VI Mexican School on QG - Lecture 2

Note Title 19/11/2004

Outline for today

- Microscopic calculation of black hole entropy (8 pp)
  - building the black hole metric in d=5 from 3 types of constituents
  - computing degeneracy of states of massless states of open strings in CFT
  - beyond leading order : beautiful agreement

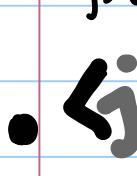
[• Gravity/gauge duality postponed to Lecture 3 ☺]

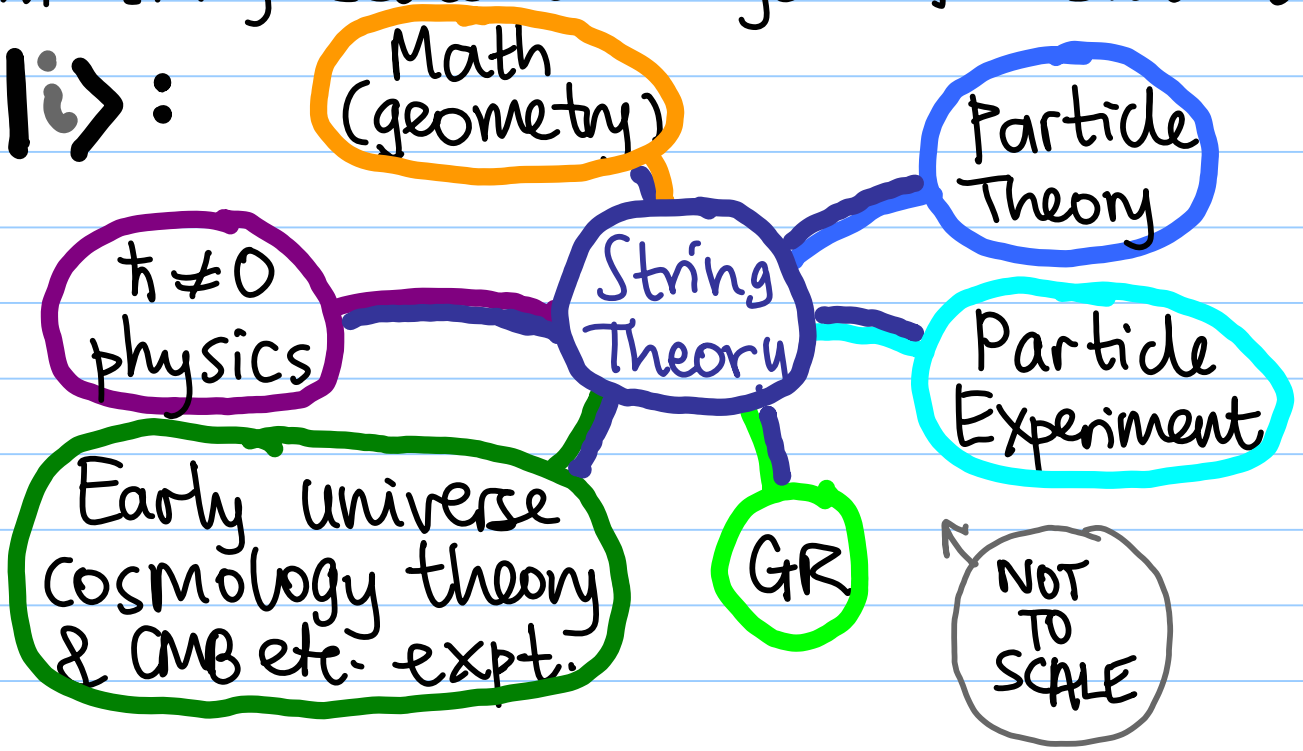
Viewpoint: String theory as an intellectual unifier

(i)

- String theorists are usually trained as physicists, although mathematicians are also interested in S.T.

- We were brought up in the tradition of Wilsonian approach which is totally natural way of comprehending universe from string scale to edges of visible universe: 10^{60}

-  :



- True interdisciplinarity is a 2-way flow of info.

- Rising tide raises all boats if lines are strong (non-threatening!)

- Proposal: think of ST as unifying intellectual force

online notes

(iii)

- These lecture notes, plus hyperlinks to some good relevant web resources (e.g. hep-th and/or review articles) related to my presentations will be ready by next Monday at

- http://www.physics.utoronto.ca/~peet/online/mx699/

  Hope you find them useful ... in general, and possibly even useful in a provocative way ☺

