Inflation as a probe of new physics

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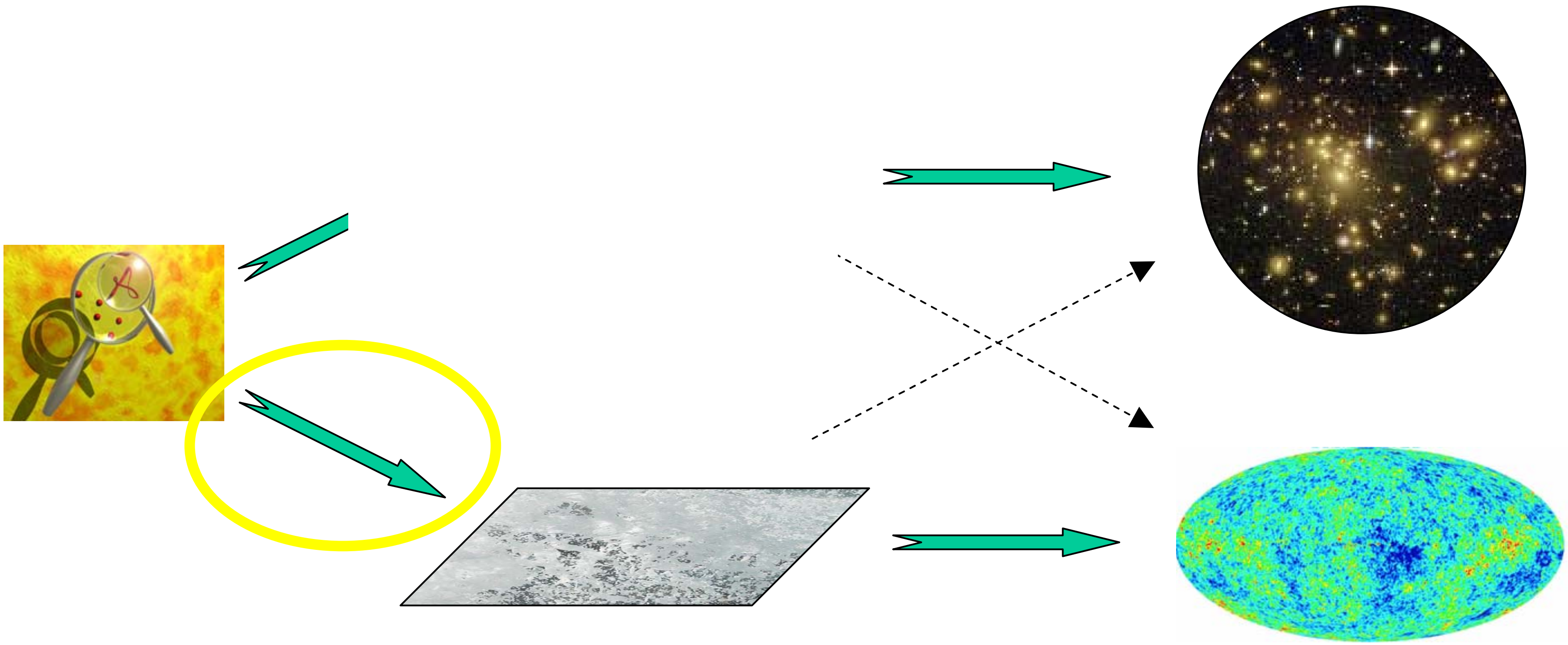
Conlusions



