Matt 16 Nov 2004

Note Title 16/11/2004

Hi Matt! – this is Amanda Peet here, using my new(ish) "gizmo" – a Toshiba Portégé M200 Tablet PC.

I really love the way writing on its screen is so easy! I hope you can read my bad handwriting ... ☺
After I've finished doing my best at answering your questions, I'm going to convert this to PDF format, which I hope you can read OK (like last time if I remember accurately).

You wrote:

1. A string vibrates, and therefore has energy. According to Einstein, energy and mass are interchangeable. Therefore, a string by definition should have some sort of mass. So, how can something with mass be the constituent part of a massless particle, like a photon?

2. What is the formula for getting the energy of massless particles like photons (instead of E=mc squared)?

(*) About your questions #1 and #2, which are nicely related to each other. You have put your finger on one of the most <u>amazing</u> pieces of quantum physics of relativistic strings!! Wow! ☺
In fact, I was just teaching this very story to the other students in my graduate string theory course a few weeks ago, after you and Mr. McColm visited with us... so your timing is excellent.

I was racking my brains about how to explain this for you, because it's not easy to do without too much inaccessible type mathematics .... But I kept thinking, because I really wanted to explain this to you ☺. Then – suddenly – one morning, in the shower, I came up with what follows. May I try it on you and see if it makes sense to you? I hope this way of explaining the central issue here is OK.

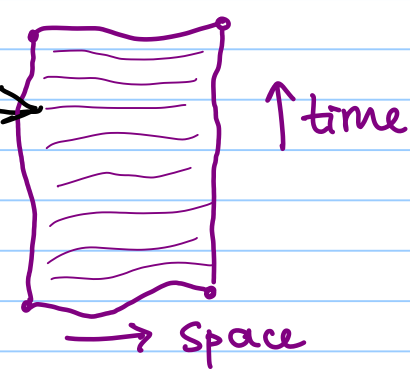
Idea #1 : World-sheets in space-time.

- We know it's possible to have two types of strings:
  - "open", i.e. has ends ;
  - "closed", i.e. has no ends .

- Typically, with any object, a physicist is most interested in knowing how the object moves : "dynamics". It's least complicated, to begin with, to assume that the space-time in which these strings move is flat – like, we're not near a black hole or anything ☺, and we've only got a few strings so they don't cause warping of space-time. OK.

- So we can wonder – when a string moves through space-time, what sort of surface does it sweep out?
- If we watch an open string, it sweeps out a shape like a (ripply) strip :

This is called the worldsheet.



(not to scale)

- What if we have a closed string? It sweeps out a (ripply) cylinder :-

This is called the worldsheet.

(not to scale)

Idea #2: Quantum Jitter

- The world is quantum – everything in the universe has a basic jitter <u>even at absolute zero</u> temperature. One way of thinking about this is via the Heisenberg Uncertainty Principle (HUP) which says: (e.g.:) you can't measure time and energy at the same time at infinitely good precision. (Also, there are other pairs of variables afflicted by this basic quantum jitter or uncertainty.) Surprisingly, an electron doesn't even <u>have</u> a definite energy if we measure it over any finite (or zero) time interval! That's kind of surprising, because we're used to saying that objects have definite qualities. But the world <u>is</u> quantum.

- Reason why quantum jitter doesn't bother you in your everyday life is that for big objects – like humans, hockey pucks and dogs – it's so small as to be completely un-noticeable. ☺

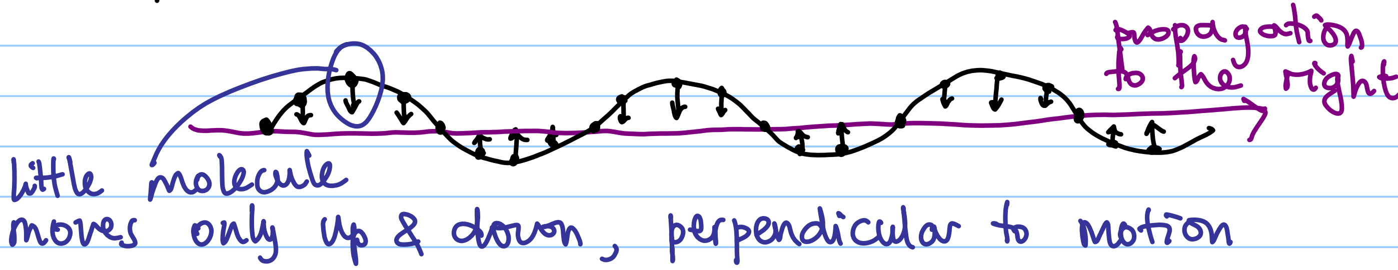
- So... why does quantum jitter affect string physics? Well, strings have ways to wiggle – the technical word for wiggles is "oscillations". And these oscillations are subject to the laws of quantum physics too! We can look separately at each of the different possible oscillation modes ("the oscillators") and ask about their quantum jitter.

