# Non-Gaussianities in String Inflation

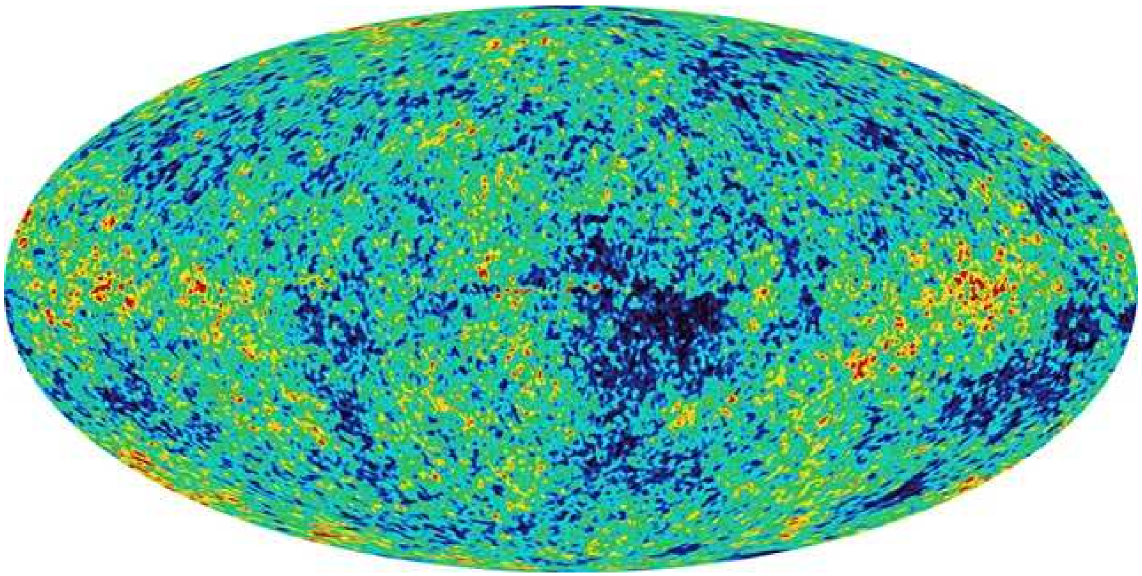
**Gary Shiu**

**University of Wisconsin, Madison**

Frontiers in String Theory Workshop
Banff, February 13, 2006

Collaborators: X.G. Chen, M.X. Huang, S. Kachru

# Introduction



from WMAP

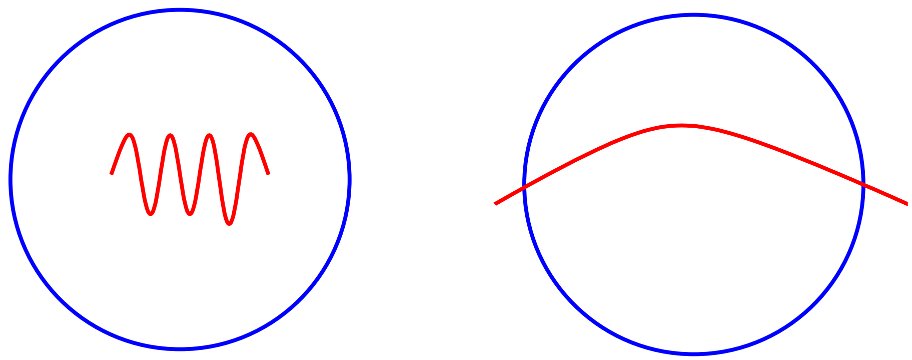
- An almost **scale invariant, adiabatic, Gaussian** primordial fluctuation predicted by **inflation** is in good agreement with CMB data.

- A tantalizing upper bound on the energy density during inflation:

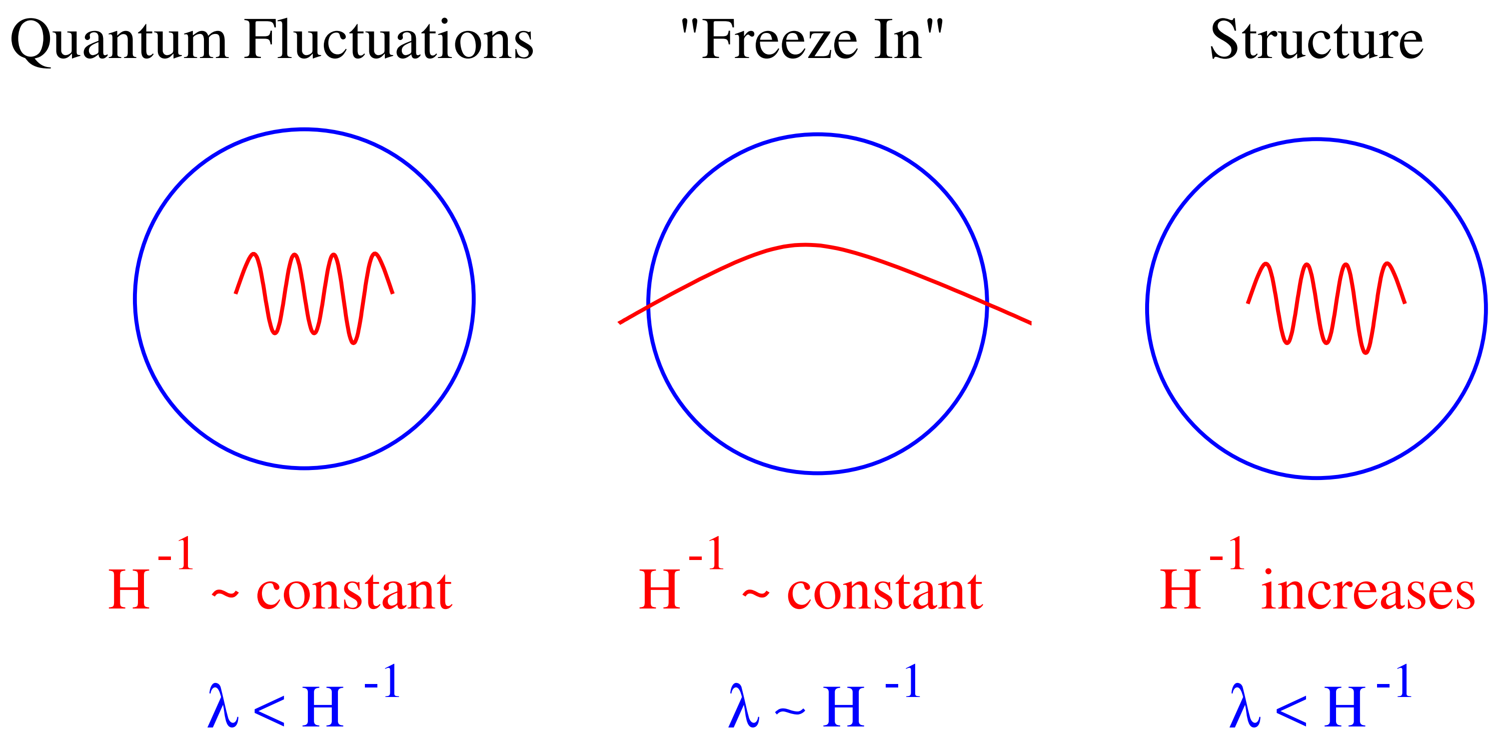
$$V \sim M_{GUT}^4 \sim (10^{16} GeV)^4 \qquad \text{i.e.,} \qquad H \sim 10^{14} GeV.$$

  The relevant energy scale is close to the scale where **stringy physics** becomes important.

