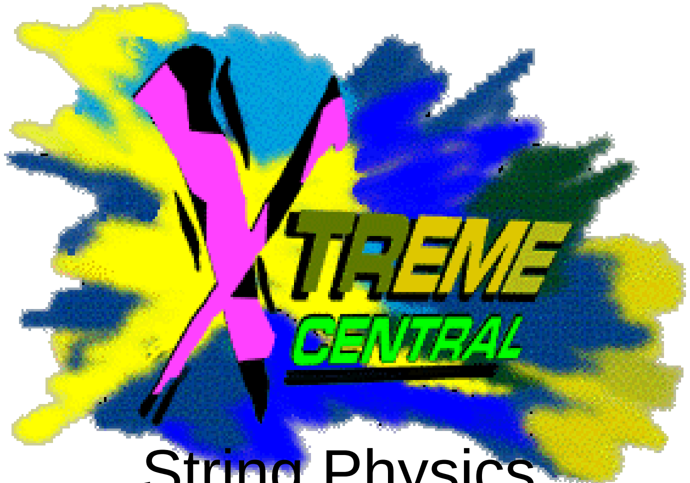


String Physics

Big Bang

← String Theorists

MIT Theory Retreat 2003 – Lecture 1 by Amanda W. Peet

Lecture 1: String Theory, and where Black Holes fit in

Lecture 2: Entropy and Holography

Lecture 3: Holography and Singularity Resolution

Focus: properties of string theory *unlike* QFT;
emphasis on central role played by black holes.

http://www.physics.utoronto.ca/~peet/online/MIT/

Other notes of increasing level of expertise:

- PiTP lectures (July 2002):
  http://www.physics.utoronto.ca/~peet/online/PiTP/ ;
- TASI-99 lectures: hep-th/0008241 ;
- CQG invited review: hep-th/9712253 (pre-AdS/CFT).

- Invitations to write RMP and LRR articles *not* taken up.

Important side notes:

1. Natural Units used throughout: $\hbar = c = k_B = 1$.

Hence,

$$[\text{time}] = [\text{length}] = \left[\frac{1}{\text{mass}}\right] = \left[\frac{1}{\text{Temperature}}\right], \quad [G_N] = [\ell]^{d-2} \tag{1}$$

2. Here we discuss only string/M theory as *the* theory of quantum gravity. Other approaches (loop quantum gravity, dynamical triangulations, causal sets, ...) have trouble producing a Newtonian limit – !. This is a gross failure of the Correspondence Principle... String theory has its own "problems", for sure, but is the most promising approach so far.

(Side note of advice: don't spend too much time trying to think deeply about quantum gravity, e.g. the origin of time. Instead use David Gross's mantra: "When in Doubt, Calculate!".)

History/Background - Wilsonian Effective Field Theory

String theory was born from particle theory (literally).
Long association with Wilsonian way of looking at Nature:–

1. Decide on a cutoff scale Λ .

2. Identify the lightest degrees of freedom – the most *relevant* – and put them in the Lagrangian.

3. Modes heavier than Λ are integrated out of the low-energy description. Their only effect is on low-energy couplings via the Wilsonian RG, which tells us $\frac{\partial}{\partial\Lambda}\alpha_{EM}$, for example.

4. The only theories we can honestly say we understand are asymptotically free (free in the UV) – they make sense in the continuum limit. For those classified "non-renormalizable", the theory gets strong in the UV, and QFT tells us nothing about degrees of freedom which must be added in the UV to make sense of the theory at high-energy.

But e.g. Osama Bin Laughlin doesn't care - because of #3!

