Radius of the Earth

In this experiment, the variation of gravity up the Burton Tower is measured with a *Sodin* gravimeter and from those measurements you can deduce the Radius of the Earth.

1. **INTRODUCTION: THE GRAVIMETER**

   The Sodin gravimeter, or gravity meter, is in essence a very sensitive balance which pivots a small mass on a torsion fiber against the torque provided by stretching a calibrated quartz spring. The device is delicate.
   Please pay attention to the following:
   - The Gravimeter cost is $\$16,000$! Be Careful!
   - Read the instructions first!
   - Handle the instrument gently!
   - Keep it in the carrying case when not in use or when you carry it,
   - Dont tilt it more than 30 degrees!
   - Dont turn the dial off scale!

   The gravity meter was developed by Texas Oil Industry. Oil shales and domes are lighter than the surrounding rocks, and anomalously low value of g will occur at the surface over such deposits. The absolute value of g is of no interest in that application, so the design has sacrificed any attempt at accuracy or absolute reading in order to increase the sensitivity to changes in g. In any case, the absolute value of g varies with the position of the sun (and moon), so geophysical prospecting uses a reference station to which the meter is returned every few hours so that all readings can be corrected for the diurnal variation. The absolute value of g as measured at the base station at the bottom of the north tower stairwell of the Burton Tower is quoted as $9.804253\frac{m}{sec^2}$. (This measurement was made at some unknown time on some unknown day about ten or twenty years ago.)

   You might be interested in estimating the change in this value due to the Sun's position at noon and midnight. (Take $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$, $M_{sun} = 2.0 \times 10^{30}$ kg and the earth to sun distance 1.50×10^8 km.) Surprised? In sum, the meter reads only changes in g, but with a very impressive sensitivity: about 0.01 mgal. A *gal*, as you might expect, is a Texan for $\frac{cm}{sec^2}$. So this is about 1 in 10^8 of g! This is sufficient sensitivity to detect readily the difference in g between the floor and the table top in the laboratory simple due to the increased distance from the center of the Earth.

2. **EXPERIMENT**

Because the gravitational force obeys an inverse square relationship to distance:

$$g = G\frac{m_{Earth}}{R^2} \tag{1}$$

then you can show that Δg, the variation of g on the scale that we exist near the surface of the Earth varies as:

$$\frac{\Delta g}{g} = -2\frac{\Delta R}{R} \tag{2}$$

You will find that $R \approx 6 \times 10^6 m$. One option you might use in investigating the Earth's gravity and deducing the Radius of the Earth from equation (2) would be to measure Δg between the lab tabletop and the floor. However, such experiment would be badly in error because of the gravitational effect of nearby masses (in this case, the floor). The floor is massive and much closer when the gravimeter is on the floor, and as the gravitational attraction of the floor goes as $g' = G\frac{m_{floor}}{R^2_{tofloor}}$ and as R becomes so small, so the change in g. Fortunately, we have a set of floors, more or less identical, called the Burton Tower (height difference between floors is 3.95 m and floor mass is $\geq 10^6$ Kg). Readings taken at each floor, with the gravimeter stationed at the same position relative to each successive floor, with then reflect the variation of gravity with elevation and the local effect of the floor will just appear as an added constant. However, there are some complications:

1) Going up one floor removes mass overhead and puts it underneath. You should estimate the magnitude of this effect and, if significant, correct your data for it. And there are two further complications to this complication:

2) The basement excavation is a missing mass. Also, the floor and the ceiling heights of the basement and the first floor are non-standard.

3) Similarly, the roof is lighter than the floors, which also gives a (smaller) anomaly.

In view of this, data taken from, say, the third to the thirteenth floors should be the most useful in your determination of the Earth's radius. However, it is interesting to see if your data can display aspects of these 3 complications. For example, you might be interested in penetrating into the basement to judge the size effect (2) but you have to stay off the roof!

Another correction must be considered. As you take your measurements, the Earth is rotating, the resulting changing gravitational force exerted by the sun is casing your readings to change. The prospectors trick of doing repeat readings (every half-hour or every hour) at some reference station should provide a basis for correction of meter readings. The reference station is usually chosen so as to be located at a place most easily accessible from the various measurement stations. You must carefully plan how you will correct for these tidal variations.

Finally, will the centrifugal force due to Earth's rotation contribute significantly to the variation of g between floors?

Plot Δg vs ΔR, where these quantities for each floor are with respect to a floor you've chosen as reference. The linear portion of your graph should give you R to about 1% or 2% of the accepted value.

Also of interest is the deviation from linearity near the basement or the fifteenth floor. What does this deviation tell you? Due to this deviation, your best bet to get an accurate result for R is to omit these floors from the fit.

Eratosthenes of Cyrene was a Greek polymath: a mathematician, geographer, poet, astronomer, and music theorist. He obtained the Radius of the Earth to $\approx$ ± 80km in 200 BC for the cost of a trip to Upper Egypt. How does this compare with the cost of a $\$16,000$ gravity meter?

$\rightarrow$ **Python Programming (PHY224/324)**: Do the data anlysis and the plots in Python. Output R and the goodness of the fit parameter.

3. EXPERIMENTAL TIPS

General

Great care must be taken in handling, operating and transporting the gravity meter.

At all time, the meter has to be carried upright and during transport securely placed in an upright position. Tilting the gravity meter more than 30 degrees off the vertical position will expose the quartz system to severe stresses which may cause loss of accuracy and permanent damage to the meter.

Setting up the meter: Level the meter using the adjustable screws at the base of the meter.

Reading the meter:

1. Pull up the light switch.

2. Look through the centre viewport and observe the pair of bubble meters. Adjust the tripod legs until the bubbles are centred, indicating the gravimeter is aligned with the gravitational field.

3. View through the eye-piece. Adjust the knob until the moving pair of black bars is centred on the crosshairs. Make the final fine adjustment with the counter always approaching in the same direction by rotating the counter knob clockwise.

4. The meter constant is expressed in milligals per division and is shown on a small plate on the counter top. The meter constant is used as the factor to convert the counter readings into milligals.

Caution: do not force the counter! Do not try readings beyond the 1000 Range.

Focusing and beam illuminator

To improve the focus on the illuminated beam, raise the eyepiece until the beam image is sharp. For maximum beam illumination rotate the lamp housing. Changing the lamp will not affect the Null Line. Switch off the light when not reading by pushing down the light switch. Immediately before or during operation, the gravity meter must not be exposed to extreme temperature changes.

4. **SPECIFICATIONS**

Table 1: Instrument Specifications

MODEL	PROSPECTOR 410
RANGE(Reset)	$3500 - 6000$ mgal
FINE COUNTER RANGE	100 div. $\times$ scale constant (≈ 100 mgal)
FINE COUNTER CONSTANT	$0.09 - 0.11$ mgal
FINE COUNTER LINEARITY	1 in 1000
ACCURACY	0.1 counter divisions
DRIFT	$0.05\frac{mgal}{day}$ or better
LEVEL SENSITIVITY	$40\frac{sec}{min}$
TEMPERATURE COEFFICIENT	less than $0.003\frac{mgal}{hr^oC}$

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