Outline

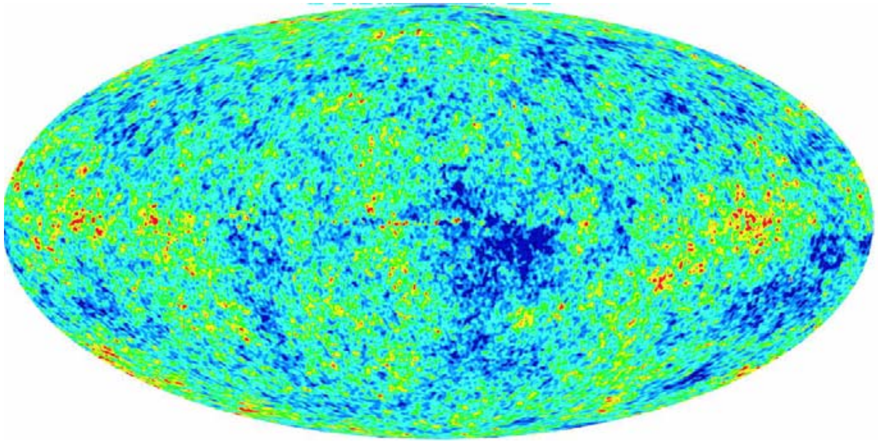
The basic idea

Effects on the CMBR

Back reaction

Both at once

Conclusions



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The basic idea

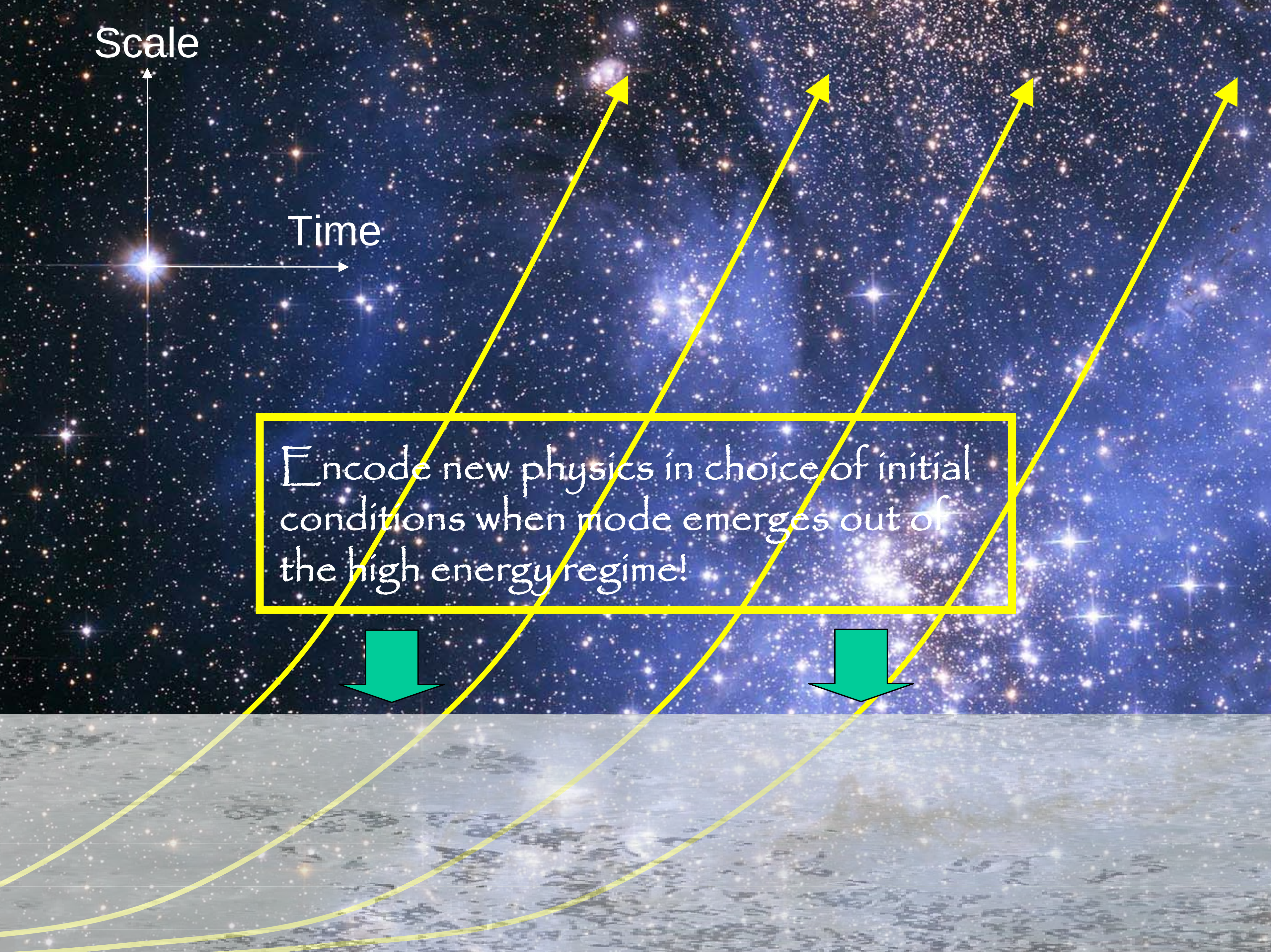
How do we construct a theory of the initial conditions of the universe?

