http://www.physics.utoronto.ca/~peet/online/pitp/

See also beautiful D-brane text of Clifford Johnson!

Lecture 4: microscopic entropy computation

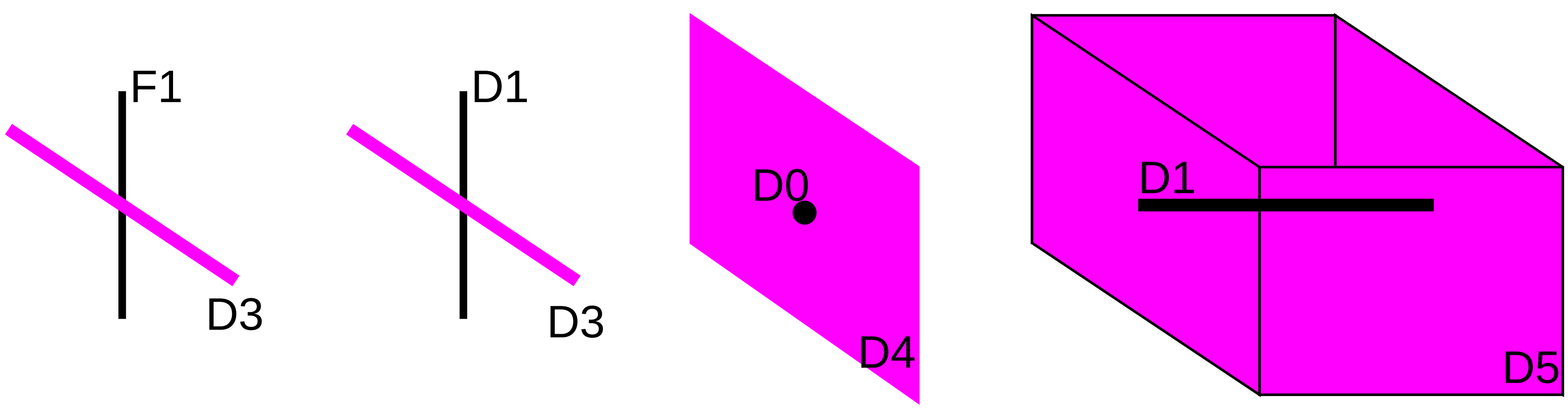
Concepts introduced :

- No-force conditions for different kinds of branes
- The harmonic function rule
- The D1 D5 system
- Adding the Wave to make 3-charge $d = 5$ BH
- Hawking temperature and Bekenstein-Hawking entropy
- Making 4-charge $d = 4$ BH - Reissner-Nordstrøm!
- The D-brane picture: open strings
- Statistical degeneracy
- How to do rotation
- Nonextremality and D-brane "Hawking radiation"

Recipe for making BPS black holes is considerably simpler than recipe for making nonextremal ones. Today, make BPS, qualitative comments only regarding nonextremal. First part of recipe is how to combine different ingredients. In other words, rules for intersecting branes.

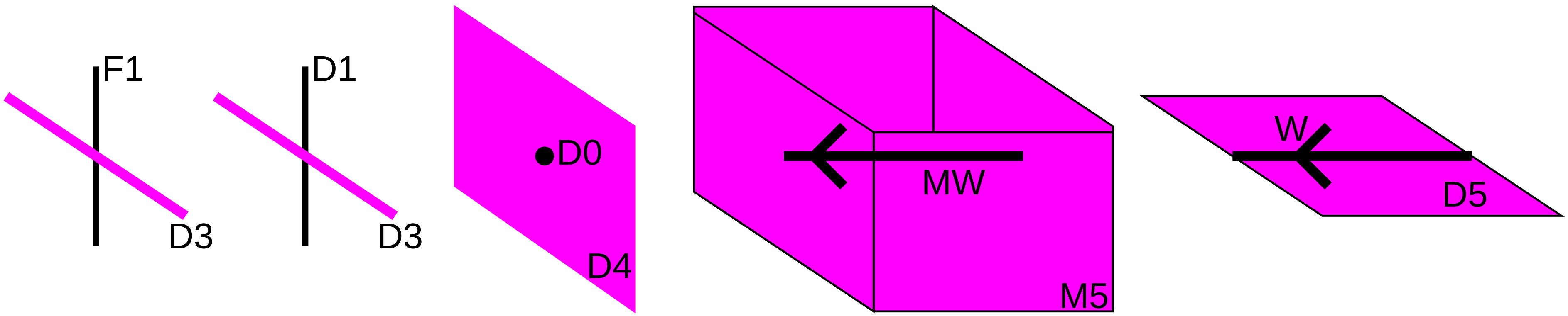
In Lecture 3: two clumps of parallel BPS p-branes in static equilibrium. Also, BPS p-branes and q-branes for some choices of p, q can be in equilibrium with each other under certain conditions. One way to find many rules is to start with the fundamental string intersecting a Dp-brane at a point, $F1 \perp Dp$, and use S- and T-duality.

