PHY357 Assignment 3, Due March 2, 2006

As was the case for the last assignment, some of the problems here require that you get look up some information in the Particle Data Group's Review of particle properties, which is available online (http://pdg.lbl.gov). I have included some of the information in the assignment, but the rest of it you will need try to to find yourself. The questions are pretty straighforward calculationally. Much of the assignment is more about finding and using information relevant to experimental particle physics.

1. Estimate the mean free path of 100 GeV positively charged kaons (K^+) in liquid hydrogen (a typical target material in the days of bubble chamber experiments). This is mostly an exercise in locating information. What do you need to know to calculate this ? All of that information can be found in the PDG listings online (and presumably elsewhere).

2. Neutrino interactions with matter proceed only via the weak interaction, since they carry no colour or electric charge. In discussing these interactions we differentiate between the "charged-current" cross-section, in which the interaction is due to the charged weak interaction (e.g. it is mediated by the $W^{\pm}$) and the neutral current interection which is due to the neutral weak interaction which is mediated by the Z^0.

   The total ν_μ-nucleon charged-current cross-section, σ_T, scales with the neutrino energy, as shown in Figure 1 which plots σ_T/E_ν as a function of E_ν.

   (a) For electron-neutrinos, ν_e, interacting with neutrons, draw Feynman diagrams for: a) the charged-current interaction; b) the neutral current interaction. Do the same for muon neutrinos ν_μ

   (b) Neutrinos, in interacting with matter, can also undergo elastic scattering from atomic electrons. For electron neutrinos, draw Feynman diagrams for the processes contributing to this (at lowest order). Do the same for muon neutrinos.

   (c) What is the mean free path λ of 100 GeV muon-neutrinos in lead (before they produce a charged lepton in an interaction) ?

   (d) If a beam of 100 GeV muon-neutrinos were incident on a block of lead of thickness T and cross-sectional area 10 m^2, with a uniform flux of 5×10^6 neutrinos/$(cm^2 \cdot s)$ how large would T have to be for there to be 10 interactions / day produced (assuming the flux to be uniform over the full cross-section of the target) ?

3. Figure 2 shows the cross-section for $e^+e^- \rightarrow$ hadrons as a function of energy as well as the ratio R of the cross-section for $e^+e^- \rightarrow$ hadrons relative to the cross-section for $e^+e^- \rightarrow \mu^+\mu^-$. We have discussed the general features of this distribution in class. Now I'd like you to describe it in more detail, including all of the "enhancements" which are due to resonances. I have said that the word resonance is most often used to describe strongly interaction states (for instance the Δ^{++} produced in $\pi^+ p$ scattering). But the word really refers to these any enhancements in a scattering cross-section, due to the production of some intermediate state.

   (a) Referring to the diagram above, describe in detail the distribution of the ratio R, including those aspects that we covered in class (e.g. the value of R away from resonances). What process dominates the scattering at low energies ? What does this allow you to say about the low-energy resonances ? What process dominates at high energies ? Figure 3 is included to give you a more detailed view of the low-energy part of the distribution. You might want to make reference to this when discussing the value of R away from the resonances. In discussing the resonances, though, you can restrict the discussion to those that are shown in Figure 2.

   (b) raw me a sketch of how you expect R to evolve at energies above those shown on the plot. What new processes contribute to $e^+e^- \rightarrow$ hadrons at these energies, at what thresholds ?

4. A beam of protons with momentum of 12 GeV/c is directed onto a liquid hydrogen target. The momenta of the (charged) reaction products are then measured in some detector via their curvature in a magnetic field. In one event there are six charged particles observed. Two of these point back to the interaction vertex and the other four correpond to two pairs of oppositely charged partices originating at vertices a few centimetres from the interaction point, indicating that two neutral particles with relatively long lifetimes were produced in the initial interaction, and they decayed after travelling some distance (of order a few cm, as stated above).

   (a) Make a rough sketch of the reaction (e.g. the tracks....include the neutral particles as dotted lines.

   (b) Discuss which mesons and baryons have lifetimes and decay modes such that they could be responsible for the two observed neutral particle decays.

