PHY357 Assignment 4, Due March 23, 2006

This is the final version of Assignment 4, in the sense that it contains all of the problems. I may still add some introductory material to question 1, but I think everything you actually need is there. I will announce it on the website if I add anything.

1. The Sudbury Neutrino Observatory (SNO) is a neutrino detector located about 2km underground in Inco's Creighton Mine in Sudbury. For part of this question you might want to look at the home page for the experiment (http://www.sno.phy.queensu.ca).

   SNO was designed to detect neutrinos produced in the nuclear processes of the sun. You can find lots of documentation about solar neutrinos at http://www.sns.ias.edu/~jnb/ if you'd like more of an explanation than I provide below.

   The nuclear processes that fuel the burning of the sun are believed to be well understood. They are testable in many ways and almost all experiments confirm the validity of the model. One of the things that the model can predict is the rate of neutrino production in the sun. From that one can predict the flux of these solar neutrinos at the earth, and if one knows the cross-sections for interactions in some detector material, one can attempt to measure this flux and thus confirm the predictions of the so-called Standard Solar Model (SSM). These experiments started in the 1960s. Experiment after experiment measured a flux that was between 1/3 and 1/2 of the predicted flux. Since other experiments testing other predictions of the SSM seemed to indicate that it worked very well,the deficit of solar neutrinos arriving at the earth was puzzling. That, in a nutshell, was the solar neutrino problem. Only electron-type neutrinos (or anti-neutrinos) are produced in the sun. Some time ago, it was suggested that a possible explanation of the solar neutrino problem is that the electron neutrinos produced in the sun "oscillate" into another type of neutrino before they reach the earth. Since all the experiments that had been done had been designed only to detect electron neutrinos, such "oscillations" might explain the observed deficit. Neutrino oscillations can only occur if neutrinos have mass, so observation of such oscillations would provide evidence for non-zero neutrino masses.

   There are a variety of processes that contribute to the neutrino production in the sun:

   - $p + p \to e^+ + \nu_e + d$
   - ${}^7_4\mathrm{Be} + e^- \to \nu_e +{}^7_3\mathrm{Li}$
   - ${}^{14}_7\mathrm{N} \to{}^{14}_6\mathrm{C} + e^+ + \nu_e$
   - ${}^{15}_8 0 \to{}^{15}_7\mathrm{N} + e^+ + \nu_e$
   - ${}^8_5\mathrm{B} \to{}^8_4\mathrm{B} + e^+ + \nu_e$

   Each of these has a characteristic neutrino energy spectrum, as shown in Figure 1 (which also includes the spectrum of "HEP" neutrinos, which are neutrinos produced in high-energy physics processes (i.e. scattering processes as opposed to nuclear decays). The total neutrino flux from each of these sources is the integral of the corresponding curve in the figure. I'll add numerical values later.

   SNO set out to solve the solar neutrino problem by designing a detector that could detect neutrinos of all types. On your last assignment I asked you to draw diagrams for the charged and neutral current scattering off of neutrons and atomic electrons. These are essentially the processes exploited by the

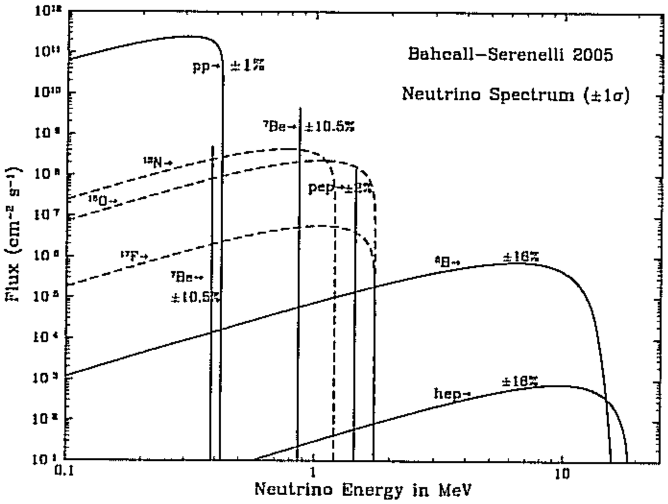


Figure 1: Energy spectrum of solar neutrinos from various nuclear processes. The integral of each curve gives the total flux for neutrinos from that source.