$$F1 \perp D3 \longrightarrow D1 \perp D3 \longrightarrow D0 \parallel D4 \longrightarrow D1 \parallel D5 \tag{1}$$

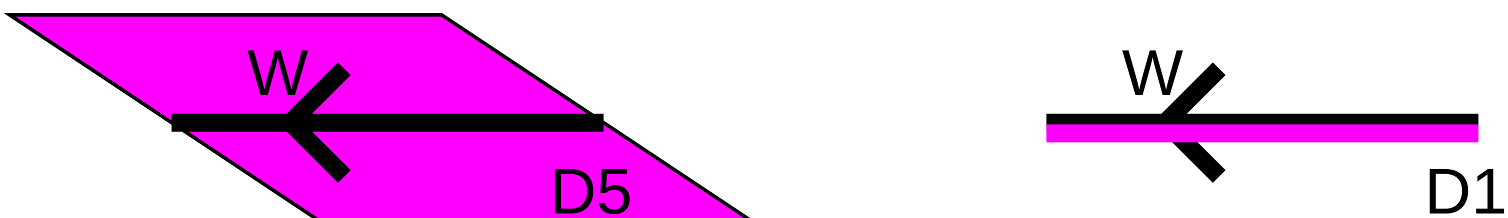


and also

$$F1 \perp D3 \longrightarrow D1 \perp D3 \longrightarrow D0 \parallel D4 \longrightarrow MW \parallel M5 \longrightarrow W \parallel D5 \qquad (2)$$



and by T-duality, $W \parallel D1$.



Therefore, $W \parallel D1 \parallel D5$ can all be in neutral equilibrium in a mutually consistent fashion.

Problems with too few ingredients

BPS black holes in dimensions $d = 4 \ldots 9$ may be constructed from BPS building blocks. Typically, however, they have zero horizon area and therefore non-macroscopic entropy. Example: consider D1-brane

$$ds^2 = H_1^{-1/2}\left(-dt^2 + dx^2\right) + H_1^{+1/2}\left(dr^2 + r^2 d\Omega_7^2\right) \tag{3}$$

where

$$H_1 = 1 + \frac{32\pi^2 g_s N_p \ell_s^6}{r^6} \tag{4}$$

Now compactify the x direction on a circle of radius R at infinity. At the horizon,

$$\frac{\mathrm{Vol}(S^1)}{(2\pi)R} = \sqrt{G_{xx}} = (H_1)^{-\frac{1}{2}} \sim r^3 \to 0 \tag{5}$$

How about Bekenstein-Hawking entropy? Transform to Einstein frame:

$$g_{\mu\nu} = \left(H_1^{1/2}\right)^{-1/2} G_{\mu\nu} = H_1^{-1/4} G_{\mu\nu} \tag{6}$$

so that

$$ds^2 = H_1^{-3/4}\left(-dt^2 + dx^2\right) + H_1^{+1/4}\left(dr^2 + r^2 d\Omega_7^2\right) \tag{7}$$

Hence (entropy same if evaluate in $d = 10$ or $d = 9$!)

$$S_{BH} = \frac{1}{4[8\pi^6 g_s^2 \ell_s^8]} \frac{16\pi^3}{15} \left(r^2 H_1^{1/4}\right)^{7/2}\Big|_{\text{horizon}} \tag{8}$$

and since $H_1 \sim r^{-6}$ near horizon,

$$S_{BH}(\text{BPS D1}) = 0. \tag{9}$$

More generally, study SUGRA field equations to find what BHs can have macroscopic entropy. Sizes of internal manifolds, plus dilaton, are scalar fields in lower-d. Horizon area depends on these scalars, which are ratios of functions of charges like H_p's.

