NON-RELATIVISTIC

SUPERSTRINGS

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TOY MODEL: CONSIDER A RELATIVISTIC, CHARGED POINT PARTICLE (m, e) IN FLAT SPACE $\left(ds^2 = -dx^{0\,2} + d\vec{x}^{\,2}\right)$

• $L = -m\sqrt{-\dot{x}^2} + e A_\mu \dot{x}^\mu$

$\Downarrow$ WORLDLINE REPARAMETRIZATION INV.

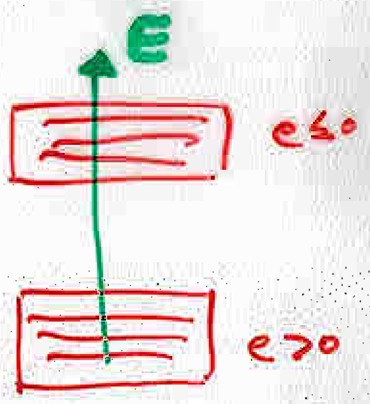
(*) $P_0 = \sqrt{m^2 + P_i P_i} + eA_0$

• CONSIDER FOLLOWING NR LIMIT

$$\begin{cases} x^0 = \omega t \Rightarrow P_0 = \frac{1}{\omega} E \\ m = M\omega \\ eA_0 = -m \end{cases}$$

$+ \quad \omega \to \infty$

$$(*) \to \boxed{E = \frac{P_i^2}{2M} + \underbrace{(|m| + eA_0)}_{\substack{\to \infty \quad e \le 0 \\ \to 0 \quad e > 0}}}$$



COMMENTS:

- GALILEAN INVARIANT SPECTRUM OF "LIGHT" PARTICLES

- FLATTENED LIGHT-CONES ($c \to \infty$)

  $\Downarrow$

  INSTANTANEOUS PROPAGATION

- ONLY $\epsilon > 0$ PARTICLES $\Rightarrow$ NO ANTIPARTICLES

CHARACTERISTIC OF NONRELATIVISTIC SYSTEMS

$$\mathcal{L}_{NRCS} = \frac{1}{2\pi}\Big[\beta\bar\partial\gamma + \bar\beta\partial\bar\gamma + \underbrace{\partial\gamma\bar\partial\bar\gamma}_{\text{INSTANTON ACTION}} + \partial X^i\bar\partial X^i\Big]$$

$$\gamma = y^0 + y^1$$
$$\bar\gamma = y^0 - y^1$$

COMMENTS

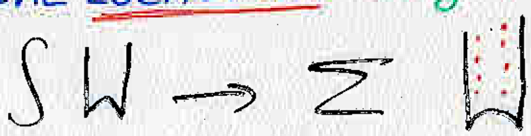
- EXTRA VARIABLES, BUT (β,γ) CFT HAS $c=2$
  - → EXACTLY SOLUBLE CFT
- ACTION INVARIANT UNDER $Gal(1,8)$
- TRIVIAL BRST COHOMOLOGY WHEN y^1 IS NONCOMPACT

$$\boxed{E = \frac{\omega R}{2\alpha'} + \frac{\alpha' p_i^2 + 2(N+\bar N)}{2\omega R}}$$

- ∄ ON SHELL GRAVITON!
- HAGEDORN SPECTRUM

$$T_H = \frac{1}{4\pi\sqrt{\alpha'}}$$

- PATH INTEGRAL "LOCALIZES" (e.g 1 LOOP FREE ENERGY)



