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Lecture 1:

Black Holes, and the Black Hole Information Problem.

Concepts introduced:

Classical:

- d = 3 + 1 warm-up: Kerr-Newman
- event horizon, singularity, ergosphere
- Penrose diagram
- d = 2 + 1 BTZ black hole
- no-hair theorems in d = 3 + 1 c.f. $d \ge 4 + 1$

Semiclassical:

- Euclidean time / Wick rotation, Rindler space
- Hawking temperature, black hole lifetime
- Reissner-Nordstrøm and $AdS_2 \times S^2$
- black hole thermodynamics
- entropy bounds and holography
- BH information problem

Natural Units used throughout: $\hbar = c = k_B = 1$. Hence,

$$[t] = [\ell] = [1/m] = [1/T] \quad [G_N] = [\ell]^{d-2}$$

However, we will *not* suppress powers of string coupling g_s , string length ℓ_s – or Newton constant G_N , as relativists do.

Here we discuss only string/M theory as theory of quantum gravity. Other approaches (loop quantum gravity, dynamical triangulations) have trouble producing a Newtonian limit - !.

Not all massive objects are black holes. To qualify, object needs its Schwarzschild radius larger than its Compton wavelength, or

 $m > m_P$

So electron, with $m_e \sim 10^{-23} m_P$, does not qualify.

String theory is *the* highest-energy physics. In very high CM energy collision, formation of big fat black holes is ubiquitous. High-energy density of states of string theory is dominated by black holes. This property makes it *completely* unlike any Lorentz-invariant QFT.

CLASSICAL BLACK HOLES

<u>Kerr-Newman</u> metric is general solution of d = 3 + 1 Einstein-Maxwell action. Stationary: no *t*-dependence. In Boyer-Lindqvist coords

$$ds^{2} = -\frac{\Delta}{\sigma^{2}} \left(dt - a \sin^{2} \theta d\phi \right)^{2} + \frac{\sigma^{2}}{\Delta} dr^{2} + \sigma^{2} d\theta^{2} + \frac{\sin^{2} \theta}{\sigma^{2}} \left[\left(r^{2} + a^{2} \right) d\phi - a dt \right]^{2}$$
(1)

where

$$\sigma^{2} \equiv r^{2} + a^{2} \cos^{2} \theta$$

$$a \equiv J/M$$

$$\Delta \equiv r^{2} - 2G_{4}Mr + a^{2} + Q^{2}$$
(2)

Electromagnetic field two-form is

$$F_{\mathcal{Z}} = Q \frac{\left(r^2 - a^2 \cos^2 \theta\right)}{\sigma^4} dr \wedge \left(dt - a \sin^2 \theta d\phi\right) + Q \frac{2ar \cos \theta \sin \theta}{\sigma^4} d\theta \wedge \left[\left(r^2 + a^2\right) d\phi - adt\right]$$
(3)

Roots of Δ occur when

$$r_{\pm} = G_4 M \pm \sqrt{(G_4 M)^2 - a^2 - Q^2}$$
(4)

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Common astrophysicist complaints:

Q1. Q is unphysical. Charged astrophysical black holes discharge on a very short timescale via Schwinger pair production.

A1: Charges on all black holes discussed herein are not carried by light elementary quanta like electrons of QED.

Q2. Astrophysical black holes formed via gravitational collapse have a lower mass limit \sim few solar masses. Smaller ones must be 'primordial'. A2: We are not size-ist.

Event horizon of a stationary black hole geometry occurs where

$$g_{rr} \to \infty$$
 (5)

Not same as $g_{tt} = 0$ condition, in general. For evolving geometry, event horizon has no definition; it is a global concept.

Although metric components blow up at horizon, this is only a coordinate singularity; see this by computing curvature invariants.

$$R^{\mu\nu}R_{\mu\nu} = \frac{4Q^4}{\left[r^2 + a^2\cos^2\theta\right]^4}$$
(6)

(For Schwarzschild, $R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma} = 48(G_4M)^2/r^6$.)

Curvature invariant at horizon of a big KN black hole is weak; blowup is at $r^2 + a^2 \cos^2 \theta = 0$ – i.e. the physical singularity.

