Domain walls, anomalies, and deconfinement

Erich Poppitz with Andrew Cox, Samuel Wong UofToronto

1909.10979

pre-anomaly work with Mohamed Anber, Tin Sulejmanpašić

Lewis&Clark Durham 1501.06773

+ with Anber on axion domain walls 2001.03631

HARVARD UNIVERSITY
CENTER OF MATHEMATICAL
SCIENCES AND APPLICATIONS

Wednesday, May 6, 2020

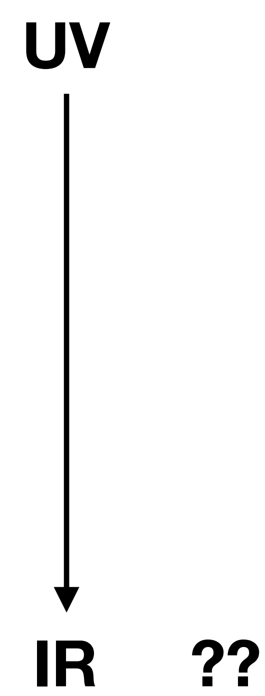
UV

$\downarrow$

IR **??**

anomaly matching

limits fantasies about IR!



thought anomaly matching was set in stone since ca. 1980
"0-form" anomalies played major role in, say, "preon" models (1980's), Seiberg dualities (1990's)

new "generalized 't Hooft anomaly matching"

Gaiotto, Kapustin, Komargodski, Seiberg, Willett … 2014-

UV

"generalized 't Hooft anomaly matching"

anomalies of global symmetries revealed by turning on
background gauge fields for global symmetries,
compatible with their faithful action
(interpret some as "gauging higher-form symmetry")

IR **??**

currently active area of research, across fields

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condensed matter, mathematical physics, high-energy theory

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general theorems

examples and dynamical implications in QFT

impossible to review!

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related (to the subject of talk) important recent work

J. Wang, Y.-Z. You and Y. Zheng, 1910.14664

I. Hason, Z. Komargodski and R. Thorngren, 1910.14039 narrow this talk's subject to:

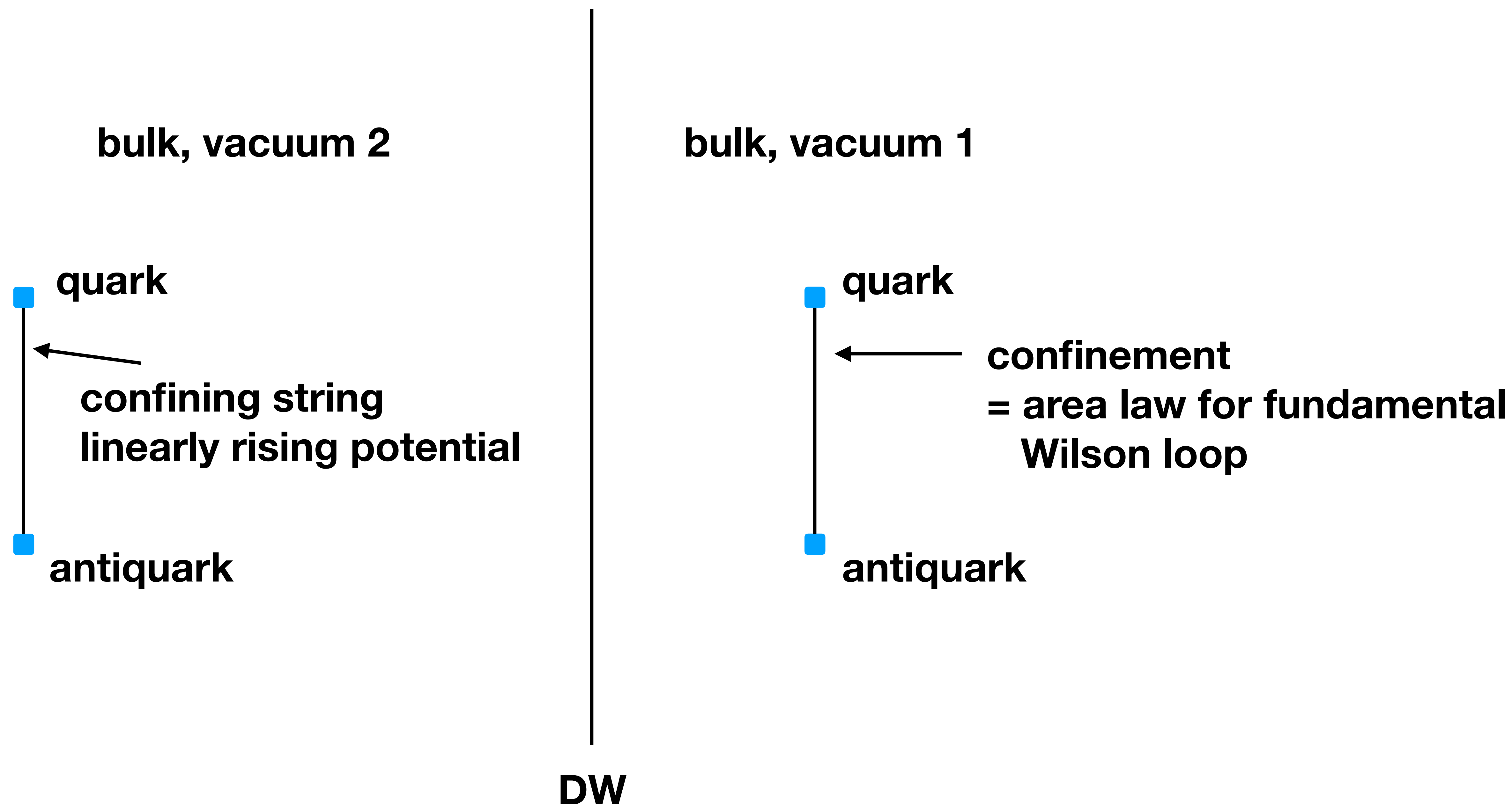
theories with a broken $\mathbb{Z}_N^{(0)}$ global symmetry
and unbroken $\mathbb{Z}_{N'}^{(1)}$ center symmetry

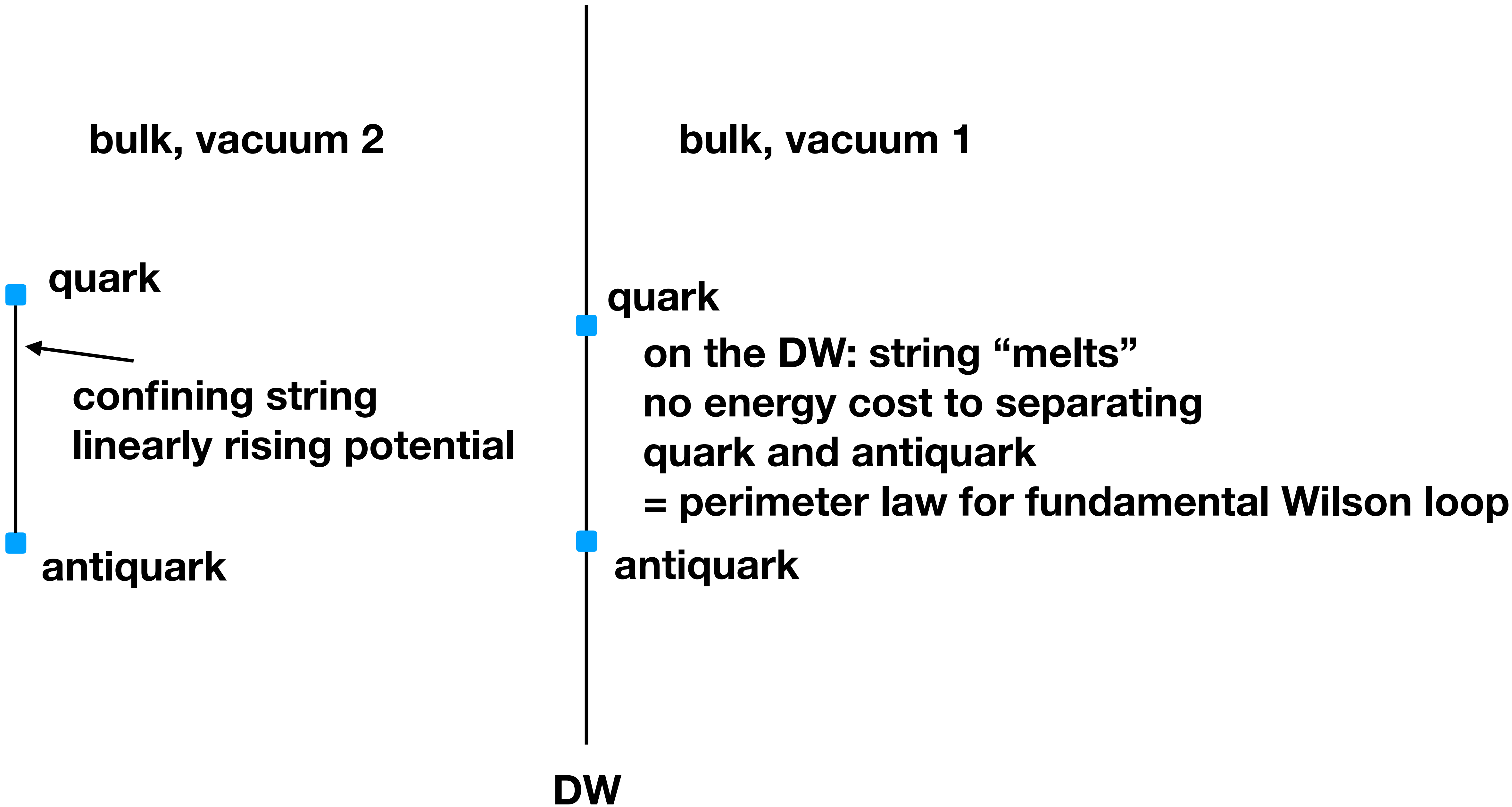
"confining theories with domain walls" (DW)

e.g.: YM (QCD) at $\theta = \pi$

QCD(adjoint) with $n_f\,(= 1{,}2{,}3{,}...5)$ massless Weyl, if…

QCD-like (vectorlike) coupled to axion


bulk, vacuum 2
bulk, vacuum 1
quark
confining string
linearly rising potential
antiquark
quark
confinement
= area law for fundamental
Wilson loop
antiquark
DW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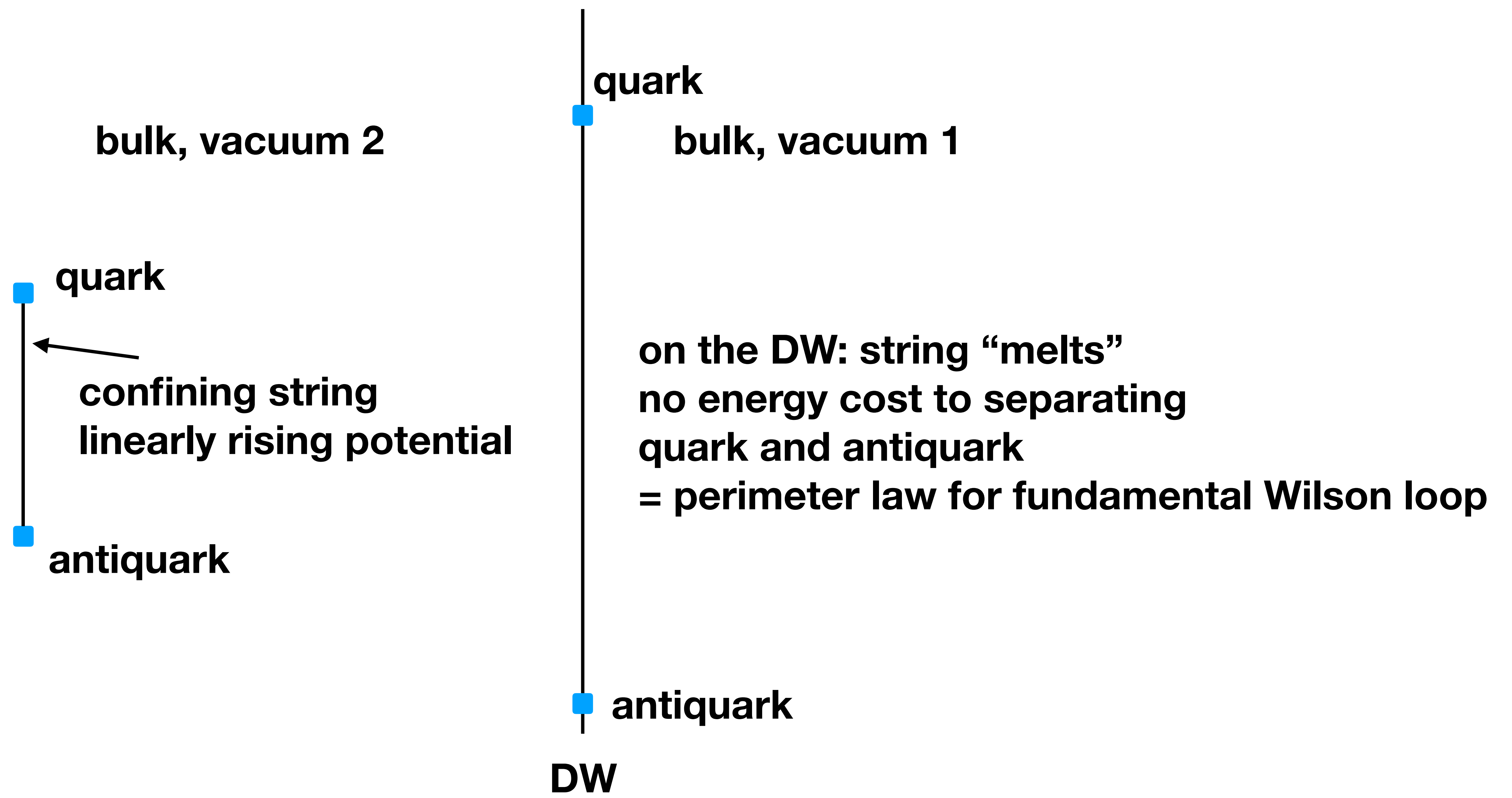
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explanations of quark liberation on DW somewhat formal

- **3d CS theory (TQFT) 'lives' on DW -** of SYM via M-theory

  [Acharya, Vafa, late 1990s]

- **MQCD picture of confining (F-) strings ending on D-(M-) walls**

  [Soo-Jong Rey, 1997; Witten, 1997;… more recently, e.g. Hsin, Lam, Seiberg 2018]

- **connection to mixed "CP** (or $\mathbb{Z}_N^{(0)}$) **-** $\mathbb{Z}_{N'}^{(1)}$ **" anomaly**

  NEW: QFT, not string

  [Gaiotto et al… 2014-]

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heuristic monopole/dyon picture -> nothing condenses on wall, so flux spreads

monopoles condense | **dyons condense**

DW

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heuristic monopole/dyon picture -> nothing condenses on wall, so flux spreads

Witten
hep-th/9706109

" the QCD monopoles themselves, are somewhat elusive "

is there a framework in QFT, where we can understand DW-deconfinement in a theoretically controllable way?

it is nice to have a more concrete physical picture

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- difficult on R^4...

entails having a theory of confinement

- possible on $R^3 \times S^1$ - a weak coupling realization of confinement and a nonperturbative semiclassical study of the vacuum is trustable! [Unsal, +..., 2007-]

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deconfinement on DWs was found in 2015 (Anber, Sulejmanpašić, EP) via honest semiclassical analysis of QFT - before relation to anomaly inflow understood - ***explain*** *and* ***extend*** *in this talk* for brevity - and elegance - narrow further talk's subject:

SU(N) QCD(adjoint) with $n_f = 1$ massless Weyl = SYM

- a broken $\mathbb{Z}_{2N}^{(0)}$ global symmetry
- unbroken $\mathbb{Z}_{N}^{(1)}$ center symmetry

with a mixed 0-form/1-form anomaly

stress that story I will tell does not require SUSY

… adjoint QCD, deformed YM, axion …

however, SUSY will help streamline the presentation…

OUTLINE

~~1. Introduction~~

2. SYM: *brief* reminder of symmetries and ’t Hooft anomaly

3. Compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- scales, and semiclassics

- EFT and symmetries
- EFT vacua and DWs
- (de)confinement and DWs

4. Conclusions

A. what I told you about

B. wish list

SYM: *brief* reminder of symmetries and 't Hooft anomaly

SU(N) + massless adjoint Weyl fermion λ_α^a $\quad a = 1,\ldots,N^2 - 1$, $\quad \alpha = 1,2 \ (SL(2,\mathbb{C}))$

center symmetry $\mathbb{Z}_N^{(1)}$ $\quad W_k(C) \to e^{\frac{2\pi i k}{N}} W_k(C)$

chiral symmetry $\mathbb{Z}_{2N}^{(0)}$ $\quad \lambda \to e^{i\alpha} \lambda$

$$\mathscr{D}\lambda \to e^{i\alpha 2NQ_{top}} \mathscr{D}\lambda$$, so $\alpha = \dfrac{2\pi}{2N}$

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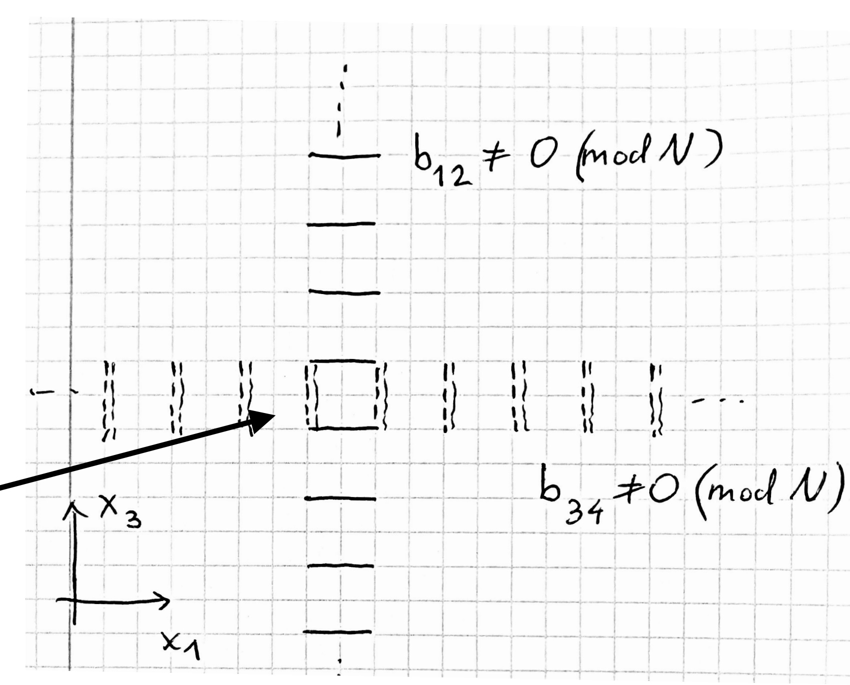
't Hooft anomaly revealed by gauging $\mathbb{Z}_N^{(1)}$ using a 2-form $\mathbb{Z}_N$ gauge field

= 't Hooft fluxes in 1-2 and 3-4

= intersecting thin center vortices

$$Q_{top} = mm'(1 - \frac{1}{N_c})$$



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mixed center/chiral 't Hooft anomaly

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5d inflow - $$S_{5d} = \frac{i2\pi}{N} \int_{M_5, \partial M_5 = M_4} \frac{2NA^{(1)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi}$$

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$\delta A^{(1)} = d\phi^0$

$$\phi^{(0)}\Big|_{M_4} = \frac{2\pi}{2N} \longrightarrow \delta S_{5d} = i\frac{2\pi}{N}\int_{M_4} \frac{NB^{(2)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi} = i\frac{2\pi}{N}$$

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$\oint \frac{2NA^{(1)}}{2\pi} = 1$ twisted chiral b.c. in one direction of M_4

chiral broken phase - domain 1-wall $M_3 \in M_4$ appears

$$S_{4d} = \frac{i2\pi}{N} \int\limits_{\hat{M}_4, \partial \hat{M}_4 = M_3} \frac{NB^{(2)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi}$$ 4d inflow on DW

= 4d inflow action for 3d CS $SU(N)_{-1}$ = candidate DW theory

hence, no confinement on DW…

SYM: *brief* reminder of symmetries and 't Hooft anomaly

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rest of talk - reveal deconfinement on DW + DW CS - semiclassically...

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- scales, and semiclassics

small-L **semiclassical** limit $NL\Lambda \ll 1$

$g^2_{4d}(\frac{1}{NL}) \ll 1$and L fixed (**NOT 3d** $g^2_{3d} = \frac{g^2_{4d}}{L}$-fixed as $L \to 0$)

holonomy: higgsing at $m_W = \frac{1}{NL}$: $SU(N) \to U(1)^{N-1}$

Cartan $U(1)^{N-1}$ weak (no charges!) at energy $\ll m_W$ $(m_W \gg \Lambda)$

weak coupling + nonperturbative: confinement, χSB, etc…

[Unsal 2007-, +…]

(generic, large class of non-SUSY theories at small-L; here: SYM**)**

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- scales, and semiclassics

due to $NL\Lambda \ll 1$ locally 4d
"remembers" 4d properties: anomalies, symmetries…

mass gap & confinement due to the proliferation of instanton-like objects - magnetic bions in SYM/QCD(adj)

- a locally-4d nontrivial generalization of Polyakov confinement!
[Unsal]

describe using a 3d EFT valid at length scales $\gg NL$

(using EFT+SUSY will help avoid many interesting details)

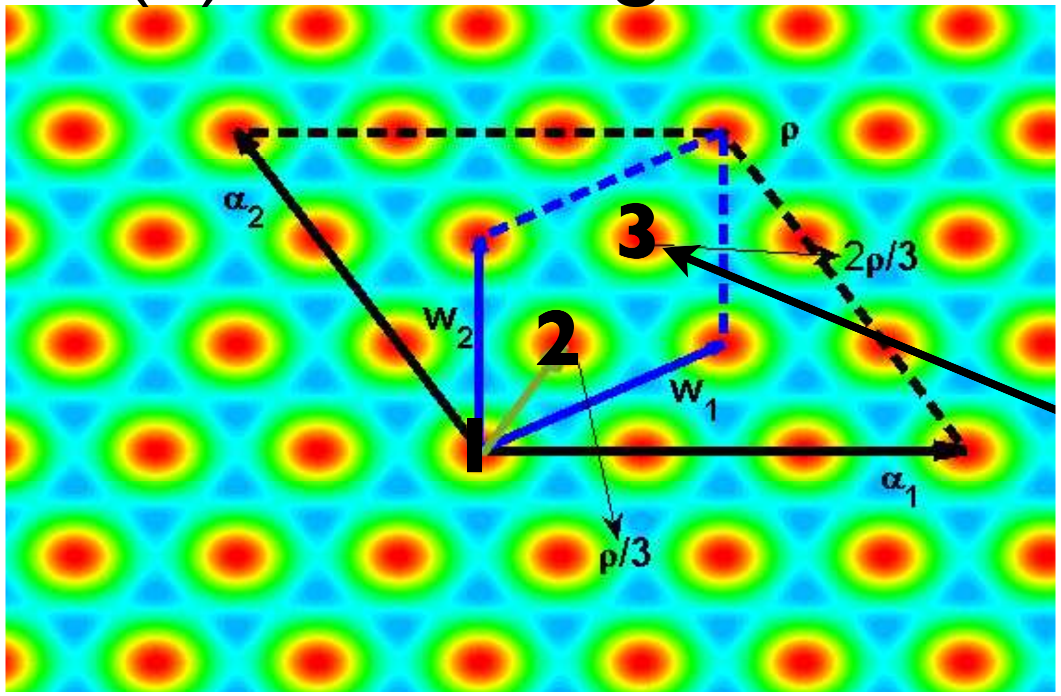
SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT and symmetries

Cartan gluons only, dualize

$$\frac{g^2}{4\pi L}\epsilon_{\mu\nu\lambda}\partial^\lambda \vec{\sigma} = \vec{F}_{\mu\nu}$$

$$\partial_x \vec{\sigma} \sim \vec{E}_y,\ \partial_y \vec{\sigma} \sim -\vec{E}_x$$

SU(3) root/weight lattices

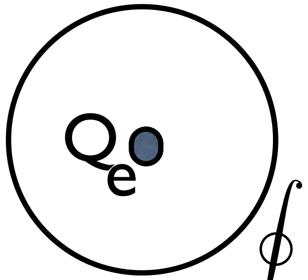


compact, unit cell of $\Gamma_{weight}(SU(N))$

$$\vec{\sigma} = \vec{\sigma} + 2\pi \vec{w}_p$$

$$\oint_{C\in\mathbb{R}^2} d\vec{\sigma} = 2\pi \vec{\lambda}$$

weight = Cartan charges inside C



$\oint d\sigma = Q_e$

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$$\partial_x \overrightarrow{\sigma} \sim \overrightarrow{E}_y,\ \partial_y \overrightarrow{\sigma} \sim -\overrightarrow{E}_x$$

$$\partial_x \overrightarrow{\phi} \sim \overrightarrow{B}_y,\ \partial_y \overrightarrow{\phi} \sim -\overrightarrow{B}_x$$

$$\langle \overrightarrow{\phi} \rangle = 0 \longleftrightarrow \langle \mathrm{Tr}\Omega_F^k \rangle = 0$$

$$\Omega_F \to e^{\frac{2\pi i}{N}}\Omega_F \qquad \Omega_F = P e^{i\oint_{\mathbb{S}^1} A_4 dx^4}$$

$\mathbb{Z}_N^{(0),c}$ “zero-form” center (along $\mathbb{S}^1$)

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$\mathbb{Z}_N^{(0),c}$ "zero-form" center (along $\mathbb{S}^1$)

$$\mathbb{Z}_N^{(0),c} : \overrightarrow{\phi} \to \mathscr{P}\overrightarrow{\phi}$$

$$\mathscr{P} = s_{\alpha_1} s_{\alpha_2} \ldots s_{\alpha_{N-1}}$$

$\mathscr{P}$ = product of Weyl reflections w.r.t all simple roots $\overrightarrow{\alpha}_k$

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT and symmetries

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$$\mathbb{Z}_N^{(0),c} : \; \vec{\phi} \to \mathscr{P} \vec{\phi}$$

$$\vec{x} = \vec{\phi} + i \vec{\sigma}$$ chiral superfield

$$\mathscr{P} = s_{\alpha_1} s_{\alpha_2} \ldots s_{\alpha_{N-1}}$$

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$\mathbb{Z}_N^{(0),c}$ "zero-form" center

$$\mathbb{Z}_N^{(0),c} : \; \vec{\phi} \to \mathscr{P} \vec{\phi}$$
$$\vec{\sigma} \to \mathscr{P} \vec{\sigma}$$

for non-SUSY, see Anber, EP 1508.00190

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$\mathbb{Z}_N^{(0)}$ **chiral symmetry**	$\mathbb{Z}_N^{(0),c}$ **"zero-form" center**
$\mathbb{Z}_N^{(0)} : \vec{\sigma} \to \vec{\sigma} + \frac{2\pi}{N}\vec{\rho}$	$\mathbb{Z}_N^{(0),c} : \vec{\phi} \to \mathscr{P}\vec{\phi}$, $\vec{\sigma} \to \mathscr{P}\vec{\sigma}$

chiral intertwined with $U(1)^{N-1}$ would-be magnetic center of dual photons, broken by monopole-instantons

($e^{i\vec{\alpha}_k \cdot \vec{\sigma}}\lambda\lambda \qquad \lambda\lambda \to e^{\frac{i2\pi}{N}}\lambda\lambda$)

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$\mathbb{Z}_N^{(0),c}$ **"zero-form" center**

$$\mathbb{Z}_N^{(0),c} : \begin{array}{l} \vec{\phi} \to \mathscr{P}\vec{\phi} \\ \vec{\sigma} \to \mathscr{P}\vec{\sigma} \end{array}$$

$\mathbb{Z}_N^{(1),c}$ **one-form center**, probed by Wilson loop $C \in \mathbb{R}^3$

$$\mathrm{Tr}_R\, P e^{i\oint_{C\in\mathbb{R}^3} A_\mu dx^\mu} \xrightarrow[\text{EFT}]{} \mathrm{Tr}_R e^{iH^a \int_C A^a_\mu dx^\mu} = \sum_{\substack{\vec{\lambda}\in R \\ \text{weights of R}}} e^{i\frac{g^2}{4\pi L}\vec{\lambda}\cdot\int_{S,\partial S=C}\partial_\mu \vec{\sigma} n^\mu d^2 s}$$

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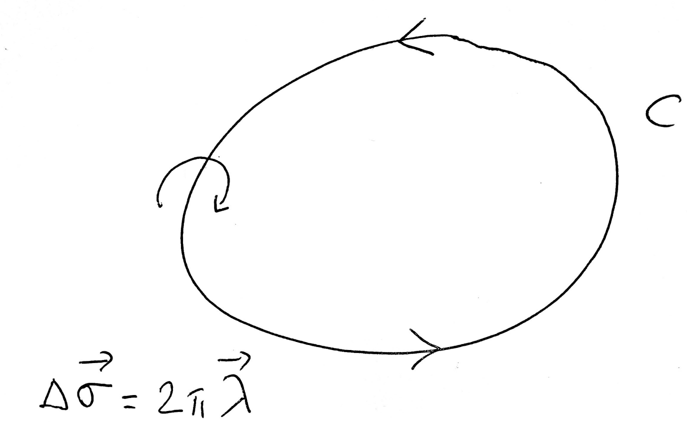
weights of R

insertion in path integral imposes $\overrightarrow{\sigma}$ monodromy $2\pi\overrightarrow{\lambda}$ around C



SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT and symmetries

$L = M\, \partial_\mu x^a g_{ab} \partial^\mu x^{*b} + \ldots$ simply $\sim tr_{Cartan}\, F_{4d}^2$ kinetic term

Kahler metric (calculated, W-loops…ignore)

$$M \sim \frac{g^2}{L} \sim m_W$$

$$g_{ab} = \delta_{ab} + \ldots$$

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nonperturbative scale

$$m \sim \frac{1}{L}\, e^{-\frac{4\pi^2}{g^2}}$$

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT and symmetries

$$L = M\, \partial_\mu x^a g_{ab} \partial^\mu x^{*b} - \frac{Mm^2}{4} \frac{\partial W(\overrightarrow{x})}{\partial x^a} g^{ab} \frac{\partial W(\overrightarrow{x}^*)}{\partial x^{*b}}$$

$$g_{ab} = \delta_{ab} + \ldots$$

$$M \sim \frac{g^2}{L} \sim m_W$$

$$m \sim \frac{1}{L}\, e^{-\frac{4\pi^2}{g^2}}$$

nonperturbative superpotential

$$W = \sum_{a=1}^{N} e^{\overrightarrow{\alpha}_a \cdot \overrightarrow{x}} \qquad \overrightarrow{\alpha}_N = -\overrightarrow{\alpha}_1 - \ldots - \overrightarrow{\alpha}_{N-1}$$

$$\left(W = X_1 + X_2 + \ldots + X_{N-1} + \frac{1}{X_1 X_2 \ldots X_{N-1}}\,, \quad X_i = e^{\overrightarrow{\alpha}_i \cdot \overrightarrow{x}_i} \right)$$

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$\mathbb{Z}_N^{(1),c}$ one-form center unbroken

