PHY357 Assignment 2, Due Feb 16, 2006

This assignment focusses on simple application of the conservation laws we have been discussing, as well as on issues related to addition of angular momenta (including the isospin formalism).

I have attached a sheet showing the Clebsch-Gordan coefficients as well as one with various particle properties. The latter is so that you can determine the quark content of the various states in question 1, which you will need, in order to distinguish between weak decays and strong interactions (for allowed processes).

Note that both particle properties related material (such as Clebsch-Gordan coefficients can be found online at the website of the Particle Data Group (http://pdg.lbl.gov). The Clebsch-Gordan coefficients can be found by clicking on Reviews, Tables, Plots, and then on Mathematical Tools. Or you can go directly to http://pdg.lbl.gov/2005/reviews/clebrpp.pdf

1. For the following reactions (decays and scattering processes) state which are allowed and which are forbidden. For those that are allowed, state which interaction dowminates. For those that are forbidden, explain why. For allowed *decays* draw me one lowest-order Feynman diagram. Note that I am writing these expressions in the convention which omits "+" signs between the particles. So $A + B \to C + D$ becomes $AB \to CD$.

   (a) $\Sigma^- \to n\ e^-\ \bar{\nu}_e$
   (b) $\pi^+\ p \to \Lambda K^0$
   (c) $\Sigma^0 \to \Lambda\gamma$
   (d) $e^- p \to \nu_e\ \pi^0$
   (e) $p\ p \to \Sigma^+\ n\ K^0\ \pi^+\ \pi^0$
   (f) $p \to e^+\gamma$
   (g) $p\ \bar{p} \to \pi^+\pi^0$
   (h) $\mu^- \to e^-\bar{\nu}_e$
   (i) $\Xi^- \to \Lambda\pi^-$
   (j) $\Omega^- \to \Lambda K^-$

2. Discuss the expected ratios of the cross-sections for the three processes below, for the cases where the I=3/2 amplitude dominates, the I=1/2 amplitude dominates and the two amplitudes are approximately equal.

   (a) $\pi^- + p \to K^0 + \Sigma^0$
   (b) $\pi^- + p \to K^+ + \Sigma^-$
   (c) $\pi^+ + p \to K^+ + \Sigma^+$

3. Suppose that you have a system consisting of a spin 1 particle and a spin 1/2 particle in a state with orbital angular momentum $\ell = 1$. If the total spin of the system is 5/2 and the z-component is 1/2, what is the probability that a measurement of the z-component of the spin 1/2 particle yields m = -1/2 ?

4. The deuteron is the simplest non-trivial nucleus, containing one proton and one neutron. Consider the general case of a system of two nucleons. Show that there are four possible states, an isotriplet ($I = 1$, which means 2(1)+1 = 3 states) and an iso-singlet ($I = 0$) [Remember this is just like adding spin 1/2 to spin 1/2, which we did in class, but here we are working in isospin space]. What are the I, I_3 assignments for the four states ? The deuteron is the ONLY nucleon-nucleon bound state. What does that tell you about which of these states it should be identified with ?

   It is assumed that this is attributable to an interaction potential between the two nucleons that is of the form $\vec{I}^{(1)} \cdot \vec{I}^{(2)}$ (where the upper indices indicates the particle, not some axis in isospin space). Evaluate this for the iso-triplet and iso-singlet cases to show that the results are of opposite sign (so attractive in one case and repulsive in the other).

5. What is the threshold energy for the protons beam(s) for the scattering process $pp \rightarrow p\Sigma^+ K^0$?

   (a) in the centre-of-mass frame

   (b) in the lab frame in which one of the protons is at rest

BARYONS (Spin $\frac{1}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
N p	uud	+1	938.280	∞	—
N n	udd	0	939.573	900	$pe\bar{\nu}_e$
Λ	uds	0	1115.6	2.63×10^{-10}	$p\pi^-$, $n\pi^0$
Σ^+	uus	+1	1189.4	0.80×10^{-10}	$p\pi^0$, $n\pi^+$
Σ^0	uds	0	1192.5	6×10^{-20}	$\Lambda\gamma$
Σ^-	dds	−1	1197.3	1.48×10^{-10}	$n\pi^-$
Ξ^0	uss	0	1314.9	2.90×10^{-10}	$\Lambda\pi^0$
Ξ^-	dss	−1	1321.3	1.64×10^{-10}	$\Lambda\pi^-$
Λ_c^+	udc	+1	2281	2×10^{-13}	not established