 $\underline{\mathsf{Ergosphere}}$ of rotating black hole is defined as region outside horizon where

$$g_{tt} = 0 \tag{7}$$

Inside ergosphere,

- Since t is spacelike, energy can be negative for real particles.
 Allows energy extraction from BH Penrose process.
- No sit-still orbits rotate in direction of J. (Extreme frame-dragging.)
- At horizon, $\Omega_H = a/(r_+^2 + a^2)$.



[Carter-]Penrose diagram gives causal structure of spacetime manifold.

Use conformal transformation to bring infinity onto the page.

Light-beams go at 45° ; timelike geodesics steeper, spacelike less so.



In gravitational collapse only part of the eternal BH diagram is present; it is matched onto a region of Minkowski space. In collapse situations there is of course no time reversal invariance, and so the physical Penrose diagram is not symmetric.

Eternal Schwarzschild \downarrow and maximal analytic extension of Ring singularity Kerr-Newman \longrightarrow at $\bar{\sigma}=0$ <11. 8 1× Singularity <"⁶ Horizon r + 1 J Ì (II B 110 1 J ,//×

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For reference, here are the Penrose diagrams for deS and AdS:



In three spacetime dimensions, Coulomb potentials are logarithmic, not power-law. BH metrics are consequently very different.

But if have a negative cosmological constant, find well-behaved black holes, the BTZ black holes. They are solutions of action

$$S = \frac{1}{16\pi G_3} \int d^3x \sqrt{-g} \left(R_g + \frac{2}{\ell^2} \right)$$
(8)

i.e. cosmological constant is $\Lambda = -1/\ell^2$. The metric is

$$ds_{\mathsf{BTZ}}^{2} = -\frac{(r^{2} - r_{+}^{2})(r^{2} - r_{-}^{2})}{\ell^{2}r^{2}}dt^{2} + \frac{\ell^{2}r^{2}}{(r^{2} - r_{+}^{2})(r^{2} - r_{-}^{2})}dr^{2} + r^{2}\left(d\varphi + \frac{r_{+}r_{-}}{\ell r^{2}}dt\right)^{2}$$
(9)

The coordinate φ is periodic, with period 2π .

The event horizons are at $r = r_{\pm}$, and quantum numbers are

$$M = \frac{(r_{+}^{2} - r_{-}^{2})}{8\ell^{2}G_{3}} \qquad J = \frac{(r_{+}r_{-})}{4\ell G_{3}} \tag{10}$$

Now consider object with special "negative mass":

$$J = 0$$
 $M = M_{\text{vac}} \equiv -\frac{1}{8\ell^2 G_3}$ (11)

This animal is not a black hole; the metric becomes

$$ds^{2} = -\frac{(r^{2}+1)}{\ell^{2}}dt^{2} + \frac{\ell^{2}}{(r^{2}+1)}dr^{2} + r^{2}d\varphi^{2}$$
(12)

This is AdS_3 in global coordinates.

Point particles in d = 2 + 1 correspond to range

$$M_{\rm Vac} < M \le 0 \tag{13}$$

These spacetimes have conical (naked) singularities.

Properties of d=3 gravity \Rightarrow BTZ spacetime is everywhere locally AdS_3 . Crux is identifications, by discrete subgroup of isometry group SO(2,2).

- For |J| > 0, no singularity, but region of CTC's $(r \le 0)$.
- For |J| = 0, "singularity" at r = 0 of Taub-NUT type.

Nonsingular behaviour does *not* persist upon coupling to matter [BTZ]. See also lectures of N.Seiberg. In general, finding new SUGRA geometries very difficult – eqns of motion nonlinear. Search aided by classical <u>no-hair theorems</u> – once conserved charges of a system are determined, spacetime geometry is unique. Essential physics:

- Specify Lagrangian, including matter couplings.
- Gravity falloffs give two conserved quantum #s: M, J.
- Gauge fields give conserved charges Q_i .
- Any other matter fields have 2nd-order PDE's in BH background. Must have solutions well-behaved both at infinity and at horizon. Not possible ⇒ uniqueness.

It is important for applicability of no-hair theorems that any black hole singularity be hidden behind an event horizon; theorems *fail* in space-times with naked singularities.

Newer results:

Emparan and Reall found new d = 4 + 1 rotating black ring solution with toroidal horizon! Throws into doubt all uniqueness theorems in d > 3 + 1!