$\mathbb{Z}_N^{(0)}$ chiral broken

$\mathbb{Z}_N^{(0),c}$ "zero-form" center unbroken

N vacua

mass gap

$\sim m$

$$\langle \overrightarrow{\phi} \rangle = 0$$

$$\langle \overrightarrow{\sigma} \rangle = \frac{2\pi k}{N} \overrightarrow{\rho}$$

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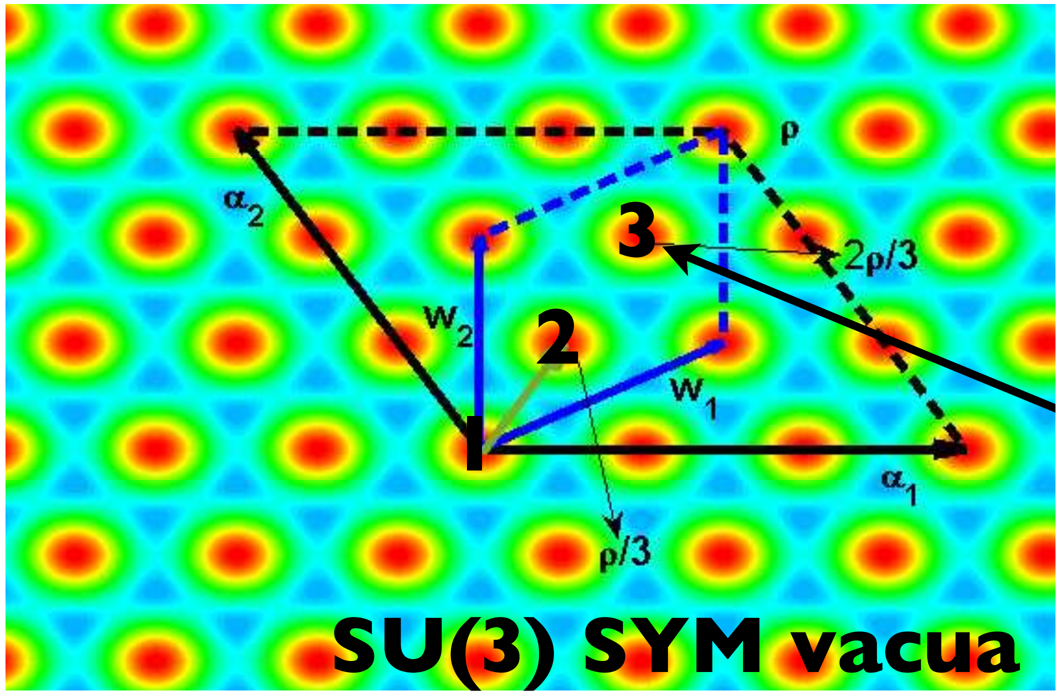


$\mathbb{Z}_N^{(1),c}$ one-form center unbroken

$\mathbb{Z}_N^{(0)}$ chiral broken

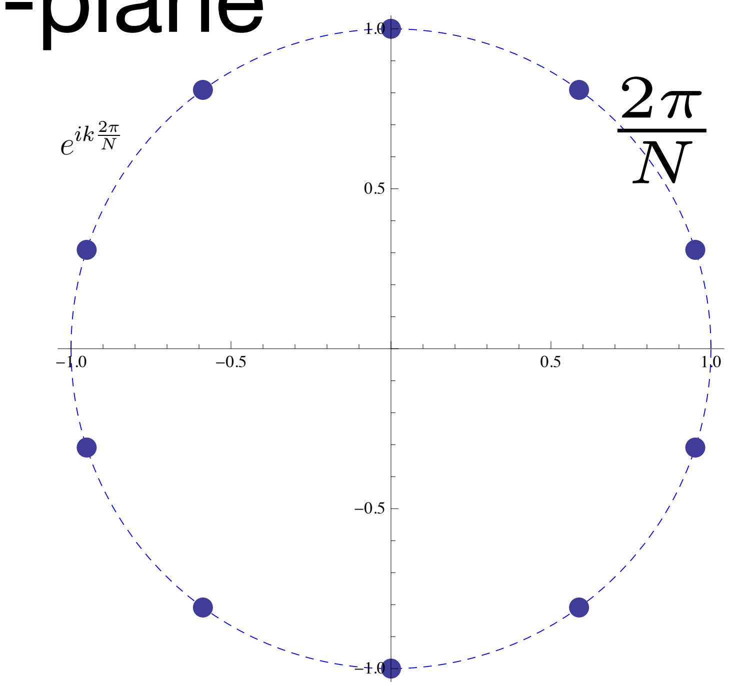
$\mathbb{Z}_N^{(0),c}$ “zero-form” center unbroken

$\overrightarrow{\sigma} \to \mathscr{P}\overrightarrow{\sigma} : 120^0$ rotation

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT vacua and DWs

N vacua $\quad \mathbb{Z}_{2N}^{(0)} \to \mathbb{Z}_2^{(0)}$

W-plane



$$\langle \overrightarrow{\phi} \rangle = 0$$

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k-walls: interpolate between vacua k units apart

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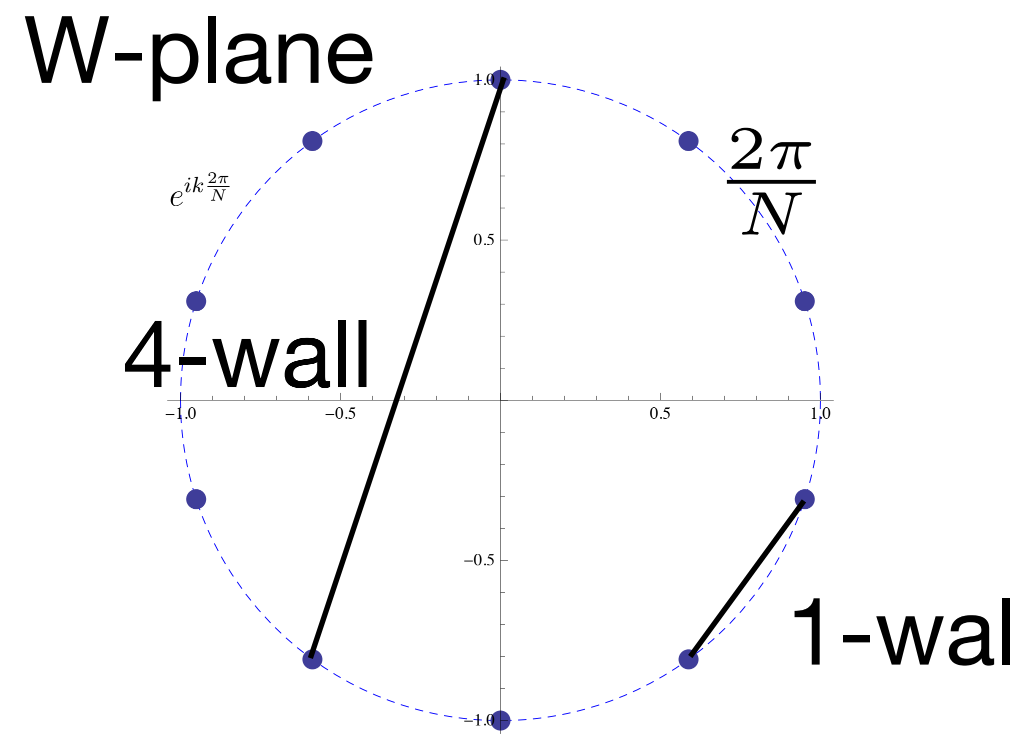

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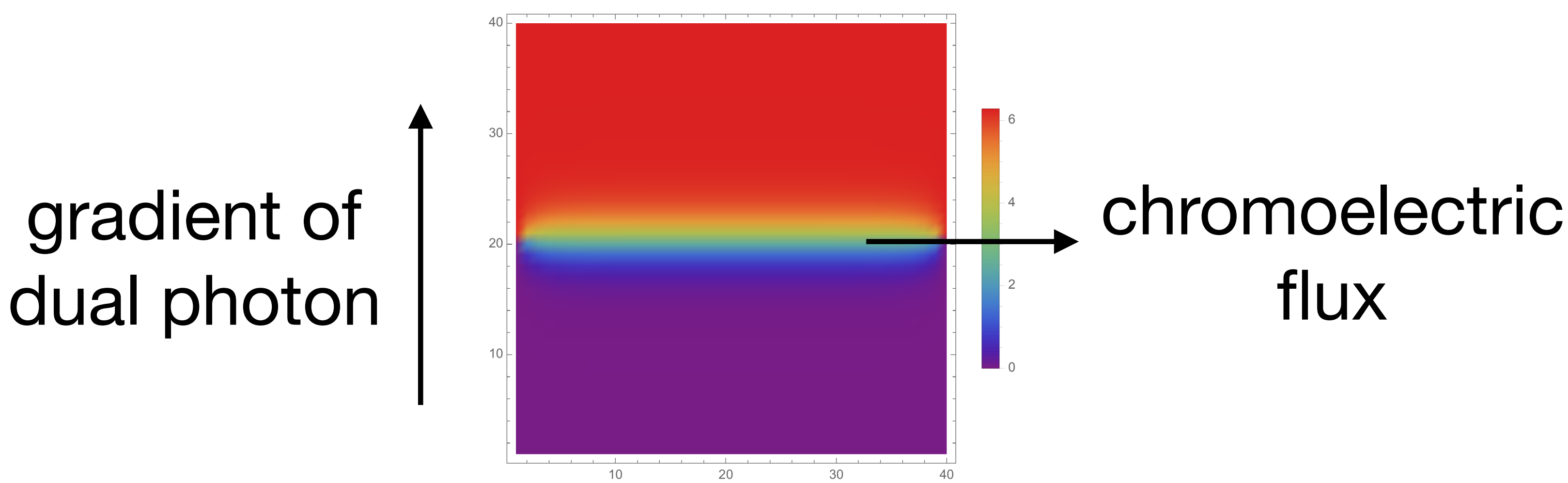
$$\Delta \vec{\sigma} \Big|_{1-wall} = \frac{2\pi}{N} \vec{\rho} \quad \text{but} \quad \partial_x \vec{\sigma} \sim \vec{F}_{0y} \sim \vec{E}_y$$

DWs (lines!) carry E-flux along worldvolume

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT vacua and DWs



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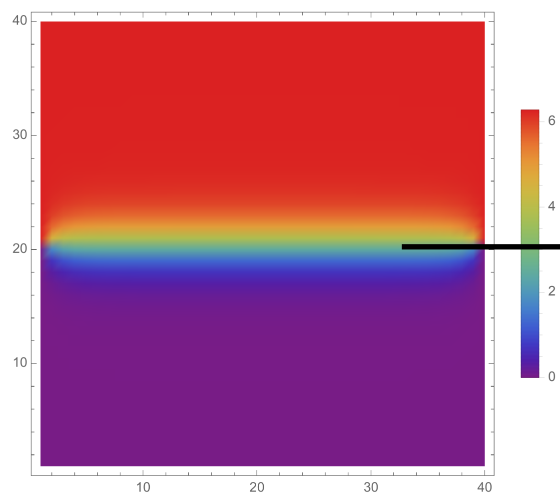
gradient of dual photon



chromoelectric flux

$$\Delta \overrightarrow{\sigma}\Big|_{1-wall} = \frac{2\pi}{N}\overrightarrow{\rho}$$

the flux, $\frac{2\pi}{N}\overrightarrow{\rho}$, does not correspond to the flux of any quark (too “small”!)

ultimately, due to the “composite” nature of magnetic bions!

“DWs” are not confining strings here (unlike Polyakov or dYM)

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT vacua and DWs

What are the electric fluxes on the lowest tension (BPS) k-walls?

“dual photons” are compact scalars, can have extra $2\pi \overrightarrow{w}_k$ monodromies across walls

important for understanding confining strings - as electric flux in the vacuum “collimates” on DW (lines)

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- EFT vacua and DWs

What are the electric fluxes on the lowest tension (BPS) k-walls?

understood for k=1 walls
Anber, Sulejmanpašić, EP
already in 1501.06773

all k>1 walls understood
+ anomaly angle
Cox, Wong, EP
1909.10979

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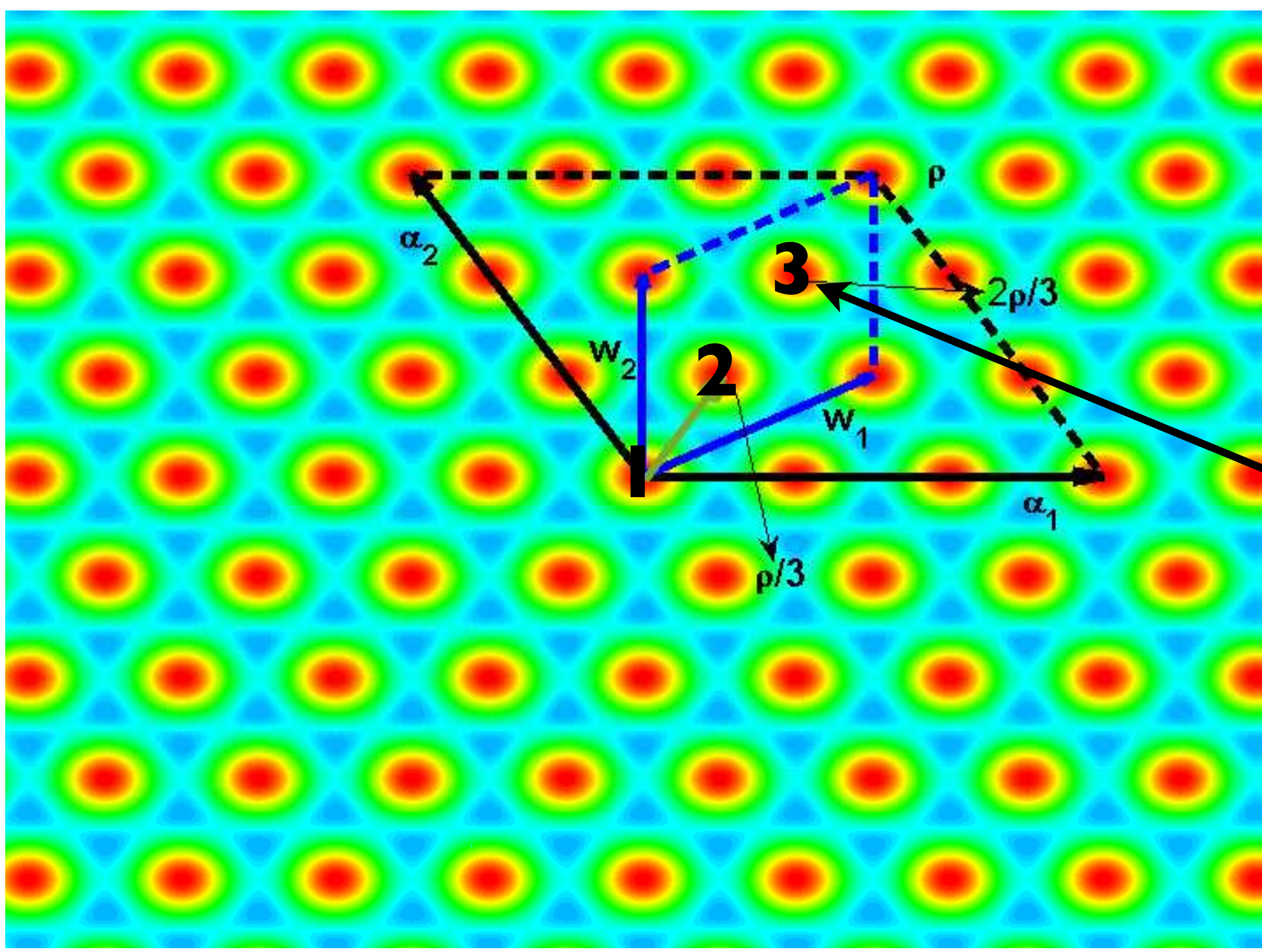

dual photon plane

periodicities:
w1, w2: weight vectors of SU(3)

