Disentangling the strong force: QCD, Factorization and the b quark

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Outline

- O. Prologue factorization and the parton model
- 1. Why study b quarks?
- 2. Effective field theory and the heavy quark expansion
- 3. Applications ... where these ideas have led
- 4. New directions

Prologue: How do we do physics at proton colliders at all? (i.e. Tevatron, LHC)





i.e. top production at Fermilab:



... this is the physics we want to study

... but protons aren't so simple ...





-> particle production! Indeterminate number of quarks in proton

So a proton looks something like this:



(Actually, it's a linear superposition of all these states ...)



... and our simple quark-level process



... is buried in the muck.

How can we calculate anything without solving QCD?

A miracle occurs "Factorization"



but then a miracle occurs "Factorization"

 $\sigma(p(P_1) + p(P_2) \to t\bar{t} + X)$

$$= \int_0^1 dx_1 dx_2 \sum_f f_f(x_1) f_{\bar{f}}(x_2) \cdot \sigma(q_f(x_1P) + \bar{q}_f(x_2P) \to t\bar{t})$$

cross section for free quarks (and gluons) - can calculate in perturbation theory



000

 $f_f(x_1)$: probability to find parton f with fraction x_1 of longitudinal momentum of proton ("parton distribution function") property of the PROTON

> - can't calculate ... but UNIVERSAL (can measure in another process)

This is not obvious!

A patently false factorization formula:



(subprocesses: travel through slits, propagate)

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This is not obvious!





Interference - can't in general disentangle the probabilities!

The proofs of factorization are long and complicated ...

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FACTORIZATION FOR SHORT DISTANCE HADRON-HADRON SCATTERING

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> Received 18 February 1985 (Revised 17 May 1985)

We show that factorization holds at leading twist in the Drell-Yan cross section $d\sigma/dQ^2 dy$ and related inclusive hadron-hadron cross sections

We review the heunstic arguments for factorization, as well as the difficulties which must be overcome in a proof. We go on give detailed arguments for the all order cancellation of soft gluons, and to show how this leads to factorization

1. Introduction

Factorization theorems [1] show that QCD incorporates the phenomenological successes of the parton model at high energy and provide a systematic way to refine parton model predictions. The term "factorization" refers to the separation of short-distance from long-distance effects in field theory. The program of factorization is to show that such a separation may be carried out order-by-order in field theoretic perturbation theory. In practice, this means analyzing the Feynman diagrams which contribute to a given process, and showing that they may be written as products of functions with the desired properties.

Such an analysis has been carried out in e^+e^- annihilation [2–4] and deeply inelastic scattering [1,5]. The purpose of this paper is to extend the analysis to



when has not a component $\frac{p_{i}^{2}(2-q^{-1}c)}{q_{i}^{2}(q_{i}^{2}-q_{i}^{2})} < \frac{q_{i}^{2}}{q_{i}^{2}(q_{i}^{2}-q_{i}^{2})} < \frac{q_{i}^{2}}{q_{i}^{2}(q_{i}^{2}-q_{i}^{2})} < \frac{q_{i}^{2}}{q_{i}^{2}(q_{i}^{2}-q_{i}^{2})}$ and $\frac{q_{i}^{2}}{q_{i}^{2}(q_{i}^{2}-q_{i}^{2})}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}^{2}}$ and $\frac{q_{i}^{2}}{q_{i}$

page 8

JC Cohe et al. / Rates haloe survey 111 If $H(Q^n)$ is a most fraction of Q^n , then only small values of s_x controls fractions maggaal, since the Former transform of a smooth fraction is sharply peaked. Even if $H(Q^n)$ is not a smooth functions [29], we may obtain a cross section that is short-altitude dominands by performing an average over Q^n [20]

$$\begin{split} \langle \mathrm{d} \sigma/\mathrm{d}^4 Q \rangle &= \int \mathrm{d}^4 Q_X (Q^* - \overline{Q}^*) \mathrm{d} \sigma/\mathrm{d}^4 Q \\ &= \int \mathrm{d}^4 x \sum [\langle i|J(0)(f) \rangle^2 \mathrm{exp} \Big[r \big[\overline{Q} + \rho_f - \rho_i \big] \ x \Big] \, g(x_s) \,, \end{split}$$

