VI MEXICAN SCHOOL ON QUANTUM GRAVITY

Note Title 14/11/2004

Aim of my 3 lectures: to introduce string theory as a remarkably successful theory of quantum spacetime, & review quantum gravity applications since "2nd superstring revolution".

Lecture 1 M@10

- String theory as quantum gravity; spectrum (pert & non.)
- Branes & their gravitational fields
- Black hole thermodynamics

Lecture 2 W@10

- Microscopic derivation of S_{BH}
  - leading order
  - next-" order
- Gravity/gauge duality
  - AdS/CFT
  - $\mathcal{N}=4$ Dp cases

Lecture 3 Θ@9

- Singularity resolution
- Black rings
- Fuzzballs
- Unstable D-branes

  \+ ...

Format / style of my lectures

★ Questions are crucial, so I can renormalize my pace to match impedances properly. (N.B.: Let me handle whether there are too many questions or unhelpful questions – I'm pretty handy at crowd control ☺.) Respect all around, please ♥

OUTLINE

String Theory as Quantum Gravity
- Perturbative spectrum
- Nonperturbative spectrum: D-branes

CFT

Loop counting: α' and g_s (not shown)

SUGRA

Wilsonian effective field theory philosophy

Hagedorn phenomenon

Schwarzschild BH
- Reissner-Nordstrøm
- No-hair theorems

Bekenstein-Hawking and the Info "Paradox"

Black p-branes

Making new solutions with algebra (not shown)

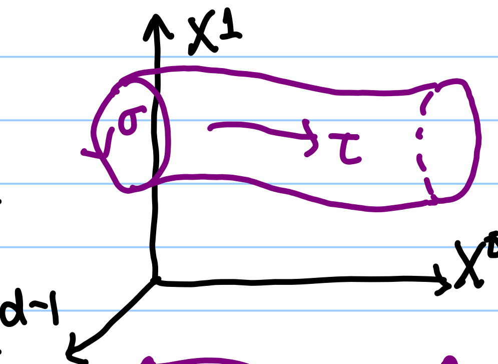
Clarification re scattering in GR & powers of G_N (not shown)

String Theory as a Theory of Quantum Gravity

- In string theory, the gravitational field is NOT fundamental, but there are well-understood régimes in which it does make sense.
- Let us choose $\boxed{\hbar = c = 1}$ BUT $\boxed{G_N \neq 1}$ (Unit system)
- For strings in flat spacetime, $g_{\mu\nu} = \eta_{\mu\nu}$, the classical string action is

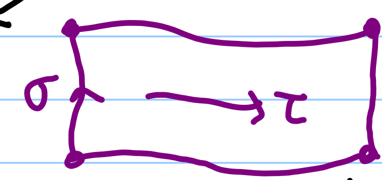
$$S_{NG} = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-\det\left(\eta_{\mu\nu}\, \partial_\alpha X^\mu \partial_\beta X^\nu \gamma^{\alpha\beta}\right)}$$

$\propto$ area swept out by string worldsheet



- Equations of motion are

$$\left[\partial_\tau^2 - \partial_\sigma^2\right] X^\mu(\tau, \sigma) = 0$$



$$\Rightarrow \quad X^\mu = X_L^\mu(\tau + \sigma) + X_R^\mu(\tau - \sigma)$$

- For closed strings, BCs $\Rightarrow$ independent L- & R-movers and

$$X_c^\mu = \sum_n \frac{a_n^+}{\sqrt{n}} e^{in(\tau+\sigma)} + \sum_m \frac{a_m^+}{\sqrt{m}} e^{im(\tau-\sigma)} + X_{cm}^\mu(\tau)$$

$$[a_m, a_n^+] = \delta_{mn}$$

- For open strings get standing waves i.e. just

$$X_o^\mu = \sum_n \frac{a_n^+}{\sqrt{n}} e^{in\tau} \cos(n\sigma) + X_{CM}^\mu(\tau)$$