# Inflation as a Probe of Stringy Physics

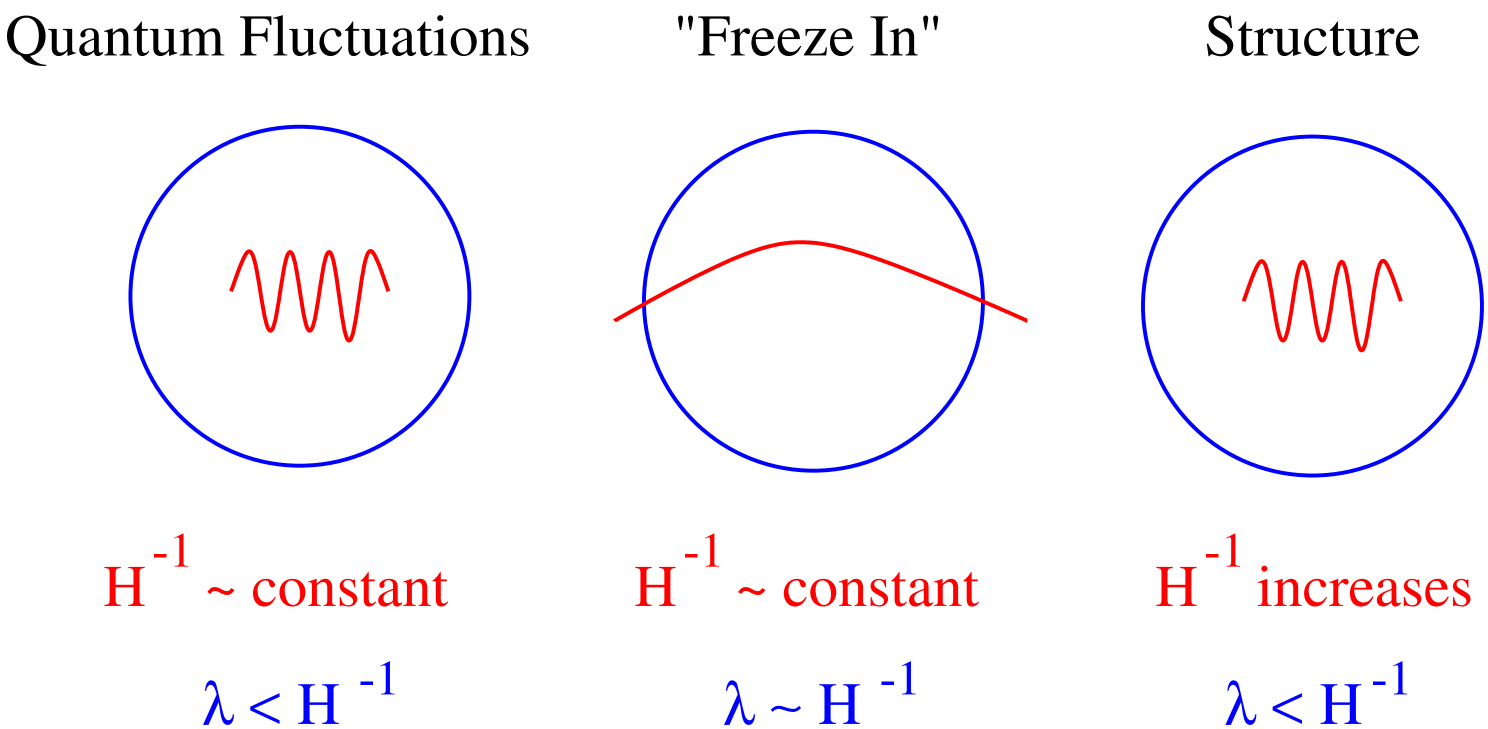


# Inflation as a Probe of Stringy Physics




- Imprints of short distance physics. [More later in this workshop]

# Inflation as a Probe of Stringy Physics




- Imprints of short distance physics. [More later in this workshop]

- Studies focused mainly on the **power spectrum**: primordial non-Gaussian fluctuations predicted by inflation are typically too small.

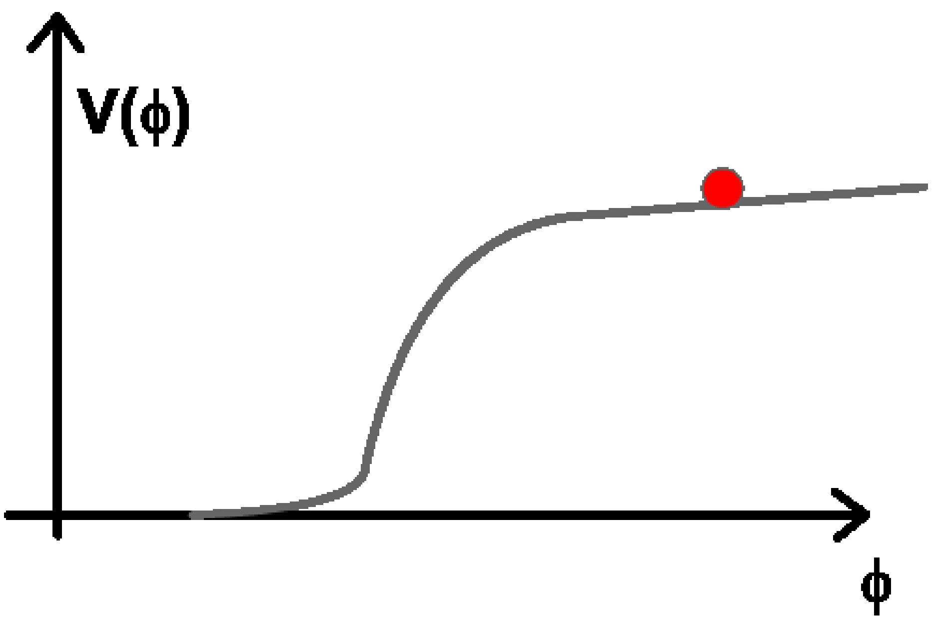
# String Inflation

A plethora of string inflationary models. **Two broad classes:**

# String Inflation

A plethora of string inflationary models. **Two broad classes:**

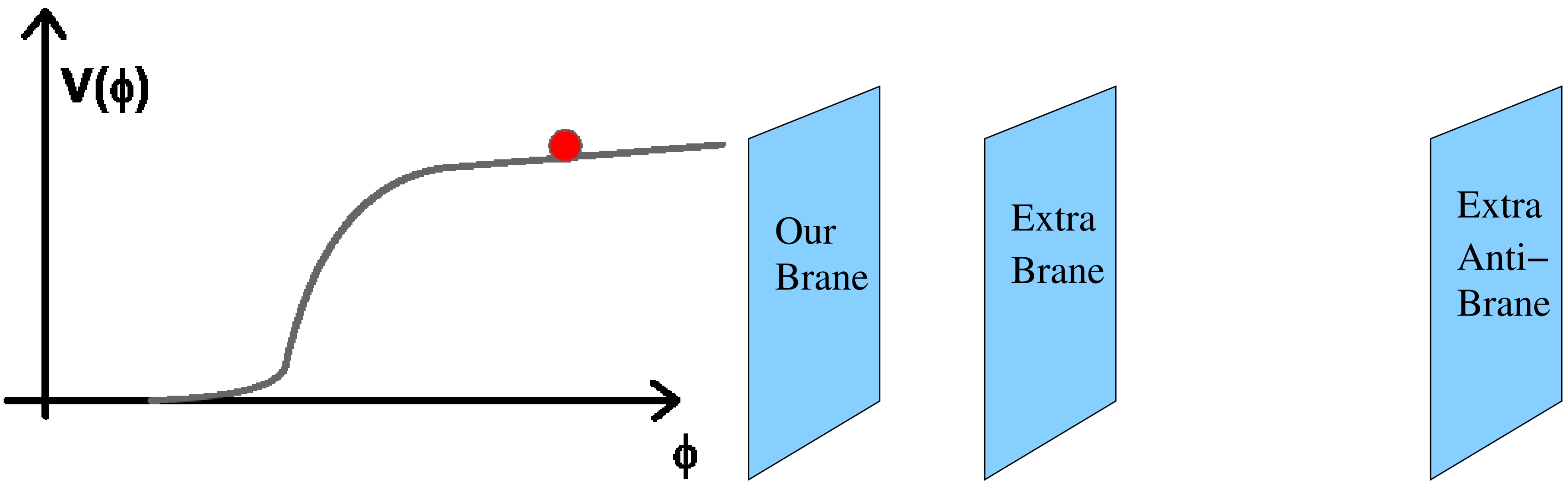
**Slow roll inflation** (e.g., KKLMMT):



