University of Toronto
ADVANCED PHYSICS LABORATORY

FAR

Faraday Waves

Revisions:

2014 September:	Very minor edit by David Bailey, including correcting typo found by student Lennart Döppenschmitt.
2011 September:	Natalia Krasnopolskaia

Image obtained by undergraduate students
Ruo Yu Gu, Timur Rvachov and Suthamathy Sathananthan (2009)

Please send any corrections, comments, or suggestions to the professor currently supervising this experiment, the author of the most recent revision above, or the Advanced Physics Lab Coordinator.

The experiment with Faraday Waves was first tested by the teams of EngSci students Ruo Yu Gu (PHY427); Timur Rvachov (PHY427); Suthamathy Sathananthan (PHY427) in 2009.

INTRODUCTION

This challenging experiment gives you an opportunity to join a cohort of experimentalists contributing into creation of scientific theory governing the behavior of vertically vibrating fluids. Your results can contribute in understanding of fundamental principles of non-linear dynamics and theory of chaos.

The objects of study are the surface standing waves in a viscose liquid called Faraday waves. Placing a container with oil on a vibrating shaker, you will observe a variety of patterns on the surface of a substance. Your goal is to find the relationship between a set of parameters of the pattern, properties of the substance (viscosity, surface tension, density) and conditions of the pattern generation (frequency and amplitude of vertical vibrations, surface area and the initial thickness of the layer).

On a cover page of this manual you can see a pattern on a surface of olive oil (Faraday waves) observed by students participated in the development of this experiment for Advanced Physics Laboratory in 2009-2010. You are encouraged to make your own search on the Internet for new discoveries or observations in the field of these physical phenomena, which are under current study by a number of laboratories world wide.

FARADAY WAVES

A fluid on a vertically vibrating plate will, upon reaching a certain frequency and acceleration, produce standing waves on its surface. These are called Faraday waves, first described by Michael Faraday in 1831. Faraday waves are still an active area of research today, more than 150 years after their initial discovery. In current research the terms “Faraday instability” and “standing gravity waves” are also used for this phenomenon.

In this experiment, Faraday waves will be generated and observed under a variety of different conditions in two kinds of Newtonian fluids: water and oil. For a Newtonian fluid, the viscosity depends only on the environmental temperature and pressure, but is independent of shear stress.

1. Theory of parametric oscillator applied to Faraday instability

The basic equation governing the behavior of a layer of fluid in a vertically vibrating system is that of the parametric oscillator. A detailed theory of parametric resonance applied to the Faraday instability is given in [1]. The full derivation of all equations shall not be included here. The main theoretical concepts however, must be understood. The parametric oscillator equation for an infinitely small fraction of the fluid surface is given by:

$$z'' + 2\mu z' + \omega_0^2[1+\alpha(t)]z = 0\,, \qquad (1)$$

where z is the vertical position of the infinitesimal fraction of the surface; μ is the damping rate [damping rate = damping ratio multiplied by ω_0], associated with viscosity of the fluid; ω_0 is the frequency of oscillation of the fraction of the surface; and $\alpha(t)$ is the dimensionless oscillating parameter function. This equation is to some extent similar to that of a damped harmonic oscillator. The oscillating parameter is the gravitational field, which in the frame of the shaker can be expressed as

$$g(t) = g + A\omega^2\cos(\omega t) = g\,[1 + \Gamma\cos(\omega t)] \quad (2)$$

if the driving force is a harmonic function; $\omega = 2\pi f$; f is the frequency of the driving force; and A is the amplitude of vibrations of the shaker. Therefore, the Faraday waves are often called the *standing gravity waves*. The ratio Γ (gamma) of the maximum linear acceleration of a shaker and the free fall acceleration is widely used under the term "*amplitude*". Equation (1) cannot be solved analytically in general, i.e. for any possible combinations of μ, A, ω_0, ω and function $\alpha(t)$. Substitution of Eq.2 into Eq.1 gives

$$z'' + 2\mu z' + \omega_0^2\left[1 + \Gamma\cos(\omega t)\right]z = 0 \qquad (3)$$

Viscosity of a fluid, finite volume of a vessel with the fluid, boundary conditions on the lateral walls and the bottom of the vessel and the real depth of the layer of the fluid, taken into account all together, demand either the numerical solution for Eq.3 or introduction of some approximations [2]. Certain combinations of Γ and ω form the resonance regions ('resonance tongues'), where z grows exponentially with time. It means that equilibrium state with $z = 0$ is unstable in these regions, and even a negligible deviation from this state can take the system into parametric resonance with an exponential growth of z.