Gibbons et al: uniqueness proven for *static* asymptotically flat spacetimes, and static charged dilatonic black holes.

Will discuss p-brane story in Lecture 3 (nonuniqueness there too...).

Consider Schwarzschild for simplicity. Asymptotically flat – see from large-r behaviour of metric. How about near-horizon region?

Change to radial coordinate of $\eta,$ proper distance, i.e. $g_{\eta\eta}=$ 1. Near $r=r_{H},$

$$\eta \sim 2\sqrt{r_H(r - r_H)} \tag{14}$$

Also, easy to rescale time,

$$\omega = \frac{t}{2r_H};\tag{15}$$

then metric becomes

$$ds^{2} \sim -\eta^{2} d\omega^{2} + d\eta^{2} + r_{H}^{2} d\Omega_{2}^{2}$$
(16)

The (η, ω) piece is in fact Rindler space.

So near-horizon region of a Schwarzschild black hole is approximately two-dim Rindler space times a constant two-sphere. This becomes exact in the limit $M \to \infty$.

True also for generic higher-d static BH, with transverse S^{d-2} .

SEMICLASSICAL BLACK HOLES

In QFT applications, Euclidean Feynman path integral formally identified with statistical mechanical partition function. Periodicity in Euclidean time is identified as inverse temperature. Had

$$ds^{2} \sim -\eta^{2} d\omega^{2} + d\eta^{2} + r_{H}^{2} d\Omega_{2}^{2}$$
(17)

Wick rotate ω . Avoid a conical singularity if we identify Euclidean time $i\omega$ with period 2π . Euclidean BH: cigar geometry in $(\eta, \omega) \longrightarrow$



Since $\omega = t/(2r_H)$, translating back to AF coord system gives

$$T_{\mathsf{H}} = \frac{1}{8\pi G_{\mathsf{4}}M}\tag{18}$$

This is <u>Hawking temperature</u> of Schwarzschild black hole. Result can easily be replicated by a multitude of other methods.

Caveat: <u>Wick rotation</u> unlikely to be a well-defined operation in quantum gravity in general. E.g. some Lorentzian spacetimes have no Euclidean counterpart. Also, smooth Euclidean spaces can turn into singular Lorentzian ones upon Wick rotation. [See Gibbons talk at Hawkingfest.] T_H is physical temperature felt by an observer at infinity. Temperature blueshifts as approach horizon; Hawking radiated particles have temperature T_P at proper distance l_P from horizon.

Additional calculable physics: BH radiates with a thermal spectrum; gravitational backscattering on way out from horizon causes wavelength-dependent filtering, and gives *greybody factors*.

Since $T_H \sim 1/M$, T_H increases as M decreases. Result: specific heat is negative. Physical effect: runaway evaporation of black hole at low mass.

BH lifetime? Radiates approx. like blackbody, so luminosity is

$$-\frac{dM}{dt} \sim (\text{Area}) T_{\text{H}}^{4} \sim (G_{4}M)^{2-4=-2} \quad \Rightarrow \quad \Delta t \sim G_{4}^{2}M^{3} \tag{19}$$

Endpoint of Hawking radiation = ?? Correspondence Principle proposes hot string state - see Lecture 3.

Mass of black hole with lifetime \sim age of Universe (\sim 14 Gyr): $\sim 10^{12}$ kg. Schwarzschild radius about a femtometre.

Taking a = 0 in Kerr-Newman obtain <u>Reissner-Nordstrøm</u> BH. Horizons at

$$r_{\pm} = \sqrt{(G_4 M)^2 - Q^2}$$
(20)

Cosmic censorship requires that singularity at r = 0 be hidden behind a horizon, i.e.

$$G_4 M \ge |Q| \tag{21}$$

Fascinatingly, this coincides with *BPS bound* when Einstein-Maxwell embedded in supergravity theory.

$$T_H = \frac{\sqrt{(G_4 M)^2 - Q^2}}{2\pi \left(G_4 M + \sqrt{(G_4 M)^2 - Q^2}\right)^2}.$$
 (22)