3 vacua - **1,2,3**
broken discrete chiral symmetry
(preserve center symmetry
120 degree rotn + w_k shift)

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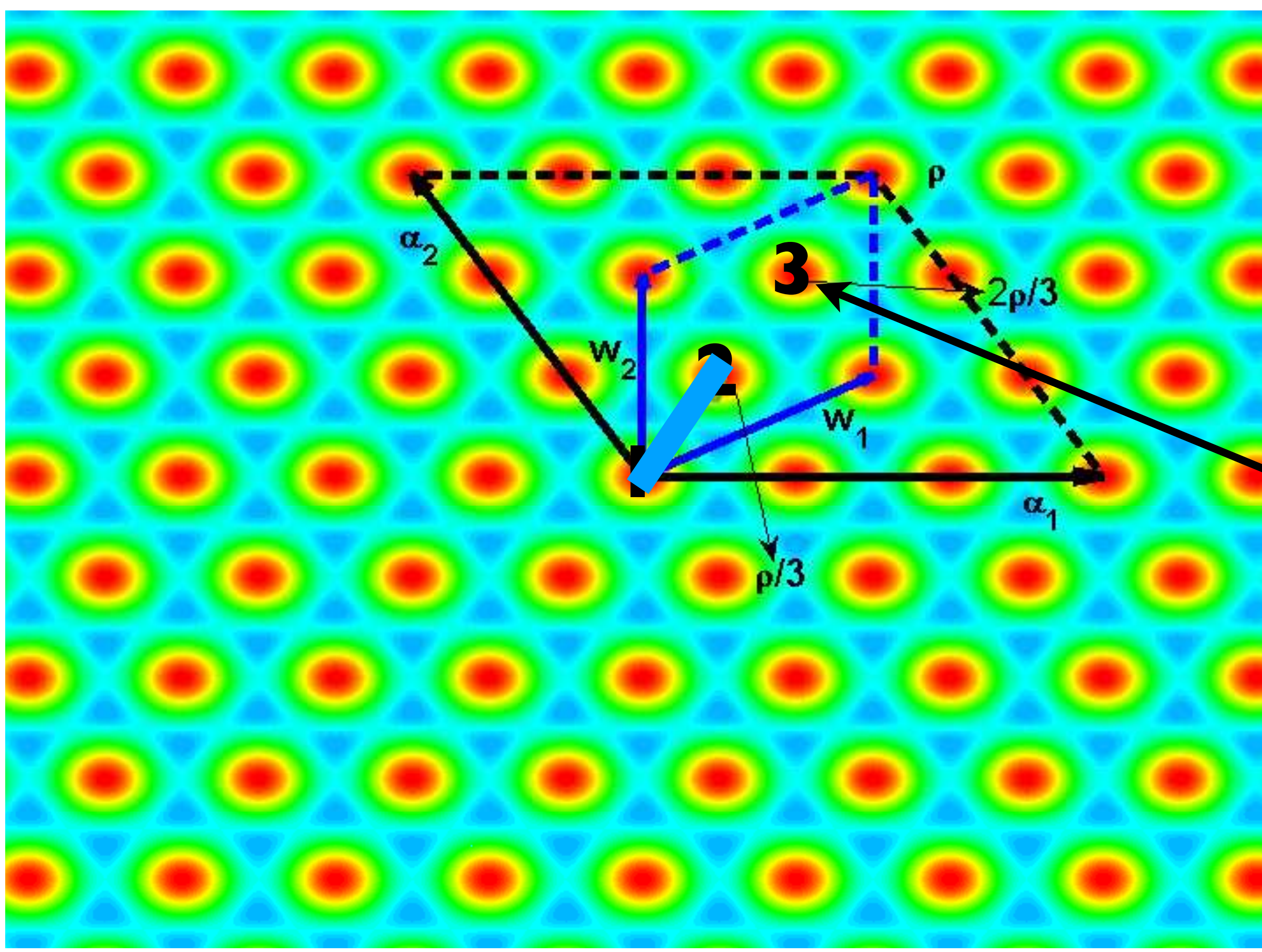

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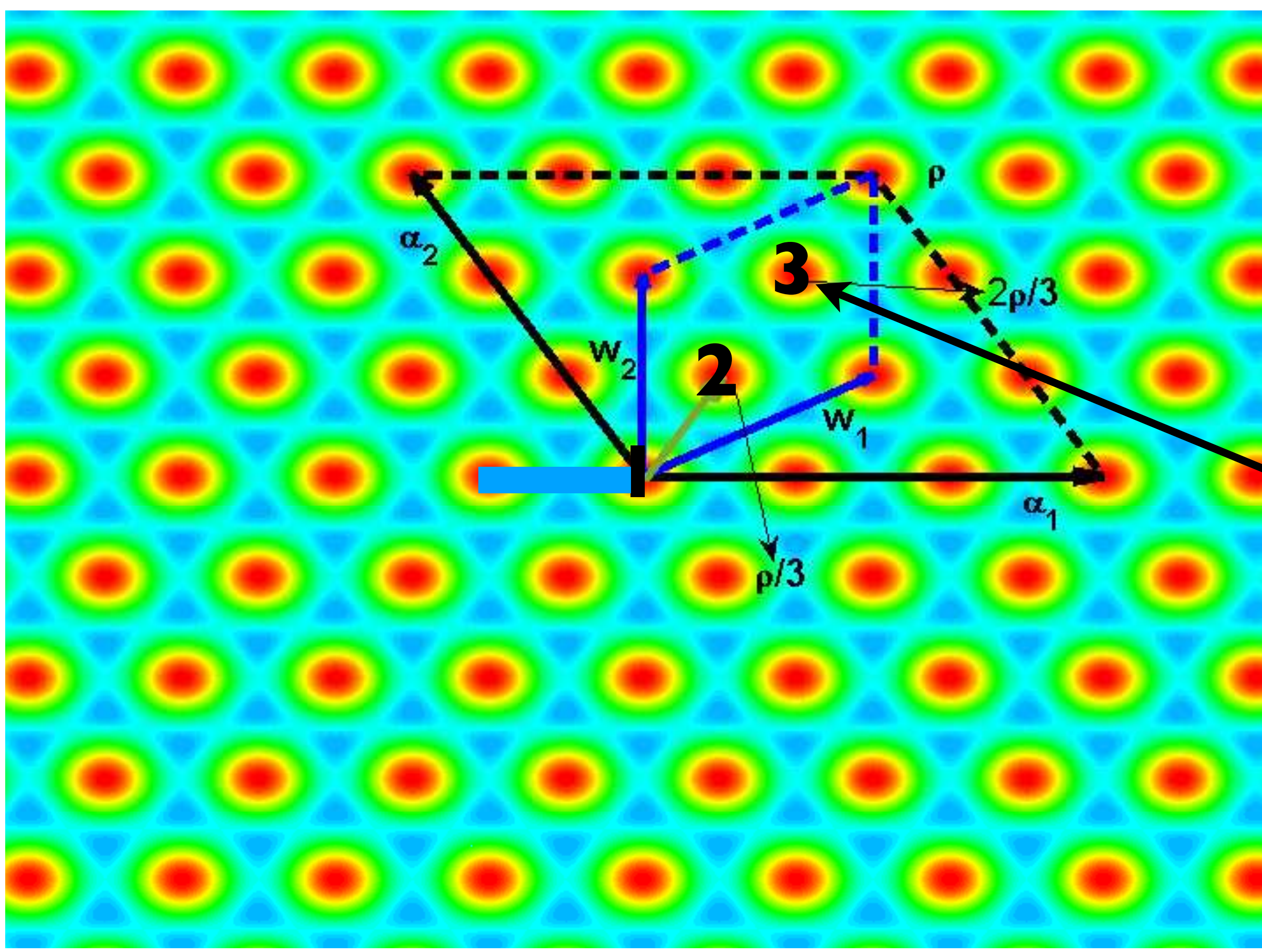

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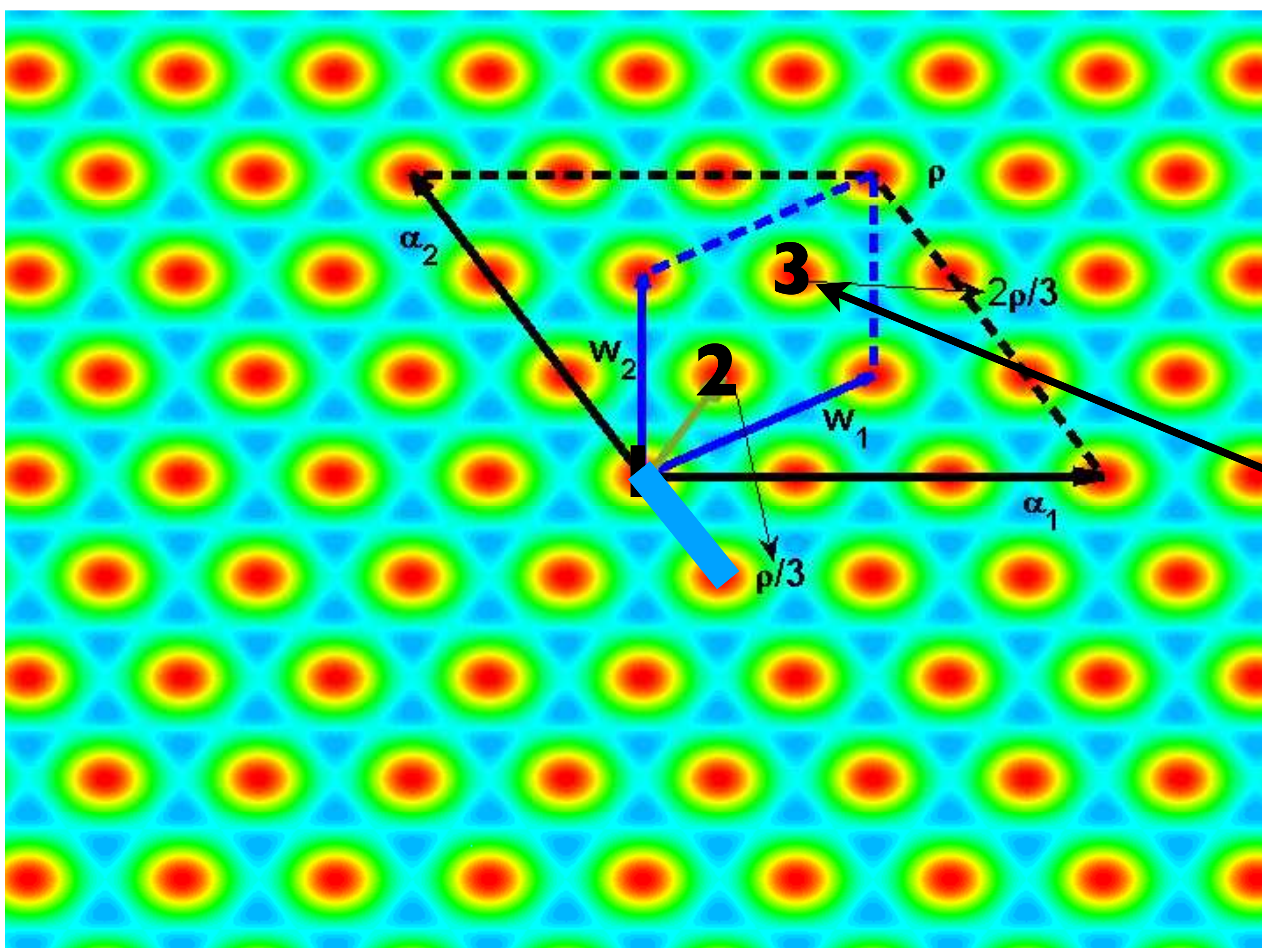

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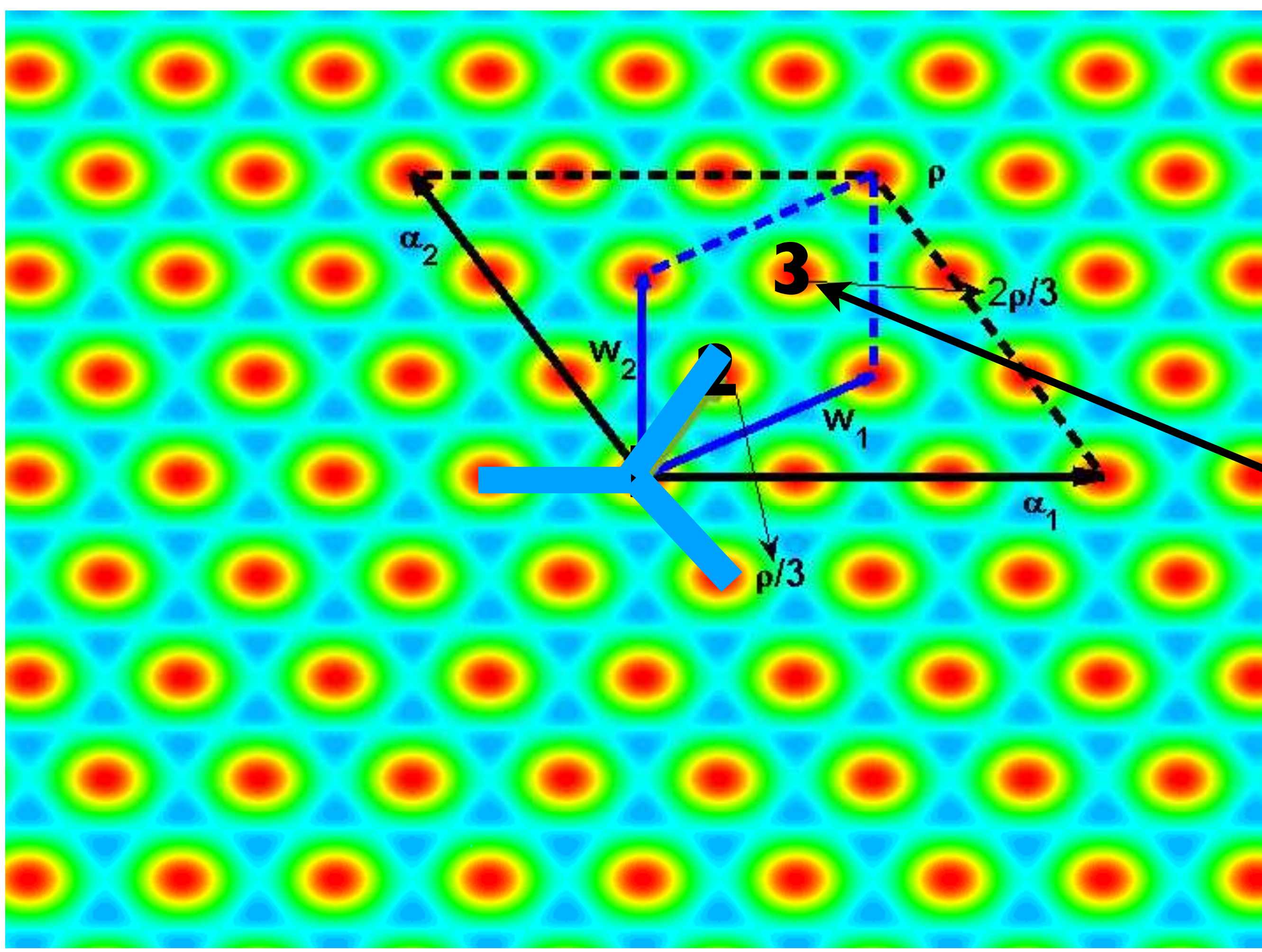