Inflation provides the answer by replacing initial conditions by dynamics.

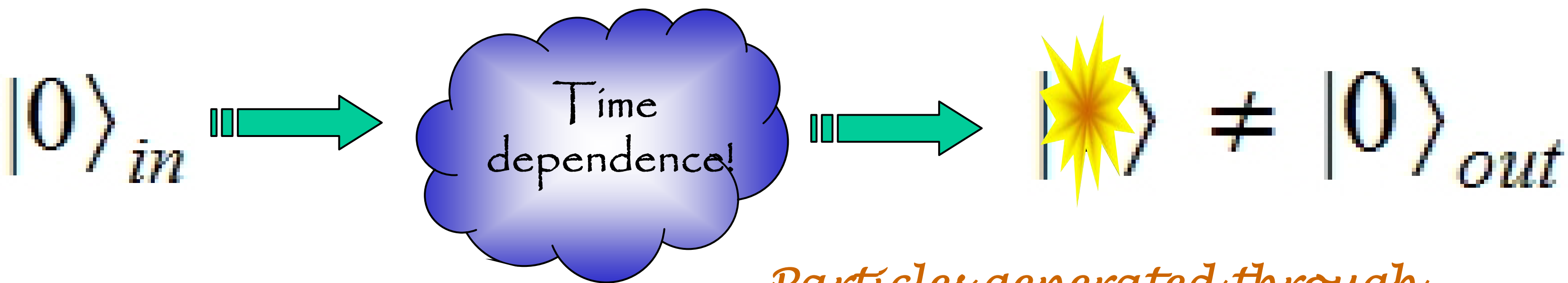
But...

What is the reason behind inflation?

Could there still be traces left of physics at earlier times and higher energies?


Scale
Time
Encode new physics in choice of initial conditions when mode emerges out of the high energy regime!

In a time dependent background (that is, no global timelike Killing vector) the definition of a vacuum is highly non-trivial...



Particles generated through Bogolubov transformations

Example: Hawking radiation

In an expanding universe, including de Sitter space, it is even trickier...

$\boxed{\text{A family of possible vacua!}}$

Luckily, when the wavelength of a mode is short enough,

$$\lambda \ll 1/H$$

... the expansion can be ignored. Hence:

$\boxed{\text{Preferred vacuum as } t \to 0 \text{ and } \lambda \to 0}$

Crucial observation:

$$\boxed{\text{Procedure limited by } \lambda \sim l_{pl}}$$

Gives estimate of span of possible vacua!

