The Cavendish Experiment

Reference: Leybold, "Directions for use: Gravitation Torsion Balance" (available from the Resource Centre)

Introduction

The Cavendish experiment was the first to allow a calculation of the gravitational constant (G) by measuring the force of gravity between two masses in a laboratory framework.

The original experiment was proposed by John Michell (1724-1793), who first constructed a torsion balance apparatus. In 1793 the apparatus passed to Henry Cavendish who carried out a series of experiments published in 1798[1]. The original apparatus included a 1.8 m wooden rod with metal spheres attached to each end, suspended from a wire. Five 350 lb (195 kg) lead balls were placed nearby. The gravitational force exerted on the end weights was enough to cause the wire to twist.

From the twisting torque on the wire and the known masses of the spheres, Cavendish was able to calculate the value of the gravitational constant.

Our experiment

In Figure 1, the position labeled *E* is the equilibrium position of the balance with the heavy masses M not on their frame. When the large masses M are in position 1, the gravitational force F_g between each pair m and M is:

$$F_g = G\frac{Mm}{x_1^2} \quad (1)$$

where x_1 is the distance between the centers of masses m and M.

Thus, the total torque Γ_g on the inner frame is:

$$\Gamma_g = G\frac{Mm}{x_1^2}2d\cos\alpha_1 \quad (2)$$

where 2d is the distance between the masses m. When the large masses M are in position 2, there is a similar equation with subscript 1 replaced by 2.

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[1] Cavendish, H. "Experiments to determine the Density of the Earth", *Philosophical Transactions of the Royal Society of London*, (part II) **88** p.469-526 (1798)

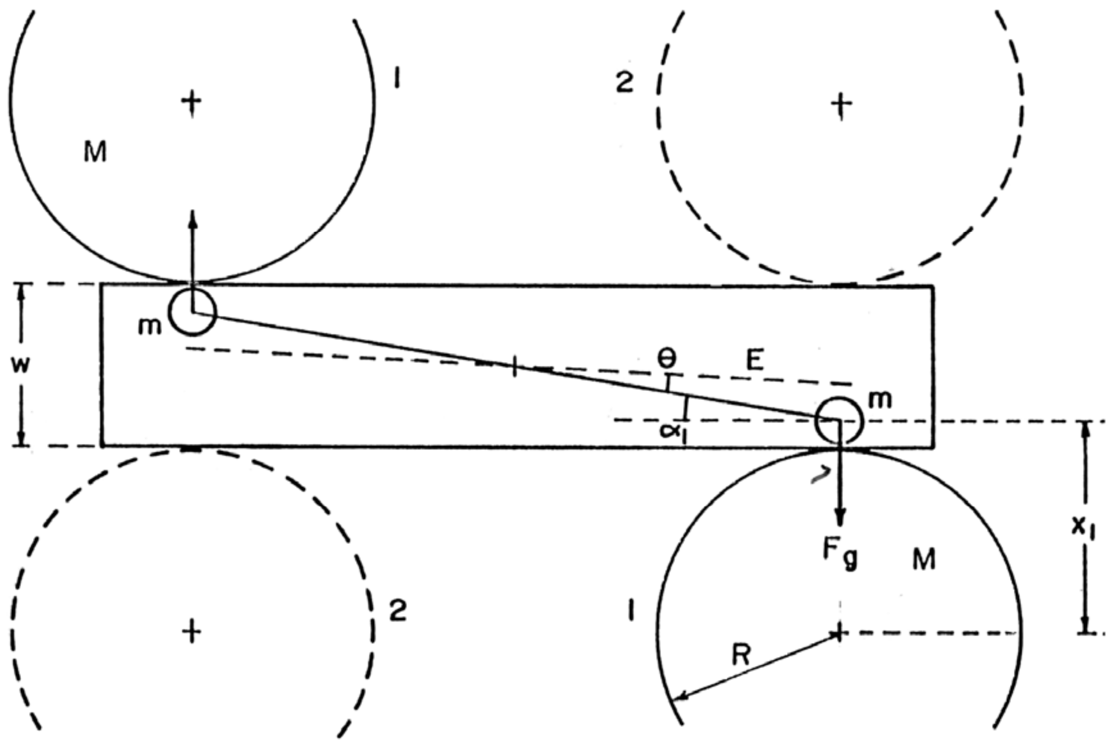

Figure 1. Schematics of the Cavendish apparatus (top view)

At equilibrium, this torque is balanced by a torque from the wire supporting the inner frame. This torque is given by $\mathbf{k\theta_1}$ (where k is the torsion constant and θ_1 is the angle of twist of the wire).

$$G\frac{Mm}{x_1^2}2d\cos\alpha_1 = k\theta_1 \tag{3}$$

The torsion constant can be determined by measuring the period *T* of oscillation as the frame approaches equilibrium.

$$T = 2\pi\left(\frac{I}{k}\right)^{1/2} \tag{4}$$

where $\boldsymbol{I}$ is the moment of inertia of the inner frame.
Assuming that the moment of inertia of the inner frame is just $2md^2$, equations 3 and 4 can be combined to eliminate m.