# String Inflation

A plethora of string inflationary models. **Two broad classes:**

**Slow roll inflation** (e.g., KKLMMT):

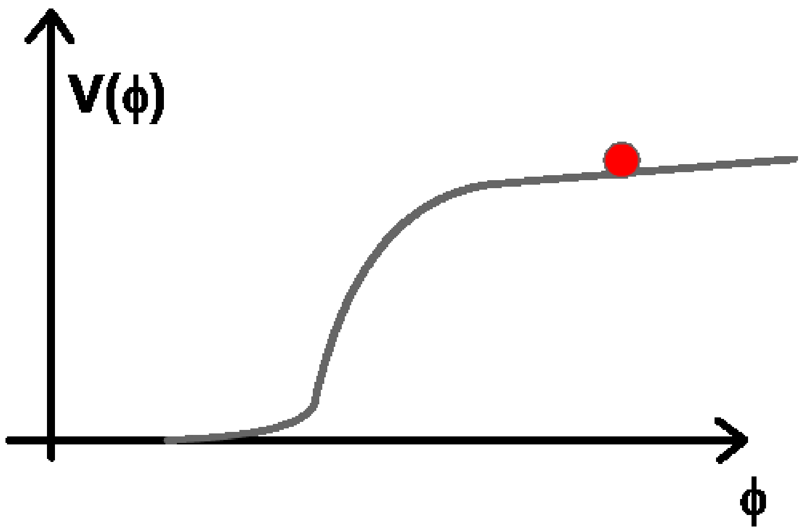


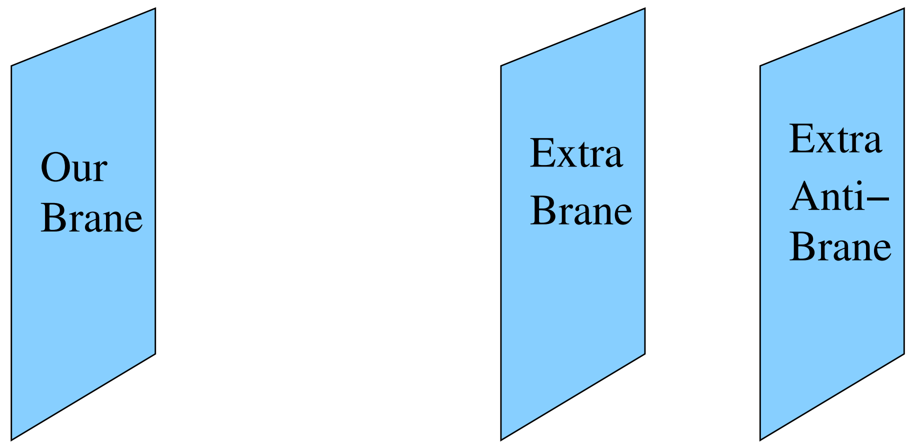

**Brane inflation** Dvali and Tye

# String Inflation

A plethora of string inflationary models. **Two broad classes:**

**Slow roll inflation** (e.g., KKLMMT):



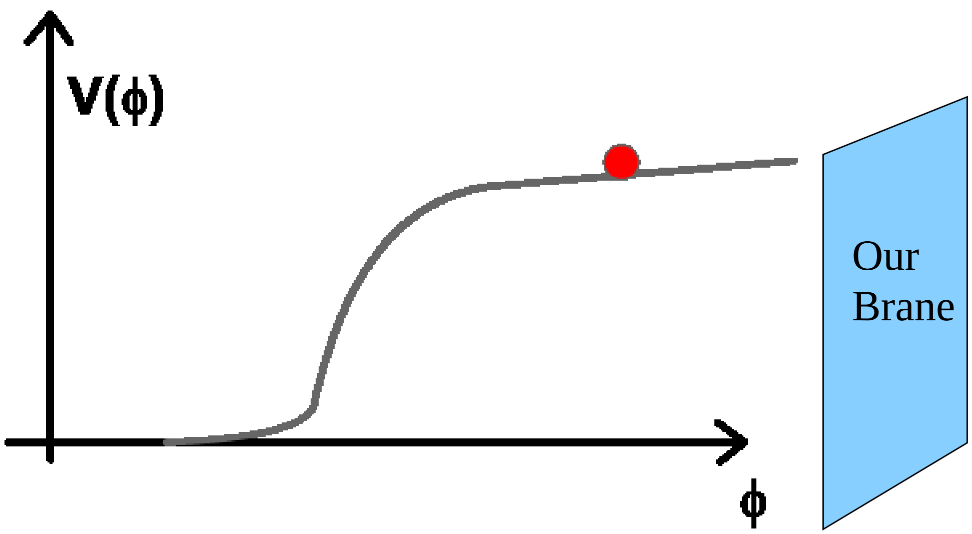



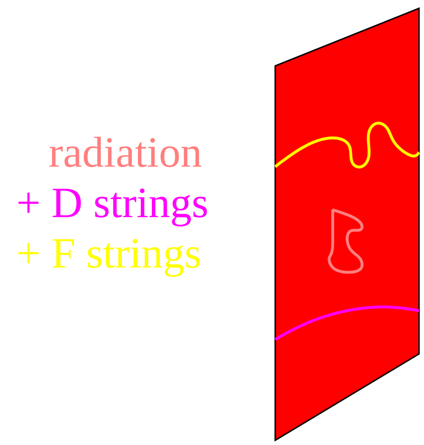

**Brane inflation** Dvali and Tye

# String Inflation

A plethora of string inflationary models. **Two broad classes:**

**Slow roll inflation** (e.g., KKLMMT):



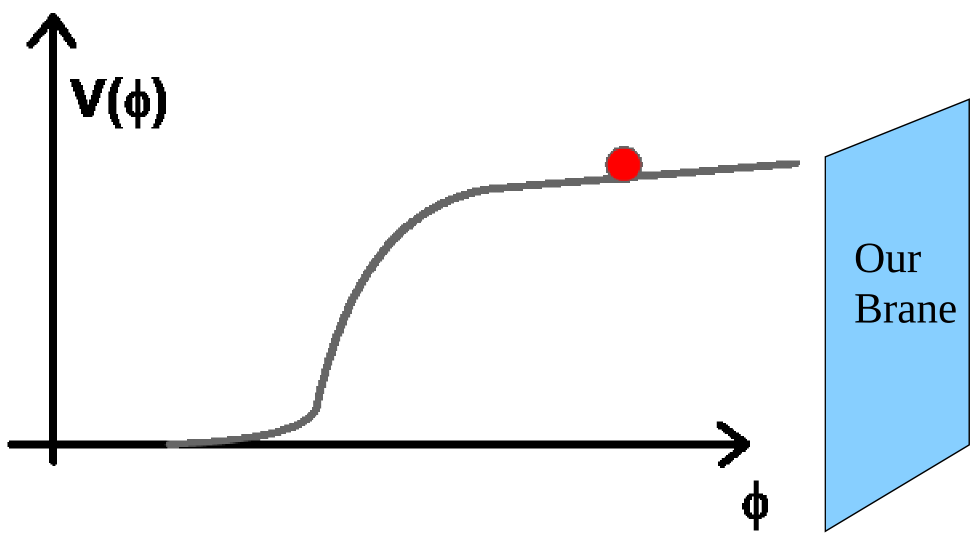

**Brane inflation**

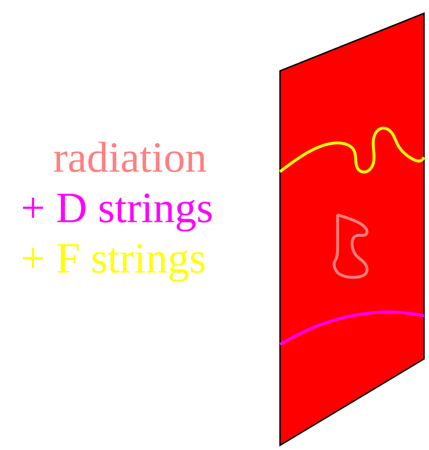
Dvali and Tye

# String Inflation

A plethora of string inflationary models. **Two broad classes:**

**Slow roll inflation** (e.g., KKLMMT):



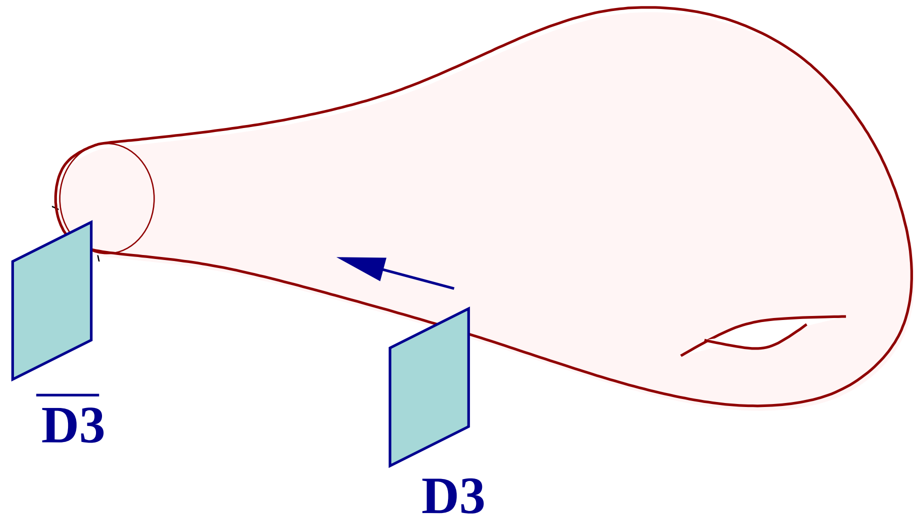




Brane inflation

Dvali and Tye

**Flux compactification**: stabilizes moduli and generates warped throats.

**Warped throats:**




- Help flatten the potential, though some degree of fine-tuning is still needed : usual $\eta$ problem.