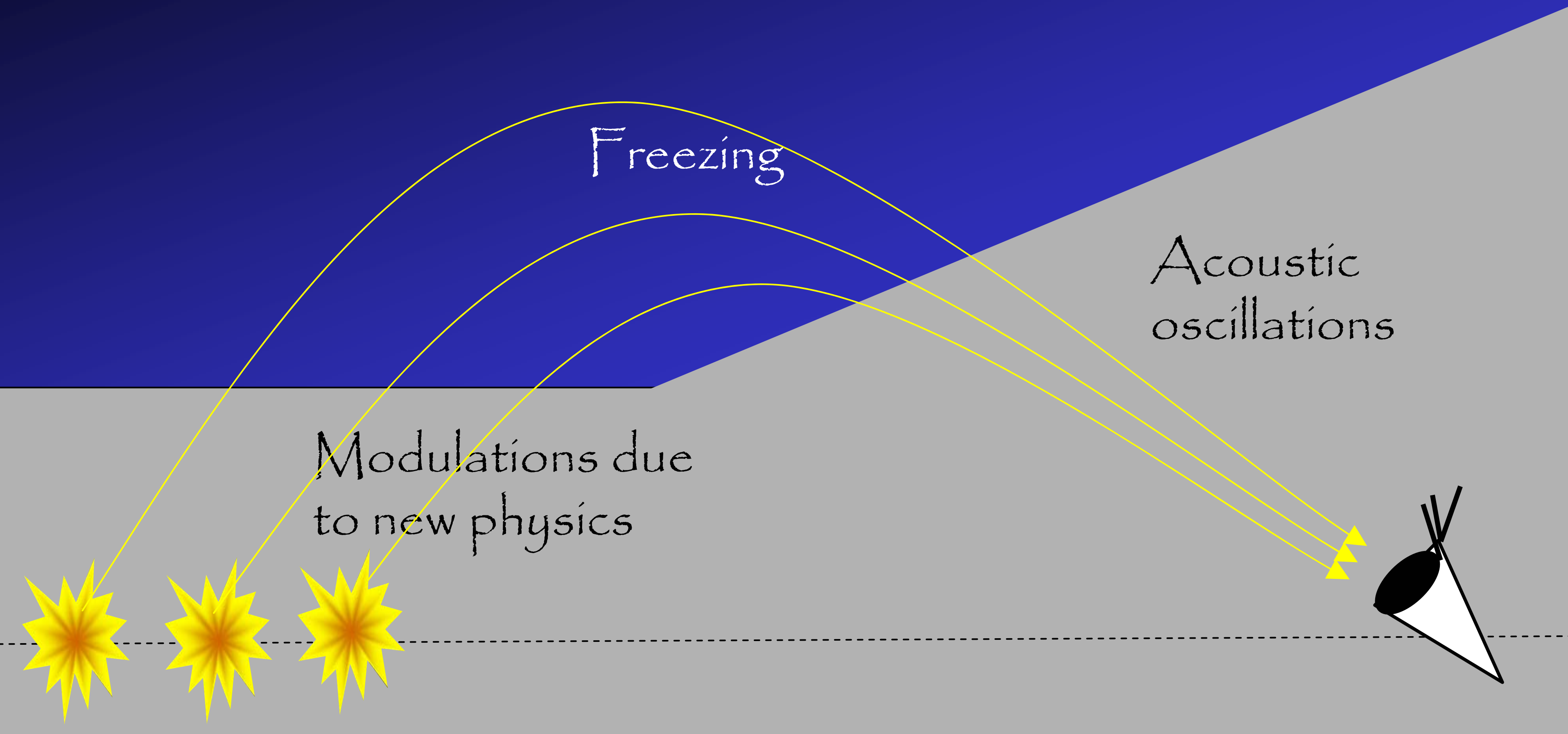
Effects on the CMBR

The generic expectation is of the form:

$$P_\phi = \left(\frac{H}{2\pi}\right)^2 \left(1 - \frac{H}{\Lambda}\sin\left(\frac{2\Lambda}{H}\right)\right)$$

… a modulated spectrum!

Summary



Freezing
Acoustic
oscillations
Modulations due
to new physics

Can it be seen?

This depends on having a large enough amplitude and the right wavelength.

Given $$\varepsilon = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

... one can show that the amplitude and wavelength are given according to

$$\frac{H}{\Lambda} \sim 4 \cdot 10^{-4}\,\frac{\sqrt{\varepsilon}}{\gamma}$$

$$\frac{\Delta k}{k} \sim 1.3 \cdot 10^{-3}\,\frac{1}{\gamma\sqrt{\varepsilon}}$$

... where $\Lambda = \gamma M_{pl}$

Example

In order to beat cosmic variance and have modulations within the scales relevant for the CMBR we need...

$$\frac{H}{\Lambda} \sim 10^{-2}$$

$$\frac{\Delta k}{k} \sim \mathcal{O}(1)$$

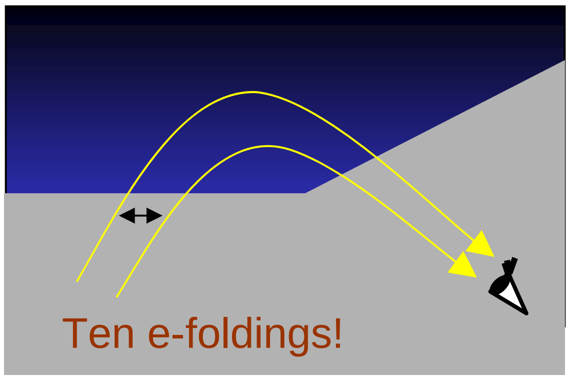
This can be achieved with

$$\frac{\sqrt{\varepsilon}}{\gamma} \sim 20 \qquad \varepsilon \sim 10^{-2}$$

Consistent with old fashioned heterotic string compactifications...

Another example…

If really slow modulation we can allow a much larger amplitude…



$$\frac{H}{\Lambda} \sim 10^{-1}$$

$$\frac{\Delta k}{k} \sim \mathcal{O}(10)$$

This can be achieved with

$$\frac{\sqrt{\varepsilon}}{\gamma} \sim 200 \qquad \varepsilon \sim 10^{-2}$$

… implying a string scale lowered by an order of magnitude.