where β_{12} , is a bit from more house gas the strength hadron $\gtrsim 0$ more showe gas the bit body but reasons making of the bits have β_{12} and β_{12} more house gas the strength have reasons the strength have reasons the strength have reasons and thave reasons and the strength have reasons and th



where e, is the on-shell energy of state a

$$\label{eq:rescaled} \begin{split} r_{e} &= \sum_{\substack{i \in W \\ i \neq w}} \{k\}, \end{split}$$
 and the symbol = < (>)^{*} means to the the (right) in a cut sum-evidened diagram, represents all integrations, normalization and numerator factors, denoted by N,. $P = \int \prod_{i=1}^{n} \frac{dN_{i}}{d(2\pi)^{-1}} \prod_{i=1}^{n} \frac{dN_{i}}{(2\pi)^{-1}} N(r). \end{split}$





page 9



from any landing parts angular surface S shoth survives the sum over outs C.
$$\begin{split} G_n &= \sum_{k} \int dK_n^* dK_n^* \prod_{j=1}^{N} \int d^k q_{ij} \prod_{j=1}^{N} d^j k_{ij}^* \\ &\times R_n^{(1)}(K_{n-1}^*) \int e^{-i k_{ij}} S^{(n)}(q_i^*, q_i^*) |_{n-1n-1-n_i} \\ &\times H^{(n)}(K_n^*, K_n^*) \times L_n^{(n)}(K_n^*, q_i^*)^{(n-n_i)} \qquad (4.10) \end{split}$$
Eq. (4.10) as represented by the 4.6, which the existent lites to which electron

page 24



Let us compare the q^{-1} margin for graphs (a) and (b) of g_{-} . 3.2 as the homomentum regard, there q^{-1} excited studies have be written as $I_{1} \approx \int \frac{1}{2q} \frac{1}{q^{-1}q^{-1}} \frac{1}{q^{-1}} \frac{1}{q^{-1$

 $\frac{\times}{2(1-x_h)k_h^n}(f_n^*-q^*)-\xi+n\xi^{N_h(q^*)}(q^*) \quad (5.2)$ Here $\xi = (k_h^2 + q^*)^2 - (l_h^2 - q^*)^2$, while ϕ_n and ϕ_n are slowly varying functions of q^* . We study the region $|q^*| < M$ Over most of the region we have $|P_n^*q^*| \gg M^2 - \xi$, (3.3)

page 13



In excert mer weis we conjunct rotation in displacitation products ground only stand-decorption by decorptions of the standard standard standard only stand-decorption by decorption line in decars, which is a space-maintained of R A. may be proved by treating the existent as borner. The difference from the representation of R (β , 5.1 protocols in a similar matter The difference from the representation of the physically displaced line β_{α} is a registered by the decord line to which the collinear glaces stands. The soft glaces are again lightlike and longutabulity physical Web real (accessed) and the consequencidentian of the standard st

page 37

(Collins, Soper, Sterman, 1980's)

... but the physics is simple



... but the physics is simple



Separation of Scales



$$\sigma(p(P_1) + p(P_2) \rightarrow t\bar{t} + X) = \int_0^1 dx_1 dx_2 \sum_f f_f(x_1) f_{\bar{f}}(x_2) \cdot \sigma(q_f(x_1P) + \bar{q}_f(x_2P) \rightarrow t\bar{t}) + O\left(\frac{\Lambda_{\rm QCD}}{2m_t}\right)$$

- form of the factorization formula (convolution over light-cone momentum fraction) is non-trivial

- final hadronic state unspecified - sum over all of them ("+X") - probability to hadronize = 1! "inclusive"

- subleading (O(Λ_{QCD}/Q)) terms ("power corrections") don't factorize in this way ... fortunately, these are small for Q~2m_t.

- to the degree that the distance scales can be separated (i.e. to LEADING ORDER in $\Lambda_{\rm QCD}/\rm Q$), hard scattering factorizes into a short-distance scattering and long distance parton distribution functions

 short-distance physics can be calculated in QCD (perturbative) .. long-distance physics is incalculable, but universal - can be measured in other processes



1970's

November 18, 2004

Caltech

-> now

Why study b quarks?



Why study b quarks?



A B (bq) meson q=u, d, s



Why study b quarks?