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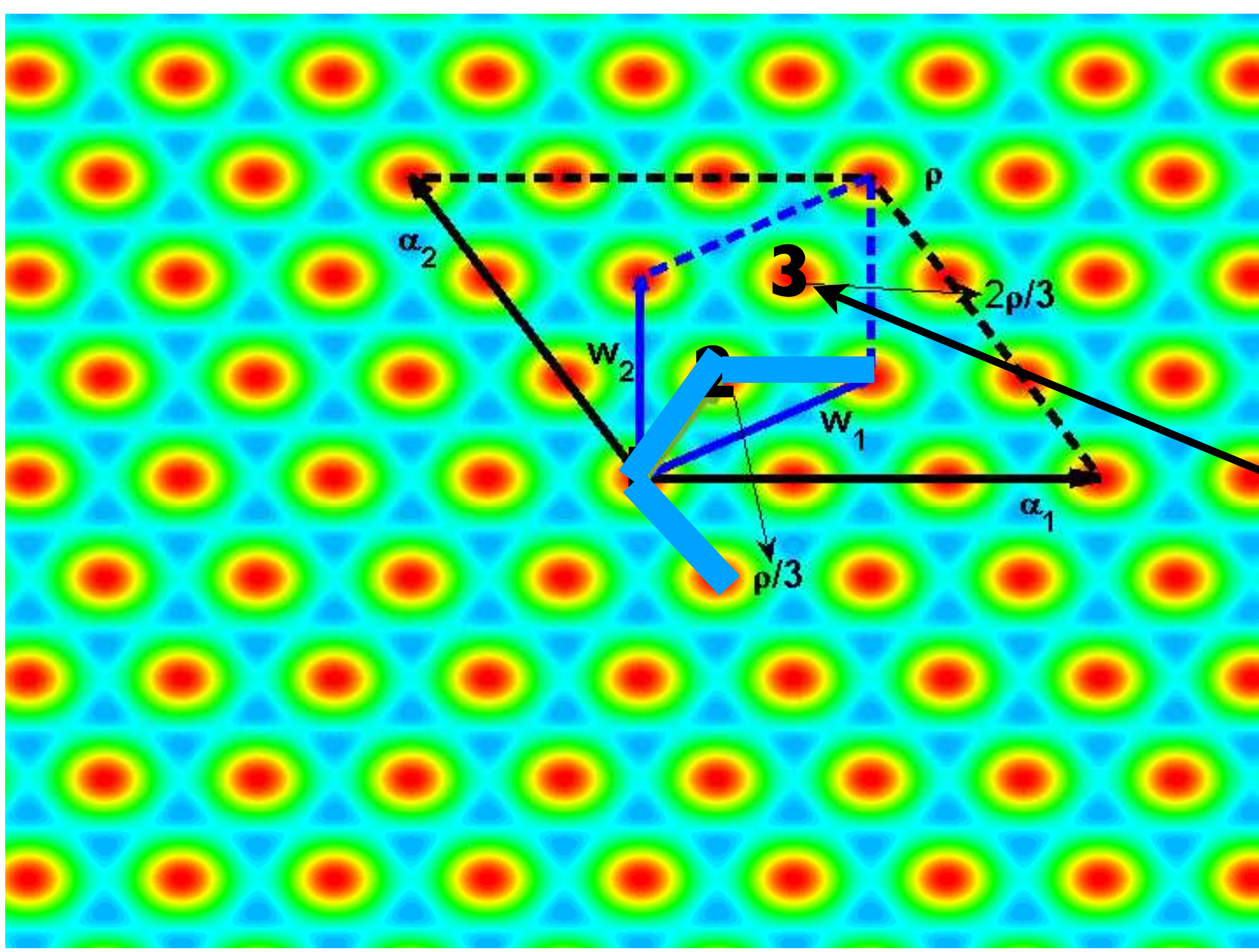

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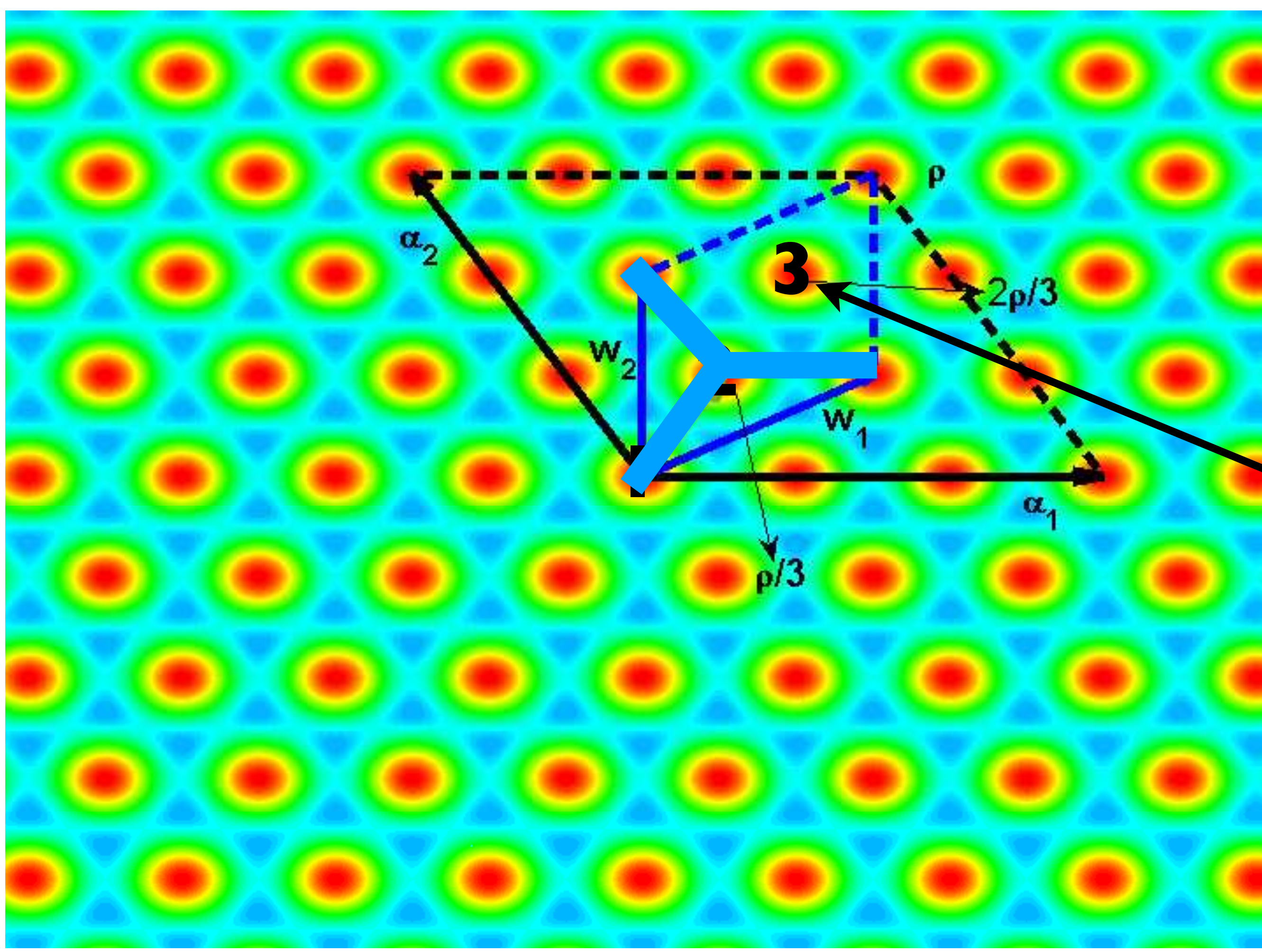
What are the electric fluxes on the lowest tension (BPS) k-walls?

BPS k=1 walls for SU(N) carry fluxes

$$\frac{2\pi}{N}\vec{\rho} - 2\pi\vec{w}_k, k = 1,..,N; \vec{w}_N = 0$$

these are **all** BPS 1-walls
1909.10979
(use Hori, Iqbal, Vafa 2000's)



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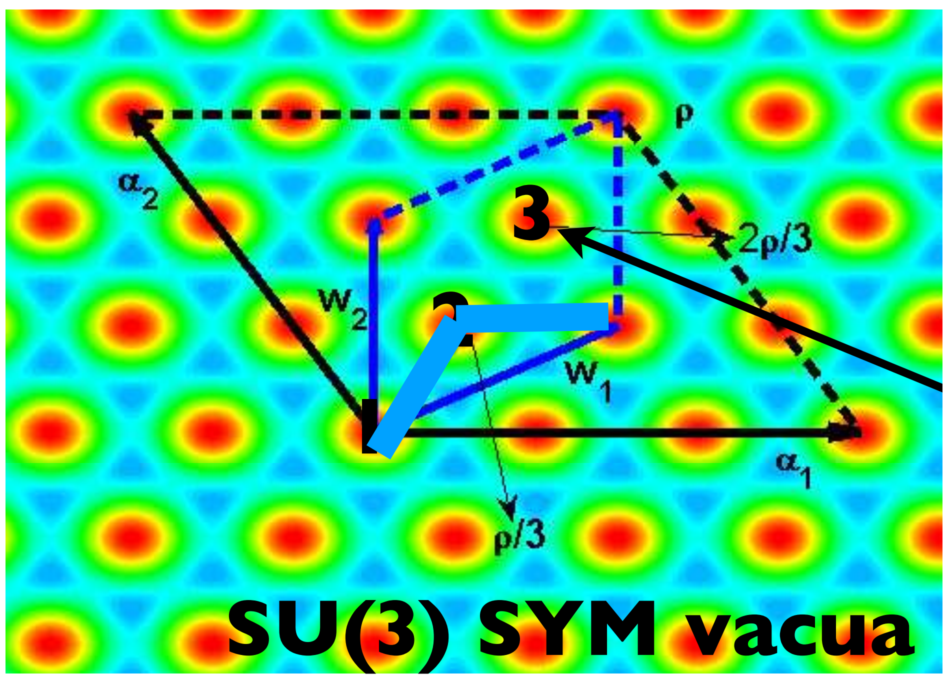

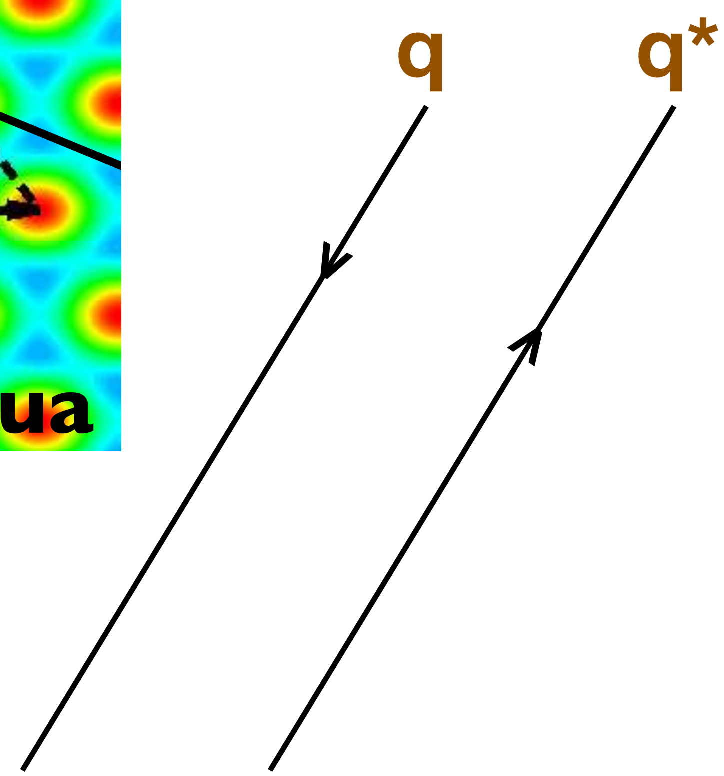

1501.06773

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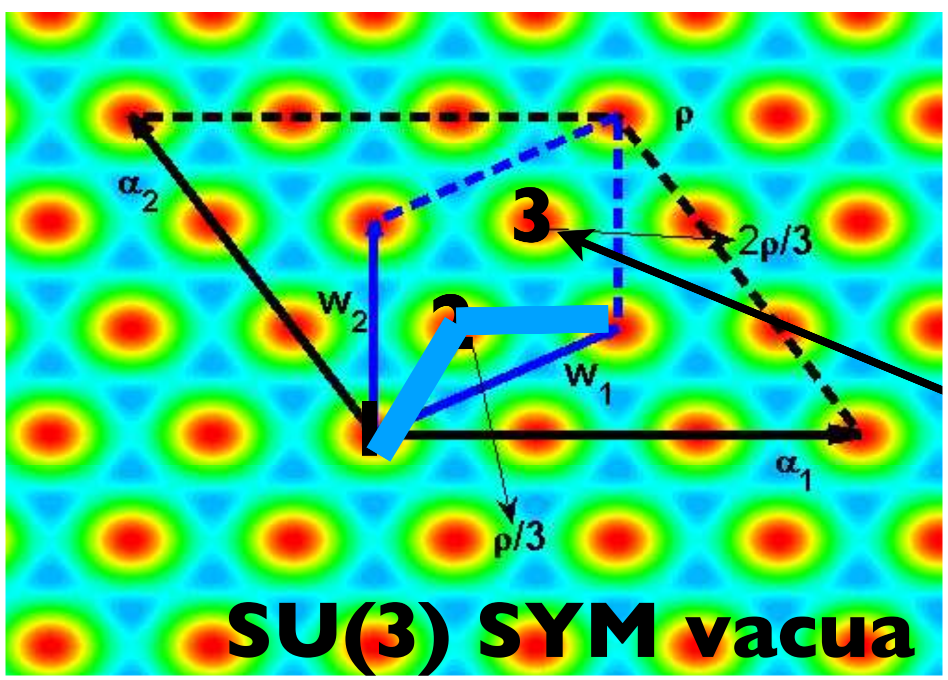

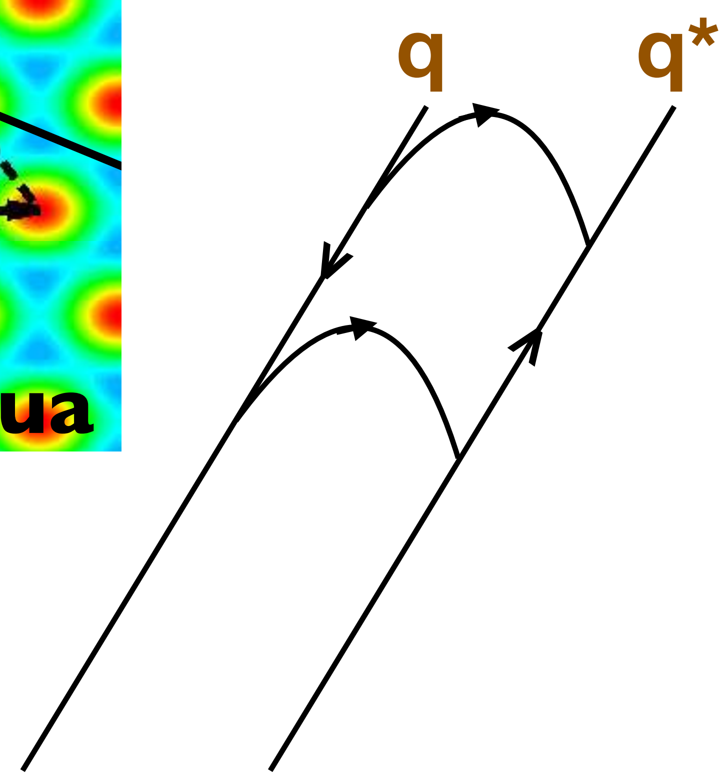

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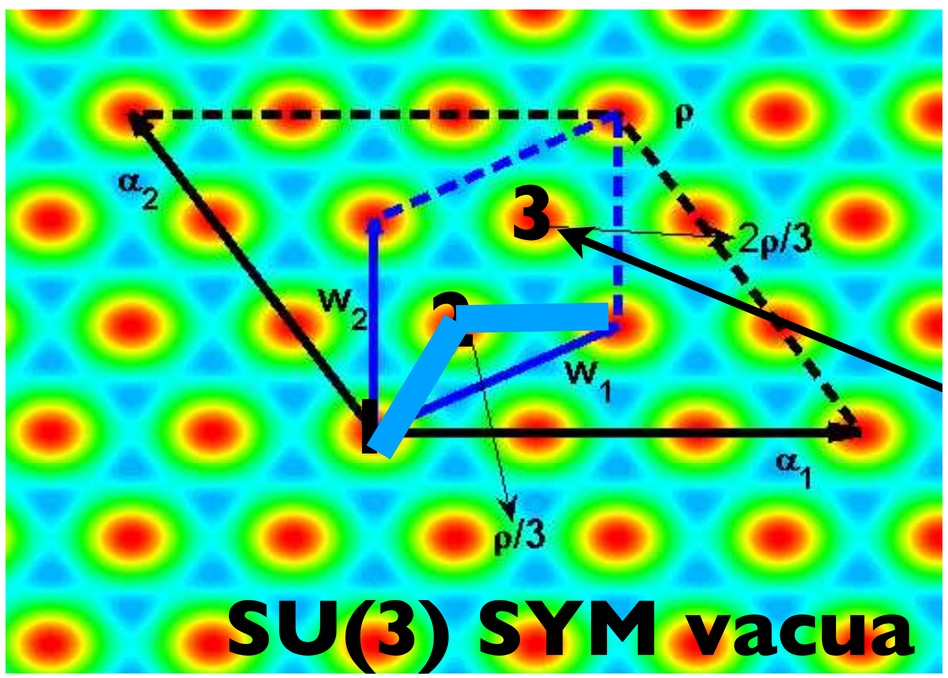