- Reheating and suppression of gravitational wave production.

Barneby, Burgess, and Cline
Kofman and Yi
Chialva, GS, and Underwood
Frey, Mazumdar, and Myers

**DBI Inflation** [Silverstein and Tong]

- **Different regime:** higher derivative terms enforce a casual speed limit.

$$S_{\mathrm{DBI}} = d^4x\sqrt{-g}\left[T\sqrt{1-\dot{\phi}^2/T} + V(\phi) - T\right]$$

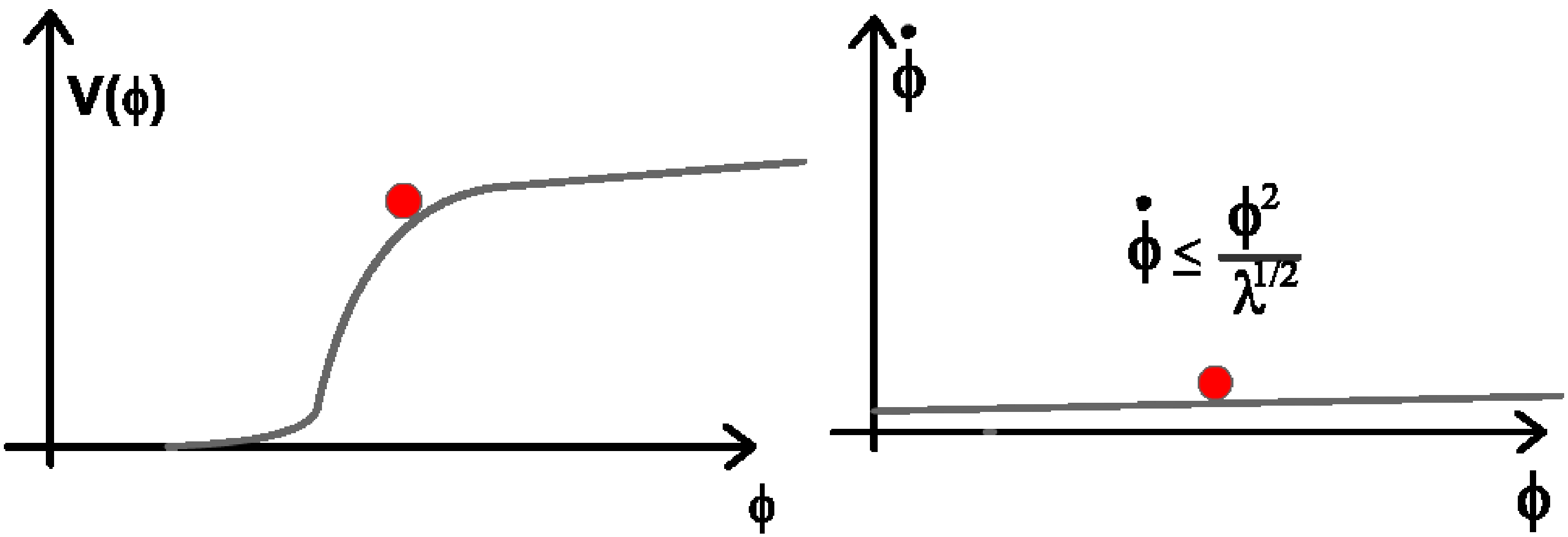
where $T = T_3 h(\phi)^4$ with $T_3$ = D3 tension, $h(\phi)$ is the warping factor.

**DBI Inflation** [Silverstein and Tong]

- **Different regime:** higher derivative terms enforce a casual speed limit.

$$S_{\text{DBI}} = d^4x\sqrt{-g}\left[T\sqrt{1-\dot{\phi}^2/T} + V(\phi) - T\right]$$

where $T = T_3 h(\phi)^4$ with $T_3 =$ D3 tension, $h(\phi)$ is the warping factor.



- **Distinctive signatures:**

Alishahiha, Silverstein, and Tong

See also: Chen

  - Large Non-Gaussianties with a characteristic **shape** of $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$:

$$f_{NL} \sim \gamma^2 \qquad \gamma = \frac{1}{\sqrt{1 - \dot{\phi}^2/T}}$$

    The numerical coefficient of $\gamma^2$ in $f_{NL}$ is 0.32. [Chen, Huang, GS]

  - Modified "consistency relation":

$$\frac{P_k^h}{P_k^\zeta} = -\frac{8}{\gamma} n_T$$

- **Distinctive signatures:** Alishahiha, Silverstein, and Tong

  See also: Chen

  - Large Non-Gaussianties with a characteristic **shape** of $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$:

$$f_{NL} \sim \gamma^2 \qquad \gamma = \frac{1}{\sqrt{1 - \dot{\phi}^2/T}}$$

    The numerical coefficient of $\gamma^2$ in $f_{NL}$ is 0.32. [Chen, Huang, GS]

  - Modified "consistency relation":

$$\frac{P_k^h}{P_k^\zeta} = -\frac{8}{\gamma} n_T$$

- **More generally:** by varying $\gamma$, one can interpolate between slow roll ($\gamma \sim 1$) to DBI inflation ($\gamma >> 1$) including intermediate regime ($\gamma \gtrsim 1$).

  Inflation in this general setup is robust. Shandera and Tye

# Our Results

- **General analysis** for an arbitrary action of the form:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(X,\phi)] \qquad X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- Our results are applicable to the **intermediate regime**, as well as in extracting **subleading** (but potentially observable) non-Gaussianities.

- We obtain all known shapes of non-Gaussianities plus more.

- Laboratory for testing the dS/CFT proposal Strominger
[Larsen, van der Schaar, Leigh]; [Maldacena]; [Larsen, McNees]; [van der Schaar]
[Bousso, Maloney, Strominger] . . .

$$\langle f_{\mathbf{k}_1} f_{\mathbf{k}_2} f_{\mathbf{k}_3}\rangle' = \frac{2Re\langle \mathcal{O}_{\mathbf{k}_1}\mathcal{O}_{\mathbf{k}_2}\mathcal{O}_{\mathbf{k}_3}\rangle'}{\prod_i(-2Re\langle \mathcal{O}_{\mathbf{k}_i}\mathcal{O}_{\mathbf{k}_i}\rangle')}$$

Maldacena, astro-ph/0210603

# The General Setup

- Consider an action of the form:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(X,\phi)]$$

whose Gaussian perturbations have been considered by Garriga and Mukhanov.

# The General Setup

- Consider an action of the form:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(X,\phi)]$$

whose Gaussian perturbations have been considered by Garriga and Mukhanov.

- Define the energy $E$ and the sound speed $c_s$ as

$$E = 2XP_{,X} - P$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

# The General Setup

- Consider an action of the form:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(X,\phi)]$$

whose Gaussian perturbations have been considered by Garriga and Mukhanov.

- Define the energy $E$ and the sound speed $c_s$ as

$$\begin{aligned} E &= 2XP_{,X} - P \\ c_s^2 &= \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} \end{aligned}$$

- Friedman equation and the continuity equation

$$\begin{aligned} 3M_{pl}^2 H^2 &= E \\ \dot{E} &= -3H(E+P) \end{aligned}$$

# The General Setup

- Consider an action of the form:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(X,\phi)]$$

whose Gaussian perturbations have been considered by Garriga and Mukhanov.

- Define the energy $E$ and the sound speed $c_s$ as

$$E = 2XP_{,X} - P$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

- Friedman equation and the continuity equation

$$3M_{pl}^2 H^2 = E$$

$$\dot{E} = -3H(E+P)$$

- For a Lagrangian with standard kinetic term: $P(X,\phi) = X - V(\phi)$ and hence $c_s = 1$. For DBI action, $c_s = 1/\gamma$.