Relevance

Gravity is a notorious example of a UV-incomplete theory. See via:

a. First, write kinetic term for field(s) of interest. Action (in natural units) must be dimensionless. Linearizing metric in Einstein's action as $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G_d} h_{\mu\nu}$ gives

$$S_{\text{free}}(\text{linearised gravity}) = \int d^d x \ \frac{1}{2}(\partial_\mu h_{\lambda\sigma})(\partial^\mu h^{\lambda\sigma}) \tag{2}$$

so $h_{\lambda\sigma}$ has mass dimension $m^{(d-2)/2}$ in spacetime dimension d.

b. Next, write interaction of interest, e.g. 3-pt vertex ($\exists$ also 4-pt)

$$S_{\text{int}} \sim \sqrt{G_d} \int d^d x (\partial^\epsilon h_{\alpha\beta})(\partial^\zeta h_{\gamma\delta})(h_{\epsilon\zeta}) \left(\eta^{\alpha\gamma}\eta^{\beta\delta} + \text{etc.}\right) \tag{3}$$

c. Do dimensional analysis on higher order processes (h.o.p.). For gravity $2 \to 2$ scattering, see that

$$\text{h.o.p.} \propto G_d E^{(d-2)} \tag{4}$$

These processes dominate at high-energy in our dimension and higher. Equally, quantum gravity appears irrelevant at low-energy!

But - just how low is low-energy?

1. Until 4-5 years ago, we assumed we were safe from ever having to think about Quantum Gravity experimentally, because the expected length scale of quantum gravity was $l_{P,4} \sim 10^{-33}$cm.

2. String theorists knew about extra dimensions, but there was no motivation for considering their size to be $L \gg \ell_{P,4}$. Why not? The argument was naturalness – what new physics we haven't seen generates the new scale?

3. Then again - we've been terribly wrong with naturalness arguments about the dark energy component of the universe! Even if there exists supersymmetry at a few TeV, we have *no* explanation for why a scale of 10^{-3}eV should exist...

4. $\sim$80 years ago, Kaluza and Klein used an extra dimension of space to unify gravity and electromagnetism – but they were disappointed because it came at the price of an extra massless scalar field.

Dimensional reduction and the Newton constant

With $g_{55} \sim e^{\#\chi}$, $g_{5\mu} \propto A_\mu$, obtain (see TASI notes)

$$\frac{1}{16\pi G_5}\int d^5x\sqrt{-g_5}R_5 = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g_4}\left(R_4 - \frac{1}{2}(\partial\chi)^2 - \frac{e^{\#\chi}}{4}F^{\mu\nu}F_{\mu\nu}\right) \tag{5}$$

where $\frac{L}{G_5} = \frac{1}{G_4}$. More generally, with $(d-4)$ extra dim's,

$$\frac{\mathrm{Vol}_{d-4}}{G_d} = \frac{1}{G_4} \tag{6}$$

We can also recall the definition of the Planck length, $G_d \sim \ell_{P,d}^{d-2}$. So

$$\ell_{P,4}^2 \sim \ell_{P,d}^2\left(\frac{\ell_{P,d}^{d-4}}{\mathrm{Vol_{d-4}}}\right) \tag{7}$$

So 4-d Planck length scale could be OK if fundamental Planck length scale is relatively big, if extra dimensions bigger! Set $\mathrm{Vol}_{d-4} = L^{d-4} \Rightarrow$

$$\ell_{P,4} \ll \ell_{P,d} \ll L \tag{8}$$

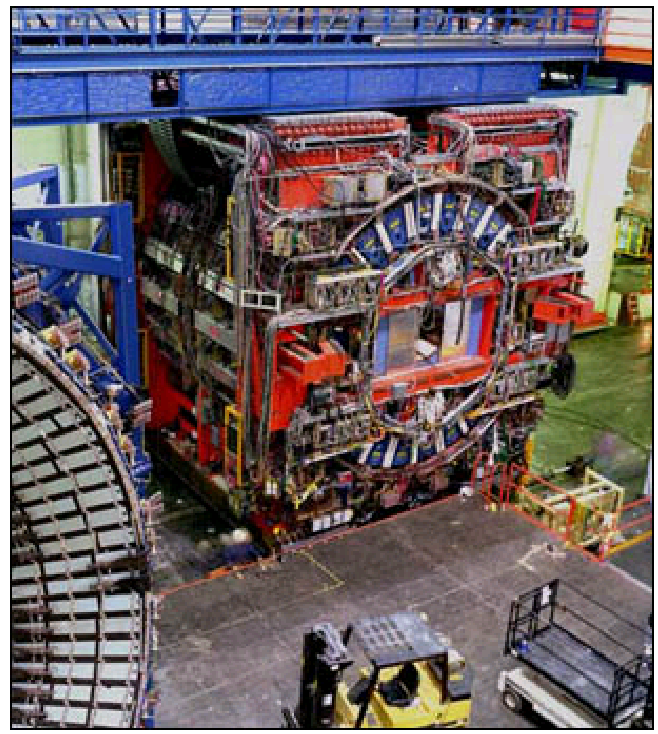
[Kane et al. hep-ph/0009145: could BH mediate p^+ decay and spoil?]