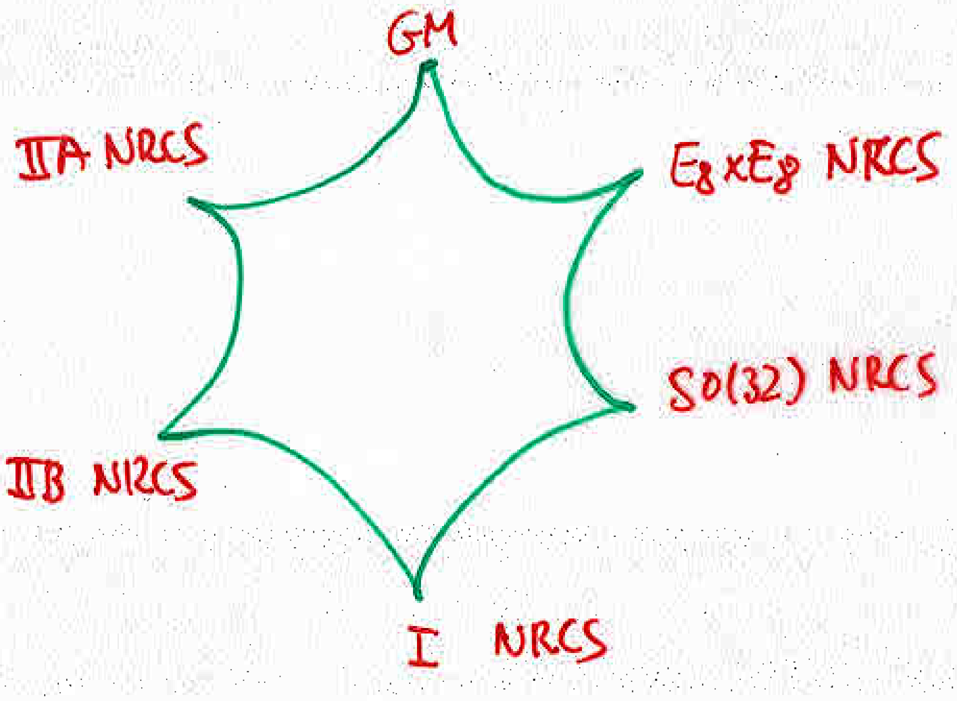
- T-DUAL TO DLCQ OF STRING THEORY (MATRIX THEORY)

• THERE IS AN ANALOGOUS DUALITY WEB IN THE NON-RELATIVISTIC SETTING:

e.g.: IIA NRCS = GM-THEORY

g_A, α' $\qquad$ $R = g_A l_s$

$$l_p = g_A^{1/3} l_s$$



• WHAT ABOUT NON-RELATIVISTIC STRINGS IN OTHER BACKGROUNDS? (EXACTLY SOLUBLE SECTOR?)

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NON-RELATIVISTIC STRINGS IN $AdS_5 \times S^5$

MOTIVATION:

- $AdS_5 \times S^5$ SIMPLEST BACKGROUND WHICH IS HOLOGRAPHICALLY DUAL TO A $D=4$ GAUGE THEORY

- NEED TO SOLVE STRING WORLDSHEET THEORY IN ORDER TO HOLOGRAPHICALLY SOLVE QCD-LIKE GAUGE THEORIES

- WORLDSHEET THEORY ON $AdS_5 \times S^5$ NOT TRACTABLE (YET)

– OUR GOAL IS TO FIND AND SOLVE THE WORLDSHEET THEORY ENCAPSULATING THE NON-RELATIVISTIC SECTOR OF TYPE IIB STRING THEORY IN $AdS_5 \times S^5$

BEFORE:

$$R^{1,9}: \; SISO(1,9) \longrightarrow SGal(1,8)$$

NOW:

$$AdS_5 \times S^5: \; SU(2,2|4) \longrightarrow \text{SUPER NEWTON-HOOKE}$$

STARTING POINT IS GREEN-SCHWARZ ACTION ON $AdS_5 \times S^5$:

$$\mathcal{L} = -\frac{T}{2}\left(\mathcal{L}^{kin} + 2\mathcal{L}^{WZ}\right) \qquad T = \frac{1}{2\pi\alpha'}$$

SYMMETRIES:

- WORLDSHEET DIFFEOMORPHISMS
- SUPERSYMMETRY OF $AdS_5 \times S^5$ [$SU(2,2|4)$].
- K-SYMMETRY

WRITE ACTION IN A MANIFESTLY SUPERSYMMETRIC WAY BY USING CARTAN'S INVARIANT ONE FORMS ON $AdS_5 \times S^5$ SUPERSPACE:

COSET SUPERSPACE: $\dfrac{SU(2,2|4)}{SO(4,1)\times SO(5)}$

$$\Downarrow$$

$$\Omega = -i g^{-1} dg = P_m L^m + P_{m'} L^{m'} + Q_{\alpha\alpha' I} L^{\alpha\alpha' I} + \frac{1}{2} M_{mn} L^{mn} + \frac{1}{2} M_{m'n'} L^{m'n'}$$

USING THE INVARIANT ONE FORMS ONE OBTAINS:

$$\cdot\ \mathcal{L}^{kin} = \sqrt{-h}\, h^{ij} \left(\eta_{mn} L_i^m L_j^n + \eta_{m'n'} L_i^{m'} L_j^{n'} \right)$$ (Metsaev Tseytlin)

$$d(\mathcal{L}^{WZ} d^2\xi) = i \bar{L} \left(\Gamma_m L^m + \Gamma_{m'} L^{m'} \right) \tau_3 L$$

WHERE:

- h_{ij} : AUXILIARY 2D METRIC
- $L_i^{m,m'}$: PULL-BACK OF ONE FORMS ON THE WORLD SHEET

IN ORDER TO FIND THE ACTION OF NONRELATIVISTIC STRINGS IN $AdS_5 \times S^5$ ONE MORE COUPLING IS RELEVANT:

$$\mathcal{L}^B = f^* B$$

WHERE

B IS A CLOSED 2-FORM ON $AdS_5 \times S^5$

$\Big\downarrow$ ANALYZE ACTION

$$\mathcal{L} = -\frac{T}{2} \left(\mathcal{L}^{kin} + 2\mathcal{L}^{WZ} + 2\mathcal{L}^B \right)$$

FROM NOW ON FIX CONFORMAL GAUGE $\sqrt{-h}\, h_{ij} = \eta_{ij}$

THE NON-RELATIVISTIC LIMIT

WE NOW SPECIFY A CHOICE OF COSET REPRESENTATIVE WHICH IS CONDUCIVE TO TAKING THE NON-RELATIVISTIC LIMIT OF $AdS_5 \times S^5$

TAKE:

$$g = \underbrace{e^{iP_1 x^1} e^{iP_0 X^0}}_{AdS_2} e^{iP_a X^a} e^{iP_{m'} X^{m'}} e^{iQ_{\alpha\alpha' I}\theta^{\alpha\alpha' I}}$$

- BOSONIC COORDINATES: $\underbrace{(X^0, X^1, X^a)}_{AdS_5}$ $\underbrace{X^{m'}}_{S^5}$
- FERMIONIC COORDINATES: $\theta^{\alpha\alpha' I}$

USING THIS PARAMETRIZATION CAN OBTAIN:

$$L^m = e^m + \text{fermions} \Rightarrow ds^2_{AdS} = e^m e^m$$

$$L^{m'} = e^{m'} + \text{fermions} \Rightarrow ds^2_{S^5} = e^{m'} e^{m'}$$

$$ds^2_{AdS} = \cosh^2\rho \overbrace{\left[-(dx^0)^2 + \cos^2\left(\frac{x^0}{R}\right)(dx^1)^2\right]}^{AdS_2} +$$

$$\left[\left(\frac{\sinh\rho}{\rho}\right)^2 (dx^a)^2 - \left(\frac{\sinh^2\rho}{\rho^2} - 1\right)\frac{(x_a dx^a)^2}{\rho^2 R^2}\right]$$

WHERE $\rho = \sqrt{x_a x^a}/R$

$$ds_5^2 = \left(\frac{\sin r}{r}\right)^2 (dx^{m'})^2 - \left(\frac{\sin^2 r}{r^2} - 1\right)\frac{(x_{m'} dx^{m'})^2}{r^2 R^2}$$

WHERE $r = \sqrt{x_{m'} x^{m'}}/R$

- WE WILL TAKE A SCALING LIMIT OF A STRING SPANNING AN AdS_2 SUBSPACE INSIDE $AdS_5 \times S^5$

- TURN ON A FLAT B-FIELD ALONG AdS_2

$$\Downarrow$$

$$\mathcal{L}_B = B_{\mu\nu} V_0^\mu V_1^\nu \qquad (B_{\mu\nu}\text{: CONSTANT } \mu,\nu = 0,1)$$

WHERE V^μ ARE AdS_2 VIELBEINS ($e^\mu = \cosh\rho\, V^\mu$)

THE LIMIT:

$$\begin{cases} X^\mu = \omega\, y^\mu \\ \vartheta = \sqrt{\omega}\,\theta_- + \dfrac{1}{\sqrt{\omega}}\,\theta_+ \\ B_{\mu\nu} = \epsilon_{\mu\nu} \\ R = \omega R_0 \end{cases} \qquad + \ \omega \to \infty$$

WHERE:

$$\Gamma_0 \Gamma_1 \tau_3 \theta_\pm = \pm \theta_\pm$$

- RESCALE LONGITUDINAL COORDINATES ($c \to \infty$)
- FLATTENS SPACE TRANSVERSE TO AdS_2
- FERMION SCALING DETERMINED BY:
  - SYMMETRIES OF BOSONIC SCALING
  - κ-SYMMETRY
  - S-NEWTON-HOOKE SYMMETRY