- Generalized slow roll parameters:

$$\begin{aligned}\epsilon &= -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{M_{pl}H^2},\\ \eta &= \frac{\dot{\epsilon}}{\epsilon H},\\ s &= \frac{\dot{c}_s}{c_s H}.\end{aligned}$$

- Generalized slow roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{M_{pl}H^2},$$
$$\eta = \frac{\dot{\epsilon}}{\epsilon H},$$
$$s = \frac{\dot{c}_s}{c_s H}.$$

- The non-Gaussianities that we found depend on **5 parameters**:

$c_s^{-2}$, $\epsilon$, $\eta$, $s$, and $\lambda/\Sigma$ (to be defined).

Potentially observable when these parameters are sufficiently large.

# Non-Gaussianities

- Primordial power spectrum:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \sim \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \frac{P_k^\zeta}{k_1^3}$$

- Non-Gaussianity contains potentially more info because of its **shape**:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

- Scaling and symmetries imply that $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is a symmetric, homogeneous function of degree $-6$.

- Primordial non-Gaussianities come from **cubic terms** in the Lagrangian.

- It is useful to work in the ADM metric formalism:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

where inflaton $\phi$ and metric $h_{ij}$ are dynamical variables, $N$ and $N^i$ are Lagrange multipliers.

- It is useful to work in the ADM metric formalism:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

where inflaton $\phi$ and metric $h_{ij}$ are dynamical variables, $N$ and $N^i$ are Lagrange multipliers.

- Focus on scalar perturbations. In the **comoving gauge**:

$$\delta\phi = 0 \qquad h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

with $\zeta$ being the gauge invariant scalar perturbation which remains constant outside horizon. Calculations greatly simplified.

- It is useful to work in the ADM metric formalism:

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

where inflaton $\phi$ and metric $h_{ij}$ are dynamical variables, $N$ and $N^i$ are Lagrange multipliers.

- Focus on scalar perturbations. In the **comoving gauge**:

$$\delta\phi = 0 \qquad h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

with $\zeta$ being the gauge invariant scalar perturbation which remains constant outside horizon. Calculations greatly simplified.

- To obtain the cubic terms: we plug the metric ansatz into the Lagragian, and substitute the Lagrangian multipliers $N$ and $N^i$ with the solutions to their equations of motion.

- It is useful to work in the ADM metric formalism:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

where inflaton $\phi$ and metric $h_{ij}$ are dynamical variables, $N$ and $N^i$ are Lagrange multipliers.

- Focus on scalar perturbations. In the **comoving gauge**:

$$\delta\phi = 0 \qquad h_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

with $\zeta$ being the gauge invariant scalar perturbation which remains constant outside horizon. Calculations greatly simplified.

- To obtain the cubic terms: we plug the metric ansatz into the Lagragian, and substitute the Lagrangian multipliers $N$ and $N^i$ with the solutions to their equations of motion.

- To compute the effective action to order $\mathcal{O}(\zeta^3)$, we need only the solutions of $N$ and $N^i$ to order $\mathcal{O}(\zeta)$. Maldacena

- The solution to the equation of motion for scalar perturbation $\zeta(t, \mathbf{k})$ at quadratic order gives the power spectrum.

- The primordial non-Gaussianities are:

$$\langle \zeta(t, \mathbf{k}_1)\zeta(t, \mathbf{k}_2)\zeta(t, \mathbf{k}_3)\rangle = -i \int_{t_0}^{t} dt' \langle [\zeta(t, \mathbf{k}_1)\zeta(t, \mathbf{k}_2)\zeta(t, \mathbf{k}_3), H_{int}(t')]\rangle \ ,$$

where $H_{int}(t)$ is the cubic Hamiltonian for the scalar perturbation $\zeta$.

- The solution to the equation of motion for scalar perturbation $\zeta(t, \mathbf{k})$ at quadratic order gives the power spectrum.

- The primordial non-Gaussianities are:

$$\langle \zeta(t, \mathbf{k}_1)\zeta(t, \mathbf{k}_2)\zeta(t, \mathbf{k}_3)\rangle = -i \int_{t_0}^{t} dt' \langle [\zeta(t, \mathbf{k}_1)\zeta(t, \mathbf{k}_2)\zeta(t, \mathbf{k}_3), H_{int}(t')]\rangle \ ,$$

where $H_{int}(t)$ is the cubic Hamiltonian for the scalar perturbation $\zeta$.

A lot of pain and sweat . . .

# Shape of Non-Gaussianities

$$F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^4 (P_k^\zeta)^2 \frac{1}{\prod_i k_i^3} \times (\mathcal{A}_\lambda + \mathcal{A}_c + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s)$$

where

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) \frac{3k_1^2 k_2^2 k_3^2}{2K^3} ,$$

$$\mathcal{A}_c = \left(\frac{1}{c_s^2} - 1\right) \left(-\frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i\neq j} k_i^2 k_j^3 + \frac{1}{8} \sum_i k_i^3\right) ,$$

$$\mathcal{A}_\epsilon = \frac{\epsilon}{c_s^2} \left(-\frac{1}{8} \sum_i k_i^3 + \frac{1}{8} \sum_{i\neq j} k_i k_j^2 + \frac{1}{K} \sum_{i>j} k_i^2 k_j^2\right) ,$$

$$\mathcal{A}_\eta = \frac{\eta}{c_s^2} \left(\frac{1}{8} \sum_i k_i^3\right) ,$$

$$\mathcal{A}_s = \frac{s}{c_s^2} \left(-\frac{1}{4} \sum_i k_i^3 - \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i\neq j} k_i^2 k_j^3\right) .$$

and $K = k_1 + k_2 + k_3$, $\Sigma = XP_{,X} + 2X^2 P_{,XX}$, $\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX}$.

# Experimental Bound

- WMAP ansatz for the primordial non-Gaussianities

$$\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{NL} (\zeta_g(x)^2 - \langle \zeta_g^2(x) \rangle$$

here $\zeta_g(x)$ is purely Gaussian with vanishing three point functions.

- The size of non-Gaussianities is measured by the parameter $f_{NL}$ in the above ansatz. Current experimental bound is

$$-58 < f_{NL} < 134 \quad \text{at 95\% C.L.}$$

Future experiments can eventually reach the sensitivity of $f_{NL} \lesssim 20$ (WMAP) and $f_{NL} \lesssim 5$ (PLANCK).

# Experimental Bound

- WMAP ansatz for the primordial non-Gaussianities

$$\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{NL} (\zeta_g(x)^2 - \langle \zeta_g^2(x) \rangle$$

  here $\zeta_g(x)$ is purely Gaussian with vanishing three point functions.

- The size of non-Gaussianities is measured by the parameter $f_{NL}$ in the above ansatz. Current experimental bound is

$$-58 < f_{NL} < 134 \quad \text{at 95\% C.L.}$$

  Future experiments can eventually reach the sensitivity of $f_{NL} \lesssim 20$ (WMAP) and $f_{NL} \lesssim 5$ (PLANCK).

- However, the experimental bound depends on the **shape** of $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$.

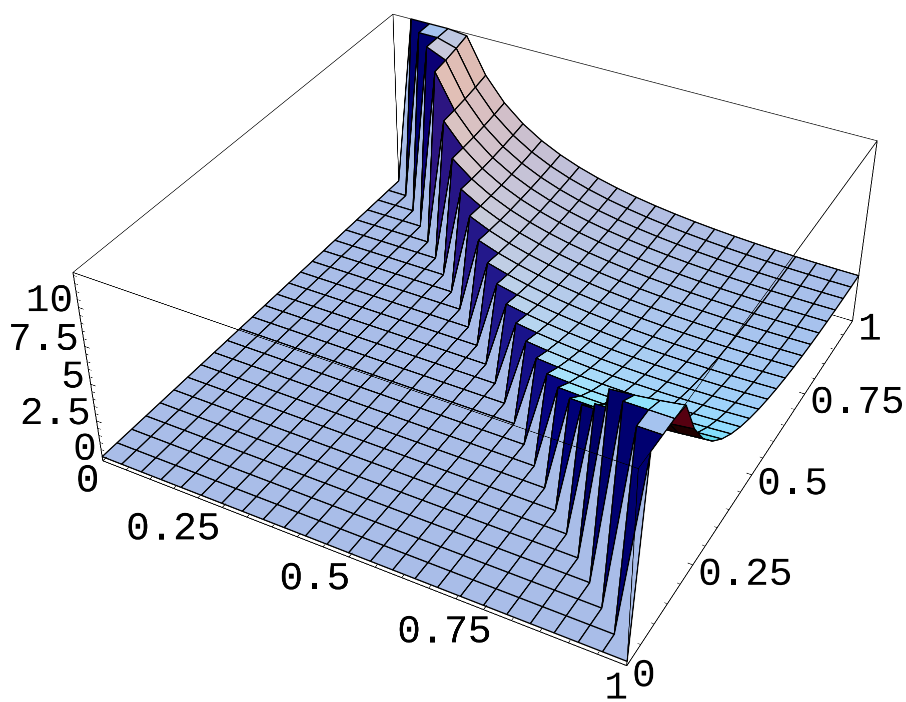
  Creminelli, Nicolis, Senatore, Tegmark, and Zaldarriaga