Why study b quarks?



b quarks are a natural microscope ... decays are determined by very SHORT distance physics, where we expect new particles/ interactions:



NB: m_W , $m_X \gg m_b$ -> seeing heavy particles VIRTUALLY

There are many possible flavour-changing interactions ...



... and by measuring as many as we can and requiring consistency with the Standard Model (highly constrained!) we can search for signs of new physics.

(NB: this sort of thing has worked in the past ...)

"B Factories" (SLAC, KEK):

- dedicated machines producing ~10⁸ bb pairs/year (on-line since 1999) - designed for high precision studies of B meson properties (also: CLEO, LEP, FNAL, ATLAS)



BaBar detector: SLAC, California



Belle detector, KEK, Japan





- LOW energy, HIGH luminosity machines (~10 GeV c.o.m. energy for virtual study of 100 GeV scale)

Consistency of the Standard Model (so far)

(from ICHEP, summer 2004)



All constraints overlap in one region ...

(1) the Standard Model
 provides the right first-order
 description of flavour-changing
 transitions

(2) discrepancies will requireprecision theory/measurementsto find (probably ...)

Precision physics with b decays is tricky ...



(and to believe small discrepancy = new physics, need model independent predictions - challenge for theory! ... cf g-2 for muon)

Does the process factorize in a useful way?



- Multiscale problem - want to unravel physics at different scales

- $\Lambda_{QCD}/m_b \sim 1/10$, so we need to understand power corrections

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The Tool: Effective Field Theory ("EFT")

"sufficient unto the day is the evil thereof" (Mt. 6:34)

Use the degrees of freedom appropriate to energy/distance scale of problem!



... to calculate projectile motion

i.e. you shouldn't use quantum gravity ...





It's HARD:

- the calculation is MUCH more complicated
- appropriate degrees of freedom are obscured in "fundamental" theory
- we don't even know what quantum gravity is



and POINTLESS:

- quantum effects are TINY (corrections ~10⁻³³ cm/r)
- if we need corrections, much simpler to expand QG in powers of r_{PLANCK}/r, take linear correction

Ex: the multipole expansion:



Physics at r~L is complicated - depends on details of charge distribution

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Ex: the multipole expansion:



BUT ... if we are interested in physics at r>>L, things are much simpler ...

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Ex: the multipole expansion:



... can replace complicated charge distribution by a POINT source with additional interactions (multipoles)...

Multipole expansion:

$$V(r) = \frac{\overrightarrow{q}}{r} + \frac{\overrightarrow{p} \cdot \overrightarrow{x}}{r^3} + \frac{1}{2} \underbrace{Q_{ij}}{r^5} \frac{x_i x_j}{r^5} + \cdots$$

$$q, p_{i}, Q_{ij}, \dots$$

$$m{q}, \ m{p}_i, \ m{Q}_{ij}, \ \dots \ :$$
 short distance quantities
 $\left\langle rac{1}{r} \right\rangle, \left\langle rac{x_i}{r^3} \right\rangle, \left\langle rac{x_i x_j}{r^5} \right\rangle, \ \dots \ :$ long distance quantities
FACTORIZATION!

higher order terms in multipole expansion suppressed by powers of (L/r) - for r»L, only need first few terms. To get more accuracy, need more parameters.

Effective Field Theory ("EFT"): more generally, any theory at momentum p<<M can be described by an effective Hamiltonian,



- C_n 's : short distance quantities (in QCD: perturbatively calculable if M>> Λ_{QCD})
- $\langle \mathcal{O}_n \rangle' s$: long distance quantities (in QCD: nonperturbative ... need to get them elsewhere)
- Effective Field Theory automatically factorizes the calculation
- by keeping more terms, can work to arbitrary accuracy in 1/M

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EFT for a b quark at low momentum transfer:



B meson dynamics

EFT for a b quark at low momentum transfer:



B meson dynamics in the limit Λ_{QCD}/m_b ->0

- at low (~ Λ_{QCD}) momentum transfers, a heavy (m_Q>> Λ_{QCD}) quark behaves as a **static colour source** .. **essentially NO dynamics** (cf. proton in H atom)

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This field became suddenly fashionable in the early 1990's ...

(Isgur, Wise; Voloshin, Shifman; Eichten, Hill; Georgi; ...)