BARYONS (Spin $\frac{3}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
Δ	uuu, uud, udd, ddd	+2, +1, 0, −1	1232	0.6×10^{-23}	$N\pi$
Σ^*	uus, uds, dds	+1, 0, −1	1385	2×10^{-23}	$\Lambda\pi$, $\Sigma\pi$
Ξ^*	uss, dss	0, −1	1533	7×10^{-23}	$\Xi\pi$
Ω^-	sss	−1	1672	0.82×10^{-10}	ΛK^-, $\Xi^0\pi^-$, $\Xi^-\pi^0$

PSEUDOSCALAR MESONS (Spin 0)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
$\pi^\pm$	$u\bar{d}$, $d\bar{u}$	+1, −1	139.569	2.60×10^{-8}	$\mu\nu_\mu$
π^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	0	134.964	8.7×10^{-17}	$\gamma\gamma$
$K^\pm$	$u\bar{s}$, $s\bar{u}$	+1, −1	493.67	1.24×10^{-8}	$\mu\nu_\mu$, $\pi^\pm\pi^0$, $\pi^\pm\pi^\pm\pi^\mp$
K^0, $\bar{K}^0$	$d\bar{s}$, $s\bar{d}$	0, 0	497.72	K_S^0 0.892×10^{-10}; K_L^0 5.18×10^{-8}	$\pi^+\pi^-$, $\pi^0\pi^0$; $\pi e\nu_e$, $\pi\mu\nu_\mu$, $\pi\pi\pi$
η	$(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$	0	548.8	7×10^{-19}	$\gamma\gamma$, $\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0$
η'	$(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$	0	957.6	3×10^{-21}	$\eta\pi\pi$, $\rho^0\gamma$
$D^\pm$	$c\bar{d}$, $d\bar{c}$	+1, −1	1869	9×10^{-13}	$K\pi\pi$
D^0, $\bar{D}^0$	$c\bar{u}$, $u\bar{c}$	0, 0	1865	4×10^{-13}	$K\pi\pi$
$F^\pm$ (now $D_s^\pm$)	$c\bar{s}$, $s\bar{c}$	+1, −1	1971	3×10^{-13}	not established
$B^\pm$	$u\bar{b}$, $b\bar{u}$	+1, −1	5271	14×10^{-13}	$D+?$
B^0, $\bar{B}^0$	$d\bar{b}$, $b\bar{d}$	0, 0	5275		
η_c	$c\bar{c}$	0	2981	6×10^{-23}	$KK\pi$, $\eta\pi\pi$, $\eta'\pi\pi$

VECTOR MESONS (Spin 1)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
ρ	$u\bar{d}$, $d\bar{u}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$	+1, −1, 0	770	0.4×10^{-23}	$\pi\pi$
K^*	$u\bar{s}$, $s\bar{u}$, $d\bar{s}$, $s\bar{d}$	+1, −1, 0, 0	892	1×10^{-23}	$K\pi$
ω	$(u\bar{u}+d\bar{d})/\sqrt{2}$	0	783	7×10^{-23}	$\pi^+\pi^-\pi^0$, $\pi^0\gamma$
ϕ	$s\bar{s}$	0	1020	20×10^{-23}	K^+K^-, $K^0\bar{K}^0$
J/ψ	$c\bar{c}$	0	3097	1×10^{-20}	e^+e^-, $\mu^+\mu^-$, 5π, 7π
D^*	$c\bar{d}$, $d\bar{c}$, $c\bar{u}$, $u\bar{c}$	+1, −1, 0, 0	2010	$>1\times 10^{-22}$	$D\pi$, $D\gamma$
Υ	$b\bar{b}$	0	9460	2×10^{-20}	$\tau^+\tau^-$, $\mu^+\mu^-$, e^+e^-

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J J ...	
	M M ...	
m_1 m_2		
m_1 m_2	Coefficients	
. .		