- Due to the symmetry and scaling property of $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, all info about the shape can be viewed by plotting [Babich, Creminelli, Zaldarriaga]

$$F(1, k_2, k_3)k_2^2 k_3^2$$

- For the WMAP ansatz:

$$F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \sim f_{NL} \left(P_k^\zeta\right)^2 \frac{k_1^3 + k_2^3 + k_3^3}{k_1^3 k_2^3 k_3^3}$$



# Slow Roll Limit

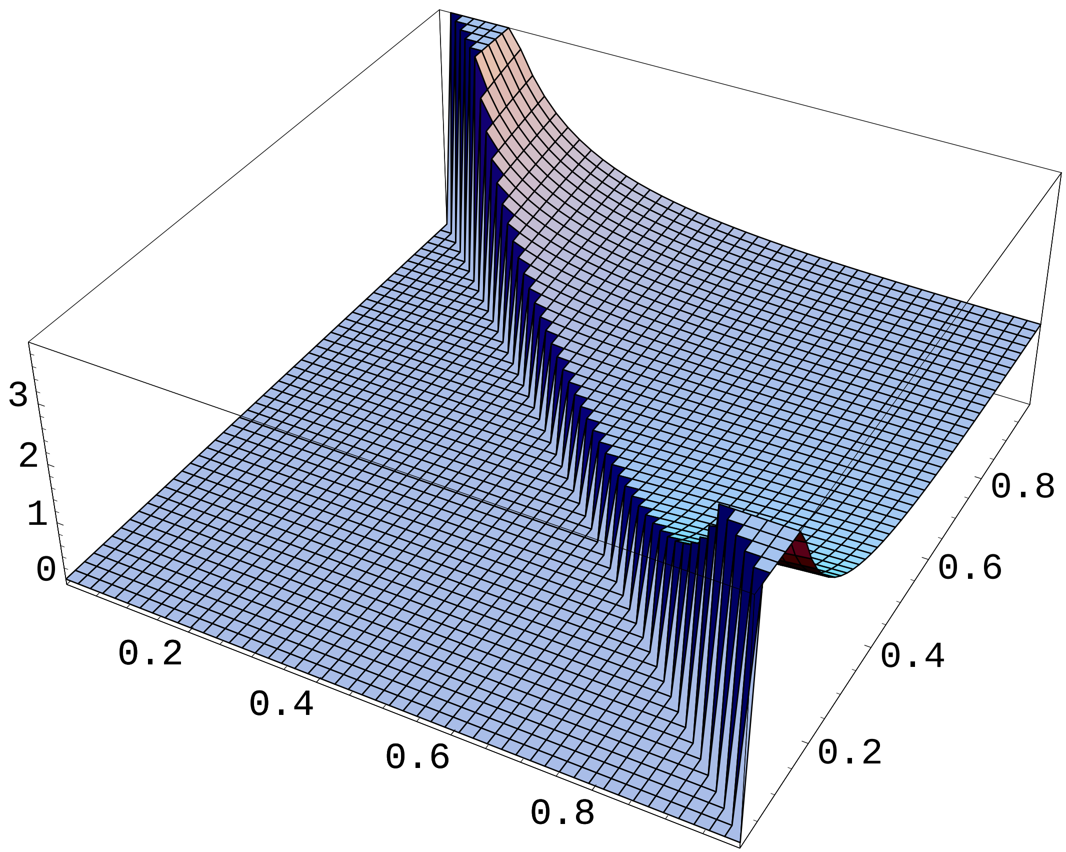
Maldacena
Seery and Lidsey
...

- Slow roll inflation predicts non-Gaussianties of order $f_{NL} \sim \epsilon \ll 1$, which is too small to be observed.

- The relevant shapes are $F(k_1, k_2, k_3) \sim \frac{1}{\prod_i k_i^3} \mathcal{A}(k_1, k_2, k_3)$ where

$$
\begin{aligned}
\mathcal{A}_\epsilon &= \frac{\epsilon}{c_s^2}\left(-\frac{1}{8}\sum_i k_i^3 + \frac{1}{8}\sum_{i\neq j} k_i k_j^2 + \frac{1}{K}\sum_{i>j} k_i^2 k_j^2\right) \ , \\
\mathcal{A}_\eta &= \frac{\eta}{c_s^2}\left(\frac{1}{8}\sum_i k_i^3\right) \ .
\end{aligned}
$$

- The shape of $\mathcal{A}_\epsilon$ is




Similar shape for $\mathcal{A}_\eta$.

- The shapes of slow roll inflation look similar to that of the WMAP ansatz: peak at the "squeeze limit".

# Consisteny Relation for Non-Gaussianities

Maldacena

- In the "squeeze triangle limit": one momentum mode is much smaller than the other two:

$$k_3 \ll k_1, k_2 \quad \mathbf{k}_1 \sim -\mathbf{k}_2$$

# Consisteny Relation for Non-Gaussianities

Maldacena

- In the "squeeze triangle limit": one momentum mode is much smaller than the other two:

$$k_3 \ll k_1, k_2 \quad \mathbf{k}_1 \sim -\mathbf{k}_2$$

- During inflation, the comoving Hubble scale decreases with time. The long wavelength mode $k_3$ crosses the horizon much earlier than the other two modes $k_1, k_2$.

# Consisteny Relation for Non-Gaussianities

Maldacena

- In the "squeeze triangle limit": one momentum mode is much smaller than the other two:

$$k_3 \ll k_1, k_2 \quad \mathbf{k}_1 \sim -\mathbf{k}_2$$

- During inflation, the comoving Hubble scale decreases with time. The long wavelength mode $k_3$ crosses the horizon much earlier than the other two modes $k_1, k_2$.

- After horizon crossing, the long wavelength mode $k_3$ acts as background whose effect is to introduce a time variation at which $k_{1,2}$ cross the horizon.

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \sim \langle \zeta_{\mathbf{k}_3} \zeta_{-\mathbf{k}_3} \rangle \frac{d}{d \ln k_1} \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \sim (n_s - 1) \frac{1}{k_1^3} \frac{1}{k_3^3}$$