Under the condition $\Gamma << 1$ (small pumping), the parametric resonance occurs at frequency ratios $\omega/\omega_0 \approx 2/n$, for $n = 1, 2, 3, \ldots$The odd-n tongues are called subharmonic, while the even-n tongues are called harmonic. Due to dissipation of energy, it is difficult to observe the tongues with $n > 2$ and in many experiments they are ignored.

Two particular cases can be studied with water (low viscosity) and oil (viscose fluid).

a) Water in a container is considered a laterally infinite and inviscid fluid ($\mu = 0$) of depth h.

Equation 3 must be modified and becomes Mathieu differential equation.
The two-dimensional surface wave consists of normal modes with the frequency, given by the following dispersion equation:

$$\omega_0^2(t) = k\left[g(t) + \frac{\gamma}{\rho}k^2\right]\tanh(kh), \qquad (4)$$

where k is the magnitude of the two-dimensional horizontal wave vector $\boldsymbol{k}(x, y)$ of the mode; $g(t)$ is given by Eq.2; γ is the fluid surface tension coefficient; and ρ is the fluid density. Introducing detuning as $\delta = \omega - 2\omega_0 << \omega_0$, an interval for existence of the Faraday instability can be calculated as in [3]

$$-\frac{\Gamma\omega_0}{2} < \delta < \frac{\Gamma\omega_0}{2},$$

or

$$2\omega_0 - \frac{\Gamma\omega_0}{2} < \omega < 2\omega_0 + \frac{\Gamma\omega_0}{2} \quad (5)$$

b) Oil in a container is considered a viscose fluid ($\mu \neq 0$) of infinite depth under the condition $\delta = \omega - 2\omega_0 << \omega_0$.

The instability boundaries are given by [3]

$$-\sqrt{\left(\frac{\Gamma\omega_0}{2}\right)^2 - 4\mu^2} < \delta < \sqrt{\left(\frac{\Gamma\omega_0}{2}\right)^2 - 4\mu^2}$$

or

$$2\omega_0 - \sqrt{\left(\frac{\Gamma\omega_0}{2}\right)^2 - 4\mu^2} < \omega < 2\omega_0 + \sqrt{\left(\frac{\Gamma\omega_0}{2}\right)^2 - 4\mu^2} \quad (6)$$

The damping rate μ is related to kinematic viscosity v and the wave number k as $\mu = 2vk^2$. The kinematic viscosity can be found experimentally or in a handbook as $v = \eta / \rho$, where η is the dynamic viscosity and ρ is the fluid density.

The pattern observed on the surface of the fluid depends upon the relationship between the wave number of each mode $k = 2\pi / \lambda$, where λ is the wavelength of the mode, and the diameter of the vessel with the fluid. The meniscus at the walls of the vessel, the possible horizontal propagation of the surface waves, and instability of the fluid-layer characteristics [2] can drastically influence observations in a real experiment. Besides, the fluctuations in the driving force frequency and amplitude can bring an existing pattern to a mixed state with a fraction of spatiotemporal chaos [4].

Safety Reminders

- Keep all body parts, hair, and clothes away from the shaker when it is turned on.
- The floor around the shaker must be keep clean and dry to prevent slip and fall injuries. Even when operating normally some fluid material may splash onto the floor. Any splashes or spills should be immediately and carefully cleaned and the floor dried.
- Immediately turn off the shaker if it starts to overheat (e.g. smoke or bad smells) or it begins making any unusual mechanical noise.