The Fundamental Issue

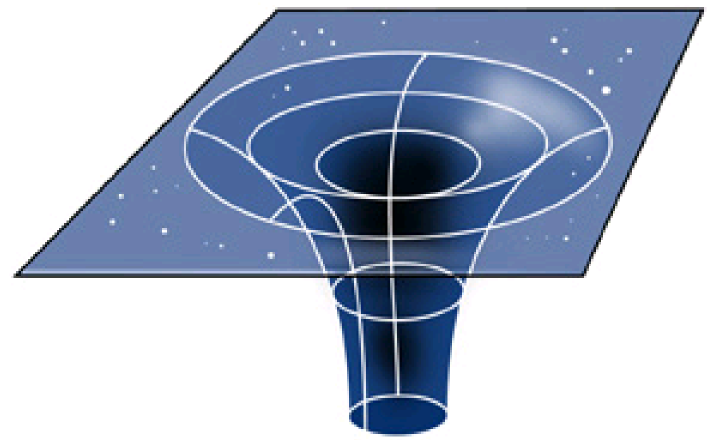
Quantum Mechanics – and Quantum Field Theory – teach us that better resolution (in x^μ space) is obtained with higher-energy probes:

$$\Delta x(QFT) \propto \frac{1}{E} \tag{9}$$

So we build bigger and bigger accelerators:



BUT classical General Relativity says: if you put too much energy into a bounded region, a black hole will form! This puts fundamental limits on even thought experiments.



Length scale associated to gravity is the Planck scale ℓ_P.
Black hole radius grows with (centre-of-mass) energy E:

$$\Delta x(GR) \propto \ell_P \, (E\ell_P)^{\frac{1}{(d-3)}} \tag{10}$$

Therefore, in "real life" (with *four* interactions),

$$\frac{\Delta x}{\ell_P} \sim \frac{1}{(E\ell_P)} + (E\ell_P)^{\frac{1}{(d-3)}} \tag{11}$$

Therefore, minimum distance scale that can possibly be probed is

$$\Delta x_{\text{min}} \sim \ell_P \tag{12}$$

The point is that a high-energy theory *cannot* ignore gravity, even though that is by far the weakest force at low-energy. Thus, QFT with Wilsonian RG may well be an incomplete framework - !!

Closed strings give gravity for free

Edward Witten says that if our civilization had evolved (very) differently, physicists might be impressed with this ↑ – but sadly, it's a *post-diction*.

How does it work? You may have heard some rough explanations, and taken away the message that they're a bit tautological, and/or require (nearly) flat space. Not!

1. Fundamental concept: since string theory is best unified Theory of "Everything", it includes spacetime and matter all rolled up into one. MTW: "Matter tells spacetime how to curve, and spacetime tells matter how to move". String equations must do both at once!

2. Classical string theory has a gigantic symmetry on its 2-d "world-sheet", the *intrinsic* surface swept out as it moves along. Conformal Symmetry.

3. Quantum theory also possesses conformal symmetry. If embed world-sheet via X^μ, ten dimensions for superstring. Also, any fields coupling to the string – like $g_{\mu\nu}$ – have equations of motion demanded by conformal symmetry. Lowest-order result: SUsy+GR+mAtter≡ SUGRA.