# DBI Limit

Alishahiha, Silverstein, and Tong

Chen

- Non-Gaussianities are generically quite large

$$f_{NL} \sim \frac{1}{c_s^2}$$

# DBI Limit

Alishahiha, Silverstein, and Tong

Chen

- Non-Gaussianities are generically quite large

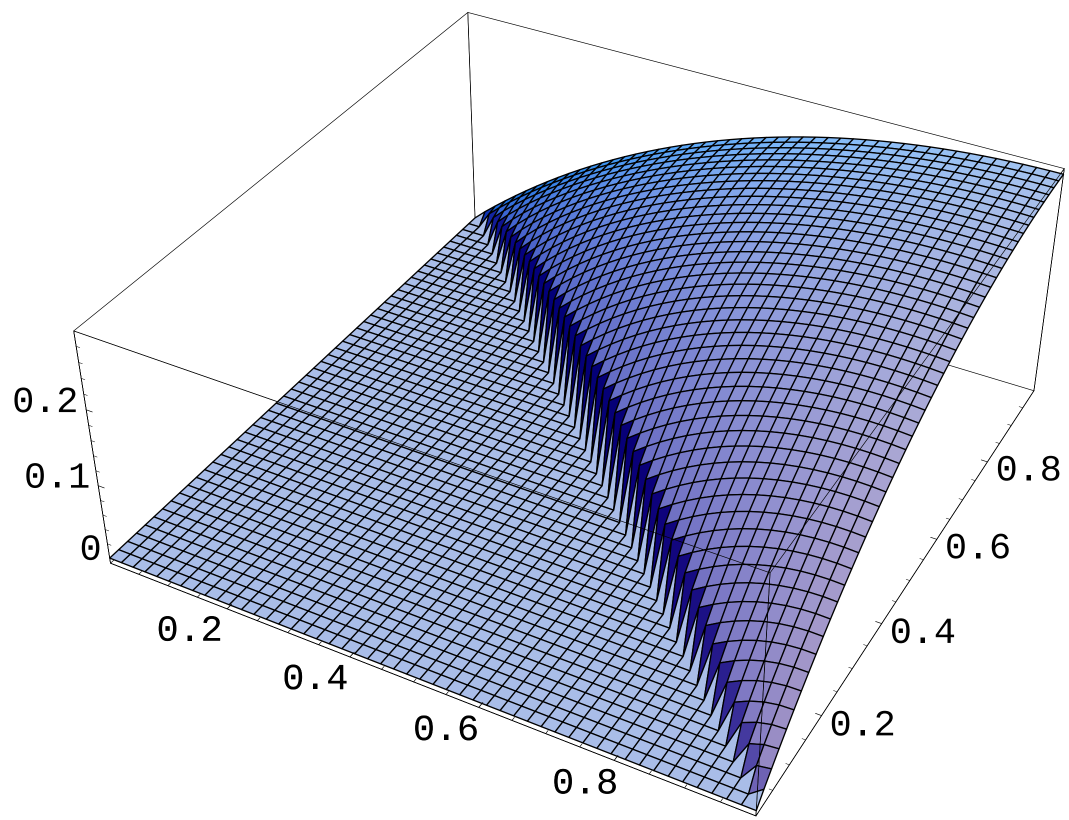
$$f_{NL} \sim \frac{1}{c_s^2}$$

- The shape of non-Gaussianities vanishes in the squeeze triangle limit $k_3 \ll k_1, k_2$. This is required by Maldacena's consistency relation:

$$F(k_1, k_2, k_3) k_1^3 k_3^3 \sim n_s - 1 \sim \epsilon$$

This contradicts that the non-Gaussianities are large, unless the shape vanishes in the squeeze limit.

- The shape of non-Gaussianities for DBI inflation




- Peak at the equilateral triangle limit.

- $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ vanishes in the squeeze limit: higher derivative interactions favor correlations between modes with comparable wavelengths.

# More Shapes

- Another shape of potentially large non-Gaussianities:

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) \frac{3k_1^2 k_2^2 k_3^2}{2(k_1 + k_2 + k_3)^3}$$

Gruzinov

# More Shapes

- Another shape of potentially large non-Gaussianities:

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) \frac{3k_1^2 k_2^2 k_3^2}{2(k_1 + k_2 + k_3)^3}$$

Gruzinov

- The Gaussianities are not multiplied by any slow roll parameter, so can be potentially large. The shape also vanishes in the squeeze limit due to the consistency relation of Maldacena.

# More Shapes

- Another shape of potentially large non-Gaussianities:

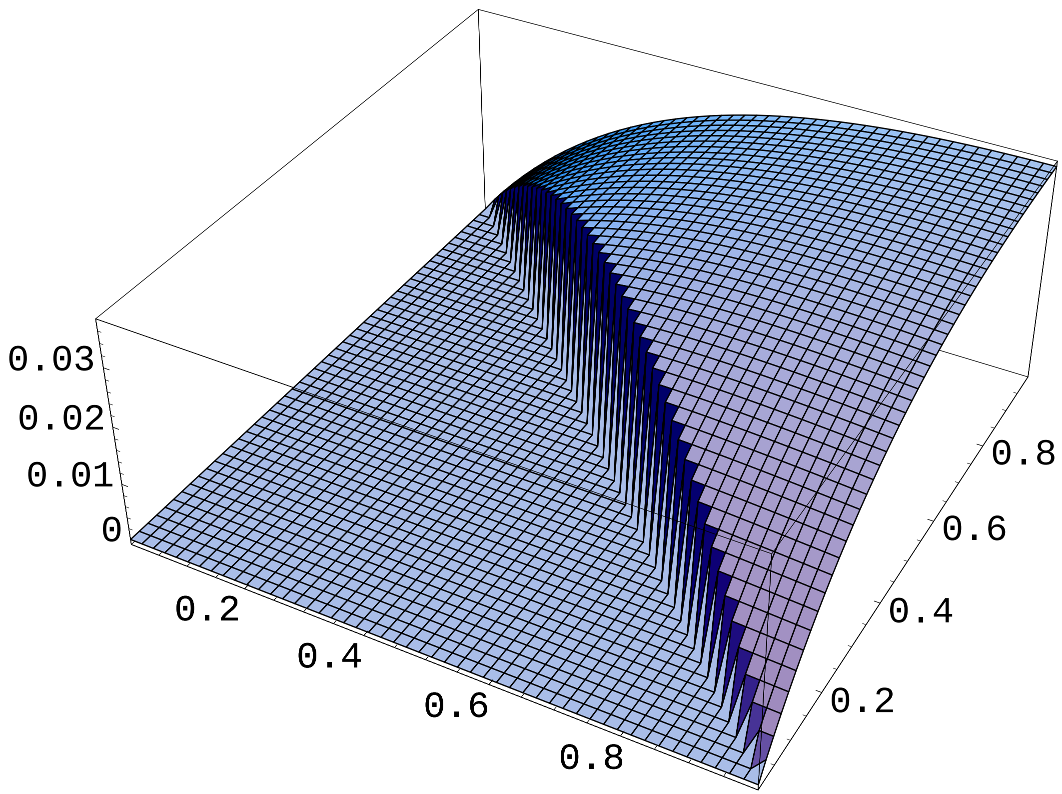
$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\frac{3k_1^2k_2^2k_3^2}{2(k_1+k_2+k_3)^3}$$

Gruzinov

- The Gaussianities are not multiplied by any slow roll parameter, so can be potentially large. The shape also vanishes in the squeeze limit due to the consistency relation of Maldacena.

- For DBI inflation, $\mathcal{A}_\lambda$ vanishes but it should be possible to construct realistic models where this shape is large.

- The shape looks similar to the DBI inflation



# Relative Sizes

- For general shapes, $f_{NL}$ is defined in the equilateral triangle limit:

$$
\begin{aligned}
f_{NL}^{\lambda} &= -\frac{5}{81}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) \ , \\
f_{NL}^{c} &= \frac{35}{108}\left(\frac{1}{c_s^2} - 1\right) \ , \\
f_{NL}^{\epsilon} &= -\frac{55}{36}\frac{\epsilon}{c_s^2} \ , \\
f_{NL}^{\eta} &= -\frac{5}{12}\frac{\eta}{c_s^2} \ , \\
f_{NL}^{s} &= \frac{85}{54}\frac{s}{c_s^2} \ .
\end{aligned}
$$

# Relative Sizes

- For general shapes, $f_{NL}$ is defined in the equilateral triangle limit:

$$\begin{aligned} f_{NL}^{\lambda} &= -\frac{5}{81}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) , \\ f_{NL}^{c} &= \frac{35}{108}\left(\frac{1}{c_s^2} - 1\right) , \\ f_{NL}^{\epsilon} &= -\frac{55}{36}\frac{\epsilon}{c_s^2} , \\ f_{NL}^{\eta} &= -\frac{5}{12}\frac{\eta}{c_s^2} , \\ f_{NL}^{s} &= \frac{85}{54}\frac{s}{c_s^2} . \end{aligned}$$

- If the sound speed is sufficiently small

$$c_s^2 \ll \epsilon, \eta$$

it is possible to observe the slow roll shapes $\mathcal{A}_\epsilon, \mathcal{A}_\eta$. This can be realized in DBI inflation, consistent with current experimental bound on $c_s$.

# Choice of Vacuum

- There have been some friendly debates on whether the initial state of inflation can deviate from the Bunch-Davies vacuum.

# Choice of Vacuum

- There have been some friendly debates on whether the initial state of inflation can deviate from the Bunch-Davies vacuum.

- A different perspective: We investigate whether there are pronounced effects of non-standard vacua to be observed in non-Gaussianities.

# Choice of Vacuum

- There have been some friendly debates on whether the initial state of inflation can deviate from the Bunch-Davies vacuum.

- A different perspective: We investigate whether there are pronounced effects of non-standard vacua to be observed in non-Gaussianities.