- in EFT, heavy quark ~ static colour source => many of its properties (mass, spin, magnetic moment, "Fermi motion") are IRRELEVANT at leading order in Λ_{QCD}/m_b ... EFT has lots of symmetry
- in a FEW cases, symmetries constrain the dynamics so strongly that at leading order there is NO unknown hadronic physics => absolute predictions!

"Classic" Application: INCLUSIVE decays (sum over all possible hadronic final states)



(Bigi, Shifman, Uraltsev, Vainshtein, Voloshin, Shifman; Chay, Georgi, Grinstein; Manohar, Wise; Falk, ML, Savage ...)

Decay: short distance (calculable)

Hadronization: long distance (nonperturbative) - but probability to hadronize (to SOMETHING) is unity - nothing to calculate!

- if all final hadronic states are included
("inclusive"), hadron decay is given by free
quark decay (at leading order in 1/m_b)

Similar to inclusive processes in proton collisions, but since the initial b quark is ~ at rest, the factorization is MUCH simpler (no convolution over momentum fraction) ... straightforward to calculate power corrections November 18, 2004

$$\begin{aligned} & \prod_{\text{(need to determine b->c weak coupling constant V_{cb})} & \prod_{k=1}^{q} \left(\frac{1}{2} + \frac{$$

 $O(\Lambda_{QCD}/m_b)$: ~20% correction $O(\Lambda_{QCD}^3/m_b^3)$: ~1-2% correction $O(\Lambda_{QCD}^2/m_b^2)$: ~5-10% correction Perturbative: ~few % -> This is now a PRECISION field!

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Nonperturbative parameters can be determined from other observables (spectral moments):





(CLEO, PRD67:072001, 2003)

 Λ , λ_1 : only unknown hadronic parameters for inclusive decays up to $O(\Lambda_{QCD}/m_b)^2$

Applications:

- spectroscopy
- semileptonic decays (measure parameters of Standard Model - calibration)
 - inclusive (sum over all hadronic states)
 - exclusive (decays to specific final states particular those with charm quarks - "Heavy Quark Symmetry")
- nonleptonic decays (lifetimes)
- rare (inclusive) decays i.e. $b
 ightarrow s\gamma$, $b
 ightarrow s\mu^+\mu^-$

All can be handled in an expansion in $\Lambda_{QCD}/m_b \sim 1/10$... remarkable success over past decade

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Much of this theory was developed in early-mid 1990's ... since then:

- 1. Much better data! (B factories, CDF, CLEO).
 - We now work to sub-sub-subleading order $(O(\Lambda_{QCD}/m_b)^3)$ in some cases
 - worry (& argue) hard about theoretical uncertainties, effects at the few % level
- 2. Effective Field Theory ideas extended to more complex situations - including much more complex forms of factorization



hadronic invariant mass moments

lepton energy moments

Caltech

0.15

0.1

0.05

0

0.01

0

-0.01

-0.02

(Bauer, Ligeti, ML, Manohar and Trott)

Global fits (summer '02 - updated '04):



(up to $1/m^3$)



V_{cb} from exclusive decays, m_b from sum rules (Hoang)

The fit also allows us to make precise predictions of other moments as a cross-check:

$$D_{3} \equiv \frac{\int_{1.6 \text{ GeV}} E_{\ell}^{0.7} \frac{d\Gamma}{dE_{\ell}} dE_{\ell}}{\int_{1.5 \text{ GeV}} E_{\ell}^{1.5} \frac{d\Gamma}{dE_{\ell}} dE_{\ell}} = \begin{cases} 0.5190 \pm 0.0007 & \text{(theory)} \\ 0.5193 \pm 0.0008 & \text{(experiment)} \end{cases}$$
$$D_{4} \equiv \frac{\int_{1.6 \text{ GeV}} E_{\ell}^{2.3} \frac{d\Gamma}{dE_{\ell}} dE_{\ell}}{\int_{1.5 \text{ GeV}} E_{\ell}^{2.9} \frac{d\Gamma}{dE_{\ell}} dE_{\ell}} = \begin{cases} 0.6034 \pm 0.0008 & \text{(theory)} \\ 0.6036 \pm 0.0006 & \text{(experiment)} \end{cases}$$

(some fractional moments of lepton spectrum are very insensitive to $O(1/m^3)$ effects, and so can be predicted very accurately) (C. Bauer and M. Trott)

NB: these were REAL PREdictions (not postdictions)

Hadronic physics with < 1% uncertainty!