   (c) if the measured momenta of the decay pairs are

      i. $|\vec{p}_+| = 0.68$ GeV/c, $|\vec{p}_-| = 0.27$ GeV/c, angle$(\vec{p}_+, \vec{p}_-) = 11°$

      ii. $|\vec{p}_+| = 0.25$ GeV/c, $|\vec{p}_-| = 2.16$ GeV/c, angle$(\vec{p}_+, \vec{p}_-) = 16°$,

      assuming that these measurements have relative errors of about 5%, which of the hypotheses from b) is compatible with the data.

   (d) Using these results, and accounting for all relevant conservation laws, produce an explanation of this event. Is there a unique solution ?

5. We will discuss colliders after the break, but we have introduced some of the concepts. We discussed luminosity in class. To recap, the particles in accelerators are accelerated in bunches, not individually. A "bunch-crossing" is when two bunches are made to cross in space (at the interaction point of a detector). This may or may not produce an interaction depending on the luminosity of the collider and the relevant interaction cross-sections.

   We saw in class that the luminosity is proportional to the number of particles in each of the bunches, and to the bunch crossing frequency f. It is inversely proportional to a quantity which is like the cross-sectional area of the beams (smaller size = tighter "focussing" = higher luminosity).

   (a) The LEP Collider was a electron-positron collider that operated at CERN from 1989 until 2001, producing collisions in the energy range from about 88-208 GeV (centre-of-mass). From 1989 until 1995, LEP operated exclusively in the region of the Z^0 resonance at a centre-of-mass energy $E_{CM} = M_{Z^0} = 91.2$ GeV/c^2. The "High-Energy Collider Parameters" in the PDG listings will give you the properties of LEP. Look up the luminosity and the bunch-crossing frequency and calculate the number of Z^0 particles produced in each bunch crossing of the machine. There are other processes that contribute to electron-positron scattering events at that energy, but assume for this question that the cross-section is dominated by Z^0 production. With that assumption what is the probability per bunch-crossing of an interaction ? What is therefore the probability of a single bunch crossing producing two interactions ?

   (b) Now repeat this exercise for the Large Hadron Collider, which is a proton-proton collider that will begin producing collisions of 7 TeV protons in 2007. This collider is currently still under construction and will occupy the same 27 km circumference ring that was used by the LEP collider. Again, using the collider parameters (and other information) provided by the PDG, calculate the average number of proton-proton interactions per bunch crossing. Note from the relevant plot in the PDG that this cross-section is almost entirely inelastic. Each of these interactions can produce charged and neutral particles that will leave energy deposits in the detector. What does this tell you about the experimental environment at the LHC relative to that at the LEP collider ?

   (c) The collisions that will take place at the LHC are not actually collisions of protons but rather collisions between consituents (sometimes called partons) of the protons. A proton can be viewed as a collection of valence quarks, gluons, and "sea" quarks, which are quark anti-quark pairs that pop in and out of existence via coupling to gluons. Figure 4 shows the momentum distribution of the proton constitutents; surprisingly, the gluons carry about 50% of the proton's momentum. The dominant collisions processes at the LHC will therefore involve quark-quark collisions, quark-gluon collisions and gluon-gluon collsions. Assuming that the cross-sections for those three processes are roughly equal, *estimate* the average centre-of-mass energy for a fundamental (parton-level) interaction at the LHC. This is what determines the scale of the physics that will be probed at the LHC. This is relatively straightforward, but not quite as simple as it might seem...try using x_1 and x_2 as the momentum fractions carried by the two interacting partons and writing things in terms of four-vectors where you can use invariants. Ask yourself what the situation is when the two fractions x_1 and x_2 are not equal (as will generally be the case). For the estimate you don't need to do a detailed calculation, but tell me how you might do a better job.