$\Rightarrow$ Perturbative spectrum of states described by SHO's ☺

Perturbative spectrum of string theory

- Find $\boxed{\alpha' m_{cl}^2 = 2\,(N_L + N_R - 2)}$, $N_L + N_R = 2n$, $n \in \mathbb{Z}^+$

  and $\boxed{N_L = N_R}$

  GSO projection

  while $\boxed{\alpha' m_{op}^2 = (N-1)}$, $N = 2n$, $n \in \mathbb{Z}^+$

  and $\boxed{D=10}$

$\Rightarrow$ massless states for string wiggles in flat space include

$s=2$, $m^2=0$: gravity field $\boxed{g_{\mu\nu}}$

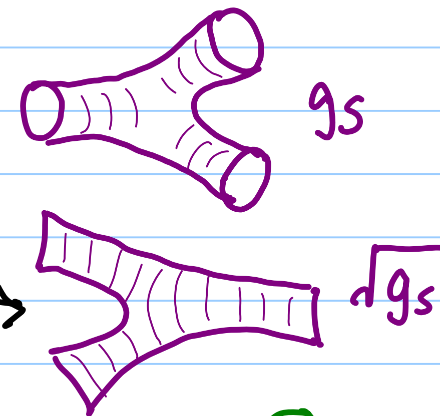
$s=1$, $m^2=0$: gauge field $\boxed{A_\mu}$

- In addition, coupling constants for these massless modes are derived:

$$\boxed{16\pi G_{10} = (2\pi)^7 g_s^2 \ell_s^8}$$

while

$$\boxed{g_{YM}^2 = (2\pi)^{p-2} g_s\, \ell_s^{p-3}}$$



- 3-string vertex $\propto$ g_s (closed)

  $\sqrt{g_s}$ (open)

- In fact, have bosonic fields : $(g_{(\mu\nu)}, B_{[2]}, \Phi)$ (A_μ)

  & fermionic superpartners: $(\psi_\mu^\alpha, \chi^\alpha)$ (λ^α)

SO(1,9) irreps

- For higher mass levels, ∞ gauge sym but no propagating ghosts

Nonperturbative spectrum : D-branes

- Fundamental string = starting point, for quantization, so far. Actually [most] string theories have D-branes, too!

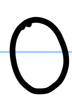
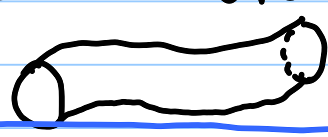
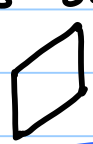
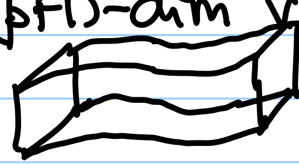
- CFT description: hypersurfaces where open strings end. Tension and dynamics of D-branes 'induced' from knowledge of perturbative string theory (!)

- Define $\boxed{\tau_{F1} = \frac{1}{2\pi\alpha'} = \frac{1}{2\pi l_s^2}}$

  Then $\boxed{\tau_{Dp} = \frac{1}{(2\pi)^p g_s l_s^{p+1}}}$

  Non-perturbative: heavy @ weak coupling.

- Moreover, Dp carry charge under fields living in D=10, Ramond-Ramond fields $C_{[n]}$ (antisymmetric tensors)

Object	Worldsheet	Coupling
Particle	1-dim. curve 	$e\int A_\mu \dot{x}^\mu d\tau = e\int A_{[1]}$
String (F1) 	2-dim. area 	$\int B_{[2]}$
Dp-brane 	(p+1)-dim volume 	$\int C_{[p+1]}$

Conformal Field Theory.

Classical string action lives in $d=1+1$ (the worldsheet) and possesses an infinite-dimensional symmetry algebra.
Easiest to see by inspecting mode expansion and defining $\left\{ \begin{array}{l} z \equiv e^{\tau_E - i\sigma} \\ \bar{z} \equiv e^{\tau_E + i\sigma} \end{array} \right\}$ [Wick rotated on worldsheet; ⇒ must also do in spacetime (!)]

so that (e.g.)

$$X^\mu_c = \sum_n \frac{\bar{a}_n^\dagger}{\sqrt{n}} z^{-n} + \sum_m \frac{a_m^\dagger}{\sqrt{m}} \bar{z}^{-n}$$

