Pulses in cables

References:

H.J. Pain: The Physics of Vibrations and Waves, 5th ed., Wiley 2002, Chapter 7 (Waves in Transmission lines)
T. R. Kuphaldt: Lessons in Electric Circuits, Volume II AC, Chapter 14 (Transmission Lines)
http://www.ibiblio.org/obp/electricCircuits/AC/AC_14.html

Introduction

Standard AC and DC circuit theory assumes that the speed of propagation of information in a circuit is infinite. If a change occurs anywhere in the circuit, then the rest of the circuit instantaneously reacts to that change. In reality pulses do not actually travel even as fast as speed of light *c* in cables, but are reduced by a factor which is related to the physical properties of the cable insulator.

The ideal transmission line

Transmission lines are discrete ladder-like networks of inductors (L) and capacitors (C) capable of storing and transmitting electric and magnetic energy. The basic unit of an ideal transmission line is an LC element of length dx and zero electric resistance (see Figure 1).

Figure 1
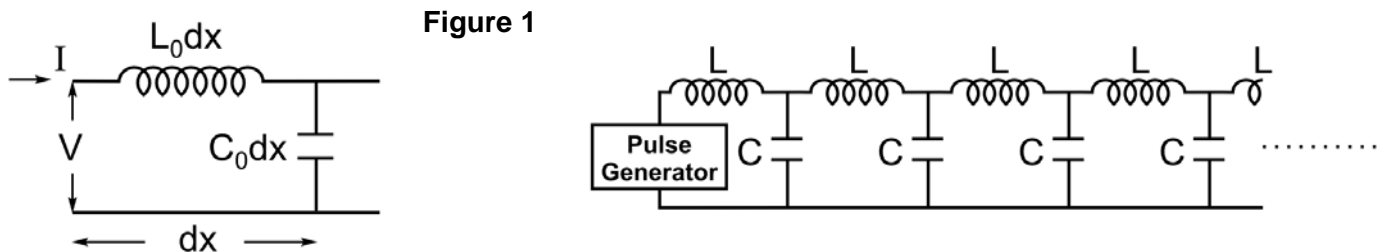
a) Basic unit of an ideal transmission line　　　**b) an ideal transmission line**

Analyses of the rates of change for both current I and voltage V take into account the self inductance of the element (L_0dx) and its capacitance (C_0dx)

$$\frac{\partial V}{\partial x} = -L_0 \frac{\partial I}{\partial t} \quad ; \qquad -\frac{\partial I}{\partial x} = C_0 \frac{\partial V}{\partial t} \qquad\qquad (1a)$$

This leads to the *wave equations* for voltage and current:

$$\frac{\partial^2 V}{\partial x^2} = L_0 C_0 \frac{\partial^2 V}{\partial t^2} \qquad\qquad (1b)$$

$$\frac{\partial^2 I}{\partial x^2} = L_0 C_0 \frac{\partial^2 I}{\partial t^2} \qquad\qquad (1c)$$

The velocity of wave propagation is defined as $v^2 = \dfrac{1}{L_0 C_0}$ (1d)

We can integrate the voltage drop along an infinitesimal unit of length dx: $\dfrac{\partial V}{\partial x} = -L_0 \dfrac{\partial I}{\partial t}$

A solution is: $V_+ = vL_0 I_+$ (2a)

where the subscript + means the positive direction of wave propagation.

$$\frac{V_+}{I_+} = vL_0 = \sqrt{\frac{L_0}{C_0}} = Z_0 \qquad\qquad (2b)$$

Z_0 is called the characteristic impedance of the line.

Coaxial cables

Figure 2
Coaxial Cable

Consider two conductor cables, separated by a dielectric material (Figure 2) with a continuous distribution of LC elements. Inductance per unit length can be written as:

$$L_0 = \frac{\mu}{2\pi} \ln \frac{R_2}{R_1} \qquad\qquad (3a)$$

where R_1 and R_2 are the radii of the inner and outer conductors, respectively; μ is the magnetic permeability of the dielectric.

Capacitance per unit length is given by:

$$C_0 = \frac{2\pi\varepsilon}{\ln \dfrac{R_2}{R_1}} \qquad (3b) \qquad \text{where } \varepsilon \text{ is the permittivity of the dielectric.}$$

It can be verified that: $\dfrac{1}{L_0 C_0} = \dfrac{1}{\varepsilon\mu} = v^2$

Knowing the velocity of signal travel in a certain coaxial cable, we can accurately calculate the travel time of an electrical signal along a given length.