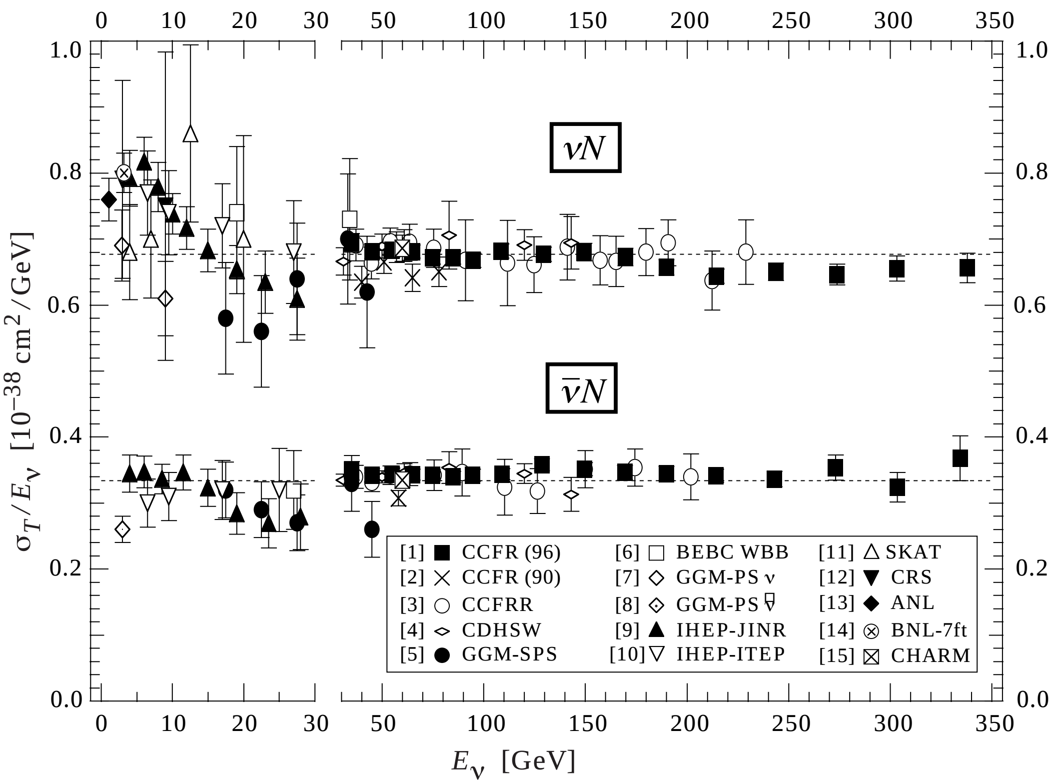

Figure 1: Plot for question 2. Total cross-sections (divided by the neutrino energy E_ν) for charged-current interactions of ν_μ and $\bar{\nu}_\mu$ with nucleons.