Virasoro algebra : holomorphic changes of coord z
anti-holo. " " " $\bar{z}$

- Quantum theory also a CFT if have central charge $\boxed{c=15}$
Can be made up of e.g. 10 dimensions of flat space via $\boxed{c = n_b + \frac{1}{2} n_f}$, or something else entirely – with no metric of any spacetime at all !! (If $c \neq 15$, pay price of extra term in string action ↔ "Liouville field".)
- If couple in massless fields $g_{\mu\nu}(X)$, $B_{\mu\nu}(X)$, $\Phi(X)$, etc. to closed string action [(−,+) signature]

$$\boxed{S_{\sigma\text{-model}} = \frac{-1}{2\pi\alpha'} \int_M d^2\sigma \left\{ \frac{\sqrt{-\gamma}}{2} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X) + \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}(X) \right. + \frac{1}{2\pi} \int_M d^2\sigma \sqrt{-\gamma}\, {}^{(2)}R \left[\Phi(X) + \ln g_s \right] + \frac{1}{2\pi\alpha'} \int_{\partial M} A_{[1]} \quad (+\text{ fermions})}$$

Loop-Counting: α' and g_s

Notice that

$$S_{\sigma\text{-model}} \supset (+\ln g_s)\left[\frac{1}{2\pi}\int_M d^2\sigma\sqrt{-\gamma}\,{}^{(2)}R\right]$$

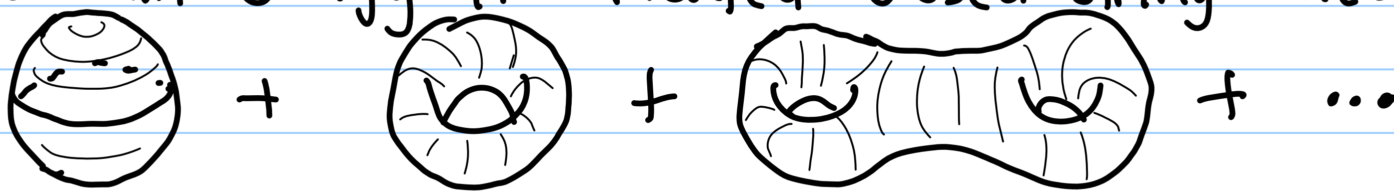
(constant)

$$= (+\ln g_s)\,[\chi] \qquad \chi = 2-2h-b-c \quad (\text{Euler } \#)$$

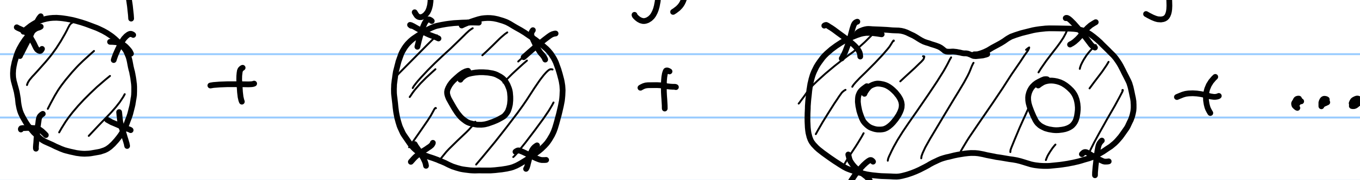
$$\Rightarrow \quad Z = \int \mathcal{D}(X,\text{etc.})\, e^{-S_\sigma} \propto g_s^{-2+2h+b+c}$$

$$\Rightarrow \boxed{\text{Loop-counting parameter is } \begin{Bmatrix} g_s^2 \text{ for closed strings} \\ g_s \text{ '' open '' } \end{Bmatrix}}$$

e.g. vacuum energy in oriented closed string theory:



e.g. in open string theory, $2\leftrightarrow 2$ scattering involves



Here, we used conformal invariance to bring our external states' ["vertex operators": 1st quant.] to finite pts.

- Doing loop diagrams beyond one-loop is <u>much</u> harder (typically) $\because$ need to avoid overcounting conformally equivalent worldsheets. Highly technical.