A useful visual analogy of a pulse traveling in the cable is that of a wave moving along a rope. We shall see that the action of the pulse at the end of the cable has a strong analogy with what

2
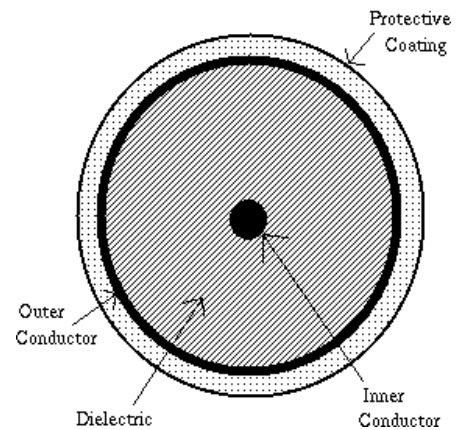
happens at the fixed end of an oscillating rope, suggesting that the math might be similar.
In the laboratory we shall see these effects by using pulses of duration ~10^{-8} seconds = 10ns, since this corresponds to a "length" of ~3m.
"Pulse length" is the real physical length of the pulse in the cable, so we can use the "length" of the pulse in either time units or length units.

The load effect

When a pulse travels along a cable and comes to an **open end** of the cable, it will reflect back and become an identical pulse moving in the opposite direction. This is illustrated in Figure 3.
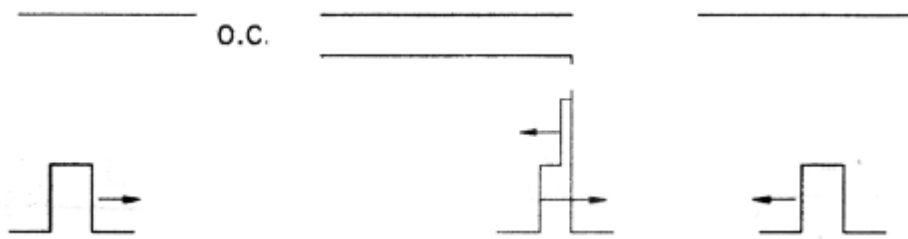

Figure 3 Pulse reflections (open end)

Assume the end is terminated by a **load of impedance Z**. If we take into account the two opposite directions of wave propagation (+, -), we may write a boundary condition at Z:

$$V_+ + V_- = V$$
$$I_+ + I_- = I$$

Which can be combined with:

$$\frac{V_+}{I_+} = Z_0 = -\frac{V_-}{I_-} \quad and \quad \frac{V}{I} = Z$$

We can now define the reflection and transmission coefficients:

$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0} \quad and \quad t = \frac{V}{V_+} = \frac{2Z_0}{Z + Z_0} \qquad (4)$$

The situation presented in Figure 3 corresponds to a reflection coefficient r = 1.0
If we replace the circuit at the end by a **short circuit**, no voltage can ever appear at the end and the pulse must reflect back with the opposite polarity r = -1. This is illustrated in Figure 4:
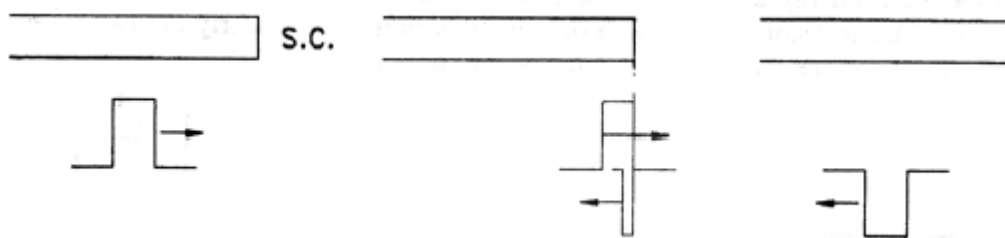

Figure 4 Pulse reflections (short circuit)

Given these two extreme cases, it is plausible to assume that for some intermediate value of resistance at the end, the reflection coefficient $r = 0$. In that case, there is no difference between putting that value of resistance there and adding another piece of cable of infinite length. In both cases as far as the first piece of cable is concerned, the pulse disappears. This value of resistance is referred to as Z_0, *the characteristic impedance* of the cable, which we defined before (Equation 2b). For our cable Z_0 is about 50Ω.

Assuming only resistive impedance (R) terminating the cable, it is reasonable to use the simplified expression:

$$r = \frac{R - Z_0}{R + Z_0} \qquad (5)$$

Voltage and current are π/2 out of phase in space and time. A **standing wave** is formed in a short-circuited transmission line of length $(2n+1)\dfrac{\lambda}{4}$ when the signal is a sinusoidal wave. At the end of the line voltage will be zero and current will be maximum. The total energy propagation will be zero.