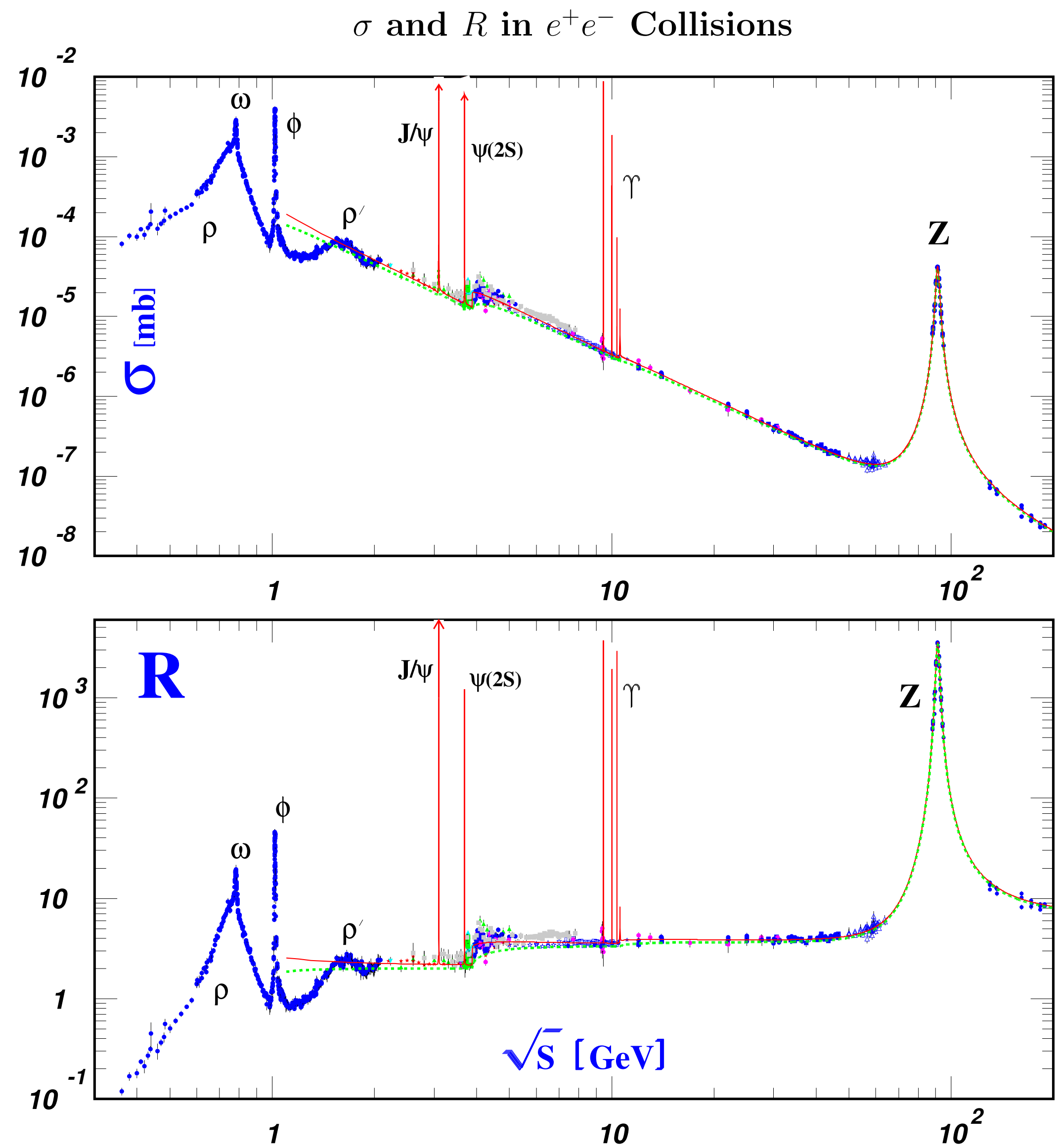

Figure 2: Plots for problem 3. The top plot shows the cross-section for the process $e^+e^- \to$ hadrons while the bottoms shows the ratio R, defined as the ratio of the cross-section into hadrons relative to the cross-section into muon pairs.