NOTE: This is not a complete list of every hazard you may encounter. We cannot warn against all possible creative stupidities, e.g. juggling cryostats. Experimenters must use common sense to assess and avoid risks, e.g. never open plugged-in electrical equipment, watch for sharp edges, don't lift too-heavy objects, ... If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. When in doubt, ask! If an accident or incident happens, you must let us know. More safety information is available at http://www.ehs.utoronto.ca/resources.htm.

2. Experiment

The experiment station includes three main parts:
I) a shaker VTS-50 with attached accelerometer MMA 1220; a vessel with fluid; a digital camera, connected to a computer; and a stroboscope (Fig.1) of one of two available models;
II) a signal generator; an amplifier; and a digital oscilloscope (Fig.2); and
III) a sample preparation station with a number of fluids; vessels; beakers; etc (Fig.3).

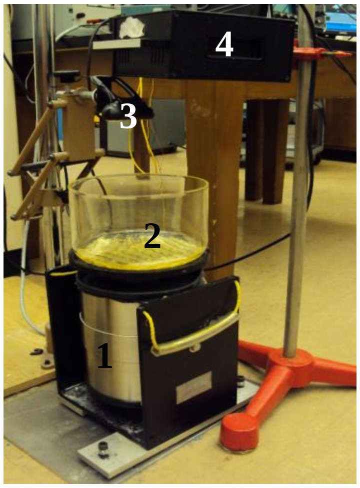

FIG.1: 1 – shaker; 2 – vessel with fluid; 3 – digital camera; 4 - stroboscope

The shaker is expensive and delicate. Students must take care to set the amplifier gains to minimum before turning on the system, gradually turning up the gain knob during use. The input frequency should never exceed 50 Hz. The range of interest for both Faraday waves in a fluid and granular patterns will be 10 Hz $\leq f \leq$ 30 Hz. When under specific conditions the beats are generated in the system, the shaker can be broken. If you see and hear the typical features of beats instead of a harmonic mode of vibration, immediately set the amplitude to zero with following adjustment of the driven frequency, just to shift the mode of vibration away from the dangerous state.

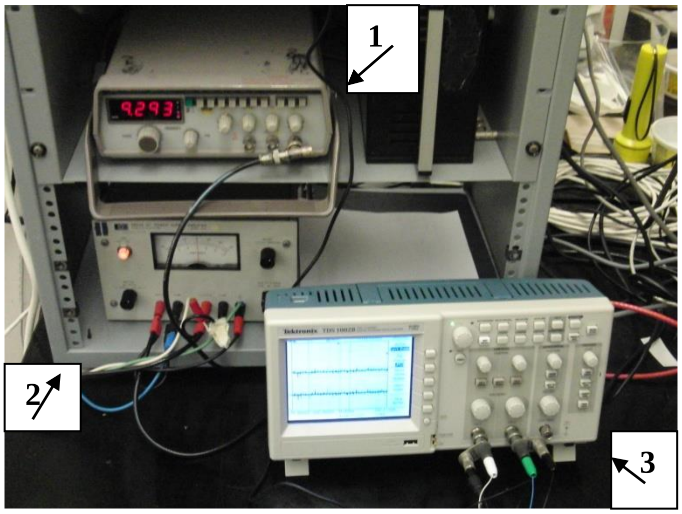

FIG.2: 1 – signal generator; 2 – amplifier; 3 – digital oscilloscope

Before measurements and observations can be made, the calibration of the accelerometer must be studied. It is recommended to always have a supply voltage V_{DD} = 5V which however can be smaller. Use a multimeter to measure V_{DD} from time to time to correctly choose the accelerometer sensitivity. The full accelerometer calibration is attached to the manual.

FIG.3: Sample preparation station

The output signal of accelerometer is measured with oscilloscope in millivolts. For a sinusoidal signal, the amplitude of acceleration a_{max} of the plate is $a_{max} = (2\pi f)^2 A$, where A is the linear amplitude of vibration of the plate of a shaker. In experiments with Faraday waves and oscillons, the function $\Gamma = (2\pi f)^2 A/g$ is used instead of the amplitude A as a parameter of vibration. Γ is a convenient measure because it can be easily obtained by dividing the amplitude a_{max} measured in millivolts by the value of sensitivity.