USING THE EXPLICIT EXPRESSION FOR THE ONE FORMS ONE HAS IN THE LIMIT:

$$\begin{cases} L^{\mu} = \omega L^{\mu(1)} + \frac{1}{\omega} L^{\mu(-1)} + \dots \\ L^{a} = L^{a(0)} + \dots \\ L^{m'} = L^{m'(0)} + \dots \\ L^{\alpha\alpha' I} = \sqrt{\omega}\, L^{\alpha\alpha' I(1/2)} + \frac{1}{\sqrt{\omega}} L^{\alpha\alpha' I(-1/2)} + \dots \end{cases}$$

PLUG INTO ACTION, ANALYZE SUPERFICIALLY DIVERGENT TERMS:

- $\mathcal{L}^{kin}_{div} = \omega^2 \eta^{ij} \eta_{\mu\nu} L_i^{\mu(1)} L_j^{\nu(1)}$

- $\frac{1}{i} d(\mathcal{L}^{WZ}_{div} d^2\xi) = \omega^2 \bar{L}^{(1/2)} \Gamma_\mu L^{\mu(1)} \tau_3 L^{(1/2)}$

- $\mathcal{L}^{B}_{div} = \omega^2 \epsilon_{\mu\nu} e_0^{\mu(1)} e_1^{\nu(1)}$, $\begin{cases} v^\mu = e^{\mu(1)} \\ L^{\mu(1)} = e^{\mu(1)} + L^{\mu(1)}_{fermi} \end{cases}$, WHERE

$$\Downarrow$$

$$\mathcal{L}_{div} = -\frac{T}{2}\left(\omega^2 \eta^{ij} \eta_{\mu\nu} L_i^{\mu(1)} L_j^{\nu(1)} + 2\mathcal{L}^{WZ}_{div} + 2\omega^2 \epsilon_{\mu\nu} e_0^{\mu(1)} e_1^{\nu(1)}\right)$$

USING THAT:

1. $\mathcal{L}^{WZ}_{div} = -\omega^2 \det (L^{\mu(1)}_{fermi})$
2. $\det (L^{\mu(1)}_{fermi}) = \det L^{\mu(1)} - \det e^{\mu(1)}$

$$\Downarrow$$

$$\mathcal{L}^{WZ}_{div} + \mathcal{L}^{B}_{div} = -\omega^2 \det L^{\mu(1)}$$

$\Rightarrow$ EVERYTHING CAN BE WRITTEN USING INVARIANT FORMS!

$$\Downarrow$$

$$\mathcal{L}_{div} = \omega^2 \frac{T}{2} \eta_{\mu\nu} (L_0^{\mu(1)} - \epsilon^{\mu}_{\rho} L_1^{\rho(1)}) (L_0^{\nu(1)} - \epsilon^{\nu}_{\sigma} L_1^{\sigma(1)})$$

MAKING A PERFECT SQUARE

CAN TAKE LAGRANGE MULTIPLIERS:

$$\mathcal{L}_{div} = \lambda_\mu (L_0^{\mu(1)} - \epsilon^{\mu\rho} \eta_{\rho\nu} L_1^{\nu(1)}) - \cancel{\frac{1}{2T\omega^2} \lambda_\mu \lambda^\mu} \rightarrow 0 \quad \text{AS } \omega \to \infty$$

GET A FINITE STRING ACTION WITH:

- SUPERSYMMETRIC NEWTON-HOOKE STRING LIKE ALGEBRA
- KAPPA SYMMETRY

$\Downarrow$

STRING ACTION :

$$\mathcal{L}^{NR} = -T\sqrt{\det g}\,\Big[g^{ij}\,\partial_i X^a \partial_j X^a + \frac{1}{R_0^2}\, X^a X^a + g^{ij}\,\partial_i X^{m'} \partial_j X^{m'} + 2\bar{\theta}_+ \Gamma^\mu e^i_\mu\, D_i \theta_+ \Big]$$

$$g_{ij} = \eta_{\mu\nu}\, e^\mu_i\, e^\nu_j \quad (AdS_2 \text{ metric})$$

$\Downarrow$

ACTION REDUCES TO FREE MASSIVE SCALARS AND FERMIONS PROPAGATING IN AdS_2

- $X^a \quad m^2 = 2/R_0^2$
- $X^{m'} \quad m^2 = 0$
- $\theta^+ \quad m^2 = 1/R_0^2$

CONCLUSIONS

- HAVE FOUND A NEW SOLUBLE SECTOR OF STRING THEORY IN $AdS_5 \times S^5$

- TO DO: QUANTIZE:
  - OPEN STRINGS
  - CLOSED STRINGS
  - INTERACTIONS

- WHAT IS THE FIELD THEORY DESCRIPTION OF THIS NON-RELATIVISTIC SECTOR?