But in any given d, have only a few independent charges on a black hole – fewer gauge fields than scalars. Too few independent charges to give all scalar fields well-behaved vevs everywhere in spacetime.

E.g. for stringy black holes made by compactifying on tori, only asymptotically flat BPS black holes with macroscopic finite-area occur with 3 charges in $d = 5$ and 4 charges in $d = 4$. The $d = 4$ case where all 4 charges are equal is Reissner-Nordstrøm. Woohoo!

The harmonic function rule

A systematic Ansatz is available for construction of SUGRA solutions corresponding to pairwise intersections of BPS branes. Known as "harmonic function rule". Ansatz: metric factorizes as product structure: simply "superpose" harmonic functions. This ansatz works for both parallel and perpendicular intersections. Important restriction: harmonic functions can depend only on overall transverse coordinates. In this way, get only "smeared" intersecting brane solutions.

Representation convention: $-$ means brane is extended in that dimension, $\cdot$ means it is pointlike, and $\sim$ says although brane is not extended in that direction *a priori*, its dependence on those coordinates has been smeared away.

E.g. D5 with D1 smeared over its worldvolume:

$$\begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \mathrm{D1} & - & - & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot & \cdot \\ \mathrm{D5} & - & - & - & - & - & - & \cdot & \cdot & \cdot & \cdot \end{array} \tag{10}$$

For D1-D5 system, let us define $r^2 \equiv x_\perp^2 = \sum_{i=6}^{9}(x^i)^2$ to be overall transverse coordinate in setup above. Then string frame metric is, using harmonic function rule,

$$
\begin{aligned}
dS_{10}^2 = \; & H_1(r)^{-\frac{1}{2}} H_5(r)^{-\frac{1}{2}} \left(-dt^2 + dx_1^2\right) + H_1(r)^{+\frac{1}{2}} H_5(r)^{-\frac{1}{2}} dx_{2\cdots 5}^2 \\
& + H_1(r)^{+\frac{1}{2}} H_5(r)^{+\frac{1}{2}} \left(dr^2 + r^2 d\Omega_3^2\right)
\end{aligned}
\tag{11}
$$

and dilaton is

$$
e^\Phi = H_1(r)^{+\frac{1}{2}} H_5(r)^{-\frac{1}{2}} \tag{12}
$$

while R-R gauge fields are as before,

$$
C_{01} = g_{\mathsf{s}}{}^{-1} H_1(r)^{-1} \qquad C_{01\ldots 5} = g_{\mathsf{s}}{}^{-1} H_5(r)^{-1} \tag{13}
$$

Independent D1 and D5 harmonic functions both go like r^{-2},

$$
H_5(r) = 1 + \frac{g_s N_5 \ell_s^2}{r^2} \qquad H_1(r) = 1 + \frac{g_s N_1 \ell_s^6 / V_4}{r^2} \tag{14}
$$

Wrap $x^2 \cdots x^5$ on T^4 to make $d = 6$ black string with two charges. Internal T^4 is finite-size at event horizon $r = 0$:

$$
\sqrt{G_{22} \cdots G_{55}} = \left(\frac{H_1}{H_5}\right)^{\frac{1}{4}4} \rightarrow \frac{N_1(\ell_s^4/V_4)}{N_5} \tag{15}
$$

Next step is to roll up direction of black string, to make black hole in $d = 5$.

Behaviour of radius of x^1 direction near horizon?

$$\sqrt{G_{11}} = (H_1 H_5)^{-1/4} \sim \frac{r}{(N_1 N_5)^{1/4}} \to 0 \tag{16}$$

Oops. Still need another quantum number to stabilize this S^1 as well as our T^4.

We can use knowledge from solution-generating to puff up this horizon to a macroscopic size by using ∞ boost in longitudinal direction x_1.