SNO detector (there can also be neutral current scattering from the protons). The SNO detector is an enormous spherical vessel 12m in diameter filled with 1kton of heavy water (borrowed from AECL). Neutrinos have a rather high cross-section for interactions with deuterium (this is just like hydrogen but with a deuteron as a nucleus instead of a single proton). Both charged and neutral current scattering of neutrinos off of the deuterons can cause the deuteron (which has a rather small binding energy as we have seen) to dissociate. In neutral current reactions, the neutron can be detected, as can the charged lepton from the charged current reaction.

(a) Estimate the minimum energy for a neutrino that can induce a deuteron to disintegrate ? Based on this, which source of solar neutrinos is SNO sensitive to ?

(b) Draw the Feynman diagram for the charged and neutral current interactions of electron neutrinos with deuterium (i.e. with a deuteron, causing disintegration).

(c) If the electron neutrinos oscillate into muon neutrinos before reaching the SNO detector, by what kind of process(es) can they induce disintegration of a deuteron ? Draw any relevant Feynman diagrams.

(d) On an attached sheet you will find the cross-sections for the relevant processes as a function of the neutrino energy in the range of the ^{8}B solar neutrinos, as well as the total cross-sections averaged over the ^{8}B neutrino spectrum (this is all you really need, I include the table just for interest sake). Based on this, calculate the expected number of charged and neutral current events in the SNO detector per day, assuming that neutrino oscillations do not occur. The ^{8}B flux prediction from the SSM is $5.8 \times 10^6/cm^2s$. For the HEP neutrino it's almost three orders of magnitude smaller, at $7.6 \times 10^3/cm^2s$

(e) If 50% of the neutrinos from the sun oscillate into muon neutrinos before they reach the SNO detector, what are the expected numbers of charged and neutral current events per day.

Charged- and neutral-current solar-neutrino cross-sections for heavy-water Cherenkov detectors, S. Ying, C. Haxton and E.M. Henley, *Physical Review* C45 (1992), 1982.

TABLE I. Total Paris potential neutral-current and charged-current cross sections for the breakup of deuterium as a function of the incident neutrino energy. The units are 10^{-42} cm^2, and $(-x)$ denotes 10^{-x}.

E_ν (MeV)	$\nu+d\to\nu'+n+p$	$\bar{\nu}+d\to\bar{\nu}'+n+p$	$\nu+d\to e^-+p+p$	$\bar{\nu}+d\to e^++n+n$
3.25	5.99(−3)	5.92(−3)	6.46(−2)	
3.50	1.10(−2)	1.08(−2)	8.84(−2)	
3.75	1.79(−2)	1.77(−2)	1.16(−1)	
4.00	2.71(−2)	2.67(−2)	1.49(−1)	
4.25	3.88(−2)	3.81(−2)	1.86(−1)	1.37(−3)
4.50	5.32(−2)	5.22(−2)	2.28(−1)	5.98(−3)
4.75	7.05(−2)	6.92(−2)	2.75(−1)	1.37(−2)
5.00	9.11(−2)	8.92(−2)	3.27(−1)	2.47(−2)
5.25	1.15(−1)	1.13(−1)	3.85(−1)	3.89(−2)
5.50	1.43(−1)	1.39(−1)	4.47(−1)	5.72(−2)
5.75	1.74(−1)	1.70(−1)	5.16(−1)	7.95(−2)
6.0	1.83(−1)	1.79(−1)	5.89(−1)	1.06(−1)
6.5	2.51(−1)	2.44(−1)	7.53(−1)	1.71(−1)
7.0	3.32(−1)	3.21(−1)	9.40(−1)	2.54(−1)
7.5	4.26(−1)	4.12(−1)	1.15(0)	3.54(−1)
8.0	5.10(−1)	4.91(−1)	1.38(0)	4.73(−1)
8.5	6.24(−1)	5.99(−1)	1.64(0)	6.10(−1)
9.0	7.51(−1)	7.20(−1)	1.92(0)	7.65(−1)
9.5	8.93(−1)	8.53(−1)	2.22(0)	9.40(−1)
10.0	1.05(0)	1.00(0)	2.55(0)	1.13(0)
10.5	1.18(0)	1.12(0)	2.91(0)	1.35(0)
11.0	1.36(0)	1.29(0)	3.29(0)	1.58(0)
11.5	1.55(0)	1.47(0)	3.69(0)	1.83(0)
12	1.76(0)	1.66(0)	4.13(0)	2.09(0)
13	2.16(0)	2.03(0)	5.07(0)	2.69(0)
14	2.66(0)	2.48(0)	6.12(0)	3.36(0)
15	3.17(0)	2.94(0)	7.29(0)	4.10(0)
16	3.77(0)	3.48(0)	8.56(0)	4.91(0)
17	4.38(0)	4.02(0)	9.95(0)	5.84(0)
18	5.09(0)	4.64(0)	1.15(1)	6.80(0)
19	5.80(0)	5.27(0)	1.31(1)	7.84(0)
20	6.61(0)	5.97(0)	1.48(1)	8.94(0)
21	7.47(0)	6.71(0)	1.67(1)	1.01(1)