1/2×1/2

m_1	m_2	M	$J=1$	$J=0$
+1/2	+1/2	+1	1	
+1/2	−1/2	0	1/2	1/2
−1/2	+1/2	0	1/2	−1/2
−1/2	−1/2	−1	1	

1×1/2

m_1	m_2	M	$J=3/2$	$J=1/2$
+1	+1/2	+3/2	1	
+1	−1/2	+1/2	1/3	2/3
0	+1/2	+1/2	2/3	−1/3
0	−1/2	−1/2	2/3	1/3
−1	+1/2	−1/2	1/3	−2/3
−1	−1/2	−3/2	1	

2×1/2

m_1	m_2	M	$J=5/2$	$J=3/2$
+2	+1/2	+5/2	1	
+2	−1/2	+3/2	1/5	4/5
+1	+1/2	+3/2	4/5	−1/5
+1	−1/2	+1/2	2/5	3/5
0	+1/2	+1/2	3/5	−2/5
0	−1/2	−1/2	3/5	2/5
−1	+1/2	−1/2	2/5	−3/5
−1	−1/2	−3/2	4/5	1/5
−2	+1/2	−3/2	1/5	−4/5
−2	−1/2	−5/2	1	

3/2×1/2

m_1	m_2	M	$J=2$	$J=1$
+3/2	+1/2	+2	1	
+3/2	−1/2	+1	1/4	3/4
+1/2	+1/2	+1	3/4	−1/4
+1/2	−1/2	0	1/2	1/2
−1/2	+1/2	0	1/2	−1/2
−1/2	−1/2	−1	3/4	1/4
−3/2	+1/2	−1	1/4	−3/4
−3/2	−1/2	−2	1	

2×1

m_1	m_2	M	$J=3$	$J=2$	$J=1$
+2	+1	+3	1		
+2	0	+2	1/3	2/3	
+1	+1	+2	2/3	−1/3	
+2	−1	+1	1/15	1/3	3/5
+1	0	+1	8/15	1/6	−3/10
0	+1	+1	2/5	−1/2	1/10
+1	−1	0	1/5	1/2	3/10
0	0	0	3/5	0	−2/5
−1	+1	0	1/5	−1/2	3/10
0	−1	−1	2/5	1/2	1/10
−1	0	−1	8/15	−1/6	−3/10
−2	+1	−1	1/15	−1/3	3/5
−1	−1	−2	2/3	1/3	
−2	0	−2	1/3	−2/3	
−2	−1	−3	1		

1×1

m_1	m_2	M	$J=2$	$J=1$	$J=0$
+1	+1	+2	1		
+1	0	+1	1/2	1/2	
0	+1	+1	1/2	−1/2	
+1	−1	0	1/6	1/2	1/3
0	0	0	2/3	0	−1/3
−1	+1	0	1/6	−1/2	1/3
0	−1	−1	1/2	1/2	
−1	0	−1	1/2	−1/2	
−1	−1	−2	1		

3/2×1

m_1	m_2	M	$J=5/2$	$J=3/2$	$J=1/2$
+3/2	+1	+5/2	1		
+3/2	0	+3/2	2/5	3/5	
+1/2	+1	+3/2	3/5	−2/5	
+3/2	−1	+1/2	1/10	2/5	1/2
+1/2	0	+1/2	3/5	1/15	−1/3
−1/2	+1	+1/2	3/10	−8/15	1/6
+1/2	−1	−1/2	3/10	8/15	1/6
−1/2	0	−1/2	3/5	−1/15	−1/3
−3/2	+1	−1/2	1/10	−2/5	1/2
−1/2	−1	−3/2	3/5	2/5	
−3/2	0	−3/2	2/5	−3/5	
−3/2	−1	−5/2	1		