Perturbative Details

Quantisating the free relativistic string essentially amounts to a bunch of harmonic oscillators.

For a closed string propagating on $\mathbb{R}^{1,9}$, quantization yields

$$m^2\ell_s^2 = 2\,(N_L + N_R - 2) \tag{13}$$

The integers $N_L\,, N_R$ quantify the number of harmonic oscillator excitations moving to the left or the right on the string. The bigger these numbers, the more massive the string mode.

Supersymmetry of the string ensures elimination of the tachyon at $m^2\ell_s^2 = -4$. GSO projection gets rid of the tachyon and every second level above that.

Hence, ground states ($m^2 = 0$) require two oscillators – spacetime field gets two legs. Lorentz irreps: $g_{\mu\nu}\,, B_{[\mu\nu]}\,, \Phi$.

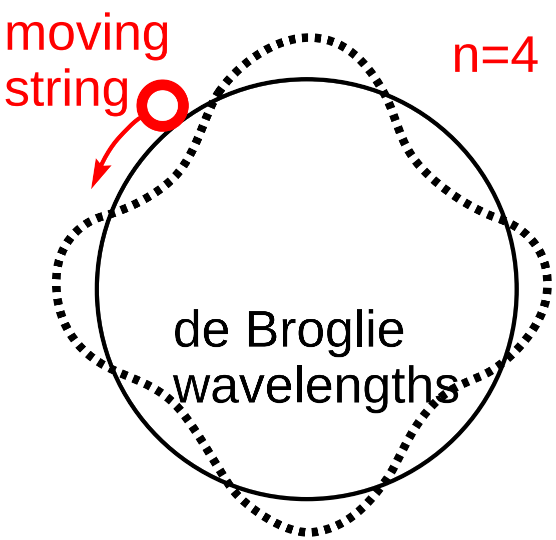
This is a cartoon version of understanding why gravity falls out of string theory for free. Interactions are necessary, but it's useful that a massless spin-two interacting particle must couple like Einstein's graviton... (Noether procedure starting with linearized theory + gauge symmetry.)

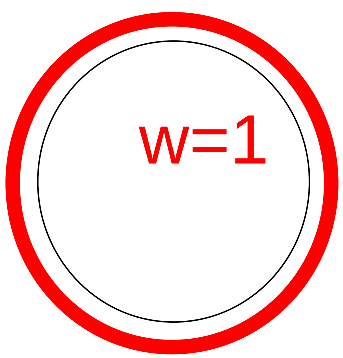
Winding modes

Interesting physics transpires if we quantise on another flat spacetime, $\mathbb{R}^{1,8} \times S^1$:

$$m^2 \ell_s^2 = \frac{n^2 l_s^2}{R^2} + \frac{w^2 R^2}{l_s^2} + 2\,(N_L + N_R - 2) \tag{14}$$

where n is the momentum number, i.e. the number of de Broglie wavelengths of the string that fit around the circle; and w is the number of times the string wraps around the circle of radius R.





Note that wrapping is peculiar to strings - particles cannot do it.

Also, at large radius, wrapping is expensive but momentum modes are cheap; at small radius the opposite is true.

$$m^2\ell_s^2 = n^2\frac{l_s^2}{R^2} + w^2\frac{R^2}{l_s^2} + 2\,(N_L + N_R - 2) \tag{15}$$

As $R \to 0$, winding modes form a continuum! More on that later. There is also a *crucial* constraint which ensures momentum balance on the string. Note that winding allows L, R oscillators unbalanced!:

$$N_L - N_R = nw \tag{16}$$

Using (15, 16) we can deduce another very important piece of string physics which is *completely* unlike particle physics.

You might think that the only massless modes for this string are obtained via $n, w = 0\,, N_L = N_R = 1$. However, there is a second set of solutions!

