A Universal Correction to the Inflationary Vacuum

Jan Pieter van der Schaar (University of Amsterdam)

with Brian Greene and Maulik Parikh

`hep-th/0512243`

ISCAP, Columbia University

Introduction and motivation

Quantum Gravity in de Sitter space?

- Even if $\Lambda \ll M_p^2$, interesting physics because of dS temperature $T_{GH} = \frac{H}{2\pi}$
  - Eternal inflation, holography, complementarity, ...
  - Cosmological constant problem?
- Moreover: observationally relevant
  - Inflation and the primordial power spectrum: $P_{\mathcal{R}}(k) \propto T_{GH}^2$
  - Current phase of acceleration

Cosmology: best bet to observationally probe string scale physics

Inflation and string scale physics

Inflation quite generally suggests the following possibilities

- CMB as a probe of the inflationary vacuum structure ($\frac{H}{M_s} > 10^{-6}$)
- CMB as a probe of effective dynamics during inflation ($\frac{H}{M_s} > 10^{-3}$)
- CMB as a probe of primordial non-gaussiannities ($c_s < 1$)
- Cosmic string production in string inflation models (?)

Not entirely hopeless if $H \sim 10^{14}$ GeV. Future prospect: the precision of the primordial scalar density perturbation might reach the 10^{-6} level [Spergel]

Worthwhile to explore every option

Probing the inflationary vacuum

Inflation is an excellent probe of the small scale vacuum structure

- Theory and observation agree: state in $\langle \mathrm{in}|\delta\phi^2|\mathrm{in}\rangle$ is very close to the BD vacuum
- Pragmatic point of view: small departures cannot be entirely excluded, but are subject to strong consistency constraints (backreaction)
- The proposals so far can lead to H/Λ corrections, but in general do not have to, and are typically either somewhat fine-tuned [BEFT] or somewhat ad-hoc [NPH]
- The proposals do lead to rather distinctive signatures (modulations)
- In general dynamical (quantum) corrections [EFT] give H^2/Λ^2 effects

Greene, Schalm, Shiu, JPvdS, `astro-ph/0503458`
Schalm, Shiu, JPvdS, `hep-th/0412288`
Kaloper, Kleban, Lawrence, Shenker, Susskind, `hep-th/0209231`
Easther, Greene, Kinney, Shiu, `hep-th/0204129`, Danielsson, `hep-th/0203198`
Martin, Brandenberger, `hep-th/0005209`

Inflationary vacuum corrections

Can we come up with calculable model-independent corrections to the inflationary vacuum?

- Proof of principle
- Obvious question: could it be observable?
- Interesting underlying physics?

One argument in favor of the BD state is that it gives rise to a thermal spectrum from the point of view of a free falling observer

Turning this argument around: do we really expect the particle spectrum to be exactly thermal?

De Sitter tunneling

Temperature in dS can be described explicitly in terms of tunneling through the dS horizon

- Energy conservation: spectrum cannot be precisely thermal
  - Energy of emitted quanta cannot exceed Narai bound
  - De Sitter horizon ‘relocates’ after emitting quanta
- Effect of backreaction can be incorporated for (dominant) s-waves

$$\Gamma_k = N\,\exp\left(-2\,\mathrm{Im}\int dt L(\omega_k)\right) = N\,\exp\Delta S(\omega_k) \approx N\,\exp\left(-\beta\omega_k\right)$$

with $L(\omega_k) = p\dot{r} - \omega_k$ and $\beta = 1/T_{GH}$

Kraus, Wilczek, `gr-qc/9408003`
Parikh, Wilczek, `hep-th/9907001`
Parikh, `hep-th/0204107`

De Sitter Vacua

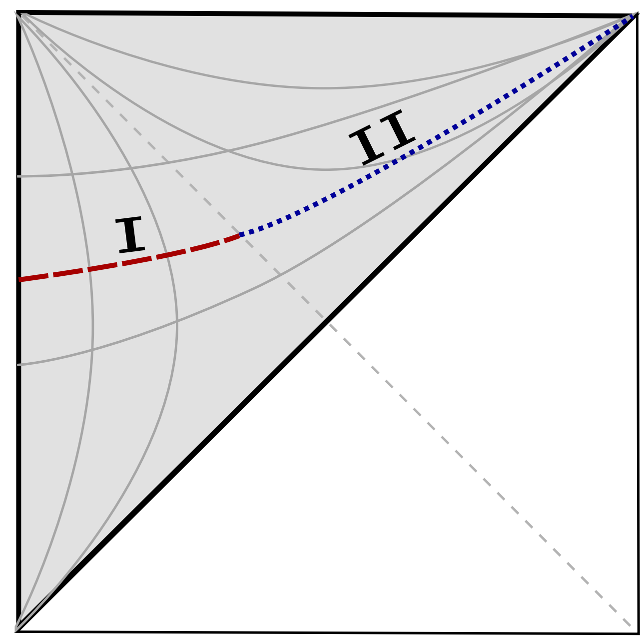
We will be assuming Painlevé coordinates covering dS

$$ds^2 = -\left(1 - H^2 r^2\right)\, dt^2 - 2Hr\, dr\, dt + \, dr^2 + r^2\, d\Omega^2$$

with $t_{\text{Painleve}} = t_{\text{planar}} = t_{\text{static}} + \frac{1}{2H}\ln(1 - H^2r^2)$ and $r_{\text{Painleve}} = r_{\text{static}} = \rho_{\text{planar}}\, e^{Ht}$