You should use different containers for water and oil. Some properties of the fluids that can be used in the experiment are given in Table 1. A digital camera is mounted above the container to get a desirable image. You can also try different regimes of illumination of the container to obtain an image with the camera. Use of stroboscope can be also helpful in measuring frequency of Faraday waves.

Turning-on procedure

a) Measure supply voltage across the accelerometer with a multimeter.
b) Before turning on the power button of a signal generator, set amplitude to a minimum by rotating the knob "AMPL" fully counterclockwise.
c) Set up the frequency range to 20 Hz (can be changed later if required).
d) Level the shaker's plate by adjusting the screws at the shaker's platform.
e) Turn on the power supply for signal generator, amplifier and accelerometer.
f) Press the button "ON" of the signal generator.
g) Set frequency to 10 Hz.
h) Turn on the amplifier.
i) Turn on an oscilloscope and set it to measure the amplitude of a signal in millivolts in the range of frequencies between 10 Hz and 30 Hz.
j) On the desktop of a computer, an icon of the camera to video recording the experiment. Set up a proper number of frames per a second. You may choose Photo or Video option.

TABLE 1. Properties of fluids at 20^{o}C.

Fluid	Density, x10^3kg/m^3	Surface tension, mN/m	Dynamic viscosity, mPa · s
Distilled water	0.998	72.8	1.00
Corn oil	0.918	33.4	65.0
Olive oil	0.920	35.8	84.0
Sunflower oil	0.920	33.0	55.8
Baby oil	0.850	30.0	1.55

Special measurements, performed by the second year student of the Department of Physics Onaizah Onaizah in a summer project SURF in 2011, gave different values for the corn oil kinetic viscosity. In her measurements, she took the kinetic viscosity of distilled water as 1 cSt (1 *centi*Stokes) at 20^{o}C. 1 cSt = 10^{-6} m^2/s. Before the oil has undergone a 5-minute shaking, she measured its kinetic viscosity to be 65.13 cSt; and after been shaken the oil viscosity has grown up to 84.03 cSt. The significant rise of the value of viscosity as a result of shaking must be taken into consideration when analyzing the values for the Faraday instability boundaries.

Onset of patterns in water

First, try to observe surface standing waves in water with different driven frequencies and amplitudes. Begin experiments with small depth $h \approx 3$ mm and increase the depth to about 1.5 – 2 cm. To measure the depth, use a beaker and measure the internal diameter of the container.

Begin slowly and carefully increasing the driving force amplitude. You should not permit splashes on the surface. Stop increasing the amplitude when the resonance is strong enough and begin to increase frequency. This must be done very slowly as every new pattern cannot become stable instantly.

> While tuning the driven frequency, watch how the amplitude of the driving force is changing. Why does it happen?

If you managed to see a clear pattern on the water surface, adjust the camera position and illumination to improve the image on the screen and make a snapshot. Also sketch the wave profile from the side view. Is it a sinusoidal wave?

FIG.4: Stable pattern on the water surface. h = 1.2 cm; f = 20 Hz; Γ = 0.15

Due to water viscosity, we can expect the Faraday instability boundaries to match the relationship (5). To verify whether the patterns appear inside the permitted boundaries for your experiment (particular depth h, Γ and ω) you must manage to measure the frequency of the standing wave ω_0. For every new pattern, measure the onset conditions. Try to plot the threshold acceleration versus driven frequency and the wavenumber of the oscillating mode versus frequency as it is done in Fig.20 of the article [2]. Some values of h may not provide you with a good evidence of the Faraday instability on the water surface. Why is it so difficult to observe the Faraday waves and measure their parameters on the water surface in your experiment?

Onset of patterns in oil

Empty and dry the vessel, as it will be used for oil. Keep station clean during the entire time of your experiment removing the traces of oil around you immediately and thoroughly.

