Higher symmetry ’t Hooft anomalies and domain walls

Erich Poppitz U of Toronto

with **Mohamed Anber** **Lewis & Clark College, Portland, OR**

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+ **many earlier related works, by us and by others**

+ may revisit past work

+ some current thoughts

1501.06773

M. Anber, Tin Sulejmanpasic, EP

Motivation: nonperturbative gauge dynamics

’t Hooft anomalies constrain possible IR behavior

Turns out some were missed in the 1980s:
the ones involving higher symmetries

- Gaiotto, Kapustin, Komargodski, Seiberg, Willett 2014 -

Old phenomena can be seen as due to these new matching conditions, e.g. “Dashen phenomenon”, while new anomaly point of view allows interesting extensions in different directions!

Outline:

1. probably most of the talk:

discrete higher form symmetries and ’t Hooft anomalies:
2d Schwinger model/4d SYM and QCD(adj)

- **simplest QFT (solvable) exhibiting them, many parallels with 4d SYM**
- **will present in detail different points of view on the anomaly**

2. time permitting:

anomaly inflow, domain walls in 4d, and recovering some string theory results from QFT

1.) new "'t Hooft anomaly" between discrete chiral symmetry and center symmetry:

the simplest example in QFT is the charge-q massless 2d Schwinger model

$$L = -\frac{1}{4e^2} f_{kl} f^{kl} + i\bar{\psi}_+ (\partial_- + iqA_-)\psi_+ + i\bar{\psi}_-(\partial_+ + iqA_+)\psi_- \qquad \partial_\pm \equiv \partial_t \pm \partial_x,\ A_\pm \equiv A_t \pm A_x,$$

"0-form" symmetries:

topological charge: $T = \frac{1}{2\pi}\int f_{12} d^2x \in \mathbb{Z}$

$U(1)_V$: $\psi_\pm \to e^{iq\alpha}\psi_\pm$ gauged

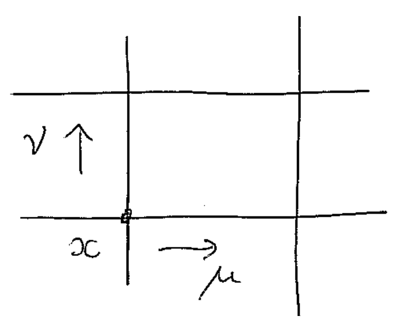
$U(1)_A$: $\psi_\pm \to e^{\pm i\chi}\psi_\pm$ anomalous: $Z_{\text{ferm.}} \to e^{i2q\chi T} Z_{\text{ferm.}}$

($\chi = \frac{2\pi}{2q}$ gives anomaly-free subgroup)

$\mathbb{Z}_{2q}^{d\chi}$: $\psi_\pm \to e^{\pm i\frac{\pi}{q}}\psi_\pm$ [similar to 4d SYM, anomaly free discrete chiral only]

"1-form" symmetry:

$\mathbb{Z}_q^C$ center symmetry



$U_{x,\mu} \to z_\mu U_{x,\mu}$

a Z_q phase, one per spacetime direction (global symmetry)

easy to see on lattice: plaquette term in action invariant, fermion hopping as well, since integer charge q>1

"1-form" symmetry only acts on line operators (hence name):

$$\mathbb{Z}_q^C : \ e^{i\oint A_x dx} \to \omega_q \, e^{i\oint A_x dx}, \quad \omega_q \equiv e^{i\frac{2\pi}{q}}.$$

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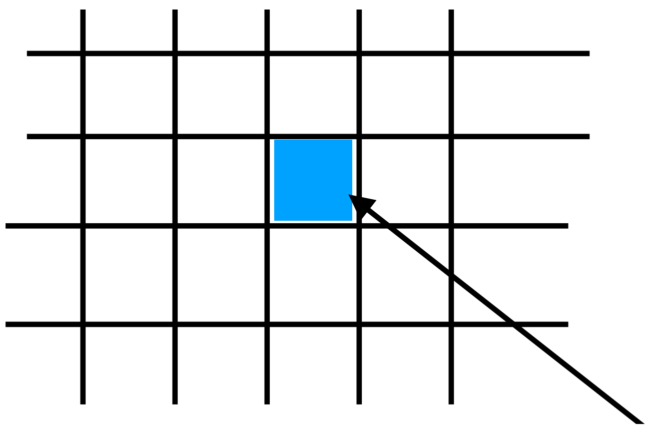
"0-form" $\mathbb{Z}_{2q}^{d\chi}$ & "1-form" $\mathbb{Z}_q^C$ have a mixed anomaly!