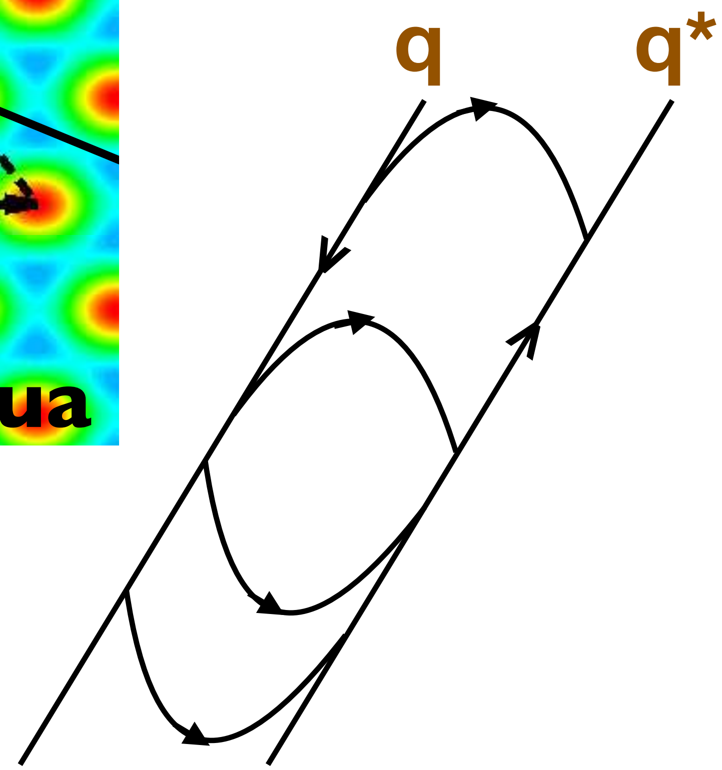

1501.06773

two-sheets of flux

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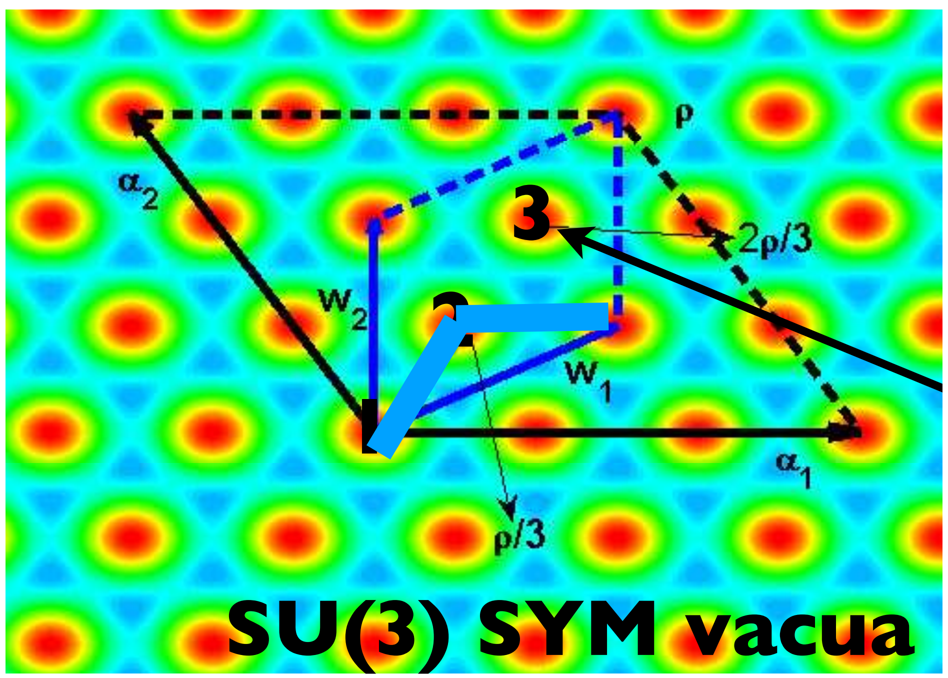
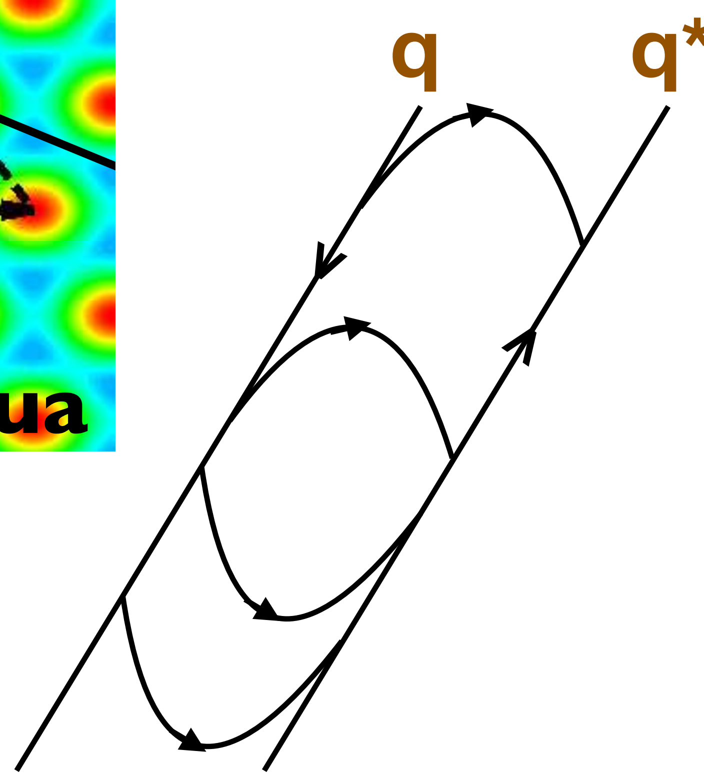
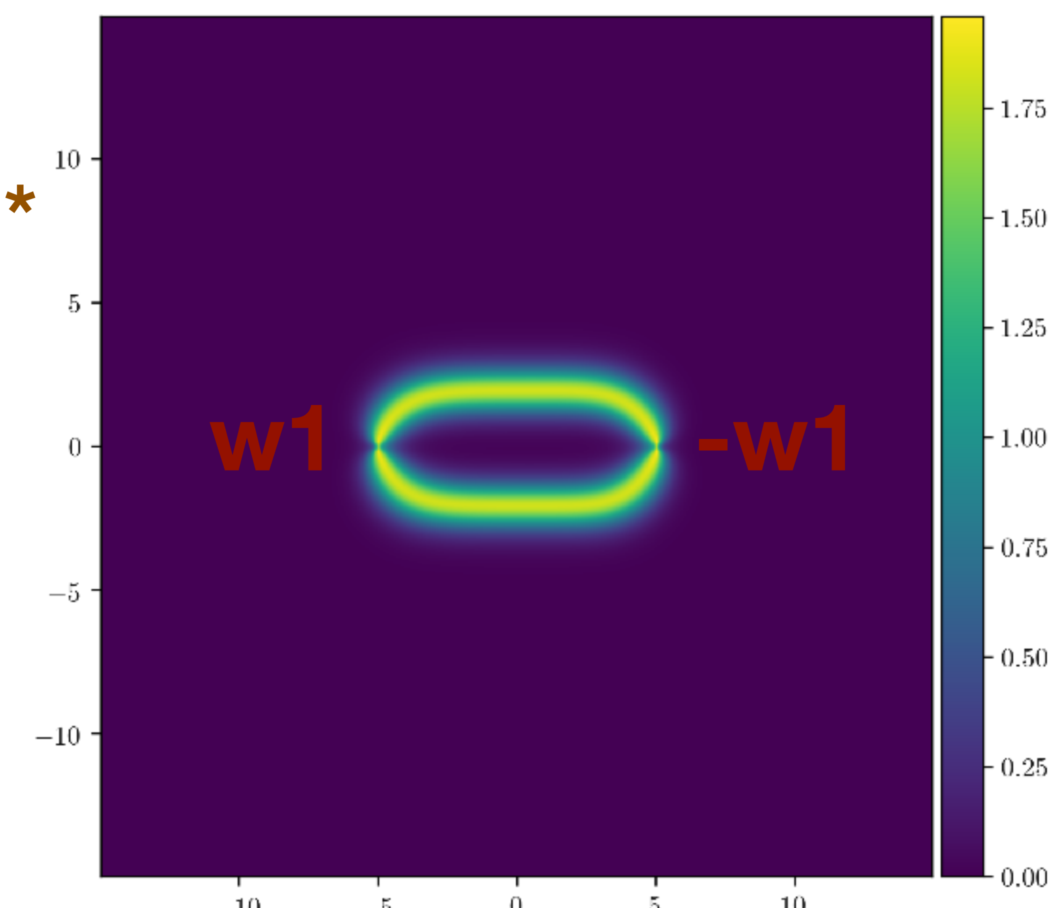
"double confining string" static configuration

vacuum 1 outside







1501.06773

two-sheets of flux

vacuum 2 inside

1909.10979

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- confinement and DWs

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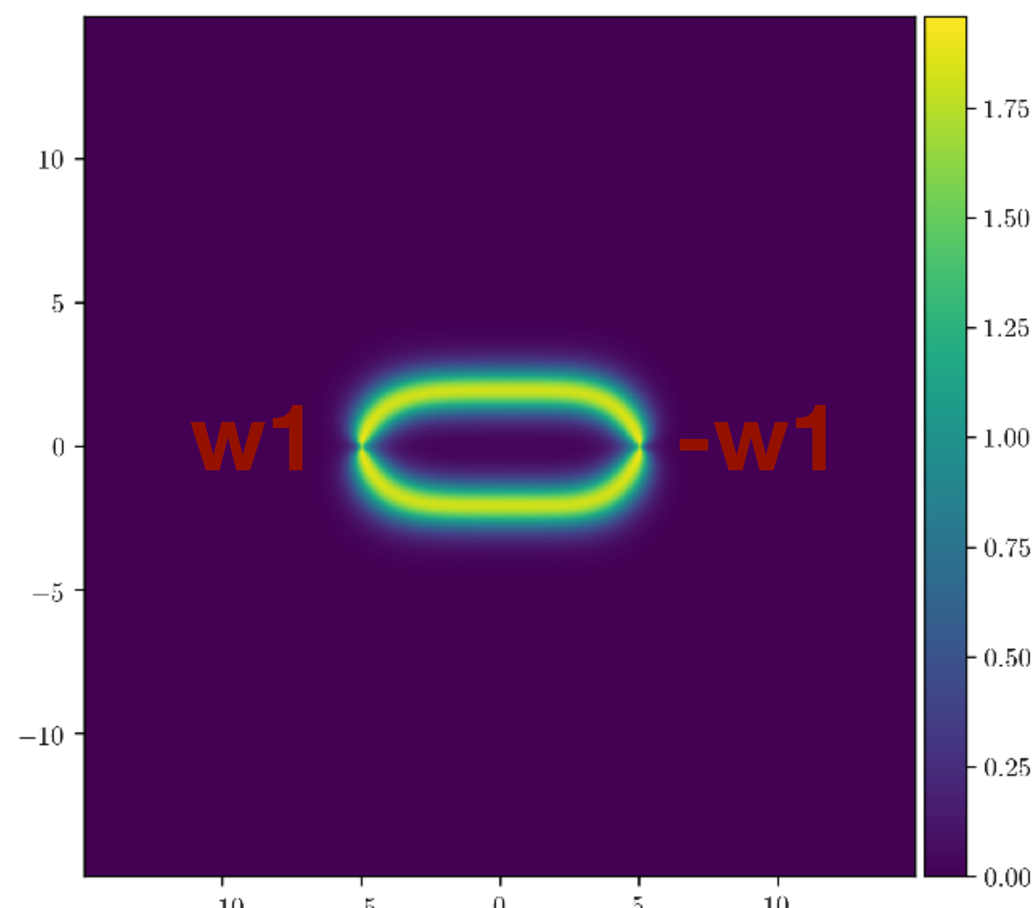
$$\frac{2\pi}{N}\overrightarrow{\rho} - 2\pi\overrightarrow{w}_k, k = 1,..,N; \overrightarrow{w}_N = 0$$

all N-ality 1 weights are confined with the same tension, due to unbroken zero-form center