Ingredients for building this black hole are then previous branes with addition of a gravitational wave W:

$$\begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \text{D1} & - & - & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot & \cdot \\ \text{D5} & - & - & - & - & - & - & \cdot & \cdot & \cdot & \cdot \\ \text{W} & - & \to & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot & \cdot \end{array} \tag{17}$$

$\to$ is direction in which gravitational Wave moves (at speed of light).

BPS metric for this system is obtained from simpler metric for plain D1-D5 system by boosting and taking extremal limit. To get rid of five dimensions to make a $d = 5$ black hole, compactify D5-brane on the T^4 of volume $(2\pi)^4 V$, and then D1 and remaining extended dimension of D5 on S^1, volume $2\pi R$. $d = 5$ Einstein frame metric becomes

$$ds_5^2 = \begin{aligned}[t] &-(H_1(r)H_5(r)\,(1+K(r)))^{-2/3}\,dt^2 \\ &+(H_1(r)H_5(r)\,(1+K(r)))^{1/3}\left[dr^2 + r^2 d\Omega_3^2\right]\end{aligned} \tag{18}$$

where harmonic functions are

$$H_1(r) = 1 + \frac{r_1^2}{r^2} \qquad H_5(r) = 1 + \frac{r_5^2}{r^2} \qquad K(r) = \frac{r_m^2}{r^2} \tag{19}$$

and (smearing for H_1 and K) we find

$$\frac{r_1^2}{\ell_s^2} = (g_{\mathrm{s}} N_1)\frac{\ell_{\mathrm{s}}^2}{V} \qquad \frac{r_5^2}{\ell_s^2} = (g_{\mathrm{s}} N_5) \qquad \frac{r_m^2}{\ell_s^2} = (g_{\mathrm{s}}^2 N_m)\frac{\ell_{\mathrm{s}}^8}{R^2 V} \tag{20}$$

This SUGRA solution has limits to its validity. For e.g. curvature, find e.g. $\mathcal{R}(d = 5) \to -2/(r_1^2 r_5^2 r_m^2)^{1/3}$ at small r; or $R^{\mu\nu}R_{\mu\nu}(d = 10) \to -24/(r_1^2 r_5^2)$. So if stringy α' corrections to geometry are to be small, need large radius parameters. Dilaton? E.g. $d = 10$ $e^{2\Phi} \to N_1/N_5$.

Suppose we keep volumes V, R fixed in string units. Therefore, need

$$g_s N_1 \gg 1 \quad g_s N_5 \gg 1 \quad g_s^2 N_p \gg 1 \tag{21}$$

Can also control closed-string loop corrections if $g_\mathrm{s} \ll 1$. These two conditions are compatible if we have large numbers of branes and large momentum number for gravitational wave W. Also note that N_p needs to be hierarchically larger than N_1, N_5.

Next properties of this spacetime to compute are thermodynamic quantities. BPS black hole is extremal and it has $T_\mathrm{H} = 0$. For Bekenstein-Hawking entropy,

$$S_\mathrm{BH} = \frac{A}{4G_5} \quad = \frac{1}{4G_5} 2\pi^2 \left\{ r^3 \left[H_1(r) H_5(r) \left(1 + K(r)\right)\right]^{3/6} \right\}_{r=0} \tag{22}$$

$$= \frac{2\pi^2}{4\left[(\pi/4) {g_\mathrm{s}}^2 {\ell_\mathrm{s}}^8/(VR)\right]} \left(r_1 r_5 r_m\right)^{1/2} \tag{23}$$

$$= \frac{2\pi V R}{{g_\mathrm{s}}^2 {\ell_\mathrm{s}}^8} \left(\frac{g_\mathrm{s} N_1 {\ell_\mathrm{s}}^6}{V} \, g_\mathrm{s} N_5 {\ell_\mathrm{s}}^2 \, \frac{{g_\mathrm{s}}^2 N_m {\ell_\mathrm{s}}^8}{R^2 V} \right)^{\frac{1}{2}} \tag{24}$$

$$= 2\pi \sqrt{N_1 N_5 N_m} \tag{25}$$

This entropy

$$S_{BH} = 2\pi\sqrt{N_1 N_5 N_m} \tag{26}$$

is macroscopically large. Notice that it is also independent of R and of V. More generally, S_{BH} for BPS guys is *independent of all moduli*. This is to be contrasted with ADM mass

$$M = \frac{N_m}{R} + \frac{N_1 R}{g_\mathrm{s}{\ell_\mathrm{s}}^2} + \frac{N_5 RV}{g_\mathrm{s}{\ell_\mathrm{s}}^6} \tag{27}$$

which depends on R, V explicitly.