also easy**(er!)** to see on the lattice! Let us gauge the 1-form center symmetry:



for symmetries acting on links (1-form center symmetry), introduce plaquette-based ("2-form") Z_q gauge field to make Z_q center symmetry local

to see the anomaly a background 2-form field suffices;
in 2d, there is no field strength of the 2-form (no cubes!);
introduce a Z_q background phase on a single plaquette =
"Z_q center vortex" [i.e. any 2-form $Z_q$background topological…
can move around shaded square by changing link variables]

now recall:

$$U_{plaquette} = \prod_{link\, \in plaquette} U_{link} = e^{ia^2 F_{plaquette}} \longrightarrow \prod_{all\ plaquettes} U_{plaquette} = e^{i flux\ thru\ T^2} = 1$$

$\longrightarrow$ $flux\ thru\ T^2 = 2\pi n,\ n \in Z$ = integer T (top. charge in continuum limit) from before!

But in the background of a single Z_q center vortex, we have instead

$$\prod_{all\ plaquettes} U_{plaquette} e^{i\frac{2\pi}{q}} = e^{i flux\ thru\ T^2} = e^{i\frac{2\pi}{q}}$$ = fractional T (top. charge)!

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So, after we introduce a center 2-form background (center vortex), we have

$$\prod_{all\ plaquettes} U_{plaquette} e^{i\frac{2\pi}{q}} = e^{i flux\ thru\ T^2} = e^{i\frac{2\pi}{q}}$$ = fractional T (top. charge)!

But recall that under a 1-form discrete chiral $\mathbb{Z}_{2q}^{d\chi}$ we have that

$Z_{ferm.} \to e^{i2\pi T} Z_{ferm.}$ and $Z_{ferm.}$ is invariant if T is integer, but not otherwise.

We conclude that if center 1-form symmetry is gauged, the discrete chiral symmetry ceases to be a symmetry, in other words, we have a $\mathbb{Z}_{2q}^{d\chi}$-$\mathbb{Z}_q^C$ 't Hooft anomaly!

$$\mathbb{Z}_{2q}^{d\chi} : \quad Z_{ferm.} \to e^{i\frac{2\pi}{q}} Z_{ferm.}$$

- phase in the chiral transform (in the center vortex bckgd) **IS** mixed 't Hooft anomaly
- phase independent on T^2 volume, RG invariant, same on all scales: UV & IR
- like for continuous symmetry 't Hooft anomalies must be matched by IR theory:
  - IR CFT, or
  - one or both symmetries should be broken ("Goldstone" mode), and/or
  - IR TQFT

Gaiotto et al, '14-'17

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The 0-form/1-form mixed anomaly was computed in 4d SYM by Gaiotto et al by turning on discrete gauge backgrounds as I showed above. A 't Hooft anomaly, however, should be a property of the theory without any backgrounds; it does not require turning on fields. Continuous symmetry 't Hooft anomalies are seen in <j j j > three-point global symmetry current correlators, as $1/q^2$ poles
[Frishman et al, Coleman et al, 1980s].

Expect the "same" should be true here. The anomalies should involve properties of the quantum operators representing the discrete symmetries. General statements are so far not known (to me) but examples exist: QM & 3d CS theory [Gaiotto et al] and 2d QFT [our work]

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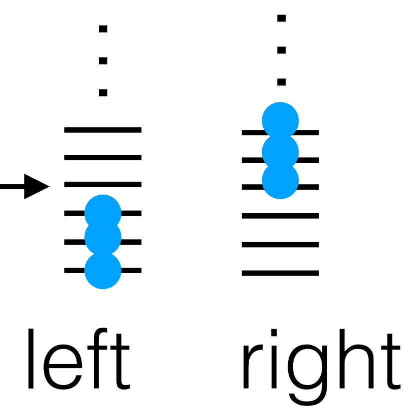
quantize using Hamiltonian $A_0=0$ on circle Manton '86; Iso,Murayama '89

$\oint A_x dx \equiv a$ - only dynamical variable from gauge sector

$Z_q^C : a \to a + \frac{2\pi}{q}$ $\quad (Z_q^C)^q$ = G large gauge trf.