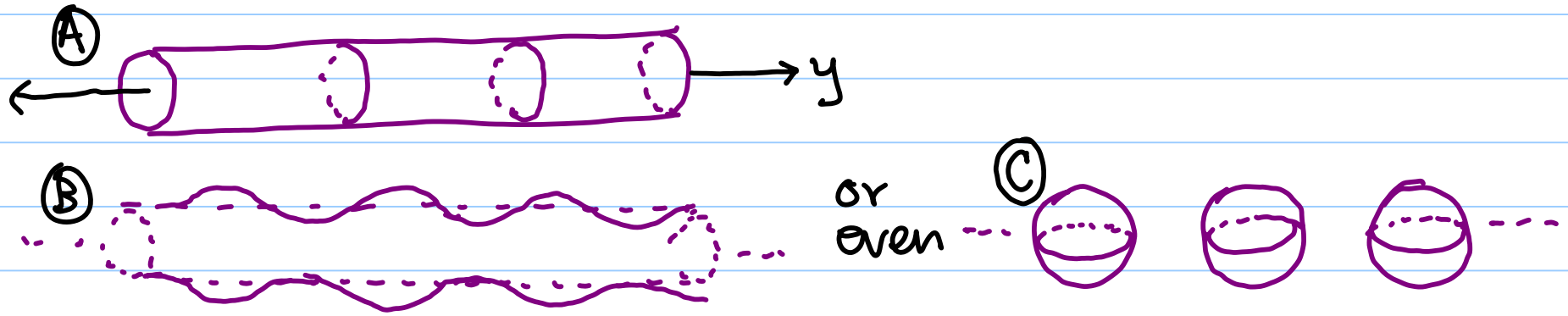
- I also maintain other online talks and sundry other stuff at http://www.physics.utoronto.ca/~peet/home/online/ and http://www.physics.utoronto.ca/~peet/home/

- I hope to do a "proper" (LaTeX) write-up as well, likely co-authored with a junior collaborator, including: material covered in these lectures, plus more recent/advanced status report on string theory as a theory of quantum gravity. ☺

Are Black p-branes Stable?

Consider black brane with translationally invariant horizon, unprotected by superpower (i.e. not BPS).

- Classical field equations possess tachyon ($m^2 \to 0$ as $\to$ BPS)
  For many (not all!) black branes, $\Leftrightarrow$ entropic instability



- Basic reason why? Sphere area $\neq$ cylinder area ☺!
  Compactify y on S^1.

$$S_{BH} \simeq \frac{A_d}{4G_d} = \frac{A_{d-1}}{4G_{d-1}} \quad \text{if } \partial_y \text{ a symmetry.}$$

$$\Rightarrow S_{BH}(A) \sim \frac{1}{G_d}(r_A^{d-3} R) \qquad \text{c.f. } S_{BH}(B) \sim \frac{1}{G_d} r_B^{d-2}$$

$\Rightarrow$ figure of merit is $\sim (r_H/R)$.

- BPS branes (conserved R-R/NS-NS charge) are stable.

"Superposing" D-branes

- BPS geometry for stack of N Dp-branes has
  $T_H = 0$, $S_{BH} = 0$
  (at lowest order in α' and g_s)

- Particular cases warranting extra attention:
  - BPS d=5 BH with 3 charges (Strominger-Vafa)
  - BPS d=4 BH with 4 charges (⊃ Reissner-Nordström)

- IF Dp-branes and Dq-branes satisfy no-force condition, metric factorizes in simplest way possible. For D1+D5,

$$ds^2 = \frac{1}{\sqrt{H_1 H_5}}(-dt^2+dy^2) + \sqrt{\frac{H_1}{H_5}}(dz^2) + \sqrt{H_1 H_5}\,(dr^2+r^2 d\Omega_3^2)$$

$$e^{\Phi} = \sqrt{H_1/H_5}$$

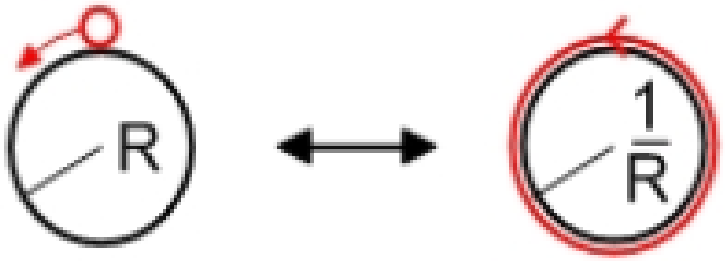
$$C_{[2]} = (H_1^{-1}-1)\, dt\wedge dy$$

$$C_{[6]} = (H_5^{-1}-1)\, dt\wedge dy\wedge dz^1\wedge\ldots\wedge dz^4$$

- Note: $g_{zz} \to$ finite @ horizon ($r=0$) but $g_{yy}\to 0$ there. Physical reason: nothing resisting tension of D1 & D5.