For entropy of black hole just constructed out of D1 D5 and W, we had $S_{\mathrm{BH}} = 2\pi\sqrt{N_1 N_5 N_m}$. More generally, for a more general black hole solution of maximal supergravity arising from compactifying Type II on T^5, it is

$$S_{\mathrm{BH}} = 2\pi\sqrt{\frac{\Delta}{48}} \tag{28}$$

where quantity Δ in surd is cubic invariant of the $E_{6,6}$ duality group,

$$\Delta = 2\sum_{i=1}^{4} \lambda_i^3 \tag{29}$$

and λ_i are eigenvalues of central charge matrix Z.

A few years ago a claim was made that all extremal black holes have zero entropy. Arguments were in Euclidean spacetime signature, and made the point that adding in surface terms at horizon was necessary to make sure Euler number of horizon was not fractional.

This result is not trustworthy in the context of string theory.

1. As we mentioned in our discussion of Third Law, there is no physical reason why zero-temperature black holes should have zero entropy.

2. Faulty nature of classical reasoning in string theory context was pointed out in Horowitz review article. In Euclidean geometry, for any periodicity in Euclidean time β at $r = \infty$, presence of extremal horizon results in a redshift which forces that periodicity to be substringy very close to horizon. Since light strings wound around this tiny circle can condense, a Hagedorn transition can occur. Classical approximation is not reliable there; in particular, arguments based on classical topology are not believable.

3. This entropy would be *hugely* smaller than entropy of very-nearly-extremal BH! Where does all the entropy *go*??

4-charge $d = 4$ black hole

Extremal Reissner-Nordström black hole can be embedded in string theory using D-branes. For extremal spacetime metric we had $H^{\pm 2}(r)$'s appearing in metric:

$$ds^2 = H^{-2}(-dt^2) + H^2(dr^2 + r^2 d\Omega^2) \quad H = 1 + r_0/r \tag{30}$$

This is to be contrasted with the $H^{\frac{1}{2}}$'s to be found in a generic p-brane metric:

$$ds^2 = H^{-1/2}(-dt^2 + dx^2_{1\ldots p}) + H^{+1/2}(dr^2 + r^2 d\Omega^2) \tag{31}$$

From this we can guess (correctly) that, in order to embed extremal RN black hole in string theory, we will need 4 independent brane constituents. Restrictions must be obeyed, however, in order for that black hole to be RN. To make more general $d = 4$ black holes with four independent charges, we simply lift these restrictions and allow charges to be anything - so long as they are large enough to permit a supergravity description.

For making $d=4$ black hole, one set of ingredients would be

$$\begin{array}{lcccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \text{D2} & - & - & - & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot \\ \text{D6} & - & - & - & - & - & - & - & \cdot & \cdot & \cdot \\ \text{NS5} & - & - & - & - & - & - & \sim & \cdot & \cdot & \cdot \\ \text{W} & - & \rightarrow & \sim & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot \end{array} \tag{32}$$

By U-duality, we could consider instead 4 mutually orthogonal D3-branes, or indeed many other more complicated arrangements.