Dirac sea states $|n\rangle \longrightarrow$



$$Q_5 |n\rangle = |n\rangle \left(2n - \frac{qa}{\pi} \right)$$

(in 1spatial dim, unlike in 4d, can solve Dirac equation for any gauge background and explicitly build "Dirac sea" states obeying Gauss law; their chiral charge depends on "a": anomalous)

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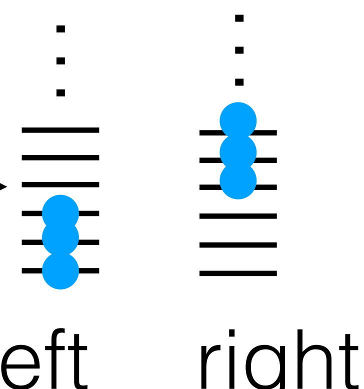

$$Q_5 |n\rangle = |n\rangle \left(2n - \frac{qa}{\pi}\right)$$

$$G|n\rangle = |n + q\rangle$$

anomaly free chiral?

$$\tilde{Q}_5 \equiv Q_5 + \frac{qa}{\pi}$$ not G invariant $\quad G : \tilde{Q}_5 \to \tilde{Q}_5 + 2q$

(like 4d, where we can add "CS" current tr(AdA+…) to make a conserved but not gauge invariant chiral charge!)

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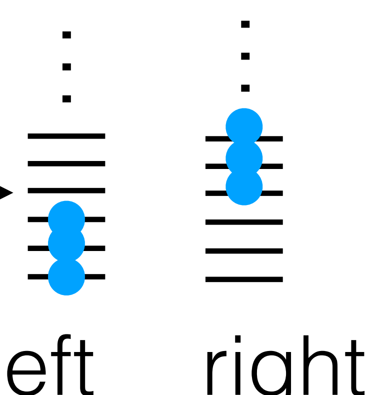

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$X_{2q} \equiv e^{i\frac{2\pi}{2q}\tilde{Q}_5}$ G invariant, generates chiral $Z_{2q}^{(0)}$

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$$X_{2q}|n\rangle = |n\rangle\, \omega_q^n \qquad (\omega_q \equiv e^{i\frac{2\pi}{q}})$$

What generates 1-form center?

$$Y_q : \quad e^{ia} \to e^{-i\frac{2\pi}{q}} e^{ia}$$ (the 2d 't Hooft "loop")

hence $Y_q = e^{i\frac{2\pi}{q}\hat{\Pi}_a}$

not commuting with $X_{2q} \equiv e^{i\frac{2\pi}{2q}\tilde{Q}_5}$

due to "CS term" in $\tilde{Q}_5 \equiv Q_5 + \frac{qa}{\pi}$

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Y_q action on Dirac sea states $\qquad (Y_q)^q = G$

$$Y_q|n\rangle = |n+1\rangle \qquad\qquad G|n\rangle = |n+q\rangle$$

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q "theta vacua"
instead of one!

$$|\theta, k\rangle \equiv \sum_{n\in\mathbb{Z}} e^{i(k+qn)\theta}|k+qn\rangle,\; k = 0, 1, \ldots, q-1$$

and their Z_q Fourier

$$|P, \theta\rangle \equiv \frac{1}{\sqrt{q}}\sum_{k=0}^{q-1}\omega_q^{kP}|\theta, k\rangle,\; P = 0, \ldots, q-1,$$

(need for clustering)

I.) new "'t Hooft anomaly" between discrete chiral symmetry and center symmetry:

the simplest example in QFT is the charge-q massless 2d Schwinger model

$$\langle P', \theta | \bar{\psi}_+(x) \psi_-(x) | P, \theta \rangle = e^{-i\theta} \omega_q^{-P} \delta_{P,P'} C$$

symmetries action on clustering vacua

$$X_{2q} \, |P, \theta\rangle = |P + 1 (\text{mod } q), \theta\rangle$$ **- discrete chiral broken**

$$Y_q \, |P, \theta\rangle = |P, \theta\rangle \, \omega_q^{-P} e^{-i\theta}$$ **- discrete E-field in each vacuum**

$$X_{2q} \, Y_q = \omega_q \, Y_q \, X_{2q} \quad (\omega_q = e^{i\frac{2\pi}{q}})$$

"central extension" of symmetry algebra, a manifestation of mixed discrete 't Hooft anomaly