The averaged total cross sections over solar ^{8}B and ^{3}He$+p$ (hep) neutrinos from the Sun are

$$\langle\sigma\rangle_{^8\mathrm{B}}=\begin{cases}4.44\times10^{-43}\ \mathrm{cm}^2, & \nu_e+d\to\nu_e+n+p\ ,\\ 1.15\times10^{-42}\ \mathrm{cm}^2, & \nu_e+d\to e^-+p+p\end{cases}$$

and

$$\langle\sigma\rangle_{\mathrm{hep}}=\begin{cases}1.24\times10^{-42}\ \mathrm{cm}^2, & \nu_e+d\to\nu_e+n+p\ ,\\ 2.97\times10^{-42}\ \mathrm{cm}^2, & \nu_e+d\to e^-+p+p\ ,\end{cases}$$

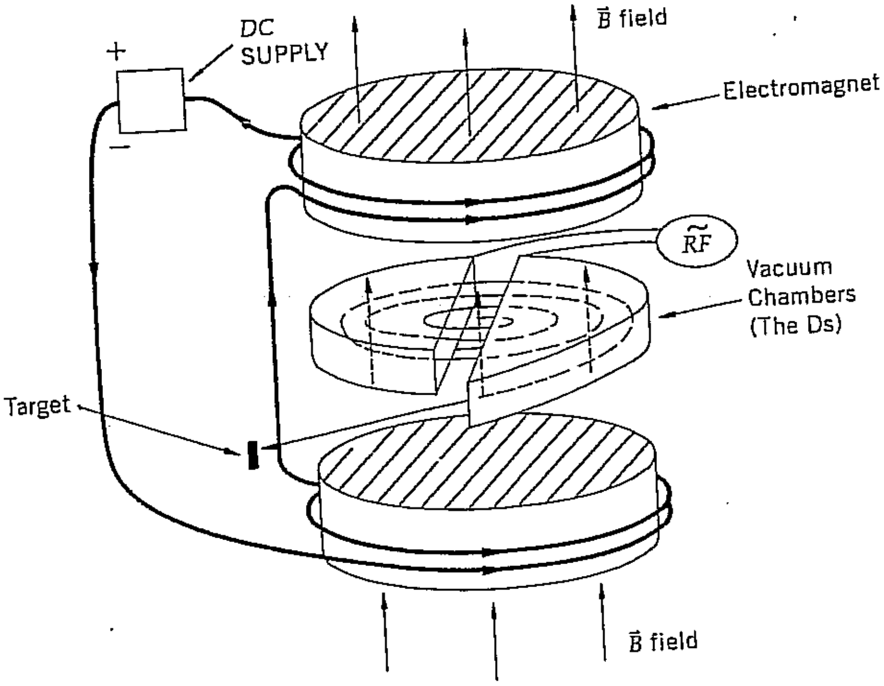
2. Here's a problems that combines some things that you've seen before but have not had to deal with at the same time. Consider a beam of pions impinging on a proton target (at rest). What is the threshold energy for the (copious) production of K^- particles in the final state ? (i.e. at what pion beam energy do the products this collision start to contain negatively charged kaons ?)