In ten dimensions we can construct BPS solution by using the harmonic function rule. So far we have not exhibited metric for NS5-branes but that can be easily obtained using D5 metric and using fact that Einstein metric is invariant under S-duality. We then have

$$\begin{aligned} dS_{10}^2 \;=\; & H_2^{-\frac{1}{2}} H_6^{-\frac{1}{2}} \left[-dt^2 + dx_1^2 + K(dt+dx_1)^2\right] \;+ H_5 H_2^{-\frac{1}{2}} H_6^{-\frac{1}{2}} (dx_2^2) \\ & + H_2^{+\frac{1}{2}} H_6^{-\frac{1}{2}} H_5 (dx_{3\ldots6}^2) + H_5 H_2^{+\frac{1}{2}} H_6^{+\frac{1}{2}} (dr^2 + r^2 d\Omega_2^2) \end{aligned} \tag{33}$$

and

$$e^{\Phi} = H_5^{+\frac{1}{2}} H_2^{+\frac{1}{4}} H_6^{-\frac{1}{4}(3)} \tag{34}$$

Smearing and Newton's constant formulæ give

$$r_2 = \frac{g_s N_2 {\ell_s}^5}{2V} \quad r_6 = \frac{g_s N_6 \ell_s}{2} \quad r_5 = \frac{N_5 {\ell_s}^2}{2R_b} \quad r_m = \frac{{g_s}^2 N_m {\ell_s}^8}{2V R_a^2 R_b} \tag{35}$$

Kaluza-Klein reduction formulæ give first a $d = 5$ black string and then finally the $d = 4$ black hole. Final Einstein metric in $d = 4$ is

$$\begin{aligned} ds^2 \; &= -dt^2 \left[\sqrt{(1 + K(r))H_2(r)H_6(r)H_5(r)}\right]^{-1} \\ &+ (dr^2 + r^2 d\Omega_2^2) \left[\sqrt{(1 + K(r))H_2(r)H_6(r)H_5(r)}\right] \end{aligned} \tag{36}$$

Reissner-Nordström black hole is obtained by setting all four gravitational radii to be identical: $r_2 = r_6 = r_5 = r_m$. Bekenstein-Hawking entropy is

$$S_{BH} = 2\pi\sqrt{N_2 N_6 N_5 N_m} \tag{37}$$

More generally, in surd is quantity $\diamondsuit/256$, where $\diamondsuit$ is quartic invariant of $E_{7,7}$

$$\diamondsuit = \sum_{i=1}^{4} |\lambda_i|^2 - 2\sum_{i<j}^{4} |\lambda_i|^2 \left|\lambda_j\right|^2 + 4\left(\overline{\lambda_1\lambda_2\lambda_3\lambda_4} + \lambda_1\lambda_2\lambda_3\lambda_4\right) \tag{38}$$

where λ_i are (complex) eigenvalues of Z.

The D-brane picture

Our setup of branes for $d = 5$ BPS BH with 3 charges was

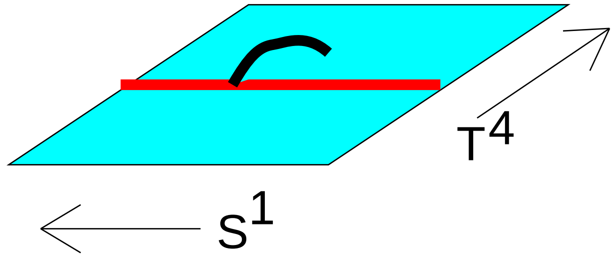
$$\begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \mathrm{D1} & - & - & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot & \cdot \\ \mathrm{D5} & - & - & - & - & - & - & \cdot & \cdot & \cdot & \cdot \\ \mathrm{W} & - & \rightarrow & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot & \cdot \end{array} \tag{39}$$

This system preserves 4 real supercharges, or $\mathcal{N} = 1$ in $d = 5$. Each constituent breaks half of SUSYs.

Necessary for SUSY to orient branes in a relatively supersymmetric way. If not, e.g. if an orientation is reversed, D-brane system corresponds to a black hole that is extremal but has no SUSY.

Beginning ingredients: D1 branes and D5 branes. What are degrees of freedom carrying momentum quantum number?

D5 branes and smeared D1 branes have a symmetry group $SO(1,1) \times SO(4)_{||} \times SO(4)_{\perp}$. This symmetry forbids (rigid) branes from carrying linear or angular momentum, so we need something else.



Obvious modes in the system to try are massless 1-1, 5-5 and 1-5 strings, which come in both bosonic and fermionic varieties.

- Momentum N_m/R carried by bosonic and fermionic strings, $1/R$ each.
- Angular momentum is carried only by fermionic strings, $\frac{1}{2}\hbar$ each.