(we explicitly constructed the q ground states; centrally extended algebra has no 1dim reps, it alone implies vacuum degeneracy)

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$$X_{2q}\, Y_q = \omega_q\, Y_q\, X_{2q} \quad (\omega_q = e^{i\frac{2\pi}{q}})$$

"central extension" manifestation of mixed discrete 't Hooft anomaly

now recall earlier discussion involving backgrounds and phases in the transforms of partition function:

- phase in the chiral transform (in the center vortex bckgd) **IS** mixed 't Hooft anomaly
- phase independent on volume, RG invariant, same on all scales: UV & IR
- like for continuous symmetry 't Hooft anomalies must be matched by IR theory:
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How does the discrete chiral breaking we found saturate anomaly?

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"central extension" manifestation of mixed discrete 't Hooft anomaly

Claim: *it does and can be seen explicitly as follows*

(relabel q->N) **consider following TQFT:**

$$S_{2-D} = i\,\frac{N}{2\pi}\int_{M_2} \varphi^{(0)} da^{(1)}$$

$$e^{i\hat{\varphi}} e^{i\hat{a}} = e^{i\frac{2\pi}{N}} e^{i\hat{a}} e^{i\hat{\varphi}}$$

$$a = \oint a^{(1)}$$

- this is the "chiral lagrangian" of the charge q=N SM
- IR theory is empty, "chiral lagrangian"=theory with N dim Hilbert space = TQFT
- *spirit similar to IR TQFT in 4d SYM with SU(N)*
- (canonically quantize…)

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algebras same

$$S_{2-D} = i\, \frac{N}{2\pi} \int_{M_2} \varphi^{(0)} da^{(1)} \qquad e^{i\hat{\varphi}} e^{i\hat{a}} = e^{i\frac{2\pi}{N}} e^{i\hat{a}} e^{i\hat{\varphi}}$$

chiral $\phi^{(0)} \to \phi^{(0)} + \frac{2\pi}{N}$ center $a^{(1)} \to a^{(1)} + \frac{1}{N}\epsilon^{(1)}$

canonically quantize, a0=0 gauge, Wilson line and constant mode of scalar are QM variables, $a = \oint a^{(1)}$, QM with vanishing Hamiltonian, has N-dim Hilbert space representing operator algebra

The solution of the S.M. represents an explicit derivation of IR TQFT from UV.
Can see matching of anomaly explicitly by introducing 2-form background for center.

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algebras same

$$S_{2-D} = i\,\frac{N}{2\pi}\int_{M_2} \varphi^{(0)} da^{(1)} \qquad e^{i\hat{\varphi}}e^{i\hat{a}} \;=\; e^{i\frac{2\pi}{N}}e^{i\hat{a}}e^{i\hat{\varphi}}$$

chiral $\phi^{(0)} \to \phi^{(0)} + \frac{2\pi}{N}$ center $a^{(1)} \to a^{(1)} + \frac{1}{N}\epsilon^{(1)}$

algebra follows

quantize:

$a_0^{(1)} = 0$ gauge, find QM $S_{\mathbb{R}_t\times\mathbb{S}_1} = \frac{N}{2\pi}\int dt\, \varphi\, \frac{da}{dt}$

QM variables $\varphi(t)$ and $a(t) \equiv \oint_{\mathbb{S}_1} a^{(1)}$ $[\hat{\varphi}, \hat{a}] = -i\frac{2\pi}{N}$

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---

gauge center symmetry: $B^{(2)} \to B^{(2)} + d\lambda^{(1)} \qquad a^{(1)} \to a^{(1)} + \lambda^{(1)}$

two form center gauge field $\oint B^{(2)} = \frac{2\pi\mathbb{Z}}{N}$ then, under chiral $\delta\varphi^{(0)} = \frac{2\pi}{N}$

$$S_{2-D} = i\,\frac{2\pi}{N}\int_{M_2} \frac{N\varphi^{(0)}}{2\pi}\,\frac{N(da^{(1)} - B^{(2)})}{2\pi} \qquad \delta_{\mathbb{Z}_N^{d\chi}} S_{2-D} = i\,\frac{2\pi}{N}\int_{M_2} \frac{N(da^{(1)} - B^{(2)})}{2\pi} = -\frac{i2\pi}{N}$$

- explicitly see anomalous transform of Z(IR)