"double confining string" static configuration

vacuum 1 outside



vacuum 2 inside

1909.10979

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- confinement and DWs

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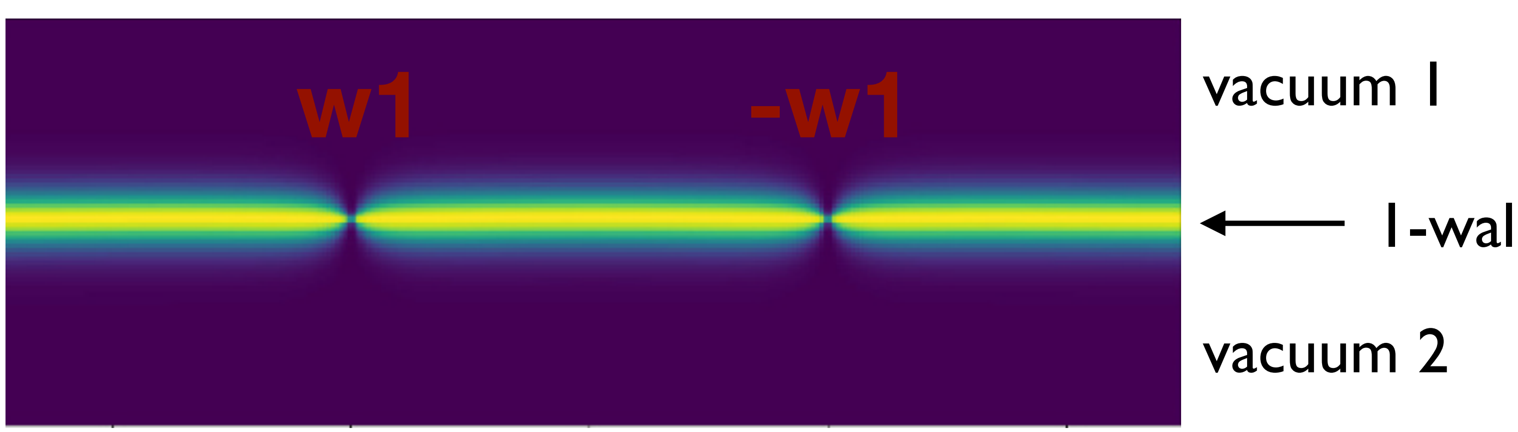
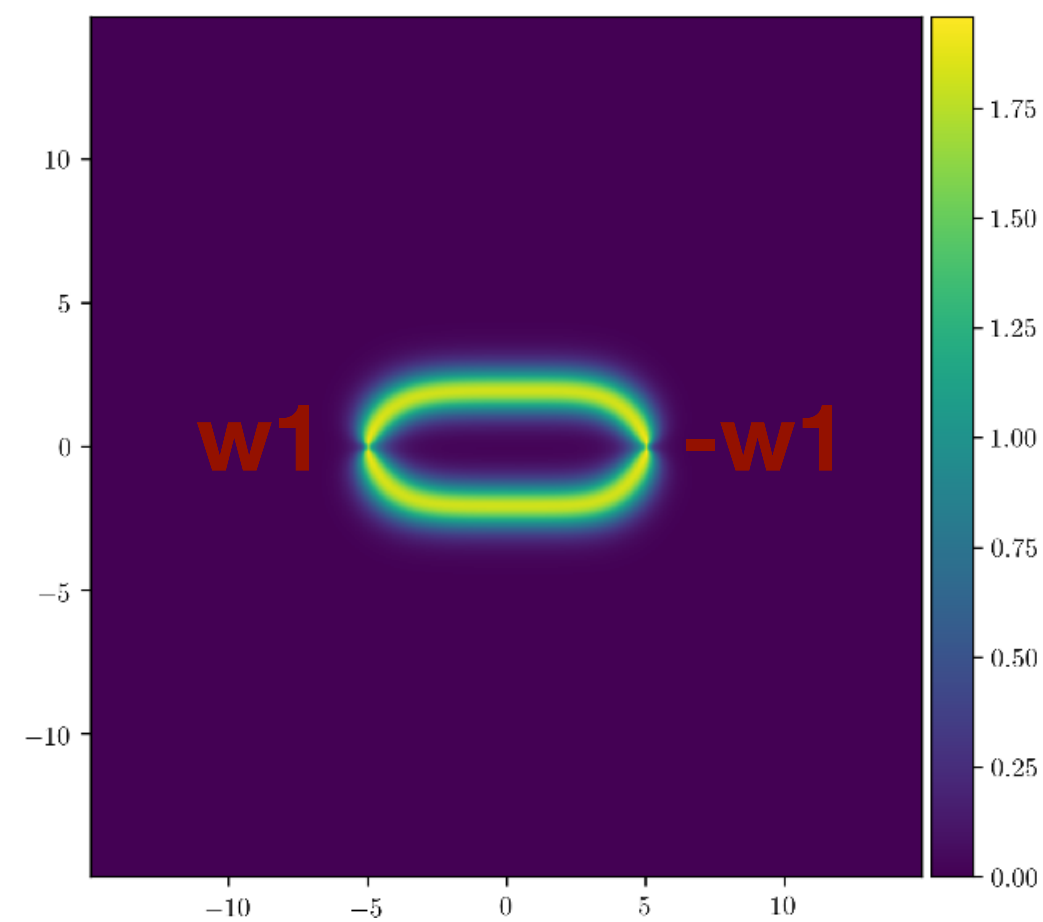
vacuum 1 outside





deconfinement on 1-wall: open up

equality of BPS tensions on L and R: no energy cost to separating

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- confinement and DWs

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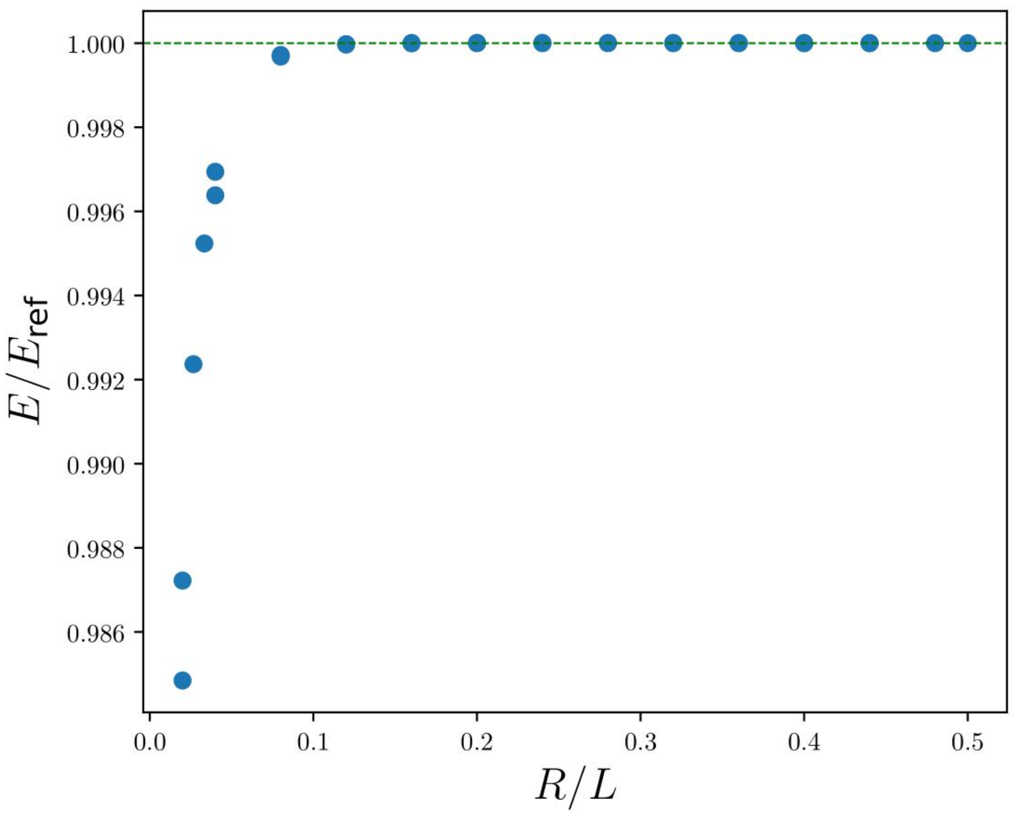
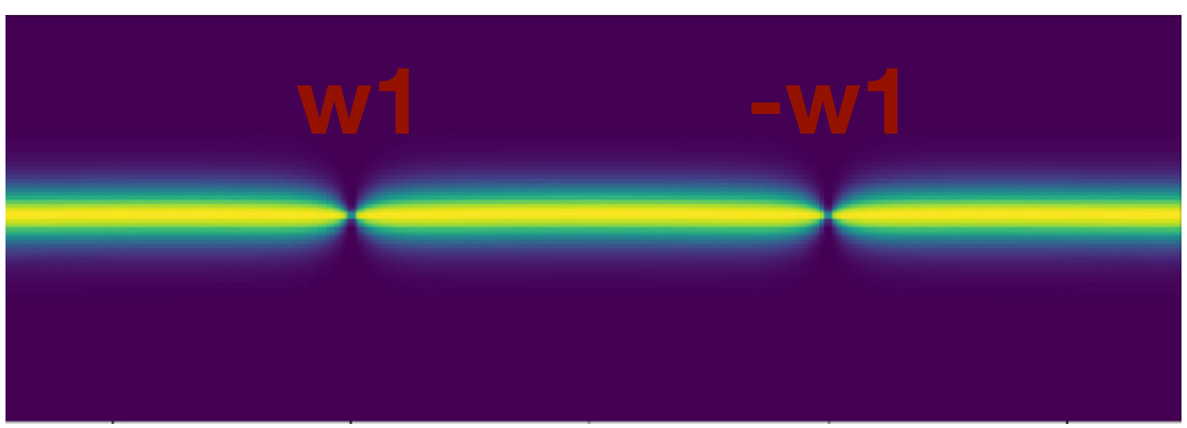




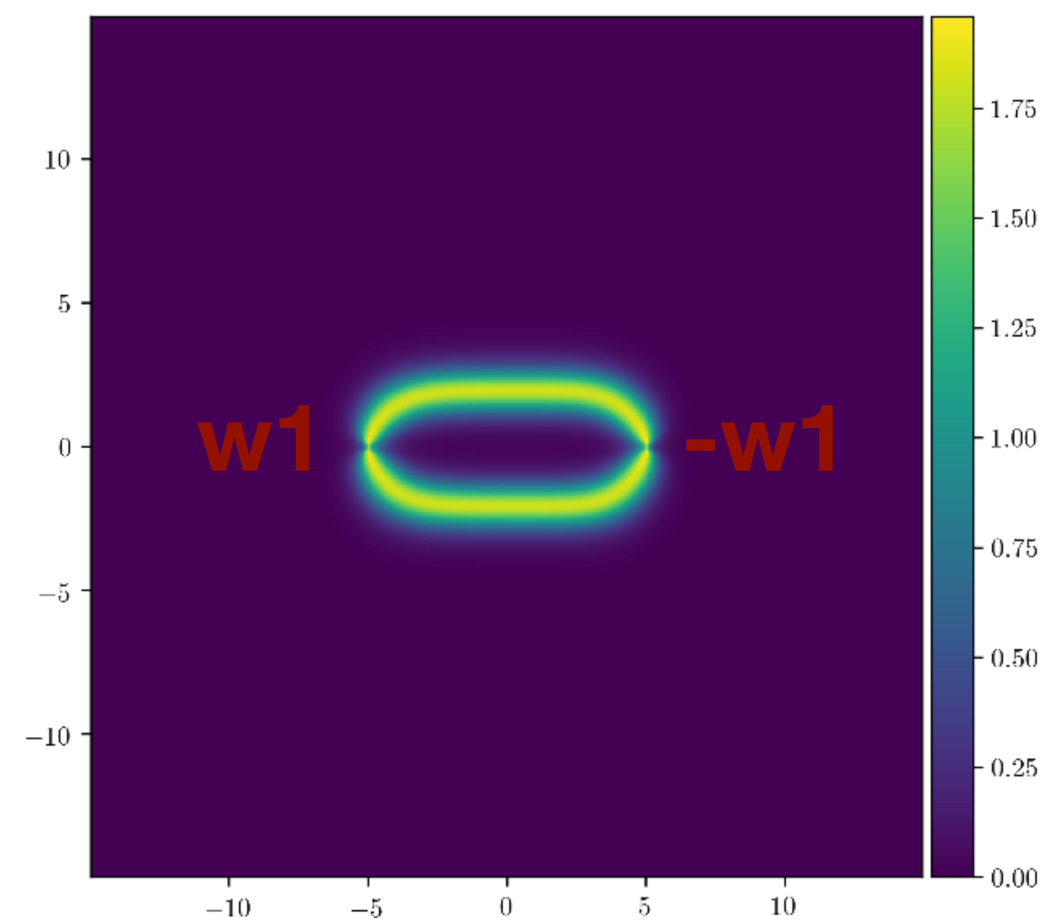
vacuum 1

1-wall

vacuum 2

"double confining string" static configuration

vacuum 1 outside



deconfinement on 1-wall: open up

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SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- confinement and DWs

promised? **deconfinement on DW**— yes, on k=1-walls
+ DW CS…— yes, on k=1-walls

BPS k=1 walls for SU(N) carry fluxes

$$\frac{2\pi}{N}\overrightarrow{\rho} - 2\pi\overrightarrow{w}_k, k = 1,..,N; \overrightarrow{w}_N = 0$$

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$$\frac{2\pi}{N}\vec{\rho} - 2\pi\vec{w}_k, k = 1,..,N; \vec{w}_N = 0$$

there are N degenerate 1-walls, related by $\mathbb{Z}_N^{(0),c}$ (unbroken in bulk)

interpret each 1-wall as a state in a worldvolume TQFT (oblivious to electric fluxes) - a 2d $\mathbb{Z}_N$ TQFT:

$$S_{k=1DW} = \frac{N}{2\pi}\int_{DW} \phi^{(0)} da^{(1)}$$

$$\mathbb{Z}_N^{(0),c} : \phi^{(0)} \to \phi^{(0)} + \frac{2\pi}{N}$$

$$\mathbb{Z}_N^{(1),c} : a^{(1)} \to a^{(1)} + \frac{1}{N}\epsilon^{(1)}, \oint \epsilon^{(1)} = 2\pi\mathbb{Z}$$

- upon gauging of 0- and 1-form centers, reproduces anomaly

(as in Anber, EP 1811.10642)

- dimensional reduction of 3d $U(1)_N$ CS

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- confinement and DWs

What are the electric fluxes on the lowest tension (BPS)k-walls?

these are ALL BPS k-walls; arguments *(initially, numerics!)* in 1909.10979 w/ Cox,Wong

the BPS k-walls' electric fluxes are:

$$2\pi\left(\boldsymbol{w}_{i_1}+\ldots+\boldsymbol{w}_{i_k}-\frac{k}{N}\boldsymbol{\rho}\right), \qquad \text{there are} \begin{pmatrix} N-1 \\ k \end{pmatrix} \text{such walls,}$$

$$2\pi\left(\boldsymbol{w}_{j_1}+\ldots+\boldsymbol{w}_{j_{k-1}}-\frac{k}{N}\boldsymbol{\rho}\right), \qquad \text{there are} \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} \text{such walls.}$$

$(i_1, \ldots, i_k)$ **and** $(j_1, \ldots, j_{k-1})$ **to be all taken different from 1…N-1**

$$\begin{pmatrix} N-1 \\ k-1 \end{pmatrix} + \begin{pmatrix} N-1 \\ k \end{pmatrix} = \begin{pmatrix} N \\ k \end{pmatrix}$$ **distinct BPS k-walls**

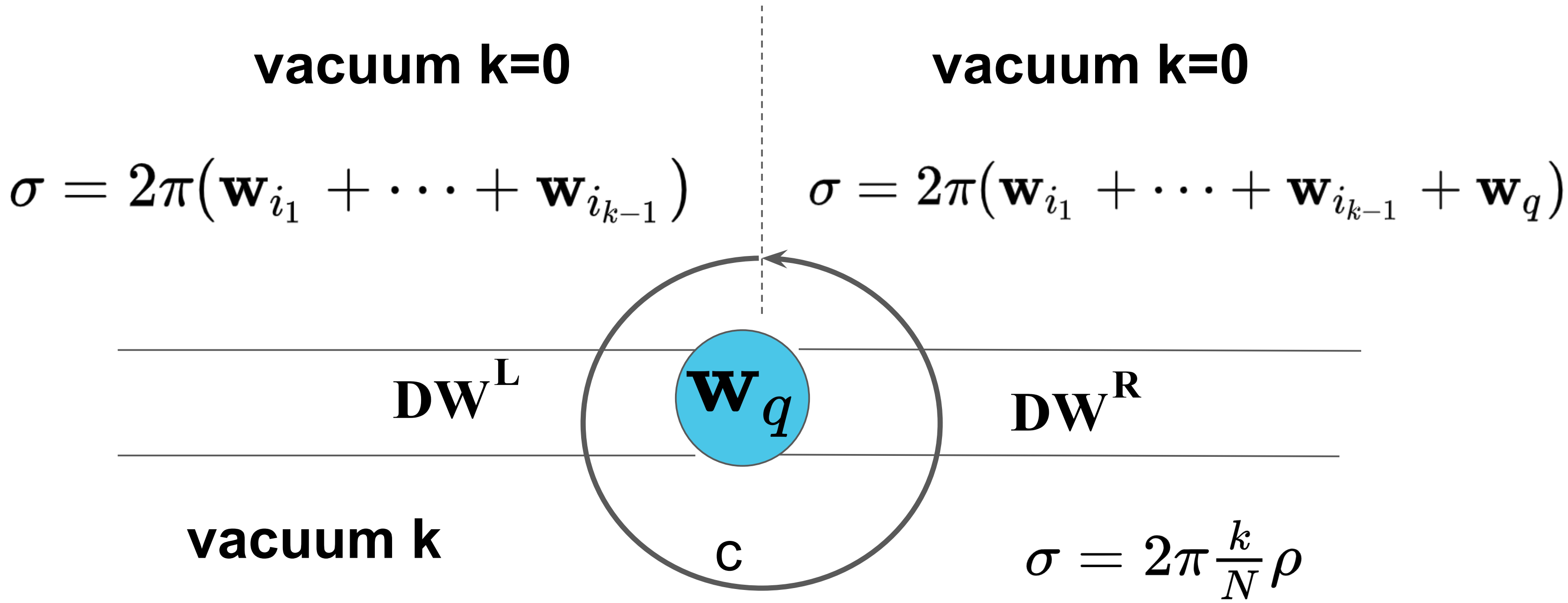
old story: **number of BPS k-walls in LG models =** $\dfrac{N!}{k!(N-k)!}$

Ceccoti-Vafa; Acharya-Vafa; Hori-Iqbal-Vafa…1990s-2000s

new story: **the electric fluxes DWs carry & relation to confinement in the bulk and deconfinement on the wall…**

SYM: compactification on $\mathbb{R}^3 \times \mathbb{S}^1$- confinement and DWs

new story: **the electric fluxes BPS DWs carry & relation to confinement in the bulk and deconfinement on the wall…**



$$\oint_c d\sigma = 2\pi\lambda \rightarrow \lambda = \sum_{a=1}^{N-1} (n_a^R - n_a^L)\mathbf{w}_a$$

any representation of N-ality *q=1,…,N-1* has $\vec{w}_q$ as a weight

there exist BPS *k*-walls of fluxes appropriate to absorb charge $\vec{w}_q$

-> perimeter law on k-walls for any representation quarks

(deconfined weight due to BPS walls “wins”)

Conclusion I:

Anomalies, vacuum structure, confinement and deconfinement on DWs are intertwined, in intricate ways.