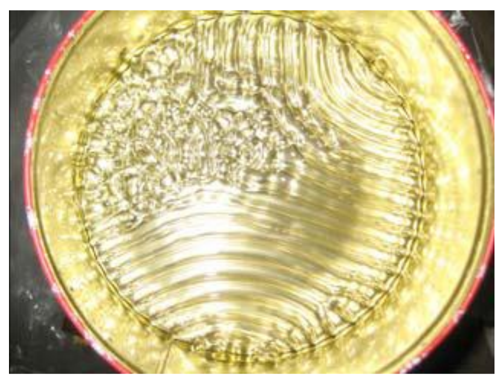

FIG.5: Stripes mixed with chaotic fraction (oil)

1) The range of depths is from 3 mm to 1.5 cm. Starting with the smallest depth, combine driven frequencies and amplitudes to observe different patterns on the oil surface. If you do not observe the patterns with h = 3 mm, add oil. You must be able to observe stripes, squares, hexagons, or mixed states, as in Fig. 5. However, not all mentioned patterns can be observed at any depth. Basing on your observations, create a graph like that in Fig.3 of the article [4] showing the regions of stable patterns of different kinds in coordinate system of "Acceleration" vs. "Frequency". Acceleration can be calculated either in m/s^2 or in units of Γ. You can also use coordinates "A" vs. "f".

2) In an experiment with oil, you should measure the Faraday instability boundaries to verify the Eq. 6 for different values of depth h. The procedure is described in detail in [2].

- Set up the driven frequency at 10 Hz. Slowly increasing the amplitude, stop at the instant the standing wave pattern appears on the surface
- Measure the standing wave frequency, amplitude (optional) and wavelength (wavenumber); verify the statement that the strongest resonance is observed for $\omega = 2\omega_0$

- If you haven't found an onset of any pattern, turn the amplitude knob to zero, set the frequency to 11 Hz and repeat the procedure. The uncertainty of your results will depend upon your measurements of f and a_{max}.
- Determine the threshold acceleration and plot Γ vs. f for instability boundaries for specific h, as in Fig.7 and 8 of the article [1]. Identify the subharmonic resonance tongue for n = 1.
- Compare your result with that, given by the relationship (6).
- Increase the depth of the oil in the container and repeat your measurements. Explain a discrepancy between the expected onset parameters and the real Γ and f in your experiment
- If time permits, repeat similar measurements and observations for different oils. What numerical results do not agree for two oils within the uncertainty range? Why?

3) Chaotic Fraction. Now that you have estimates of acceleration at which a pattern first appears, you are able to observe how these patterns change as the driven amplitude is slowly increased. Replicate your results from one of the previous experiments to obtain a clear pattern of stripes and slowly increase the acceleration. You should observe this pattern changing slowly and only in certain areas. By taking photographs, estimate the fraction of the pattern that has changed (the chaotic fraction) at each value of acceleration, keeping the frequency constant. Compare to the results reported in [4], which are reproduced in Fig.6.

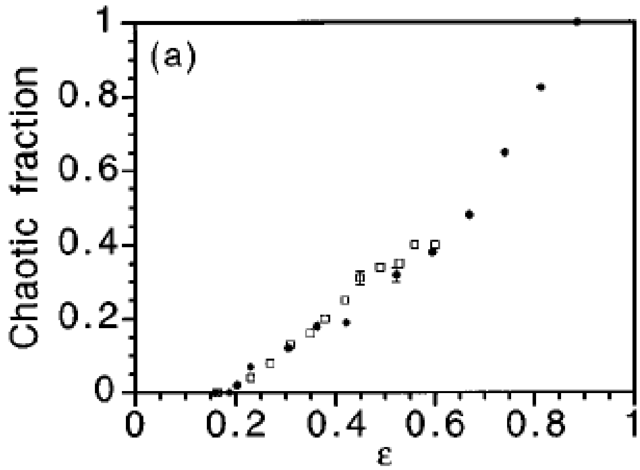

FIG.6: Chaotic fraction in the mixed state [4]. $\varepsilon = A / A_C - 1$, where A is the vibration amplitude and A_C is the onset amplitude

Faraday waves were recently discovered in Bose-Einstein condensate [5].

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