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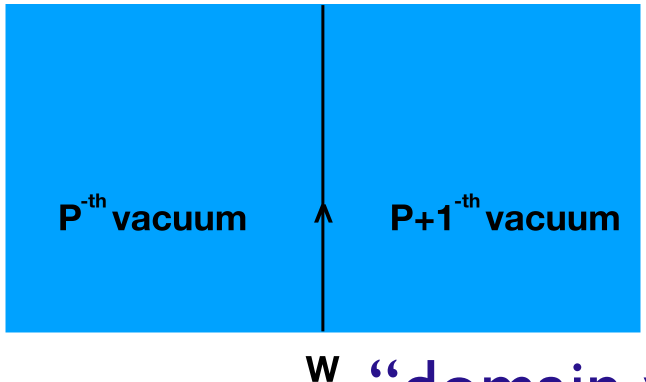
SUMMARY SO FAR lattice-y, Hamiltonian, Euclidean path integral/bosonization (skipped) in

charge-q 2d massless Schwinger model
discrete chiral Z_q
discrete 1-form center Z_q
mixed 't Hooft anomaly RG invariant
matched in IR by a TQFT describing q
vacua of broken discrete chiral

$$S_{2-D} = i\,\frac{N}{2\pi}\int\limits_{M_2} \varphi^{(0)} da^{(1)}$$

(q→ N)

no dynamical DWs (Gaussian, vacua don't "talk")



"domain wall"= charge-1 **external** static charge

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($q \rightarrow N$)

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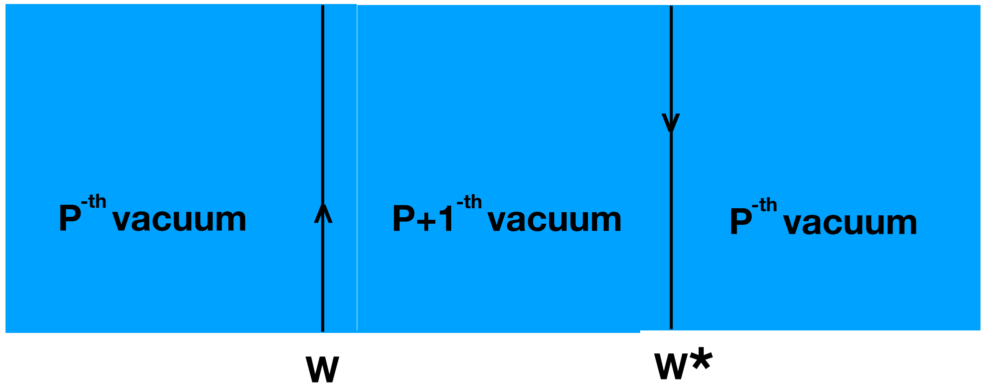

charge-1 probe is deconfined
perimeter law or "broken center"
picture reminiscent of theta=pi 2d QED

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COMPARE WITH 4D SYM

charge-q 2d massless Schwinger model
discrete chiral Z_q
discrete 1-form center Z_q
mixed 't Hooft anomaly RG invariant
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SU(N) 4d SYM
discrete chiral Z_N
discrete 1-form center Z_N
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matched in IR by a TQFT describing N
vacua of broken discrete chiral

$$S_{2-D} = i \frac{N}{2\pi} \int_{M_2} \varphi^{(0)} da^{(1)}$$

(q→N)

→ similar…

$$S_{4d} = i \frac{N}{2\pi} \int_{M_4} \phi^{(0)} da^{(3)}$$

no dynamical DWs (Gaussian…)

charge-1 probes deconfined
(perimeter law)

dynamical DWs exist

fundamental probes confined
(area law)

now,
turn to a study of DWs here…

2.) domain walls in 4d SYM/QCD(adj) & anomaly inflow…
- re-obtaining some stringy results

some further motivation:

- turns out some of the DW worldvolume theories are related to the simplest study case of QFT with mixed 0-form/1-form 't Hooft anomalies - our solvable 2d ex.