If the equilibrium position E is symmetric, then $\theta_1 = \theta_2 = \alpha_l = \alpha_2 \equiv \theta$. Also, the distances between the masses m and M are equal: $x_1 = x_2 = x$, so it can be shown by simple geometry that:

$$x = R + \frac{w}{2} - d\sin\theta \tag{5}$$

where w is the width of the case and 2d is the distance between masses m (2d = 10.00 cm). Including all the above approximations, the value of 'G' can be calculated.

Apparatus notes

Our apparatus consists of three main parts:
- The Cavendish balance itself. Details of the balance may be found in the manufacturer's (Leybold) notes;
- Detector and associated electronics;
- Acquisition software and computer.

A 2-element photodiode is mounted such as to replace the pen of the recorder. The output of the photodiode is connected to the *Cavendish Controller*, which outputs a voltage which is fed into the inputs of the recorder. Thus, the recorder will follow the laser beam reflected from the mirror of the Cavendish balance. The controller also contains a circuit which will cause the photodiode to seek for the beam.
The output signal is fed into a second controller (Pasco), digitized and sent to the computer. The software used for collection of data and analysis is *LabVIEW 8.2* (National Instruments)

Experimental notes

In this experiment, you will determine the equilibrium position of the reflected beam when:

1. The two large lead balls are not mounted on the frame of the Cavendish balance.
2. The two large lead balls are mounted and positioned fully clockwise on the balance.
3. The two large lead balls are mounted and positioned fully counter-clockwise on the balance.

You will determine these equilibrium positions by analyzing the oscillations of the balance: it is not necessary to wait for the system to come to steady equilibrium. The recording will allow you to determine the period of oscillation of the balance, and therefore calculate the torsion constant of the wire support.
After setting up the system, it will take about 45 minutes for you to take your data for each of these situations. Thus, you should *carefully* read this section and begin taking data as soon as possible.

First insure that the balance is level, and that the inner frame holding the two small lead balls is swinging freely. The two knurled knobs under the balance support two pans which can be raised to arrest the motion of the balls. **Do not use these unless absolutely necessary**.

The metal disc and thumb screw on the top of the balance adjusts the equilibrium position of wire supporting the inner frame. It should not need adjustment. If you suspect it does need adjustment **do not attempt to do so yourself unless you are positive you know what you are doing!**

One difficulty of the experiment is aligning the laser diode beam. The incident and reflected beam **must** lie in a plane that is perpendicular to the axis of rotation of the balance. Further, the reflected spot must strike the photodiode directly in the centre. One way to align the system is to leave the recorder off; then you may slide the recorder on the support bench until the photodiodes are horizontally aligned with the beam. Now you may gently adjust the laser diode until the reflected beam strikes the photodiodes. In the course of making this adjustment, the balance will probably be disturbed and will start vibrating: wait a few minutes for it to settle down before proceeding. Check the alignment by turning on the recorder and the controller to see if the photodiode head tracks the beam.

The control on the lower-right of the recorder panel is labeled (roughly) **<|0|>** and controls the zero-offset of photodiode arm. Do not adjust it once you started taking data. Similarly, do not adjust the laser between sets of data.

When the inner frame is oscillating back and forth you want to position the recorder so that the entire path of the oscillations is being tracked.

Data acquisition

To open the data acquisition program, click on the ‘Cavendish’ shortcut, located on the desktop. START/STOP allows you to begin/end a recording. The default acquisition end time is 1 hour, but the usual recording time could be ~45 minutes. You may rescale the graph only during data acquisition.

When you finish a recording and you are sure the equilibrium has been reached, you may stop the acquisition and take time and amplitude measurements, using the four cursors (2 vertical, 2 horizontal).

Note: *There is a 1:1 correspondence between the vertical units of amplitude on the display and the units on the recorder.*

OPEN/PRINT GRAPH allows viewing the graph and sending to printer. Data file is saved by default on D:\Student’s Documents in .txt format. If a new recording is started, the data file will be overwritten. *Make sure you upload each data file in your memory stick before you start a new recording!*

Note: *LabVIEW is a powerful graphical development environment, used for signal acquisition, measurement analysis, and data presentation. If you want to find out more about it, use the LabView 8.2 shortcut from the desktop, which allows you to browse Resources and Documentation. By accessing the National Instruments web page, you may also get more facts about LabVIEW:*
http://www.ni.com/labview/

Questions

1) Is the equilibrium position E really symmetric?
2) The torsion pendulum is clearly a damped harmonic oscillator. Is the motion *simple harmonic*? What effect does the damping have on the determination of the period?
3) Each mass m will also be attracted by the remote second mass M. What effect does this have on your result?
4) What is the dominant error in the determination of G? Does the approximation being made have an effect comparable to this dominant error? If yes, what can be done about it?

This experiment was revised in 2007 by Ruxandra M. Serbanescu. Previous versions: DH – 1987-88. Thanks to Phil Scolieri for interfacing the Cavendish Controller with LabVIEW.