Studied a weakly-coupled semiclassically tractable example of the implications of anomaly inflow for the 0-form/1-form anomalies.

Physical picture appealing, comforting, based on our detailed understanding of the "double-string" confinement mechanism on $R^3 \times S^1$.

Applies also to various non-SUSY YM ($\theta = \pi$), QCD(adj),...
No time to go into detail, but whenever there is an anomaly, confinement due to a double-strings (DWs of same tension, as in SYM).

(e.g. axion domain walls w/ Anber 2001.03631)

Conclusion II (wish list):

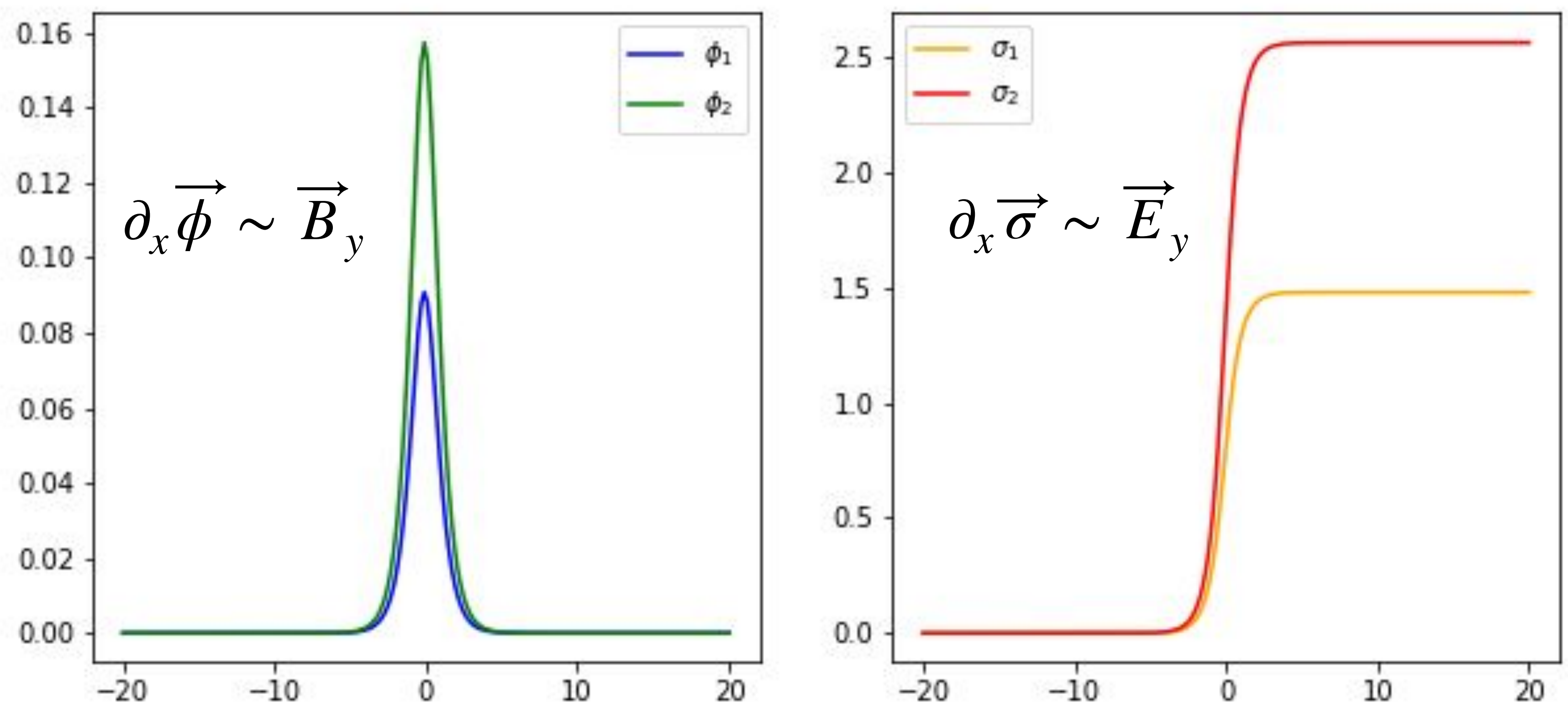
1. Symmetry/anomaly often not enough to fix the DW "worldvolume TQFT". In the case at hand (we) only understand the TQFT on the k=1 walls. For k>1 DWs on $R^3 \times S^1$ open… related to combinatorics of fluxes?

2. All other gauge groups - with or without center - also tractable at small-L. Repeat… worldvolume TQFTs?

Conclusion III (wish list):

3. Solutions reveal that DWs also carry magnetic fields - no net magnetic flux; due to nonlinear coupling of “ $\overrightarrow{E}$, $\overrightarrow{B}$” due to magnetic/neutral bions

After running algorithm: *SU(3)* example, *k=1*



distance across DW in units of m^{-1}

numerically, then analytically found:

“magnetless” solutions only for k=N/2 walls in SU(N-even)

Conclusion IV (wish list)**:**

3. Solutions reveal that DWs also carry magnetic fields -
no net magnetic flux; due to nonlinear coupling of " $\overrightarrow{E}, \overrightarrow{B}$ " due to magnetic/neutral bions

showed "magnetless" solutions only for k=N/2 walls in SU(N-even)

(π rotation: $k \leftrightarrow N - k$ walls, reversal of worldvolume flux)

out of the $\begin{pmatrix} N \\ N/2 \end{pmatrix}$ BPS $k = N/2$-walls

only 2 magnetless walls if N not divisible by 4
only 6 magnetless if N divisible by 4

(further, all can be constructed from analytic SU(2)-wall solution!)

Conclusion IV (wish list):

3. Solutions reveal that DWs also carry magnetic fields -
no net magnetic flux; due to nonlinear coupling of " $\vec{E}, \vec{B}$ " due
to magnetic/neutral bions

showed "magnetless" solutions only for k=N/2 walls in SU(N-even)

(π rotation: $k \leftrightarrow N-k$ walls, reversal of worldvolume flux)

out of the $\binom{N}{N/2}$ BPS $k = N/2$-walls

only 2 magnetless walls if N not divisible by 4
only 6 magnetless if N divisible by 4

(further, all can be constructed from
analytic SU(2)-wall solution!)

begs for
a symmetry
explanation?

J. Wang, Y.-Z. You and Y. Zheng, 1910.14664

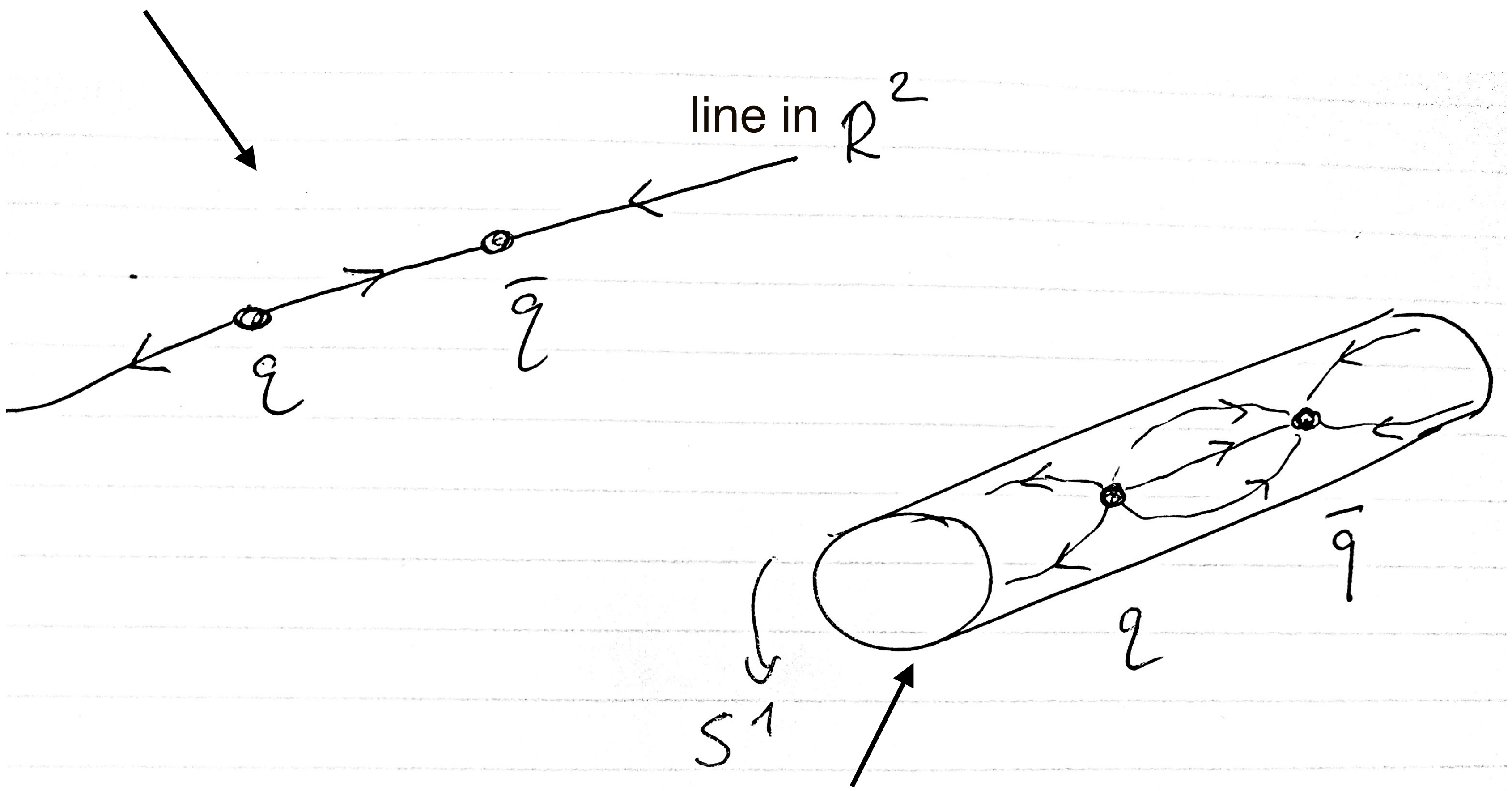
I. Hason, Z. Komargodski and R. Thorngren 1910.14039

Conclusion V (wish list):

4. *An excursion to* $\mathbb{R}^4$?

CS and other arguments (eg Hsin, Lam, Seiberg 2018) imply "anyonic" nature of deconfined quarks on DWs (braiding). In our 2d DW worldvolume setup braiding not visible, as quarks have to pass through each other.

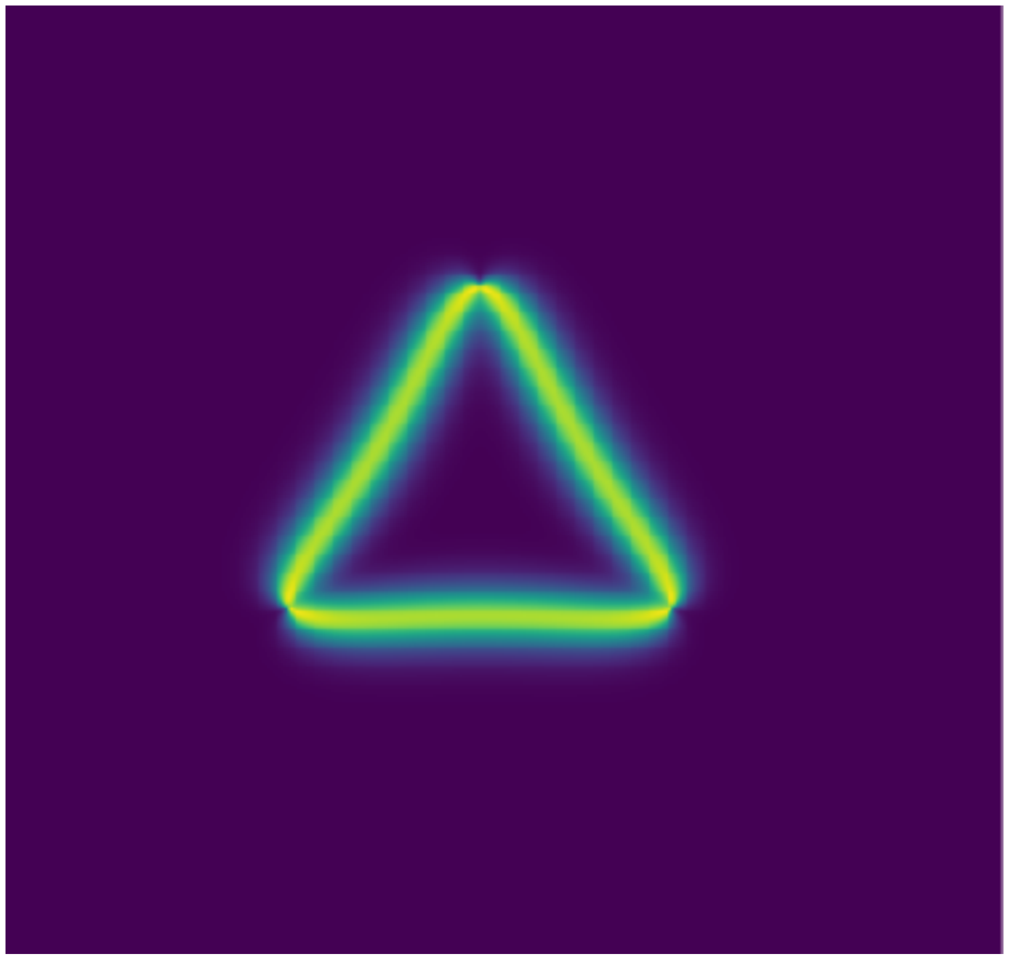
our discussion, ignoring x_4, $\mathbb{S}^1$-coordinate dependence



in reality, our DWs are wrapped and our q's localized on $\mathbb{S}^1$

our $\mathbb{S}^1$ is small but finite, and theory weakly coupled at all scales: hope?

... describe without 3d duality!



heavy “baryon” in SU(3) SYM

“Color field,” Mark Rothko