Extremal case, $G_4M = |Q|$, has zero temperature. (Similarly for BTZ extremal, where $r_+ = r_-$.) Stable! Related fact: specific heat at constant charge c_Q is not monotonic:

$$c_Q > 0 \quad \text{for} \quad G_4 M - |Q| \ll |Q|$$

$$c_Q < 0 \quad \text{for} \quad G_4 M \gg |Q| \qquad \qquad \text{like Schwarzschild} \qquad (23)$$

Consider extremal RN geometry, and let double horizon be at r_0 . Changing radial coordinate to $\hat{r} := r - r_0$, metric becomes

$$ds_{\text{ext}}^{2} = -H(\hat{r})^{-2}dt^{2} + H(\hat{r})^{2} \left(d\hat{r}^{2} + \hat{r}^{2}d\Omega_{2}\right)^{2}$$
(24)

where H is harmonic function

$$H(\hat{r}) = 1 + \frac{r_0}{\hat{r}}$$
 (25)

Isotropic coordinates: manifest SO(3) symmetry.

Extremal black hole geometry has an additional special property. Near horizon $\hat{r}=0,$

$$ds^{2} = -\left(\frac{\hat{r}}{\hat{r}+r_{0}}\right)^{2} dt^{2} + \left(1+\frac{r_{0}}{\hat{r}}\right)^{2} \left(d\hat{r}^{2}+\hat{r}^{2} d\Omega_{2}\right)^{2}$$

$$\to \underbrace{-\frac{\hat{r}^{2}}{r_{0}^{2}} dt^{2}+\frac{r_{0}^{2}}{\hat{r}^{2}} d\hat{r}^{2}}_{AdS_{2}} + \underbrace{r_{0}^{2} d\Omega^{2}}_{\times S^{2}} \qquad (26)$$

Bertotti-Robinson.

Reissner-Nordstrøm spacetime is also asymptotically flat. Thus, it interpolates between two maximally symmetric spacetimes. No-hair theorems indicate that we know *very* little about a BH by looking from outside. Only quantum numbers conserved because of a gauge symmetry survive. This suggests that a black hole will possess a degeneracy of states, and hence an entropy, as a function of its conserved quantum numbers:

$$S(M, J, Q) \tag{27}$$

In late 1960's and early 1970's, laws of classical black hole mechanics were discovered. Striking resemblance to laws of thermodynamics.

Zeroth black hole law says that surface gravity $\hat{\kappa}$ is constant over the horizon of a stationary black hole.

First law is

$$dM = \hat{\kappa} \frac{dA}{8\pi} + \omega_{\mathsf{H}} dJ + \Phi_{\mathsf{e}} dQ \tag{28}$$

where $\omega_{\rm H}$ is angular velocity at horizon and $\Phi_{\rm e}$ electrostatic potential. Second law says that horizon area A must be nondecreasing in any classical process. (Singularity theorems: horizons don't bifurcate.) Third law says that it is impossible to achieve $\hat{\kappa}=0$ via a physical process such as emission of photons. After doing many Gedankenexperiments, Bekenstein proposed that entropy of black hole should be proportional to area of event horizon. Hawking's semiclassical calculation of black hole temperature

$$T_H = \frac{\hbar \hat{\kappa}}{2\pi} \tag{29}$$

made entropy-area identification precise by fixing the coefficient. (In semiclassical approximation, spacetime is treated classically, while matter fields interacting with it are treated quantum-mechanically.)

<u>BH entropy</u> (BH stands for Bekenstein-Hawking or Black Hole) is in any spacetime dimension d

$$S_{\mathsf{BH}} = \frac{A_d}{4\hbar G_d} \tag{30}$$

where A_d is area of event horizon, and G_d is Newton constant (dimensions of (length)^{d-2}) This is a *universal* result for any black hole, applicable to any theory with Einstein gravity as its classical action.

Enormous entropy: for Earth-mass BH, $r_H \sim 1$ cm and $S_{BH} \sim 10^{66}$.

Extremal RN BH has finite entropy and zero Hawking temperature. Note that this does *not* imply a violation of third law of BH thermodynamics! Contrary to beliefs of some, there is no requirement in fundamental laws of thermodynamics that entropy should be zero at zero temperature. (That version of the third law is just a statement about equations of state for ordinary types of matter.)