Let us rewrite the mass formula as follows:

$$m^2\ell_s^2 = \left(\frac{n\ell_s}{R} + \frac{wR}{\ell_s}\right)^2 + 4\,(N_R - 1) = \left(\frac{n\ell_s}{R} - \frac{wR}{\ell_s}\right)^2 + 4\,(N_L - 1) \tag{17}$$

Enhanced gauge symmetry

$$m^2\ell_s^2 = \left(\frac{n\ell_s}{R} + \frac{wR}{\ell_s}\right)^2 + 4\,(N_R - 1) = \left(\frac{n\ell_s}{R} - \frac{wR}{\ell_s}\right)^2 + 4\,(N_L - 1) \quad (18)$$

Now we can see that additional massless modes appear at the special radius $R = \ell_s$ – these require the special choices

$$n = -w = \pm 1\,, N_R = 1\,, N_L = 0\,, \qquad n = w = \pm 1\,, N_L = 1\,, N_R = 0\,. \quad (19)$$

Note that these are the *only* choices consistent with the momentum constraint $N_L - N_R = nw$. For example, $n = 2, w = 1, N_L = 1, N_R = 0, R/\ell_s = \sqrt{2}$ does not give a massless state (although $(\cdots)^2 = 0$, momentum constraint says $N_R = 0, N_L = 1$ inconsistent with $n = 2, w = 1$).

When the oscillator points in a non-compact direction, obtain gauge bosons of enhanced $SU(2)_{R,L}$. When it points in the compact direction, get scalar Higgses of $SU(2)_{R,L}$ – these are only massless at $R = \ell_s$.

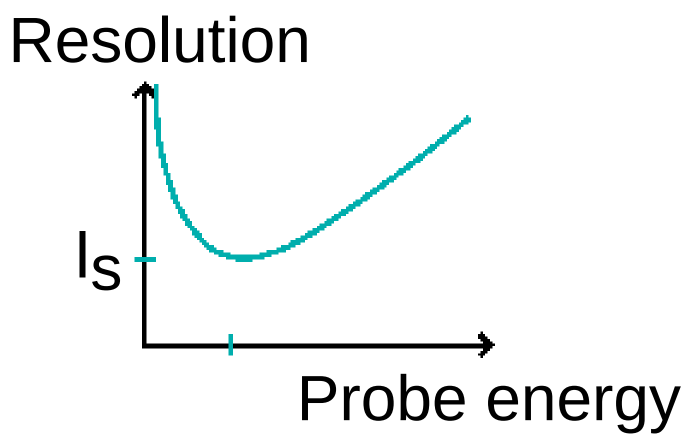
When the circle radius moves away from the self-dual point, the enhanced gauge group is Higgsed.

NB: Even if the circle radius becomes smaller than the string scale, those modes become massive again, *not* tachyonic.

The first duality

Also, the physics of strings propagating on $\mathbb{R}^{1,8} \times S^1$ is completely equivalent if we perform the switch $R/\ell_s \leftrightarrow \ell_s/R$ if $n \leftrightarrow w$. This phenomenon is known as T-duality.

From this, we draw the startling conclusion that there is no physical meaning to a distance smaller than the string scale!



Physically – energy beats tension and string stretches.

This is true to all orders in perturbation theory – so long as the probes which we use are strings themselves.

If however we use other string theory ingredients called D-branes, then (sometimes) shorter distance scales can be probed at weak coupling.

Degeneracy of States

As the mode numbers increase, the number of ways of making a mode of a given mass increases – exponentially!

See qualitative idea by looking at free massive states on $\mathbb{R}^{1,9}$: $N_L = N_R$ and

$$m^2\ell_s^2 = 2\,(N_L + N_R - 2) \tag{20}$$

For groundstate, two directions to point oscillators in: few ways of doing this. For N oscillators, rapidly increasing number of ways. Exponential.

[If we do not specify the spacetime spin, the degeneracy is larger because we can point the oscillators in any direction in spacetime.
On the other hand, if we do specify the spacetime spin, then the degeneracy is still exponentially large, but is smaller.]