- There are two pieces of physics, for string oscillator quantum jitter, that make the story a bit more intricate than you might guess:
  - ⊛ only $(d-2)$ oscillation directions count, when there are d possible directions of space-time;
  - ⊛ the string oscillations can't spread out as much as they want to, and this gives a negative contribution to the mass-squared.

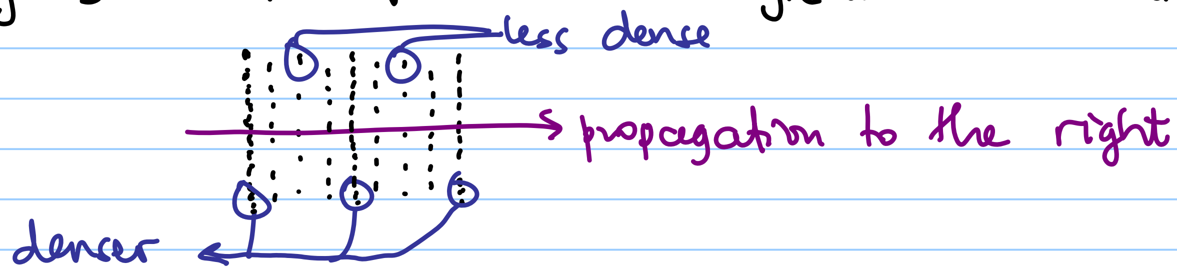
Idea #3 : No longitudinal oscillations of string

Let's think about sound waves. When we send sound through e.g. air or water, there are two qualitatively different types of air-molecule motions that transmit sound: longitudinal and transverse.

Transverse: If we look at a few of the air molecules participating, we see that they move perpendicular to the direction in which the sound wave is propagating, kind of like this :-



Longitudinal: Here, the sound wave is propagated through the medium (like air or water) because there can be regions of the medium with greater or smaller density:



For strings, which are [thought to be] the most fundamental things in the universe, there is <u>no medium</u> in which string oscillations propagate! So there are actually <u>no physical longitudinal modes of strings.</u>

Next, let's move to thinking about that basic jitter of every quantum thingy in the universe, even at absolute zero, which is called the <u>zero-point energy</u>.

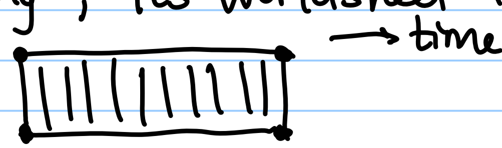
Idea #4 : "Casimir energy"

One of the things about quantum jitter is that everything wants to do the jittering: electrons, photons; particles; strings ... everything! The other relevant thing is that quantum thingys (like electron-particles, or strings) actually are very messy at heart and they like to spread out their ability to jitter all over space.

Moreover, if you arrange for some piece of space to be excluded (somehow) from the quantum jitter options of any thingy, it gives an energy deficit, which is given the name "Casimir energy" (after some old white dead guy called Casimir who discovered the effect ☺). Basically, this energy deficit ΔE is roughly

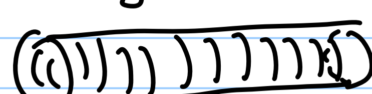
$$\Delta E = -\begin{pmatrix}\text{the volume of}\\ \text{excluded space}\end{pmatrix} \times \begin{pmatrix}\text{the zero-point energy}\\ \text{density for the thingy}\end{pmatrix}.$$

Now, for an open string, its worldsheet in space-time looks like a strip:



and, since this worldsheet only covers a limited region of space, it has a deficit energy.

Similarly, the closed string also has only a restricted region of space covered by its worldsheet:



so it gets a deficit energy too. (It turns out to be a slightly bigger deficit.)

Now we can write the mass formula! Using a calculation of the Casimir energy, our knowledge of the lack of longitudinal oscillations, and quantum physics of relativistic strings gives (da-dah! ☺) :-

$$\boxed{\left(\frac{mc^2}{T}\right)^2_{closed} = 2(N-2)}$$ $N=2n \geq 2$ for closed string (n = positive integer)

or

$$\boxed{\left(\frac{mc^2}{T}\right)^2_{open} = (N-1)}$$, $N \geq 1$ for open string (N = positive integer)

In these expressions,
N is the oscillator energy in the string, in units of T,
(mc^2) is the mass of the string &
T is the tension energy of the string.

Let's do a couple of examples!

1) Closed: massless modes with $mc^2=0$ require $N=2$. The two directions in which these two oscillators point signify a spin-two object i.e. the graviton.

2) Open: massless $mc^2=0$ requires $N=1$. The direction in which this single oscillator points signifies a spin-one object, i.e. the photon.

The fact that open & closed strings can give rise to massless gravitons & photons is part of why we call it a Unified Theory. ☺

(*) One more thing. The Casimir energy story actually also forces us to have, for consistency,
transverse dimensions = (8)

and since # dimensions = # transverse + # longitudinal
$= \underbrace{8}_{\text{transverse}} + \underbrace{1+1}_{\text{space \& time in string worldsheet}}$

$\Rightarrow$ **# dimensions = 10** ☺ Cool!

→ All the best,
– Prof. Peet ☺