Microstates of D1-D5-P-KK monopole System*

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Plan

1. Mathur Conjecture and the D1-D5 system

2. Adding KK monopoles to D1-D5-P:

   - The idea

   - Formalism of minimal supergravity

   - Choice of base metric

   - Singularity Analysis

3. Properties of the solution

4. Open questions and future directions

Mathur Conjecture

- The strong form: States in the CFT which admit a supergravity dual do so via geometries which are smooth upto acceptable orbifold singularities. In particular they do not have horizons.

- Motivation for the conjecture arose from the study of the Ramond ground states of D1-D5 CFT.

- Mathur and Lunin constructed a large class of smooth classical solutions describing the bound states of D1 and D5 branes in terms of chiral null models.

- The simplest and most important of these states had been found earlier (Balasubramanian *et al.*, Maldacena and Maoz). This was the gravity dual (of the spectral flow) of the NS vacuum. (BdKRMM solution!)

- Evidence from D1-D5 with momentum: Perturbative calculations had suggested that atleast some states of D1-D5-P might be smooth (Mathur,A.S. & Srivastava).

- Some explicit solutions were later found which described multiple spectral flows of the NS vacuum and carried a large amount of momentum and angular momentum (Lunin, GMS).

- Progress thus far had been restricted to D1-D5. Bena and Kraus (2005) produced the first example of a microstate for the four dimensional system with D1, D5 and KK monopole charges.

- The Bena-Kraus solution is the four dimensional analogue of the BdKRMM solution. It is smooth (in 10D) upto orbifold singularities.

- A natural question is whether one can generate solutions carrying both momentum and KK-monopoles (microstates of D1-D5-KK-P).

Adding KK to D1-D5-P

- The trick is to use the connection between 5D and 4D BPS objects proposed by Gaiotto, Strominger and Yin.

- In this way, one can think of the BK geometry(4D) as being the BdKRMM geometry(5D) wearing a Taub-NUT "hat".

- The way to add momentum to D1-D5-KK is then to start with the known D1-D5-P solution and embed it in Taub-NUT (and hope fervently that it does not produce severe singularities).

- The tools one needs to do this are provided by the known BPS solutions of minimal supergravity in D=6 (Gutowski, Martelli & Reall).

BPS solutions of minimal sugra

- For time independent solutions in minimal supergravity the equations of motion simplify considerably. The ansatz for the metric is

$$ds^2 = -\frac{1}{Z_1 Z_P}(dt+k)^2+\frac{Z_P}{Z_1}\left(dy+(1-Z_P^{-1})dt+\omega_P-\frac{k}{Z_P}\right)^2+Z_1 h_{mn}dx^m d$$

- Here $h_{mn}dx^m dx^n$ is the metric on a four dimensional hyperkahler manifold $\mathcal{M}$. Z_1, Z_P are functions while k and ω_P are one forms on $\mathcal{M}$.

- We will choose the base metric to be of the Gibbons-Hawking form. We set

$$ds^2_{\mathcal{M}} = V^{-1}(dz+\chi)^2+V(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2)$$

- However it is important that we admit base metrics which are not strictly GH. We will allow the function V to become negative in some compact region of $\mathcal{M}$. This kind of behavior is needed to describe the known D1-D5-P solution.

- This relaxation in general will lead to severly singular base metrics. If V can change sign in some region, the metric on $\mathcal{M}$ changes signatures from (4,0) to (0,4)!

- This singularity of the base metric will be very nicely resolved in the full six dimensional geometry.

- In case the full 6D solution preserves the isometry along z, the equations of motion can be completely solved in terms of harmonic functions on *three* dimensional flat space.

- Most of the solutions considered so far satisfy this property. The BdKRMM geometry needs a flat base space ($V = 1/r$) while the BK geometry has Taub-NUT, $V = 1 + Q/r$.

- For Taub-NUT the extra 1 in V allows it to interpolate between $\mathbb{R}^4$ in the interior and $\mathbb{R}^3 \times S^1$ asymptotically. In this sense the interior of BK is identical to the interior of BdKRMM.

- In the D1-D5-P geometry the base metric is more complicated. One finds (Giusto & Mathur) $V = \gamma_1/r + \gamma_2/r_c$ where $r_c = \sqrt{r^2 + c^2 + 2cr\cos\theta}$.

- We expect that in order to add KK-monopoles we keep the inner behaviour of V same while altering the asymptotics by adding 1.

- With this choice of base metric one needs to recalculate all the functions appearing in the metric.

- The pole structure of the various harmonic functions is chosen to coincide with the ones from D1-D5-P solution while allowing for the residues to be modified.

- This leads to a seventeen parameter family of solutions in general! However, demanding that the solutions be non-singular fixes all the free parameters upto a discrete choice.

- The resulting solutions are parametrized by two integers ($n \in \mathbb{Z}$ and $m \in \mathbb{Z}_+$), charges Q_1, Q_5 and Q_K. The momentum charge is not arbitrary and is fixed in terms of the above parameters.

- The case $n = 0$ corresponds to the geometry found by Bena and Kraus.

Properties of the solution

Orbifold Singularities

- If m is relatively prime to $N_K(n+1)$ where N_K is the number of KK monopoles, there are no singularities at $r = 0$. Otherwise there are orbifold singularities of order l where l is the highest common factor of m and $N_K(n+1)$.

- Similarly, at $r_c = 0$ there are orbifold singularities if m has common factors with nN_K.

- Absence of horizons: After reduction to four dimensions we find that g^{rr} vanishes only at $r, r_c = 0$. However, a full analysis of the metric around these points reveals that there are only the above mentioned orbifold singularities and no horizons.

Charges and Angular Momentum

- Demanding regularity of the solution fixes the momentum charge to be $Q_P = \frac{4n(n+1)Q_1^2 Q_K}{m^2 R_y^2}$. Note that for fixed values of Q_1, Q_K and R_y the momentum charge is allowed to take only a discrete set of values.

- The angular momentum is fixed to be $J = -\frac{2Q_1^2 Q_K}{mR_y}$. It is independent of n.

- The solution also posseses a KK electric charge. The value of this charge in the decoupling limit is $Q_e = \frac{2(2n+1)Q_1^2}{mR_y}$.

- The core of the geometry is $AdS_3 \times (S^3/\mathbb{Z}_{N_K})$ for $m = 1$ as would be expected for N_K coincident KK monopoles.

Summary, Open Questions and Future Directions

- We have succeeded in constructing a smooth, rotating four charge solution carrying D1, D5, KK monopole and momentum charges.

- However, we do not have a good CFT understanding of the underlying state. The $(0, 4)$ CFT underlying the D1-D5-KK system is not known explicitly.

- If the geometry we have constructed is truly a bound state of the branes one can learn a bit more about the CFT from the properties of the solution. In particular it seems that the nonsupersymmetric side of the CFT might admit a "spectral flow" with the role of R-charge being played by the KK electric charge.

- The BK geometry as well as the geometry constructed here are axially symmetric. It would be nice to be able to find more general solutions having less symmetry. This problem would be more tractable for the case without momentum.