Find (tricky combinatorics, or Prof. Zwiebach's upcoming textbook!):

$$\text{At large } m\,, \qquad \rho_{\text{string}} \sim \exp\left(\ell_s m\right) \sim \exp\left(\ell_s\sqrt{N_L + N_R}\right) \tag{21}$$

Thermodynamics

Suppose we consider a gas of strings in the canonical ensemble:

$$Z(T) = \int dE\, \rho_{\text{string}}(E)\, e^{-E/T} \sim \int dE\, (\cdots) e^{E\ell_s}\, e^{-E/T} \tag{22}$$

Immediately, we notice that this integral does not make sense above the temperature

$$T_H \sim \frac{1}{\ell_s} \tag{23}$$

This is known as the Hagedorn temperature. It is the limiting temperature for an ensemble of strings; if more energy is pumped into the system, the energy goes into producing longer and longer pieces of string – which also actually interact with each other a great deal at this temperature – rather than raising the temperature.

Actually, the high-energy density of states of string theory is really *dominated by black holes* – the BH density of states ρ_{BH} grows more quickly at higher energy than ρ_{string} – proof in Lecture 2. This property makes it *completely* unlike any Lorentz-invariant QFT.

Black Holes - defining characteristic

Black holes are ubiquitous in string theory at very high energy. But not all massive objects should be thought of as black holes!!

Defining characteristic of black hole: *event horizon*. For stationary BH (metric components independent of t), event horizon occurs at*

$$g_{rr} \to \infty \tag{24}$$

To qualify, object needs its Schwarzschild radius larger than its Compton wavelength – so we can consider it truly corralled within its event horizon. In d-dimensions,

$$(Gm)^{\frac{1}{(d-3)}} > \frac{1}{m}, \quad \text{i.e.} \quad m > m_P\,. \tag{25}$$

In $d = 4$, $m_P \sim 10^{-5}$g, so the electron, with $m_e \sim 10^{-23} m_P$, does not qualify as a BH.

In general, finding new gravity geometries very difficult – eqns of motion horribly nonlinear from math p.o.v..

* *Not* same as $g_{tt} = 0$ condition, in general. Also, for evolving geometry, event horizon doesn't even have a local definition; it is a global concept.

Finding New Black Hole Geometries

Search aided in the past by classical *no-hair theorems* – once conserved charges of a system are determined, spacetime geometry is unique. Essential physics:

- Specify Lagrangian, including matter couplings.
- Gravity falloffs give two conserved quantum #s: M, J.
- Gauge fields give conserved charges Q_i.
- Any other matter fields have 2nd-order PDE's in BH background. Two integration constants, so need two BC's. Must have solutions well-behaved both at infinity and at horizon. "Hair" forced to zero $\Rightarrow$ uniqueness. (& Non-abelian hair unstable.)

(*) Condition: any black hole singularity must be hidden behind event horizon; theorems *fail* in spacetimes with naked singularities.

(*) Newer results: Life is harder with rotation in higher-d!
Emparan and Reall found new $d = 4 + 1$ rotating black ring solution with toroidal horizon! Threw into doubt all uniqueness theorems in higher dimensions. Can do proofs for static BHs; also current research on compactified black branes.

Yeah, yeah

Common astrophysicist complaints about BH studied in string theory:
Q1. Q is unphysical. Charged astrophysical black holes discharge on a very short timescale via Schwinger pair production.
A1: Charges on all black holes discussed herein are not carried by light elementary quanta like electrons of QED. (Topological, e.g.)
Q2. Astrophysical black holes formed via gravitational collapse have a lower mass limit $\sim$ few solar masses. Smaller ones must be 'primordial'.
A2: We are not size-ist. (Yeah, yeah, I know about inflation...)

Singularities – Lecture 3

Although metric components blow up at horizon, this is only a coordinate singularity; see this by computing curvature invariants.

Find

$$R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma} \sim \frac{1}{r^4}\left(\frac{G_d m}{r^{d-3}}\right)^2 \tag{26}$$

For GR, herein lies a *disaster* at $r = 0$: theory predicts seeds of its own destruction!