Supergravity

- The spacetime fields $g_{\mu\nu}$, $B_{[2]}$, Φ are functions of X^μ and hence should be thought of as coupling functions!
- They should not run with energy, in order to get a CFT.
  $\Rightarrow$ "beta-function equations" $\beta_\Phi = 0$; $\beta_{g_{\mu\nu}} = 0$; $\beta_{B_{[2]}} = 0$; $\Leftrightarrow$ spacetime equations of motion!
- Reconstruct action: for common sector have

$$\boxed{S^{(NS-NS)}_{SUGRA} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g}\left\{ e^{-2\Phi}\left[R + 4(\nabla\Phi)^2 - \frac{1}{2}|dB_{[2]}|^2\right\} \ (+\text{fermions})}$$

- Transform to Einstein frame from string metric $g_{\mu\nu}$:

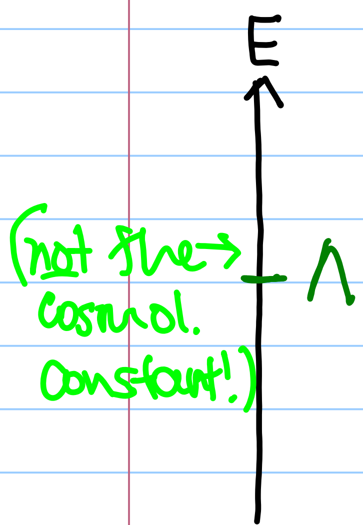
$$\tilde{g}_{\mu\nu} = e^{-4\Phi/(D-2)} g_{\mu\nu} \qquad \text{(works } \forall D\text{, in fact)}$$

$$\Rightarrow S^{(NS-NS)}_{SUGRA} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-\tilde{g}}\left\{ \tilde{R} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - \frac{1}{2}e^{-\Phi}|dB_{[2]}|^2\right\}$$

- Corrections to this lowest-order physics can be computed using standard algorithms of QFT. First nonzero corrections are invariant under field redefinitions (not true at higher orders). For maximally supersymmetric cases (Type II) first nontrivial corrections start at $(\alpha')^3$.
  $\Rightarrow$ two perturbation expansions : $\boxed{(\alpha' ; g_s)}$
- Existence of Newtonian limit assured in this approach $\Rightarrow$ this quantization of gravity does match onto real-world physics.

Wilsonian point of view & Low-Energy String Theory

- Work philosophically within framework of Wilsonian approach to QFT and the universe:



  - Imagine having full fundamental action $S[\Phi^A]$. Set UV cutoff scale Λ and integrate out modes with momentum higher than Λ. Get
  $$S_{eff}[\phi^a;\Lambda] = \sum_n \int d^Dx \; g_n \mathcal{O}_n \cdot \frac{1}{\Lambda^{D-n}}$$
  (n = mass dim. of $\mathcal{O}_n$)
  - Only effect of UV physics is Λ-dependence of dimensionless couplings g_n for light modes $\{\phi^a\}$
  $\Rightarrow$ RG equations $\frac{\partial}{\partial \ln\Lambda} g_n = f_n(\{g_i\};\Lambda)$

- We quantized the string action precisely according to this recipe.
- Free strings in $\eta_{\mu\nu}$ $\Rightarrow$ got infinite set of SHO's @ $\hbar\omega_n = n\hbar\omega_1$, & an ∞-dimensional symmetry algebra (a pair, for closed strings).
- We insisted on conformal invariance and got RG flow equations for the worldsheet QFT $\Rightarrow$ spacetime eqns of motion.
- SUGRA action for $g_{\mu\nu}$ (etc.) is a $S_{eff}[\phi^a;\Lambda\sim 1/\ell_s]$. Then α' and g_s corrections are thought of as higher-dimension operators in $S_{eff}[\phi^a]$.

Hagedorn phenomenon

- Strings in highly excited states have many possible ways of partitioning oscillator energy N

  e.g. $N=2$ can be apportioned as $2\omega_1$ or $1\omega_2$
  $N=3$ $3\omega_1$, $1\omega_2 + 1\omega_1$, $1\omega_3$

  and so forth.

- At large N, Hardy-Ramanujan formula gives approximation to # ways of partitioning an integer N

  $$\rho_{st}(m) \propto \exp\left(\frac{+m}{T_{Hag}}\right)$$

  where $T_{Hag} = \mathcal{O}(l_s^{-1})$ and depends on the string theory.