Spherical harmonics:

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\,Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2×3/2

m_1	m_2	M	$J=3$	$J=2$	$J=1$	$J=0$
+3/2	+3/2	+3	1			
+3/2	+1/2	+2	1/2	1/2		
+1/2	+3/2	+2	1/2	−1/2		
+3/2	−1/2	+1	1/5	1/2	3/10	
+1/2	+1/2	+1	3/5	0	−2/5	
−1/2	+3/2	+1	1/5	−1/2	3/10	
+3/2	−3/2	0	1/20	1/4	9/20	1/4
+1/2	−1/2	0	9/20	1/4	−1/20	−1/4
−1/2	+1/2	0	9/20	−1/4	−1/20	1/4
−3/2	+3/2	0	1/20	−1/4	9/20	−1/4
+1/2	−3/2	−1	1/5	1/2	3/10	
−1/2	−1/2	−1	3/5	0	−2/5	
−3/2	+1/2	−1	1/5	−1/2	3/10	
−1/2	−3/2	−2	1/2	1/2		
−3/2	−1/2	−2	1/2	−1/2		
−3/2	−3/2	−3	1			

2×3/2

m_1	m_2	M	$J=7/2$	$J=5/2$	$J=3/2$	$J=1/2$
+2	+3/2	+7/2	1			
+2	+1/2	+5/2	3/7	4/7		
+1	+3/2	+5/2	4/7	−3/7		
+2	−1/2	+3/2	1/7	16/35	2/5	
+1	+1/2	+3/2	4/7	1/35	−2/5	
0	+3/2	+3/2	2/7	−18/35	1/5	
+2	−3/2	+1/2	1/35	6/35	2/5	2/5
+1	−1/2	+1/2	12/35	5/14	0	−3/10
0	+1/2	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	+1/2	4/35	−27/70	2/5	−1/10
+1	−3/2	−1/2	4/35	27/70	2/5	1/10
0	−1/2	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	−1/2	12/35	−5/14	0	3/10
−2	+3/2	−1/2	1/35	−6/35	2/5	−2/5
0	−3/2	−3/2	2/7	18/35	1/5	
−1	−1/2	−3/2	4/7	−1/35	−2/5	
−2	+1/2	−3/2	1/7	−16/35	2/5	
−1	−3/2	−5/2	4/7	3/7		
−2	−1/2	−5/2	3/7	−4/7		
−2	−3/2	−7/2	1			

2×2

m_1	m_2	M	$J=4$	$J=3$	$J=2$	$J=1$	$J=0$
+2	+2	+4	1				
+2	+1	+3	1/2	1/2			
+1	+2	+3	1/2	−1/2			
+2	0	+2	3/14	1/2	2/7		
+1	+1	+2	4/7	0	−3/7		
0	+2	+2	3/14	−1/2	2/7		
+2	−1	+1	1/14	3/10	3/7	1/5	
+1	0	+1	3/7	1/5	−1/14	−3/10	
0	+1	+1	3/7	−1/5	−1/14	3/10	
−1	+2	+1	1/14	−3/10	3/7	−1/5	
+2	−2	0	1/70	1/10	2/7	2/5	1/5
+1	−1	0	8/35	2/5	1/14	−1/10	−1/5
0	0	0	18/35	0	−2/7	0	1/5
−1	+1	0	8/35	−2/5	1/14	1/10	−1/5
−2	+2	0	1/70	−1/10	2/7	−2/5	1/5
+1	−2	−1	1/14	3/10	3/7	1/5	
0	−1	−1	3/7	1/5	−1/14	−3/10	
−1	0	−1	3/7	−1/5	−1/14	3/10	
−2	+1	−1	1/14	−3/10	3/7	−1/5	
0	−2	−2	3/14	1/2	2/7		
−1	−1	−2	4/7	0	−3/7		
−2	0	−2	3/14	−1/2	2/7		
−1	−2	−3	1/2	1/2			
−2	−1	−3	1/2	−1/2			
−2	−2	−4	1				

d functions:

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2 \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1) \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.