The "Ur" Potential Function In The Sky

Why did we end up in a universe with this set of dimensionless parameters - coupling constants, mass ratios, etc.? (It was mighty convenient, for us, because with small changes to them, we wouldn't have bound Hydrogen or satisfied a number of other conditions necessary for life. It's way cool being made up of supernova ejecta. However, I *hate* the anthropic "principle"! - only post hoc logic.)

String theory currently has an embarrassment of riches: too many roughly-Standard-Model vacua. Two potential answers:

1: The Cosmic Arrogance Hypothesis: we had to be here!
This is arrogant for 2 reasons :
a: As all Star Trek fans know, we humans aren't so special;
b: String theorists will be smart enough to eventually *calculate* for you why we live in this universe! (all other "vacua" in our current naive story will turn out to be wrong - e.g. not true minima).

2. The Cosmic Joke Hypothesis: we landed here by accident!

“Ur” Wanderings - Bubble Trouble!

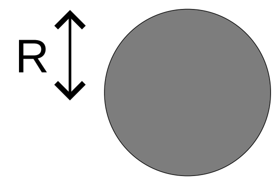
Mechanics of understanding either #1 or #2 are aided considerably if we can, in the pre-Big-Bang universe, wander around between different “vacua” – then what we need is to find the “Ur” theory: the dynamical principle telling us how to select “vacua”, perhaps by minimizing some giant Ur-potential.

(The bummer, of course, experimentally, is that we’ve got only one Universe. Cosmic Variance, argh!)

Important paper by Tom Banks (hep-th/0011255) points out that Ur-wanderings are however *fundamentally and severely limited* – by those old devils the black holes! How so? Let’s discuss a cartoon version.

1. Consider “vacua” of string/M (“Ur”) theory with no more than four supercharges - i.e. $\mathcal{N} = 1$ supersymmetry in $d = 4$. These must be isolated vacua - there are no flat directions at all (ways to go between vacua that are essentially zero-cost in energy).

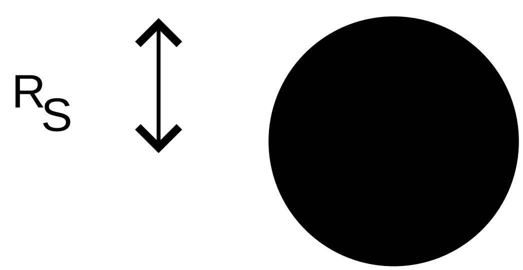
2. In local QFT, there is no problem considering any two such vacua being in the same overall theory! We can, by choice of probe, get from one vacuum to the other. This kind of probe is called a big *bubble*.



We do have to pay a cost in energy, $\propto R^2$ (in $d = 4$), and big bubbles want to shrink, but causality tells us that one cannot collapse on a timescale less than $(c)R$... So we can do experiments creating bubbles of the other vacuum and study it.

3. When gravity is present, this scenario is ruled out! Why?

Because we will instead make a black hole!!



Give the bubble surface tension σ. Then the energy required to create a big bubble of radius R is

$$E \sim \sigma R^2 \tag{27}$$

This configuration has a Schwarzschild radius given by

$$r_S \sim G_4 \sigma R^2 \sim \ell_P^2 \sigma R^2 \tag{28}$$

This means that the bubble will be inside its own Schwarzschild radius (be a black hole) if

$$\ell_P^2 \sigma R^2 > R\,, \quad \text{i.e. } \frac{R}{\ell_P} > \frac{1}{(\ell_P^3 \sigma)} \tag{29}$$

Unless the bubble tension is extremely tiny in Planck units, an experimenter trying to make a big bubble will instead make a black hole!

E.g. bubble of size 10cm, not a black hole, would require tension $\sigma < 10^{-34} m_P^3$, or $\sqrt[3]{\sigma} < 10^8$GeV. We are in no danger at TeV energies... but in the very early universe we would have to keep energy scale of tension down by $\sim$ 11 orders of magnitude from the ambient energy! Even with bubble tension at the inflation scale we will make a BH.