- Now consider canonical ensemble for strings. Define partition function

  $$Z(\beta) \hat{=} \int dm.\, \rho_{st}(m).\underbrace{e^{-\beta m}}_{\text{Boltzmann factor}}$$

  $$= \int dm \exp\left[\left(-\frac{1}{T} + \frac{1}{T_{Hag}}\right)m\right]$$

  $$\to +\infty \quad \text{if} \quad T \geq T_{Hag}$$

- What happens?
  - Pumping in more energy just makes longer and longer pieces of floppy string, rather than raising T.

  (& Interactions matter.)

Schwarzschild Black Holes

- SUGRA $\supset$ Einstein gravity; $\Rightarrow$ should have BH in string theory. Black holes in string theory not simply "nice luxury"; BH appear naturally and play crucial role. ☺

$$ds^2 = -\left[1-\left(\frac{r_H}{r}\right)^{d-3}\right]dt^2 + \left[1-\left(\frac{r_H}{r}\right)^{d-3}\right]^{-1}dr^2 + r^2 d\Omega^2_{d-2}$$

ADM mass : $\boxed{GM \propto r_H^{\,d-3}}$

Near-horizon geometry is Rindler space; to avoid conical singularity in Euclidean continuation identify $\beta \Rightarrow$ (not always available for any given Lorentzian spacetime!)

$$T_H \propto \frac{1}{r_H}$$

- Bekenstein-Hawking entropy: get (e.g.) ex $dM = T_H \, dS_{BH}$

$$S_{BH} = \frac{A_d}{4G_d} \propto \frac{1}{G_d} r_H^{\,d-2} \propto \frac{1}{G_d}(G_d M)^{\frac{(d-2)}{(d-3)}}$$

so $\boxed{\rho_{BH}(M) \propto \exp\left[\frac{1}{G_d}(G_d M)^{\frac{d-2}{d-3}}\right]}$

Using $G_d \sim g_s^2 \, \ell_s^{d-2}$, see $\boxed{\frac{S_{BH}}{S_{St}} \sim \left(\frac{r_H}{\ell_s}\right)}$

Black Hole Correspondence Principle

Reissner-Nordstrom and $AdS_2 \times S^2$ $(d=4)$

- $$ds^2 = -\left(1-\frac{2GM}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2GM}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

  $$F_{[2]} = +\frac{Q}{r^2}\, dt\wedge dr$$

  Horizons at $r_\pm = GM \pm \sqrt{G^2M^2-Q^2}$

  Extremal geometry when $GM = |Q|$; then $r_\pm = GM$.

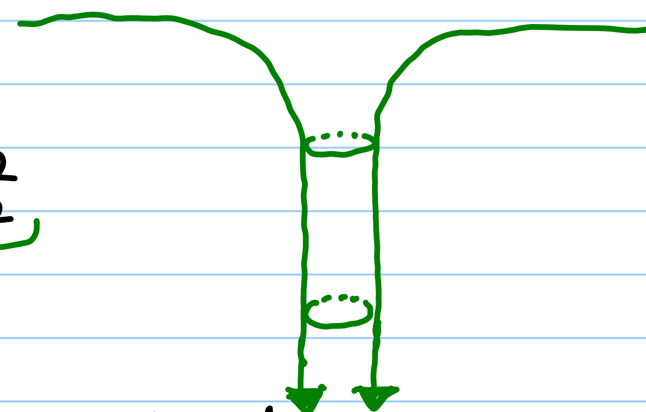
- Change to isotropic coords $R = r-|Q|$

  $$\Rightarrow \quad ds^2 = -H^{-2}dt^2 + H^2\left(dR^2 + R^2 d\Omega_2^2\right)$$

  where $H = 1+\frac{|Q|}{R}$

- In region $R\to 0$, have

  $$ds^2 = \underbrace{-\frac{R^2}{Q^2}dt^2 + \frac{Q^2}{R^2}dR^2}_{AdS_2} \;\times\; \underbrace{+\; Q^2 d\Omega_2^2}_{S^2}$$



- Wilsonian reasoning $\Rightarrow$ don't trust classical topology near horizon $\because$ string winding modes around $S^1_\beta \longrightarrow$ massless!