BH entropy is 1/4 area of event horizon in Planck units. So... S_{BH} scales like area rather than volume! Violates our QFT intuition about extensivity of thermodynamic entropy. Central idea behind holography.

There are several versions of holography...

Elevation to principle occurred with 't Hooft and then Susskind.

Basic idea: since entropy scales like area rather than volume, fundamental degrees of freedom describing quantum BH are characterised by a QFT with one fewer space dimensions and with Planck-scale UV cutoff.

AdS/CFT correspondence provides very explicit and precise example of this idea. Details of this are to be discussed by other lecturers.

Subtlety suppressed so far: asymptotically flat black hole cannot really be in equilibrium with a heat bath; problematic if we want to use canonical ensemble.

Trouble is Jeans instability: even a low-density gas distributed throughout flat spacetime cannot be static: it undergoes gravitational collapse.

Technical ways around this problem have been devised, e.g. put black hole in a box and keep walls of box at finite temperature via proverbial reservoir.

Why it works: the box introduces IR cutoff which gets rid of Jeans problem. It also alters relation between black hole energy and box boundary temperature. Results in positive specific heat for black hole.

For large box, which affects properties of spacetime as little as possible, black hole is always entropically preferred state. For a small enough box, hot flat space results. Classically, black hole horizon never gets smaller. Hawking radiation results in loss of mass for black hole, therefore violates classical area theorem. Worse, appears to violate second law.

On the other hand, thermal Hawking radiation contributes to entropy.

Defined generalized entropy of black hole plus other stuff such as Hawking radiation,

$$S_{\text{tot}} = S_{\text{BH}} + S_{\text{other}} \ge 0 \tag{31}$$

Bekenstein argued that this fixes up the second law.

In addition, Bekenstein did many Gedankenexperiments involving various things falling into black holes. Argued for bound on entropy, given by entropy of black hole with horizon bounding volume of interest.

<u>Bekenstein bound</u> is *not* completely general, however. Bekenstein himself made this clear – his bound applies only to systems with "limited self-gravity". System to which bound is applied must also be an entire closed system – not an open subsystem. Examples of systems not satisfying bound include:

- closed FRW universe
- super-horizon region in flat FRW universe

In these situations, cosmological expansion drives overall dynamics, selfgravity is not limited. Pick large enough volume, violate bound.

Bousso formulated a more general, covariant, entropy bound.

New ingredient in this construction: use null hypersurfaces bounded by area A. Surfaces used are *light-sheets*: surfaces generated by light rays leaving A which have nonpositive expansion everywhere on sheet.

(Recall Raychaudhuri equation for expansion θ of a bunch of null geodesics with tangent vector field k^{μ} if

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - 8\pi T_{\mu\nu}k^{\mu}k^{\nu} + \omega_{\mu\nu}\omega^{\mu\nu}$$
(32)

where λ is affine parameter, and $\omega_{\mu\nu}, \sigma_{\mu\nu}$ are the shear and vorticity.)

$$S_{\leq}\frac{A}{4} \tag{33}$$

Bousso bound has been rigorously proven, if certain conditions are imposed on entropy flux across light sheets. Conditions physically reasonable for normal matter in semiclassical regimes, below Planck scale.

Bousso bound not general: can be violated.

However, Bousso bound holds up whenever light sheets make sense in physical situation of interest. No known Gedankenexperiments which violate it whilst semiclassical approximation is used *self-consistently*.

Think of Bousso bound as semiclassical proxy for fundamental law.

Kindergarten thermodynamics: have entropy, need quantum statistical mechanical partition function and hence degeneracy of states.

For black hole, how to compute degeneracy of states?

Intimately related problem: Hawking's information paradox.

Semiclassical computation says that spectrum of Hawking radiation is exactly thermal. This computation is apparently remarkably robust – order one changes in Planck scale physics of quantum fields near the horizon result in only exponentially small corrections to Hawking spectrum.

Suspicion: full quantum gravity will be necessary!

Loss of information happens because in falling book and vacuum cleaner of same mass give rise to identical Hawking radiation. Closely connected to no-hair theorems: observers at asymptotic infinity can see only longrange hair, which is very limited.

In quantum field theory, global symmetries are possible. Black holes gobble these up. This may not be a problem in string theory, since there are no global symmetries, only gauge symmetries.