"Gravitational waves" [and T-duality]

- BPS bound can be saturated by objects with $M = 0 = |Q|$.
  Moves at speed of light along one direction y.
  $ds^2 = -dt^2 + dy^2 + (H-1)(dt+dy)^2 + d\vec{x}_\perp^2$ "W"
- [Related to geometry of fundamental strings by
  T-duality symmetry of string theory



Swap $n \leftrightarrow w$ $\quad (n\lambda_{deB} = 2\pi R)$

$\frac{R}{l_s} \leftrightarrow \frac{l_s}{R}$

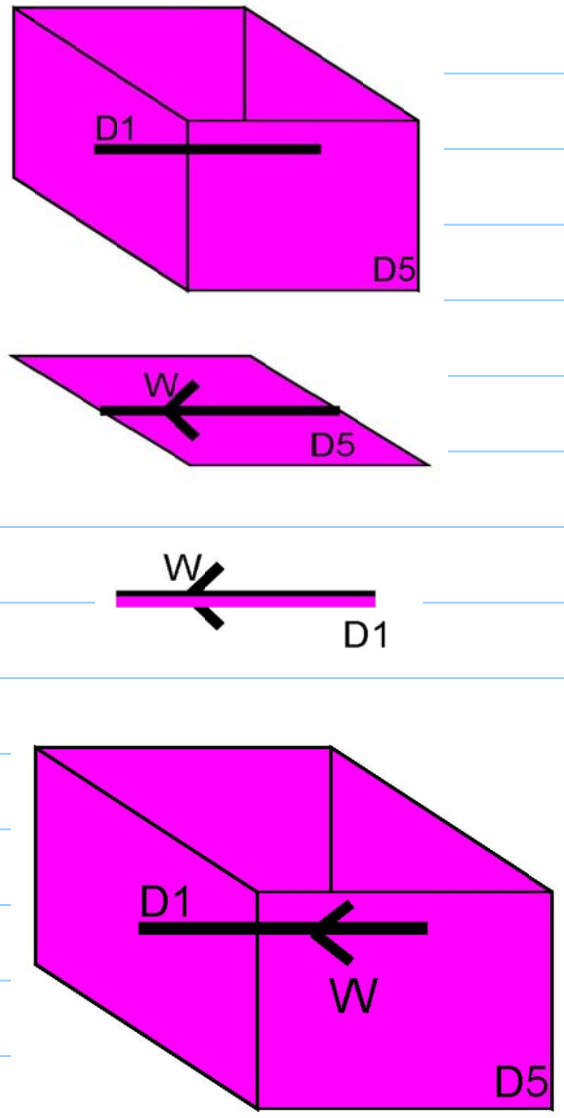
Realized also @ SUGRA level.

- Special states of string theory on $\mathbb{R}^{1,8} \times S^1$ become massless at $R = l_s$ ("self-dual radius")!
  Mass formula: $m^2 l_s^2 = 2(N_L + N_R - 2) + n^2 \frac{l_s^2}{R^2} + w^2 \frac{R^2}{l_s^2}$

  momentum constraint: $N_R - N_L = nw$
  $\Rightarrow \{N_R, N_L; n, w\} = \{|+1, 0; \pm 1, \pm 1\rangle, |0, +1; \pm 1, \mp 1\rangle\}$
  Enhanced gauge symmetry $SU(2)$; Higgsed at $R \neq l_s$]

Cooking the Strominger-Vafa confection

D1, D5 and W satisfy pairwise no-force condition ⇒ marginally bound system. W puffs out S¹ @ r→0 ☺



- Calculate SUGRA geometry ⇒ entropy

$$S_{BH} = 2\pi\sqrt{\Delta}\,, \quad \Delta = N_1 N_5 N_W$$

Independent of $T^4 \times S^1$ moduli
Δ depends only on monopoles.
(cf. BPS black rings!)

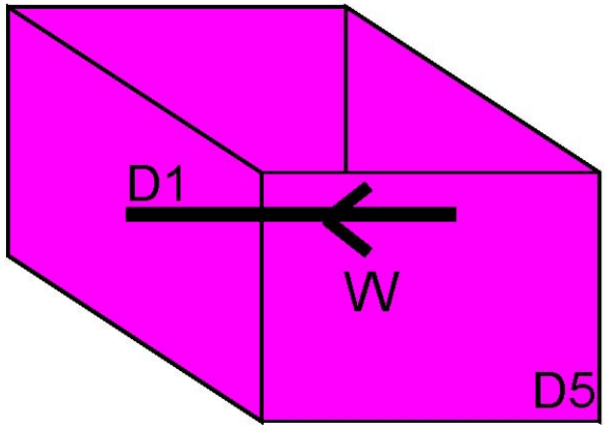
- Adding J_φ and J_ψ [d=5] found $J_\varphi = -J_\psi$ (see directly during algebraic solution-generating) &

$$S_{BH} = 2\pi\sqrt{N_1 N_5 N_W - J^2}$$

- Corrections to this result @ NLO in α' are computable— tough, but beautiful! See p-8 ↓

Microscopic computation of S

We know our configuration of D-branes, and we want to count S_{micro} (tree level).