Observable signature is running of the spectral parameter between CMBR and large scale structure

Back reaction

The presence of the non-standard vacuum gives rise to an extra energy density.

A rough estimate of the energy density gives

$$\rho_\Lambda \sim \int_0^\Lambda dp\, p^3 |\beta|^2 \sim \Lambda^4 |\beta|^2 \sim \Lambda^2 H^2$$

What about back reaction?

Small if $\Lambda^4 |\beta|^2 \lesssim M_p^2 H^2$ That is $|\beta|^2 \lesssim \dfrac{M_p^2 H^2}{\Lambda^4}$

... but where does the energy come from?

Introduce a

$$\boxed{\text{Source term!}}$$

How?

Let's consider some of the equations used to describe cosmology in FRW-coordinates...

$$\cancel{H^2 = \frac{8\pi}{3M_p^2}\rho}$$

$$\dot{H} = -\frac{4\pi}{M_p^2}(\rho + p)$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

We will use

$$\dot{H} = -\frac{4\pi}{M_p^2}(\rho + p)$$

together with

$$\dot{\rho} + 3H(\rho + p) = 0$$

... from which we can also obtain

$$H^2 = \frac{8\pi}{3M_p^2}\rho + \frac{8\pi}{3M_p^2}\rho_{cc}$$

... up to a constant of integration...

The cosmological constant!

What if we introduce a source term?

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = q$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0$$

with

$$p_\Lambda = w_\Lambda \rho_\Lambda$$

$$p_m = w_m \rho_m$$

Using

$$\dot{H} = -\frac{4\pi}{M_p^2}(\rho_\Lambda + p_\Lambda + \rho_m + p_m)$$

... we find

$$H^2 = \frac{8\pi}{3M_p^2}(\rho_\Lambda + \rho_m) - \frac{8\pi}{3M_p^2}\int^t q\,dt$$

The source term is determined from

$$\rho_\Lambda(a) = \frac{1}{2\pi^2}\int_\varepsilon^\Lambda dpp^3 \frac{H^2\left(\frac{ap}{\Lambda}\right)}{\Lambda^2} = \frac{1}{2\pi^2}\frac{\Lambda^2}{a^4}\int_{a_i}^a dxx^3 H^2(x)$$

... this yields

$$q = \frac{1}{2\pi^2}\Lambda^2 H^3$$

It is straightforward to solve the modified Friedmann equations giving the result

$$\boxed{H^2 = C_1 a^{-2n_1} + C_2 a^{-2n_2} + \frac{8\pi}{3M_p^2}\frac{(1+w_m)(1-3w_m)}{(1+w_m)(1-3w_m)-\frac{16\Lambda^2}{9\pi M_p^2}}\rho_m}$$

where

$$n_{1,2} = 1 \pm \sqrt{1 - \frac{4\Lambda^2}{3\pi M_p^2}}$$

More general:

$$\rho_\Lambda(a) = \frac{1}{2\pi^2}\int_\varepsilon^\Lambda dpp^3 g\left(\frac{H\left(\frac{ap}{\Lambda}\right)}{\Lambda}\right) = \frac{1}{2\pi^2}\frac{\Lambda^4}{a^4}\int_{a_i}^a dxx^3 g\left(\frac{H(x)}{\Lambda}\right)$$

Assuming $g = h_n f^n$, where $f = f(z) = \dfrac{H^2}{\Lambda^2}$ and $z = a^{-4}$

... we find: $\boxed{f'' = -\dfrac{\Lambda^2}{3\pi M_p^2} h_n \dfrac{1}{z^2} f^n}$

This is the Emden-Fowler equation.

The case n=1 was solved above. The case n=0 can also be solved exactly with the result:

$$H^2 = C_1 a^{-4} + C_2 - \frac{4\Lambda^2}{3\pi M_p^2} h_n \ln a$$

Back to our favorite case...

Consider for simplicity no other matter than radiation.

We then have:

Resupplied radiation decaying slower than usual.

$$H^2 = C_1 a^{-2n_1} + C_2 a^{-2n_2} + \frac{8\pi}{3M_p^2} \frac{(1+w_m)(1-3w_m)}{(1+w_m)(1-3w_m) - \frac{16\Lambda^2}{9\pi M_p^2}} \rho_m$$

A rolling cosmological constant

where

$$n_{1,2} = 1 \pm \sqrt{1 - \frac{4\Lambda^2}{3\pi M_p^2}}$$

The conclusion is...