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There are lots of other theoretical issues arising in this game ...

- Ø phase space boundaries experimental cuts can ruin usual 1/m expansion
 - in some restricted regions, infinite series can be summed into nonperturbative "shape function" (Bigi, Uraltsev, Shifman, Vainsthein; Neubert)
 - recently shown to generalize to all orders in 1/m (cf subleading twist parton distribution functions)
 (Bauer, ML and Mannel; Leibovich, Ligeti and Wise)
- perturbation theory



- "renormalons" (apparently bad behaviour when unphysical parameters used) (Bigi et. al., Beneke, ML, Manohar and Savage, Neubert and Sachrajda)
- enhanced 1/m³ corrections ("weak annihilation")

(Bigi and Uraltsev, Voloshin)

- Iong-distance physics fragmentation, light quark loops
- 🧭 "quark-hadron duality"

I ..

... useful as this is, in the B factory era it only touches a small fraction of the interesting decays

We'd like to understand more complex situations (particularly 2 body, nonleptonic decays - important for CP violation studies) Ex: want to measure the COMPLEX PHASE of the b-u coupling (this is the kind of measurement the B Factories were built to make)



Ex: want to measure the COMPLEX PHASE of the b-u coupling (this is the kind of measurement the B Factories were built to make)



The best place to get this is in B -> $\pi\pi$ decays. None of the preceding allows us to pull this apart into anything simpler.

In addition, other short-distance contributions contribute to the same decay! ("penguin pollution") - need to disentangle



The best place to get this is in B -> $\pi\pi$ decays. None of the preceding allows us to pull this apart into anything simpler.

"QCD Factorization" proposal (not an EFT)



"Soft-Collinear Effective Theory" (SCET) Bauer, ML, Fleming, Pirjol, Stewart, ... $\overbrace{\pi}^{} \longleftarrow \underbrace{\mathbb{B}}^{} \longrightarrow \overbrace{\pi}^{}$

- pions have LARGE energy ($\sim m_b/2 \gg \Lambda_{QCD}$), LOW mass ($\sim \Lambda_{QCD}$)

"SOFT" constituents $p^{\mu} = (p^+, p^-, p^{\perp}) \sim (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}})$ "Collinear" constituents $p^{\mu} = (p^+, p^-, p^{\perp}) \sim \left(\frac{\Lambda_{\text{QCD}}^2}{m_b}, m_b, \Lambda_{\text{QCD}}\right)$

SCET is a "Large energy" expansion - complicated because of extra scales ..



Factorization in B Decays (c. 1994):



$$\frac{1}{\Gamma_0}\Gamma(B \to X_c \ell \bar{\nu}_\ell) = 0.369 \left[1 - 1.54 \frac{\alpha_s(m_b)}{\pi} + 3.35 \frac{\bar{\Lambda}}{m_B} + 5.81 \frac{\bar{\Lambda}^2}{m_B^2} - 5.69 \frac{\lambda_1}{m_B^2} - 7.47 \frac{\lambda_2}{m_B^2} + O\left(\frac{\Lambda_{\rm QCD}}{m_B}\right)^3 \right]$$

Factorization in B Decays (c. 2004):

(Beneke, Buchalla, Neubert Sachraja; Bauer, Pirjol, Rothstein, Stewart)



Final Comment

This is always going to be with us ... need to factorize problems for nonperturbative lattice QCD calculations as well!





- need L>1 fm to simulate proton
 need a<1/Q to simulate short-
 distance physics w/momentum Q
- extremely inefficient to simulate short-distance (perturbative) physics on the lattice!

Factorization -> do short-distance physics analytically, longdistance physics numerically with lattice spacing a>>1/Q

Summary:

- Factorization allows us to separate short-distance (interesting) physics from long-distance QCD in a model-independent way
- effective field theory systematizes the calculation
- in the heavy quark limit, exact results can be proven which allow us to finesse nonperturbative QCD for b decays (in some cases)
- this is now a precision field limiting effects are at the $O(1/m^3)$ level (few percent in many cases)
- new approaches to EFT are allowing us to study more complicated situations



NRQCD: "Non-relativistic QCD" - EFT for systems with two heavy quarks (i.e. $b\bar{b}$ bound states) (more complicated due to correlated scales)

- $b\overline{b}$, $c\overline{c}$ production and decay (fixed huge discrepancy with exp't)
- b quark mass to 50-100 MeV

Ex: tt production near threshold



NRQCD

NRQED: EFT simplifies high precision QED calculations - can get state-of-the-art results with a few Feynman diagrams ...