Both linear and angular momenta can be built up to macroscopic levels.

Next step: identify degeneracy of states of this system. Simplification made by [Strominger-Vafa] is to choose the four-volume small by comparison to circle radius

$$V^{\frac{1}{4}} \ll R \tag{40}$$

Makes theory on D-branes a $d = 1 + 1$ theory. This theory has $(4, 4)$ SUSY in $d = 1 + 1$ language.

$d = 1 + 1$ partition function of a number n of boson fields and an equal number of fermion fields is

$$Z = \left[\prod_{N_m=1}^{\infty} \frac{1 + w^{N_m}}{1 - w^{N_m}} \right]^n \equiv \sum \Omega(N_m) w^{N_m} \tag{41}$$

where $\Omega(N_m)$ is degeneracy of states at $d = 1 + 1$ energy $E = N_m/R$. At large-degeneracies, which happen with big quantum numbers like we have here, we can use Cardy formula

$$\Omega(N_m) \sim \exp \sqrt{\frac{\pi\, c\, E\, (2\pi R)}{3}} = \exp \left(2\pi \sqrt{\frac{c}{6}\, E R} \right) \tag{42}$$

(*Technical note:* This formula assumes that lowest eigenvalue of energy operator is zero, as it is in our system. Otherwise must use instead $c_{\text{eff}} = c - 24\Delta_0$, where Δ_0 is ground state energy.)

We know R, radius of circular dimension. Need c and E.

Central charge

$$c = n_{\text{bose}} + \tfrac{1}{2} n_{\text{fermi}} \tag{43}$$

How many bosons (and fermions) do we have???

Boson and fermion count in system of D1, D5 and open strings?
Can be done rigorously; here is the basic physics:

- $N_1 N_5$ 1-5 strings that can move in 4 directions of torus, hence $c = 6N_1N_5$.
- Alternatively, we can use neat fact that D1-branes are instantons in D5-brane theory. Have N_1 instantons in $U(N_5)$ gauge theory, and N_5 orientations to point them in. Etc...

Now, how about energy E? System is supersymmetric, and since no Z's down here in this $d = 1 + 1$ story, need $P^\mu P_\mu = 0$. So $E = |P|$. In $d = 1 + 1$ things can move only to R or L. Our sign conventions make us have R-moving groundstate, and put all the action in L-movers. Momentum was $P = \pm N_m/R$, so $E = N_m/R$.

Cardy said

$$\Omega(N_m) \sim \exp\sqrt{\frac{\pi\, c\, E\,(2\pi R)}{3}} = \exp\left(2\pi\sqrt{\frac{c}{6}\,ER}\right) \tag{44}$$

Therefore

$$S_{\text{micro}} = 2\pi\sqrt{N_1\, N_5\, N_m} \tag{45}$$

This agrees exactly with black hole result!!

Rotation

In $d = 5$ there are two independent angular momentum parameters, because rotation group transverse to D1's and D5's splits up as

$$SO(4)_\perp \simeq SU(2) \otimes SU(2) \tag{46}$$

Angular momentum *is* consistent with $d = 5$ superalgebra.

Metrics for general rotating black holes are algebraically rather messy, we will not write them here. We will simply quote result for BPS entropy:

$$S_{\mathrm{BH}} = 2\pi\sqrt{N_1 N_5 N_m - J^2} \tag{47}$$

BPS black holes have a nonextremal generalisation, in which the two angular momenta are independent. However, in extremal limit something interesting happens: two angular momenta are forced to be equal and opposite, $J_\phi = -J_\psi \equiv J$. There is also a bound on angular momentum,

$$|J_{\mathrm{max}}| = \sqrt{N_1 N_5 N_m} \tag{48}$$

Beyond J_{max}, closed timelike curves develop, and entropy walks off into complex plane.

Another notable feature of this BPS black hole: those funny cross-terms in R-R sector of the SUGRA Lagrangian are turned on. So this black hole is *not* a solution of $d = 5$ Einstein-Maxwell theory! Gauge charges are unmodified by funny cross-terms because they fall off too quickly to contribute to surface integrals.