In the presence of sources one can not assign an unambiguous value to the cosmological constant!

To be more precise...

Any fixed dimensionful cosmological constant, is effectively replaced by a dimensionless parameter determining the running, given by the ratio of a fundamental scale and the Planck scale.

Putting things together

Can both kind of effects be relevant at the same time?

Let us consider the case of a slow roll inflaton...

$$3aH^2\phi' = -\frac{dV}{d\phi}$$

This leads to...

$$\frac{d}{da}(a^5 HH') = -\frac{8n\Lambda^2}{3\pi M_p^2} a^3 H^2 - \frac{4\pi}{M_p^2}\frac{d}{da}\left(a^4 (aH\phi')^2\right)$$

Here is a specific example...

$$V = \tfrac{1}{2} m^2 \phi^2 +$$

Solving the equation of motion for the inflaton yields...

$$\phi = \phi_0 a^{-\frac{m^2}{3H^2}}$$

$$\boxed{\varepsilon_{\inf} = \frac{4\pi}{9} \frac{\phi^2}{M_p^2} \frac{m^4}{H^4}}$$

... and furthermore

$$\frac{d}{da}(a^5 H H') = -\frac{8n\Lambda^2}{3\pi M_p^2} a^3 H^2 - \frac{16\pi}{9M_p^2}\phi^2 \frac{m^4}{H^4} a^3 H^2$$

$$\boxed{\varepsilon = \frac{2n\gamma^2}{3\pi}}$$

There is now a decoupling between the slow roll of the inflaton, determined by $\varepsilon_{\rm inf}$, and the slow roll of the Hubble constant determined by ε ....

$$\boxed{\begin{aligned} \frac{H}{\Lambda} &\sim 4 \cdot 10^{-4} \frac{\sqrt{\varepsilon_{\rm inf}}}{\gamma} \\ \frac{\Delta k}{k} &\sim 1.3 \cdot 10^{-3} \frac{\sqrt{\varepsilon_{\rm inf}}}{\gamma \varepsilon} \end{aligned}}$$

We see that we need $\varepsilon_{\rm inf} \sim 400\gamma^2$

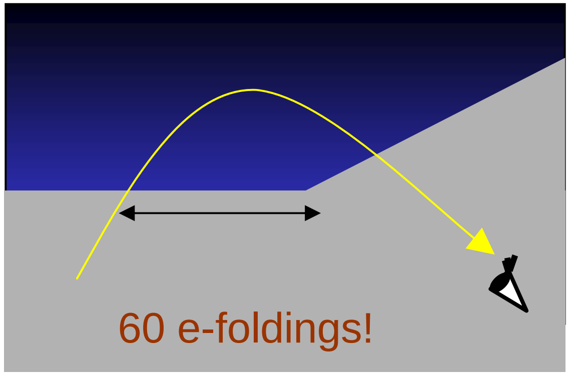
Comparing with $\varepsilon = \frac{2n\gamma^2}{3\pi}$

... we typically would like $\boxed{n \sim 10^3}$

Back to our example where we have

$$\varepsilon_{\rm inf} \sim \frac{4\pi}{9}\frac{\phi_0^2}{M_p^2}\frac{m^4}{H_0^4}e^{4N\varepsilon - \frac{2m^2}{3H_0^2}Ne^{2N\varepsilon}}$$

... where is ***N*** the number of e-foldings before the end of inflation...



With $N = 60$ and $\varepsilon \sim 10^{-2}$ ...

$$e^{4N\varepsilon - \frac{2m^2}{3H_0^2}Ne^{2N\varepsilon}} < e^{4N\varepsilon} \lesssim 10$$

... it is somewhat difficult to have a small $\varepsilon_{\rm inf}$ that come to dominate and end inflation...

Another example...

$$V = -\frac{1}{2}m^2\phi^2 +$$

In this case the rolling goes the other way.... $\phi = \phi_0 a^{\frac{m^2}{3H^2}}$

... and we find $$\varepsilon_{\inf} \sim \frac{4\pi}{9}\frac{\phi_0^2}{M_p^2}\frac{m^4}{H_0^4}e^{4N\varepsilon + \frac{2m^2}{3H_0^2}Ne^{2N\varepsilon}}$$

Implementation in string theory?

Conlusions