String theory is a fully unified theory, including quantum gravity. Therefore, no degrees of freedom (information) should go missing. Information loss must therefore be an artifact of the semiclassical approximation.

As a result, information must be returned via subtle correlations of outgoing Hawking radiation particles. This point of view was espoused early on by Don Page, Gerard 't Hooft, Leonard Susskind,, and collaborators including me. Information return requires a quantum gravity theory with very subtle nonlocality, which is apparently impossible to see at the semiclassical level.

In the case of the AdS/CFT correspondence, there is a precise equivalence between a quantum gravity setup and a quantum field theory. In this case, it is clear that information must be returned, because quantum field theories are certainly unitary. The difficulty is translated into understanding properties of field theories in the limit of extremely strong coupling, which is itself a very hard problem. It does, however, give a proof of principle. The question of what happens to black hole information was extremely contentious a decade ago. There was a conference at ITP on Quantum Aspects of Black Holes in 1993, and participants believed in various scenarios for solving the information problem. There was even a vote!

1. Hawking still believes that information is just lost in quantum gravity. Most high-energy theorists cannot stomach this, because unitarity goes out the window. Even so, this option usually violates energy conservation, although a refinement is possible in which clustering is violated instead.

2. Several relativists believe that all information regarding anything that ever felt in to a black hole resides in a *remnant* of Planckian size. From the point of view of an outside observer, this seems impossible from the point of view of information theory: it takes energy to encode information. In addition, the required density of states would be enormous, and this is incompatible with what we know about string theory objects. It also presents dangers if remnants are allowed to circulate in quantum loops. Remnants have also been shown to cause trouble in the thermal atmosphere of big black holes... A possibility for remnants which relativists like is the idea that they could be baby universes. Before I was 'born', people thought about this in Euclidean space, looking for effects on the big universe via a condensate of Planckian sized wormholes. Arguments were made that tiny wormholes lead to no *observable* loss of quantum coherence in our universe. The problem is still there in principle, however. Also, the physics depends on selection of the wave function of the universe. Perhaps worst of all is that it is difficult to be sure that only small wormholes exist. Keep in mind also that caveat about Euclidean quantum gravity.

3. Information return scenario.

All but a small part of black hole spacetime near the singularity can be foliated by Cauchy surfaces called "nice slices". Property: both infalling matter and outgoing Hawking radiation have low energy in local frame of slice. Adiabatic argument shows that information return within framework of local QFT is difficult to reconcile with existence of nice slices. One possibility: singularity plays important – nonlocal! – role in returning information. Information return champions show that this scenario can be shown to be inconsistent only if assumptions are made about physics above the Planck scale. An important example is is diffeomorphism invariance – something sacrosanct to relativists.

Example Gedankenexperiment: the correlated spins. Prepare entangled spins a and b. Let a fall into the black hole with an apparatus A capable of measuring the spin's z-component and sending signals. Keep the other spin b outside the horizon with apparatus B. Wait long enough that the information regarding the first spin comes out in hawking radiation, as spin h. Now a and h are both supposed to be correlated with b! You might think there is actually no contradiction, as it looks like aand h are measured by different observers. But the observer outside the black hole can now fall in, and also collect signals from A. This would surely violate the *no quantum Xerox machine* theorem. The reason why there is still no contradiction is that it would take so much energy for Ato communicate its message to B that back reaction would wreck the black hole.

It is not good enough to try to wait until the black hole is nearly evaporated to return all the information, though because it would not have enough energy. Black holes also have a significant information retention time; this can be seen by doing computations with entropy of entanglement between BH and Hawking radiation. (In the literature there is significant confusion on entanglement entropy in black hole Gedanken-experiments. Let me clear this up: in general, entanglement entropy is *not* the statistical entropy of the black hole, although in the BH + Hawking radiation system it is bounded above by it.)

Subtle nonlocality may end up being crucial to information return. Makes particle theorists queasy, though – do not want to mess up known low-energy physics. Perturbative string theory does obey cluster decomposition... but what about nonperturbative M-theory? Possibly we will find a subtle nondecoupling of the UV and IR in quantum gravity. Or perhaps we will have to give up some fundamentals of quantum theory. Either way, or if something completely different happens, it will be new physics. Ω