- The quantum state of inflaton is

  $$u_k = u(\tau, \mathbf{k}) = \frac{iH}{\sqrt{4\epsilon c_s k^3}}(C_+(1 + ikc_s\tau)e^{-ikc_s\tau} + C_-(1 - ikc_s\tau)e^{ikc_s\tau})$$

  where $C_+$ and $C_-$ satisfy the normalization condition $|C_+|^2 - |C_-|^2 = 1$. Bunch-Davies vacuum corresponds to $C_+ = 1$, $C_- = 0$.

- Two potentially observable contributions $\tilde{A}_\lambda$ and $\tilde{A}_c$ due to deviation from Bunch-Davies vacuum. The size of the non-Gaussianities are:

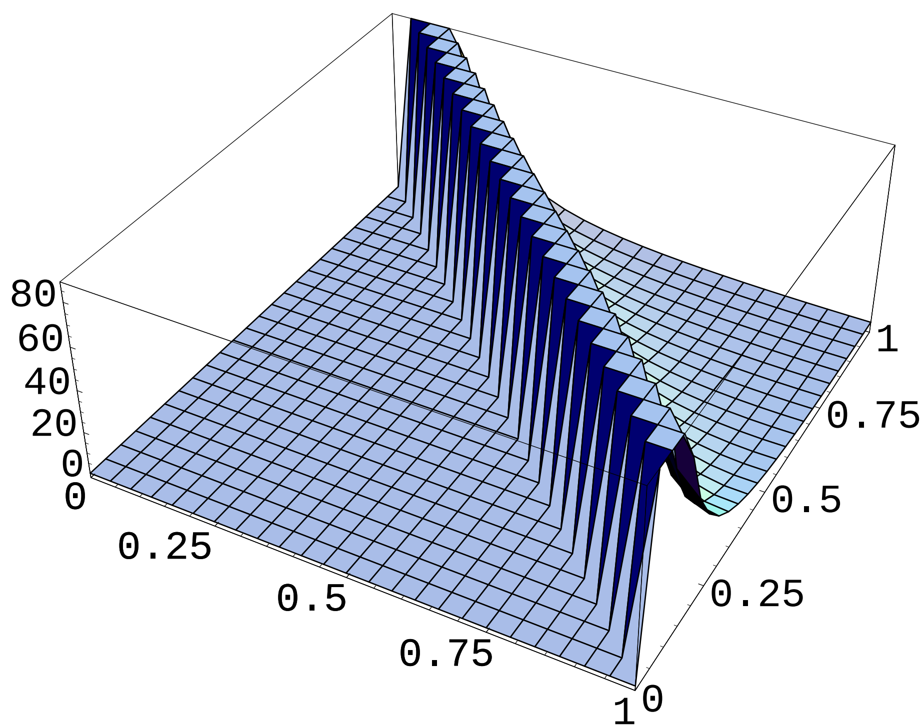
$$\tilde{f}^{\lambda}_{NL} = -5Re(C_-)(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma})$$

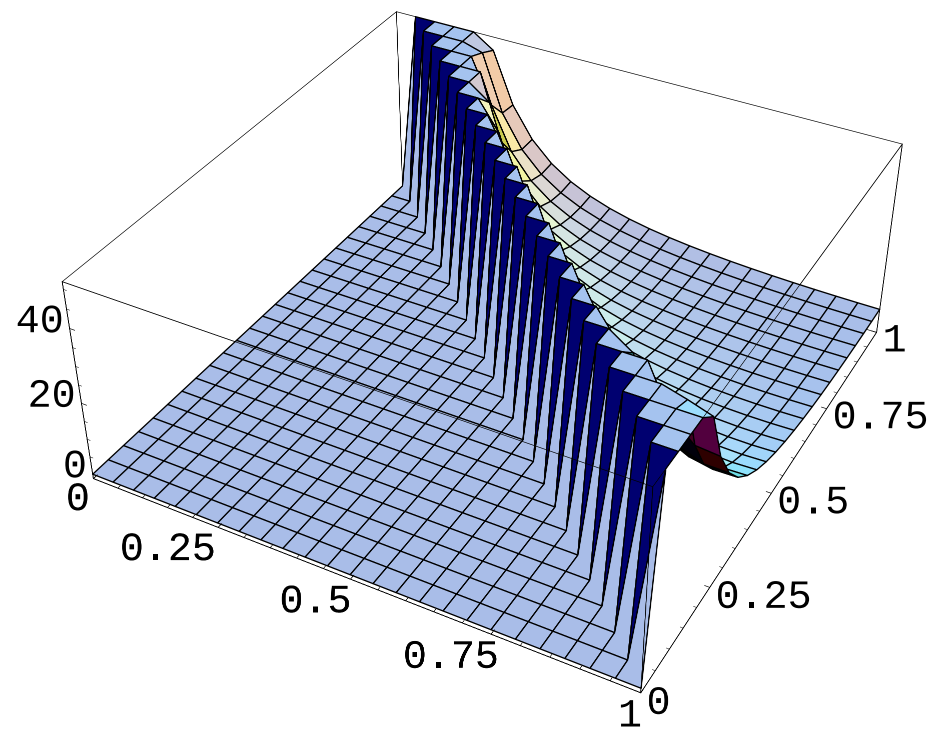
$$\tilde{f}^{c}_{NL} = \frac{25}{4}Re(C_-)(\frac{1}{c_s^2} - 1)$$

- Two potentially observable contributions $\tilde{A}_\lambda$ and $\tilde{A}_c$ due to deviation from Bunch-Davies vacuum. The size of the non-Gaussianities are:

$$\tilde{f}^{\lambda}_{NL} = -5Re(C_-)(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma})$$
$$\tilde{f}^{c}_{NL} = \frac{25}{4}Re(C_-)(\frac{1}{c_s^2} - 1)$$

- More importantly, the **shapes** are very **distinctive**:



- These shapes are peaked at the **”folded triangle”** limit:

$$k_1 = k_2 + k_3$$

for arbitrary $k_2$ and $k_3$.

- A feature not shared by other sources of non-Gaussianities, potentially more pronounced than modulation in power spectrum.

# dS/CFT

Strominger

- Unlike AdS/CFT, the dS/CFT proposal suffers from many objections: absence of supersymmetry, no concrete string example, and other conceptual issues.

- Nevertheless, it is useful to recast problems in (approximate) dS space in terms of a dual 3D CFT as the results are sometimes a consequence of the underlying symmetries.

- The two point functions and three point functions of the inflaton $f$ are related to the correlators of the CFT operators by the following

$$
\begin{aligned}
\langle f_{\mathbf{k}} f_{-\mathbf{k}} \rangle' &= -\frac{1}{2Re\langle \mathcal{O}_{\mathbf{k}} \mathcal{O}_{-\mathbf{k}} \rangle'} \\
\langle f_{\mathbf{k}_1} f_{\mathbf{k}_2} f_{\mathbf{k}_3} \rangle' &= \frac{2Re\langle \mathcal{O}_{\mathbf{k}_1} \mathcal{O}_{\mathbf{k}_2} \mathcal{O}_{\mathbf{k}_3} \rangle'}{\prod_i (-2Re\langle \mathcal{O}_{\mathbf{k}_i} \mathcal{O}_{\mathbf{k}_i} \rangle')}
\end{aligned}
$$

Maldacena

- In conformal field theory, the two point and three point correlation functions are constrained by conformal symmetry.

$$\langle \mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{y})\rangle \sim \frac{1}{|\mathbf{x}-\mathbf{y}|^{2\Delta}}$$
$$\langle \mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{y})\mathcal{O}(\mathbf{z})\rangle \sim \frac{1}{|\mathbf{x}-\mathbf{y}|^{\Delta}|\mathbf{x}-\mathbf{z}|^{\Delta}|\mathbf{y}-\mathbf{z}|^{\Delta}}$$

- For $\Delta \simeq 3$, the two point function on the CFT gives the correct scaling for the powers spectrum. [Larsen, McNees];[van der Schaar]

- We are testing whether conformal symmetries can determine the universal shapes of non-Gaussianities.

# Summary

- Observational signatures and non-Gaussianities of general single field inflation.

- Size and Shape of non-Gaussianities depend on 5 parameters:

$$c_s^{-2}, \epsilon, \eta, s, \lambda/\Sigma$$

- By varying these parameters, can recover all known shapes and more.

- Deviation from Bunch-Davies vacuum can lead to pronounced signatures in non-Gaussianities.

- Interesting to see whether the dS/CFT proposal can shed light on the universality of the shape of non-Gausianities. This will be very useful especially for multi-field inflation.

Thank You