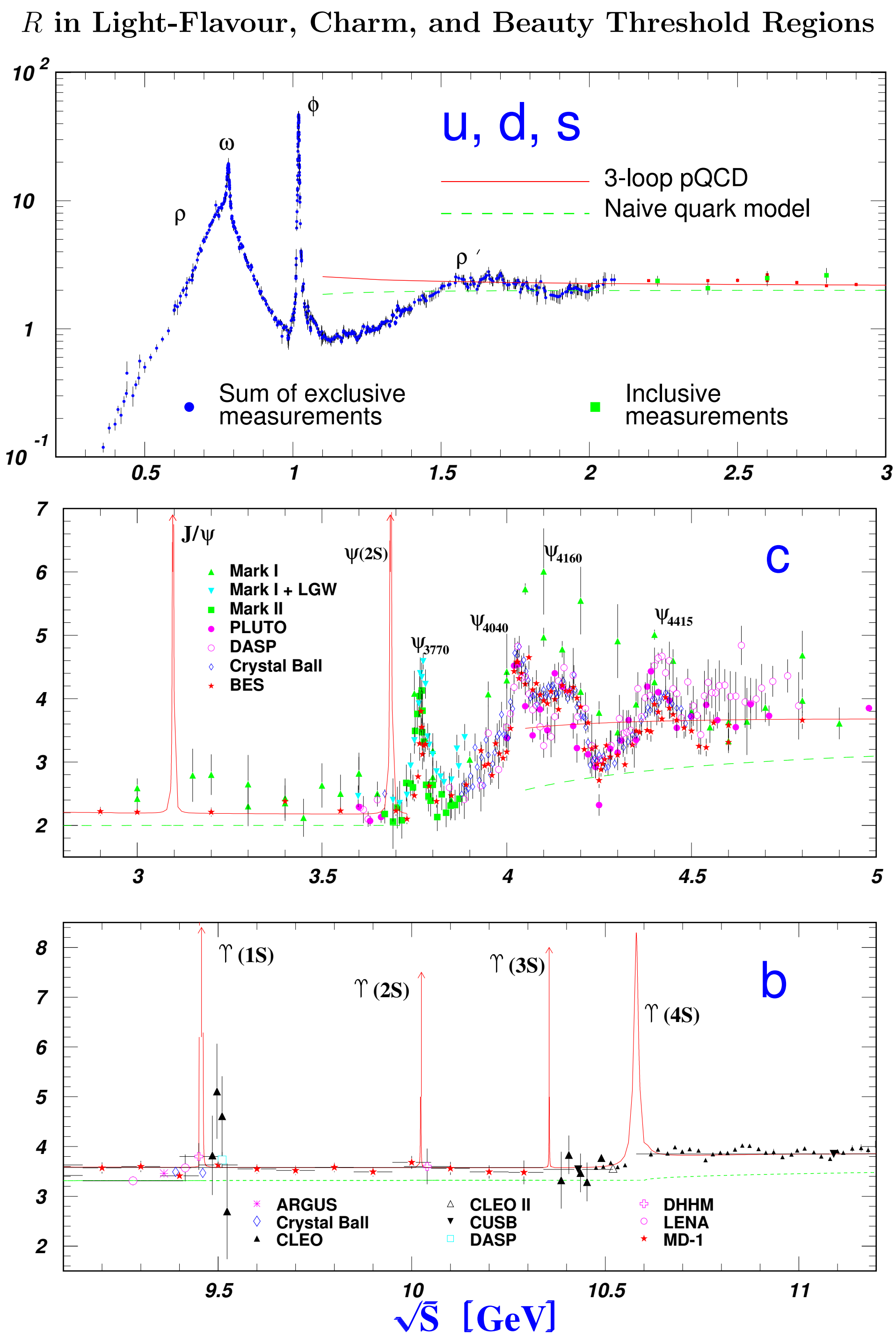

Figure 3: Plots for problem 3. These show the ratio R in three regions of energy covering the range from 0-11 GeV.

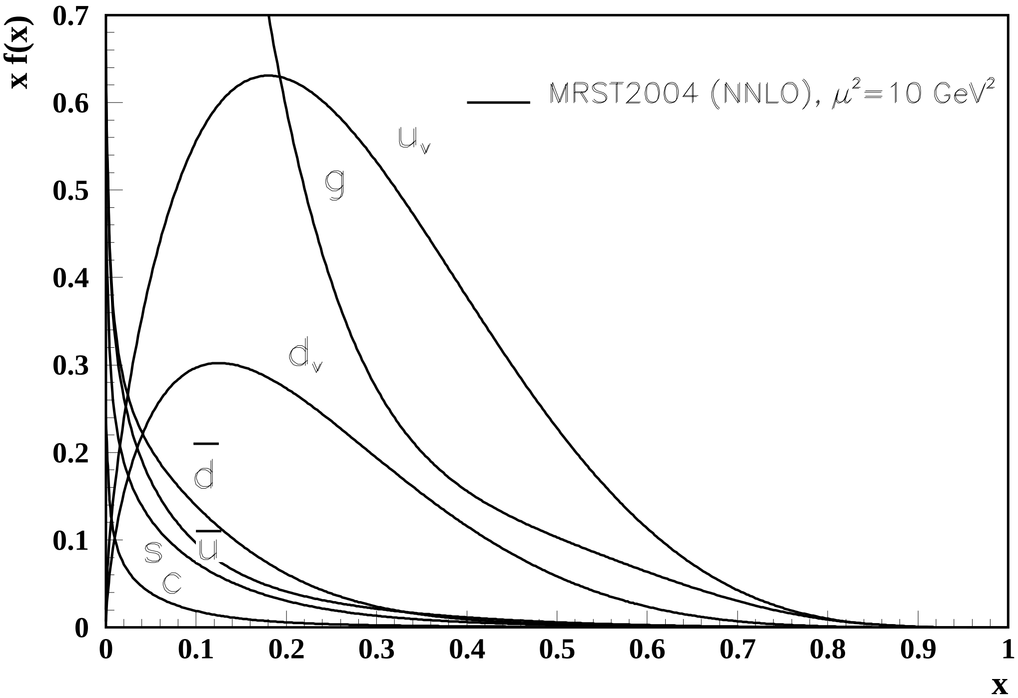

Figure 4: For question 4c. This shows the fraction of the proton energy that is carried by individual constituents. As you see, a large fraction (about 50%) is carried by gluons. The rest is mostly carried by the valence quarks, though some is carried by so-called "sea" quarks.