrams for the scattering process $AA \rightarrow BB$

6. Using the interaction vertices that were shown in class, write down the leading order diagrams for the following scattering processes.

   (a) $e^-\gamma \rightarrow e^-\gamma$ (Compton scattering)

   (b) $e^+e^- \rightarrow \tau^+\tau^-$

   (c) $e^+e^- \rightarrow W^+W^-$

   (d) $e^+e^- \rightarrow Z^0Z^0$