Signals are not propagated without losses. **Attenuation** is the most visible defect of any cable:

$$Attenuation(dB) = 10\log_{10}\frac{Output\ Intensity\ (W)}{Input\ Intensity\ (W)} = 10\log_{10}\left(\frac{V_{reflected}}{V_{initial}}\right)^2 \qquad (6)$$

Attenuation is also a function of signal frequency and cable length ℓ :

Apparatus notes

We have two pulse generators: one for the study of the transmission line (SIGLENT SDG805) and another one (U of T Physics) for coaxial cable studies. We also have a digital oscilloscope which can display the pulses. We need to connect the two together with the transmission line or cable on which we are going to observe the transmission of the pulses. We do this with a TEE piece as shown in Figure 6.
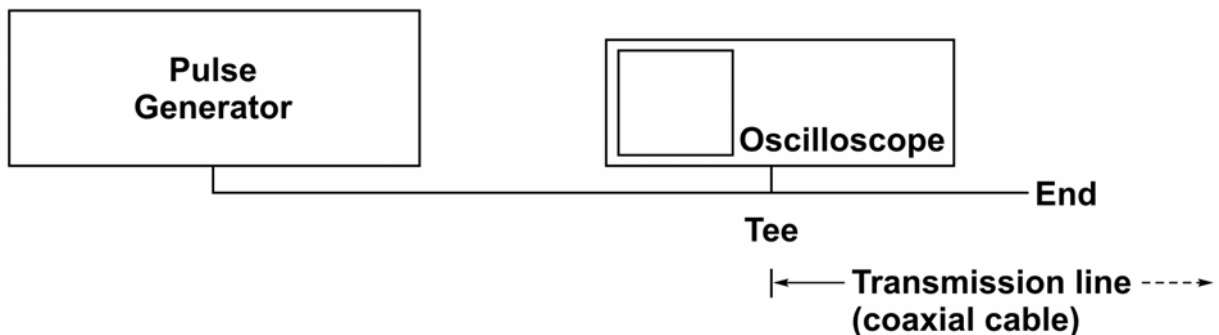

Figure 6 Circuit used to study transmission lines and coaxial cables

The TEE piece is symmetrical in all three arms. The pulse generator has an impedance of Z_0 and the oscilloscope has very large impedance. Can we safely ignore the presence of the oscilloscope as far as reflections go?

You should now be able to determine what comes back to the oscilloscope for any termination

of the long cable, particularly for **short-circuit** and **open-circuit** cases. As a check consider that any result must also be true as we increase the length of the pulses so that they overlap and eventually go to the DC levels on the cable. For example, if the cable is short-circuited in the DC case, there can never be any potential difference in the circuit. What will it be the case for an open circuit?

Exercise 1: A transmission line

Use the 41 LC units transmission line (TL), the unit pulse generator and power supply (SIGLENT SDG805). *Hint: start with pulse width of 75 µs and frequency of 300 Hz.*
With no load, connect the pulse generator to TL and to Ch1 of oscilloscope using a TEE. The TL output will be connected to Ch2. Vary the pulse duration and frequency until you observe the returned pulse on both Ch1 and Ch2. Explain what each channel measures.

Insert a resistive load across the end of the line. Vary R until the returned pulse cancels. This will be the impedance match between the source and the load (matching allows a maximum power transfer to the circuit). Use the MATH menu (CH1+CH2) of oscilloscope.
Measure the delay times between the transmitted pulse and the reflected one every 5 LC units. Plot *delay time (µs)* vs. *number of units*, estimate the speed of pulse propagation along the line and compare with calculated value from Equation 1d.
Some TL constants: C_0 = 0.01 µF; L_0 = 1.5 mH (these are values per LC unit). Speed of propagation will have units of 'LC unit' per second.
➔ **Python Requirement 1 (PHY224/324 students only):** Plot the above dependencies using Python. Your program would output the speed of pulse propagation

Exercise 2: Coaxial Cables
Use the "U of T Physics" pulse generator with various cable lengths, terminations and pulse widths. You should be able to investigate the open circuit, short circuit and several resistive terminations.
Measure the time of flight of the pulse and compare it to the result using the speed of light. Pulses do not actually travel as fast as *c* in the cable, but are reduced by a factor which is related to the dielectric constant of the insulator.
Measure the attenuation factor of the cable. Express your answer in dB/m.
Hint: use an open circuit so that the entire pulse is reflected.
Find the value of Z_0 for the cable.
➔ **Python Requirement 1 (PHY224/324 students only):** Do all the analysis of coaxial cables from Exercise 2 using Python.

Some cable constants: ε = 2.25; µ = 1 (polyethylene)
Geometrical characteristics (radii) can be found at: bwcat.belden.com and www.amphenol.co.za