No-Hair Theorems

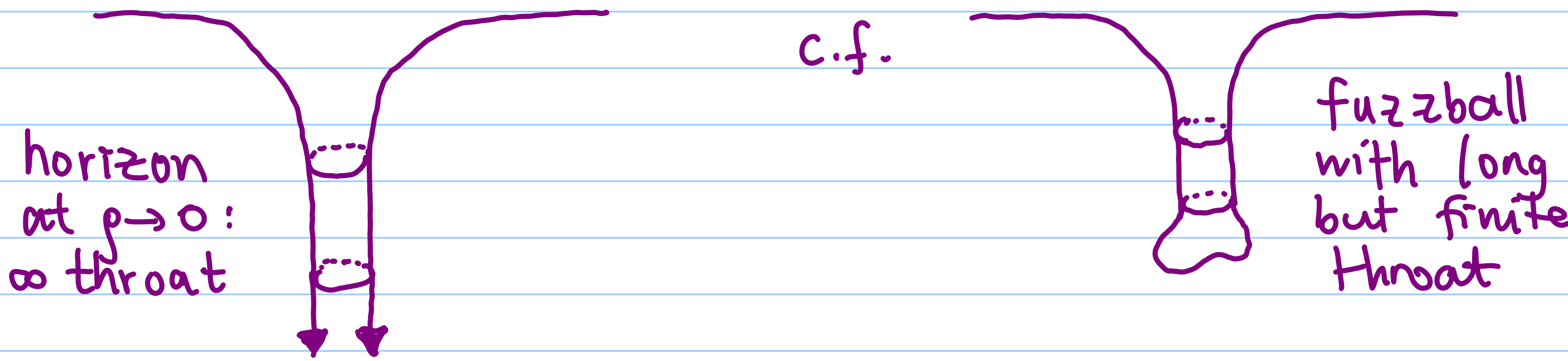
- In general, finding new solutions of {GR + matter} difficult because of nonlinearity of field equations.
  Aided in $d=4$ by no-hair theorem
  (1) Choose $\mathcal{L}_{matter}(\phi)$
  (2) Find eqn for ϕ ; 2nd order PDE
  (3) Demand regularity at ∞ (asymptotic region) and at the event horizon (if it exists!)
  (4) Only nontrivial "hair" = Noether charges
- BH in $d=4$ characterized by (M, J, Q) only.
- Cosmic censorship conjecture: BH singularity hidden behind event horizon(s).
- $d=5$ surprise !! Black ring solutions with toroidal horizon.
  Simple intuition: $\left\{\begin{matrix}\text{gravitational}\\ \text{attraction}\end{matrix}\right\}$ balances $\left\{\begin{matrix}\text{centrifugal}\\ \text{repulsion}\end{matrix}\right\}$
  Dominate entropically over (Myers-Perry) rotating BH with spherical horizons for some regimes of parameters.
- Can prove no-hair theorems in higher-d for static cases.

Bekenstein-Hawking & the Information Paradox

- Hawking recently publicly capitulated on an old bet: he now believes (like the entire string theory community) that BH do <u>not</u> destroy information. But as yet, no calculable model in which information explicitly seen to be fully returned! (Perturbative approaches don't work...)

- "Info paradox" comes about from assuming that the horizon cleanly cuts off external observer from interior info.
  - BH Complementarity ⇒ no global event horizon; only apparent horizon. Nonetheless, trapped surfaces.

* Is a black hole really a black hole, or just a grey fuzzball, like a lump of coal?



Black p-branes

- SUGRAs with extended supersymmetry may possess "BPS states" preserving fraction ν of the supersymmetries; for simplest D-brane configurations $\nu = \frac{1}{2}$.
- Mass positivity bound

$$\boxed{M \geq c(g_s)\,|Q|}$$

(depends on the theory and the object type.)

saturated for BPS states.