	$lpha^8 \ln^3 lpha$	Lamb	Н	agree/new
			$\mu^+e^-,~e^+e^-$	new
SNY		(no h.f.s)		agree
	$lpha^4 \ln^3 lpha$	(no $\Delta\Gamma/\Gamma$)	agree
	$lpha^7 \ln^2 lpha$	Lamb	$H,\ \mu^+e^-,\ e^+e^-$	agree
		h.f.s.	$H,\ \mu^+e^-,\ e^+e^-$	agree
San S	$lpha^3 \ln^2 lpha$	$\Delta\Gamma/\Gamma$	e^+e^- ortho and para	agree
	$lpha^6 \ln lpha$	Lamb, h.f.s.	$H,\ \mu^+e^-,\ e^+e^-$	agree
	$\alpha^2 \ln \alpha$	$\Delta\Gamma/\Gamma$	e^+e^- ortho and para	agree

(from A. Manohar, Ringberg Workshop '03)

NRQCD NRQED

Lattice QCD: NONPERTURBATIVE (numerical) - but hard to handle multiscale problems! (need fine lattice spacing $\sim 1/m_b < 1/\Lambda_{QCD}$ - computationally demanding) - EFT removes short-distance dynamics so it doesn't have to be simulated



64

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NRQCD NRQED Lattice QCD

Nuclear Physics: NN scattering, model-independently - renormalization and counterterms instead of potential models, offshell ambiguities, ...

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Ex: np->dy at NNLO:
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(Savage, Scaldeferri and Wise, Nucl. Phys. A652:273-286,1999)



"Classic" Application: Heavy Quark Symmetry in B->D*e ν decay

$$|\mathsf{B}\rangle = |\langle | | \rangle - | | \rangle | \rangle | |$$

Isgur & Wise, 1989



- at zero recoil kinematic point, brown muck doesn't know decay has occurred! - form factor is ONE (fixed by symmetry) What does this buy us?

- "turn-the-crank" FACTORIZATION
- calculation organized as a power series in Λ_{QCD}/m_b ("power counting") $\Lambda_{QCD}/m_b \sim 1/10$, so higher order corrections essential for precision (good expansion parameter for theorists!)
- virtual excitations (at all energy scales) are systematically included ("renormalization")

EFT is allowing us to do as much of the problem as we can, and isolate the nonperturbative physics

B decay requires a hierarchy of **effective theories** ... at each threshold, degrees of freedom are "integrated out" and a new theory is constructed:



Global fits (summer '02 - updated '04):

(up to $1/m^3$)

- lepton energy and hadronic invariant mass moments $(ar B o X_c\ellar
 u)$, photon energy spectrum moments $(ar B o X_s\gamma)$
- measured with varying cutoffs by DELPHI, CLEO, CDF, BABAR and BELLE
- simultaneously fit for hadronic matrix elements, mb, Vcb

$$egin{aligned} R_0(E_0,E_1) &= rac{\int_{E_1} rac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_0} rac{d\Gamma}{dE_\ell} dE_\ell}, & R_n(E_0) &= rac{\int_{E_0} E_\ell^n rac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_0} rac{d\Gamma}{dE_\ell} dE_\ell}, & n = 1, \ 2 \ \end{array} \ S_1(E_0) &= ig\langle m_X^2 - ar m_D^2 ig
angle \Big|_{E_\ell > E_0}, & S_2(E_0) &= ig\langle (m_X^2 - ig\langle m_X^2 ig
angle)^2 ig
angle \Big|_{E_\ell > E_0} \ T_1(E_0) &= ig\langle E_\gamma ig
angle \Big|_{E_\gamma > E_0}, & T_2(E_0) &= ig\langle (E_\gamma - ig\langle E_\gamma ig
angle)^2 ig
angle \Big|_{E_\gamma > E_0} \end{aligned}$$