3. In class we covered the principles behind cyclotrons. The picture from the lecture is reproduced below. Remember that the particles circulate until they are extracted at some radius, as shown. Assume that you have a cyclotron accelerating protons. If the extraction radius of the particles is $40cm$, and the magnetic field strength is 1.5 Tesla answer the following questions:

   (a) What is the required RF frequency ?

   (b) What is the kinetic energy of the extracted protons ?

   Comment on how energetic a proton beam one might expect to create using a cyclotron ? What is the limiting factor ?



4. Part of this question is done somewhere in one of Professor Orr's old assignments. Please try to do it on your own. It's not difficult and it is (I hope) a useful exercise.

   In class we stated that the Born approximation for the scattering of particles from a weak potential $V(\vec{r})$ gives us the scattering amplitude as

$$f(\vec{q}) = -\frac{m}{2\pi\hbar^2}\int V(\vec{r})e^{\frac{i}{\hbar}\vec{q}\cdot\vec{r}}d^3\vec{r}$$

   (a) Show that for a spherically symmetric potential this corresponds to

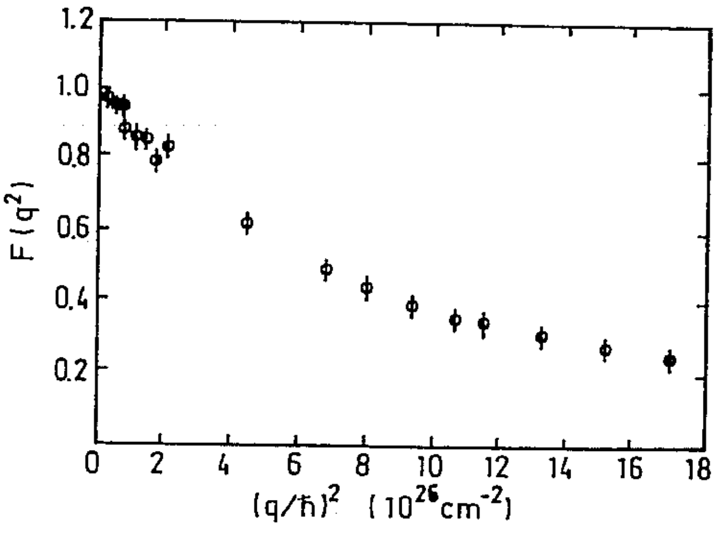
$$f(q^2) = -\frac{2m}{q\hbar}\int_0^\infty dr r \sin\frac{qr}{\hbar}V(r)$$

   (b) Show that for a spherically symmetric charge distribution

$$\left.\frac{\mathrm{d}F(q^2)}{\mathrm{d}q^2}\right|_{q^2=0} = -\frac{<r^2>}{6\hbar^2}$$

   where $< r^2 >$ is the mean-square of the electric charge distribution.

   (c) The data shown in the figure below comes from scattering off protons. We've ignored spin in the above treatment, but assuming our calculation to be applicable to this data, estimate the root-mean square proton radius.



5. In class we discussed the anomalous magnetic moments of the proton and neutron. If these were pointlike Dirac particles (i.e. spin-1/2) they would have g factors of 2. Instead, they have values of 2.79 for the proton and -1.91. Show that the ratio these two values is well explained in the quark model.

   I don't expect you to do this from scratch. This is just a little bit of self-study on something we might not get to in class. You can find this described in many (but not all) texts on introductory nuclear and particle physics; there is, for instance, a relatively straightforward treatment in the book *Particles and Nuclei* by Povh, Rith, Scholz and Zetsche. As part of this you will need to deal with the wavefunction of the proton, which we have not discussed, but I think this is relatively strightforward. Let me know if you feel this is not the case.