- Why believe that $S_{BH} = S_{micro}$?

  (a) supersymmetric renormalization theorem
  (b) correspondence transition not at a point, but along a locus of lower codimension!
  For 2 or 3 R-R charges, amazing 'conspiracy' ⇒ no phase diagram with different degrees of freedom for different M (etc.) as for Schwarzschild!
  In fact:

$g_s N_1 \ll 1$	same system,	$g_s N_1 \gg 1$
$g_s N_5 \ll 1$	just different	$g_s N_5 \gg 1$
$g_s^2 N_p \ll 1$	d.o.f. in	$g_s^2 N_p \gg 1$
perturbative	← different →	black
strings	regimes.	hole

The CFT story

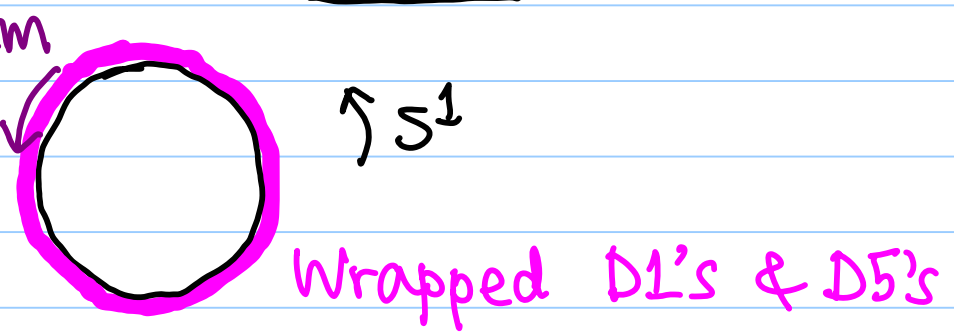
- Strominger-Vafa realized that, for small vol(T^4), theory involving D-branes is a CFT.
- General Cardy formula says

$$\rho_{CFT} \cong \exp\left(2\pi\sqrt{\frac{c}{6}ER}\right)$$

where c = central charge of $d=1+1$ theory $(n_b + \frac{1}{2}n_f)$
E = energy in $d=1+1$
R = size of spatial dimension

- Our system with D1+D5+W has momentum around S^1. But BPS D1 or D5 cannot carry P ∴ boost-invariant.
⇒ carried by light fundamental strings between D-branes
BPS state with $E_{d=1+1} = |P|$ ⇒ $\boxed{E = \frac{N_p}{R}}$

- Now need to know central charge (c).



Strominger-Vafa

- Theory involving massless open strings starting/ending on N_1 D1 & N_5 D5 can be regarded as a $d=5+1$ gauge theory with gauge group $U(N_5)$ and with instanton # N_1 ... or various other ways.
  Implement the F- and D-term constraints (gauge sym..)

  $\Rightarrow$ $\boxed{C = (4+\frac{4}{2})\, N_1 N_5}$

  Roughly: $m^2=0$ open strings starting on D1 & ending on D5

  $\Rightarrow$ $\boxed{S_{micro} = 2\pi\sqrt{N_1 N_5 N_p}}$ 3-charge $d=5$ system

  (+ linear corrections)

- If want to do 4-charge $d=4$ system: Consider e.g. D2 // D6 // W ⊥ NS5 (or D3 ⊥ D3 ⊥ D3 ⊥ D3, etc.)
  D-branes can end on NS5 $\Rightarrow$ N_5 as many $m^2=0$ strings

  $\Rightarrow$ $\boxed{S_{micro} = 2\pi\sqrt{\diamondsuit}\ ,\ \diamondsuit = N_{D2} N_{D6} N_p N_{NS5}}$ (+ quadratic corrections)

Wald's formula and α' corrections

- Entropy $\neq$ area of event horizon in generic gravity theory!
  Wald & Iyer : when have Killing horizon, S is a Noether charge

$$S = \frac{1}{16}\oint_{S^1} \epsilon_{ab}\epsilon_{cd}\frac{\delta(8\pi\mathcal{L})}{\delta R_{abcd}}$$

$\leftarrow [a,b,c,d = 0,1]$

- Must recognize this to properly match black hole entropy to string/D-brane-y microscopic prediction, at sub-leading order (resolved earlier contradiction!)
  $\Rightarrow$ joint triumph for relativists & string theorists ♥

- α' corrected black hole geometries obtained via beautiful and tour-de-force computations of deWit and others ($d=4$, $\mathcal{N}=2$; special geometry) R^2 terms

- $$S_{BH} = 2\pi\sqrt{\frac{1}{6}|q_0|\left(C_{ABC}\,p^A p^B p^C + c_{2A}\,p^A\right)}$$

  charge vector ($d=5$)

  triple intersection CY 4-cycles; 2nd Chern class