- Supersymmetric Dp-brane geometries:

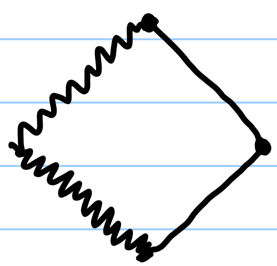
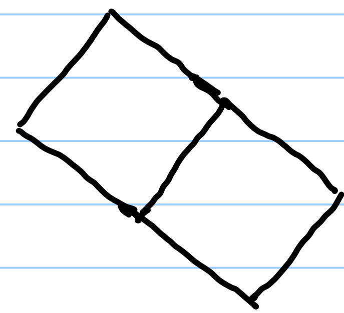
$$\boxed{\begin{aligned} ds^2 &= H^{-1/2}\left(-dt^2 + d\vec{x}_{\parallel}^{\,2}\right) + H^{+1/2}\left(dr^2 + r^2 d\Omega^2_{8-p}\right) \\ e^{\Phi} &= H^{(3-p)/4} \\ C_{[p+1]} &= \left(H^{-1} - 1\right) dt \wedge dx^1 \wedge \ldots \wedge dx^p \end{aligned}}$$

where

$$\Box H = 0 \quad \Rightarrow \quad \boxed{H = 1 + c_p\, g_s\, N \left(\frac{\ell_s}{r}\right)^{7-p}}$$

Above-BPS geometries also known!

Penrose diagrams in $d=10$:-

Dp≠3  ← (SUGRA says) → D3 

Making New Solutions with algebra

e.g. #1 : D0-brane solution in $d=10$.

- Begin with $d=10$ neutral black hole (in IIA SUGRA)
- Lift to $d=11$.
- Boost along 11th dimension (Solution-generating transformation)
- Compactify again along x^4;
  $R_4 = g_s l_s$
- Get solution with charge $Q_0 \propto \sinh(2\gamma)$ [where γ = rapidity parameter for boost] and mass.
- Extremal limit obtained via
  $m \to 0$, $\gamma \to \infty$; $e^{2\gamma} m = \text{finite}$.

Used SUGRA dimensional reduction expression

$$ds^2 = e^{-2\Phi/3} g_{\mu\nu} dx^\mu dx^\nu + e^{4\Phi/3} |dx^4 + C_{[1]}|^2$$

$$A_{[3]} = C_{[3]} + B_{[2]} \wedge dx^4$$

- Very useful solutions : Myers-Perry (rotating BH in $d>4$) $[(d-1)/2]$ J_i's have e.g. $dt\,d\varphi$ components in ds^2 and a boost can nicely hook onto those.
- Recently, very interesting solutions (Mathur, Saxena, +c.) obtained by taking clever limits of boosted rotating BH.

[Clarification re: a question in Lecture 1

- The Newton constant G_N IS obtained via study of scattering amplitudes in our lowest-energy theory ie. SUGRA ⊃ GR. Clear via quantum / Wilsonian principles!

$$S_{GR} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g}\, R_g$$

- Expand $g_{\mu\nu} \cong \eta_{\mu\nu} + \sqrt{8\pi G_D}\, h_{\mu\nu}$ so that the free part of the action is canonically normalized:

$$S_{GR} = S_{free} + S_3 + S_4 + \cdots$$

where $S_{free} = \int d^D x \left\{ -\frac{1}{2} \partial_\mu h_{\alpha\beta} \partial^\mu h^{\alpha\beta} \right\}$

and (e.g.) $S_3 = \sqrt{G_D} \int d^D x \left\{ \partial^\mu h_{\alpha\beta} \partial^\nu h_{\gamma\delta} h_{\mu\nu} \left(\eta^{\alpha\gamma} \eta^{\beta\delta} + \text{etc.} \right) \right\}$

- $S \Rightarrow 2 \to 2$ scattering of gravitons has cross-section

$$\sigma_{2\to2} \sim (\text{tree}) \left[1 + G_D E^{D-2} + \mathcal{O}(G_D E^{d-2})^2 + \cdots \right]$$

∴ quantum corrections out of control at $\boxed{E \gtrsim E_p}$

$[16\pi G_D \equiv (2\pi)^{D-3} \ell_p^{D-2}$ & $E_p \equiv \ell_p^{-1}]$

non-renormalizable]