These coordinates are useful for several reasons

- Cover the full planar patch
- Between static and planar
  - Planar sections of constant t
  - Static sections of constant r
- Stationary



De Sitter Vacua

Introduce $|\mathrm{out}\rangle = |I\rangle \otimes |II\rangle$, $|\mathrm{BD}\rangle$ and $|\mathrm{BD}'\rangle$, then with $b_k|\mathrm{out}\rangle = 0$ the probability for particle creation can be written as

$$\left| \frac{\langle \mathrm{out}|\, b_k\, b_{-k}\, |\mathrm{BD}\rangle}{\langle \mathrm{out}|\mathrm{BD}\rangle} \right|^2 = e^{-\beta\omega_k}\,, \quad \left| \frac{\langle \mathrm{out}|\, b_k\, b_{-k}\, |\mathrm{BD}'\rangle}{\langle \mathrm{out}|\mathrm{BD}'\rangle} \right|^2 = e^{\Delta S(\omega_k)}$$

- The state $|I\rangle$ should be thought of as the static vacuum
  - Consequently $|II\rangle$ corresponds to a 2nd static vacuum
  - Annihilation operators each act on different sides

Exercise: determine the Bogolyubov rotation between the $|\mathrm{BD}\rangle$ and $|\mathrm{BD}'\rangle$ state

$$a'_{\,k} = \alpha_k^*\, a_k - \beta_k^*\, a^\dagger_{-k} \quad \text{with} \quad a_k|\mathrm{BD}\rangle = a'_k|\mathrm{BD}\rangle = 0$$

- Transformations are diagonal because all states are defined in the same patch
- The Bogolyubov transformations wrt the $|\mathrm{out}\rangle$ state are known

Relating the states

$$\tilde{\gamma}_k \equiv -\frac{\tilde{\beta}_k^*}{\tilde{\alpha}_k} = \frac{\langle \mathrm{out}|\, b_k\, b_{-k}\, |\mathrm{BD}\rangle}{\langle \mathrm{out}|\mathrm{BD}\rangle} \quad , \quad \tilde{\gamma}_k' \equiv -\frac{\tilde{\beta}_k'^*}{\tilde{\alpha}_k'} = \frac{\langle \mathrm{out}|\, b_k\, b_{-k}\, |\mathrm{BD}'\rangle}{\langle \mathrm{out}|\mathrm{BD}'\rangle}$$

After some basic linear manipulations one obtains

$$\alpha_k^* = \tilde{\alpha}_k' \tilde{\alpha}_k^* - \tilde{\beta}_k'^* \tilde{\beta}_k \quad , \quad \beta_k^* = \tilde{\alpha}_k' \tilde{\beta}_k^* - \tilde{\beta}_k'^* \tilde{\alpha}_k$$

Defining $\gamma_k = -\frac{\beta_k^*}{\alpha_k}$ then gives $\gamma_k = \frac{\tilde{\gamma}_k - \tilde{\gamma}_k'}{1 - \tilde{\gamma}_k \tilde{\gamma}_k'}$

- Nontrivial phases are absent
- Consequence of tunneling description and consistent with α-state calculations

Since $|\tilde{\gamma}_k|^2 = e^{-\beta\omega_k}$ and $|\tilde{\gamma}_k'|^2 = e^{\Delta S(\omega_k)}$ we find, after expanding to leading order,

$$|\gamma_k| \approx \frac{e^{\pi \frac{\omega_k}{H}}}{e^{2\pi \frac{\omega_k}{H}} - 1} \, \frac{\omega_k H}{8 M_p^2}$$

Primordial spectrum

Our claim: the state $|\mathrm{BD}'\rangle$ is the 'true' inflationary vacuum

For small corrections we then immediately find

$$P(k) \approx P_{BD}\left[1 + \frac{1}{4e^{\pi}}\left(\frac{H(k)}{M_p}\right)^2\right]$$

- Evaluated at time when a mode crosses the horizon, i.e when $\omega_k = H$
- Absence of k-dependent phases, so no modulation
- Tiny effect, already constrained at the $H^2/M_p^2 \lesssim 10^{-10}$ level
- Indistinguishable from a (specific) dimension 6 operator correction

Final thoughts

Some remarks and questions

- Obviously too small an effect to be observable
- Nevertheless formally interesting
  - Model independent, i.e. universal
  - Backreaction effect
  - Explicitly calculable
- Effective field theory interpretation?
- Breaks de Sitter $SO(1,4)$ isometries (spontaneously)
  - Consequences for de Sitter stability?

Finding models that can make testable predictions remains an important challenge