Reduced entropy can be understood rigorously in D-brane field theory.

But basic physics is simple: aligning $\frac{1}{2}\hbar$'s all in a row to build up macroscopic angular momentum *costs oscillator degeneracy*. Energy is reduced as

$$\frac{N_m}{R} \longrightarrow \frac{1}{R}\left[N_m - \frac{J^2}{N_1 N_5}\right] \tag{49}$$

So entropy reduced to

$$S_{\text{micro}} = 2\pi\sqrt{N_1 N_5 N_m - J^2} \tag{50}$$

Agrees with black hole calculation again.

Also, find $J_\phi = -J_\psi$ from SUSY.

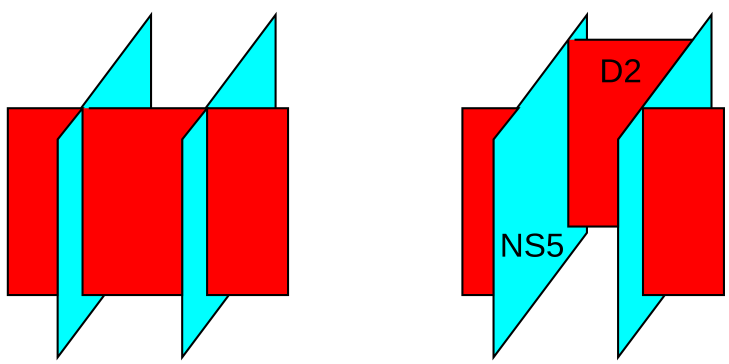
$d = 4$ entropy counting

A canonical set of ingredients for building $d = 4$ system is what we had previously in building black hole:

$$
\begin{array}{c|cccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\mathrm{D2} & - & - & - & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot \\
\mathrm{D6} & - & - & - & - & - & - & - & \cdot & \cdot & \cdot \\
\mathrm{NS5} & - & - & - & - & - & - & \sim & \cdot & \cdot & \cdot \\
\mathrm{W} & - & \rightarrow & \sim & \sim & \sim & \sim & \sim & \cdot & \cdot & \cdot
\end{array} \tag{51}
$$

First three ingredients are simply T-dual to our (D1, D5, W) system.

New feature: NS5-branes. New physics: D2-branes can end on NS5-branes. It costs *zero* energy to break up a D2-brane like so:



These extra massless degrees of freedom in system lead to an extra label on 2-6 strings, giving rise to an extra factor of N_{NS5} in degeneracy. Entropy counting proceeds just as before, and yields

$$S_{\mathsf{micro}} = 2\pi\sqrt{N_2 N_6 N_{\mathsf{NS5}} N_m} \tag{52}$$

which again agrees exactly with Bekenstein-Hawking black hole entropy. A major difference between this and $d = 5$ case is that the single rotation rotation parameter is incompatible with supersymmetry.

Nonextremality

New ingredient: add extra energy (but no other charges) to system of D-branes (and NS-branes) and open strings carrying linear and angular momenta.

SUGRA: nonextremal branes cannot be in static equilibrium with each other – they want to fall towards each other, and they do *not* satisfy simple harmonic function superposition rule.

Least confusing way to construct nonextremal multi-charge solutions is to start with appropriate higher-d neutral Schwarzschild or Kerr type solution, and to use multiple boostings and duality transformations to generate required charges.

For *nearly BPS* systems, D-brane pictures for $(D1, D5, W)$ and $(D2, D6, NS5, W)$ stay in $d = 1 + 1$.

Physics: new energy adds a small number of *R-movers* as well as L-movers. (Breaks BPS condition.)

Think of R-movers and L-movers as dilute gases, interacting only very infrequently. Energy and momentum are additive, and so is entropy.

Amazingly, entropy agrees with near-extremal black brane entropy. Why? - no theorem protecting degeneracy of non-BPS states. What is going on physically is that conformal symmetry possessed by the $d = 1 + 1$ theory is sufficiently restrictive, even when it is broken by finite temperature, for black hole entropy to be reproduced by field theory.

Multi-parameter agreement. $\uparrow\downarrow$

Also greybody factors can be computed. Mindbogglingly, D-brane story gives same answer!!