- physics on the high-T DW (2d) shares features of the low-T theory, both bulk (4d) and DW (2d/3d)

- high-T DW are a semiclassical counterpart to "center vortices," field configurations thought to be responsible for area law of Wilson loop at low-T in pure YM

(not theoretically controllable; seen in lattice simulations)

[Greensite+…; 'D Elia, de Forcrand;… 1998-]

2.) domain walls in 4d SYM/QCD(adj) & anomaly inflow…
- re-obtaining some stringy results

$SU(2)$ Yang-Mills theory endowed with n_f adjoint Weyl fermions $T \gg \Lambda_{QCD}$

$$S_{3D}^{\rm boson} = \frac{\beta}{g^2}\int_{\mathbb{R}^3}\left(\frac{1}{2}{\rm tr}\left(F_{ij}F_{ij}\right) + {\rm tr}\left(D_i A_4\right)^2 + g^2 V(A_4) + \mathcal{O}\left(g^4\right)\right)$$

$$V(A_4) = -\frac{1}{12\pi\beta^4}\left[-6\pi\left(\beta A_4^3\right)^2 + 4\left(\beta A_4^3\right)^3\right], \quad \text{for } \beta A_4^3 \in [0,\pi]$$

(shown for nf=1 SYM)

- **two vacua** $\beta A_4^3 = 0, 2\pi$ broken center ("0-form", along x_4) $\frac{1}{2}\langle {\rm Tr}_F \exp\left[i\oint_{\mathbb{S}^1_\beta} A_4\right]\rangle = \pm 1$

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- Z_2 "domain walls", or "interfaces", or "center vortices" of width ~1/gT — Bhattacharya et al 1991 …
- Z_2 0-form center restored on DW

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- re-obtaining some stringy results

$SU(2)$ Yang-Mills theory endowed with n_f adjoint Weyl fermions $T \gg \Lambda_{QCD}$

$$S_{3D}^{\text{boson}} = \frac{\beta}{g^2} \int_{\mathbb{R}^3} \left(\frac{1}{2} \text{tr}\left(F_{ij} F_{ij}\right) + \text{tr}\left(D_i A_4\right)^2 + g^2 V(A_4) + \mathcal{O}\left(g^4\right) \right)$$

$$V(A_4) = -\frac{1}{12\pi\beta^4} \left[-6\pi \left(\beta A_4^3\right)^2 + 4\left(\beta A_4^3\right)^3 \right], \quad \text{for } \beta A_4^3 \in [0, \pi]$$

(shown for nf=1 SYM)

- two vacua $\beta A_4^3 = 0, 2\pi$ broken center ("0-form", along x_4) $\frac{1}{2} \langle \text{Tr}_F \exp \left[i \oint_{\mathbb{S}^1_\beta} A_4 \right] \rangle = \pm 1$
- Z_2 "domain walls", or "interfaces", or "center vortices" of width ~1/gT — Bhattacharya et al 1991 …
- Z_2 0-form center restored on DW
- U(1)unbroken on wall (Polyakov loop not ~1) Cartan of SU(2) massless; W-boson mass ~T
  localized 2d U(1) on wall — not very interesting except $\theta = \pi$ pure YM! — Gaiotto et al 2017

- and even richer in SYM and QCD(adj)!

2.) domain walls in 4d SYM/QCD(adj) & anomaly inflow…
- re-obtaining some stringy results

- SYM has Z_2 center 1-form and Z_4 chiral 0-form w/ mixed 't Hooft anomaly!
  story similar as in Schwinger model: gauging center=fractional topological charge in 4d (here: 1/2) background of two center vortices intersecting at a point *(one along x1-x2, the other along x3-x4)*

- at $T<T_c$, Z_4 chiral broken to Z_2, matching the anomaly (assume Tchi= Tc)
- at $T>T_c$, Z_2 center broken, matching the anomaly (…> or =)

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- adjoint fermions at high-T have zero modes on the wall
  for nf=1, two are normalizable, leading to worldvolume L:

$$\mathcal{L}^{\rm axial}_{DW} = \frac{1}{4e^2} F_{kl}F_{kl} + i\bar{\lambda}_+ \left[\partial_1 + i\partial_2 - i2(A_1 + iA_2)\right]\lambda_+ + i\bar{\lambda}_- \left[\partial_1 - i\partial_2 + i2(A_1 - iA_2)\right]\lambda_-$$

← axial Schwinger model of charge-2! L and R have opposite charge

- In 2d axial and vector easily mapped to each other: Z_4 chiral symmetry and Z_2 center.
  From q=2 Schwinger model results, chiral and center broken, so:
- **nonzero fermion condensate -**on DW in chirally restored phase **+**
  Wilson loop perimeter law on the high-T "center vortex" [for lattice!]

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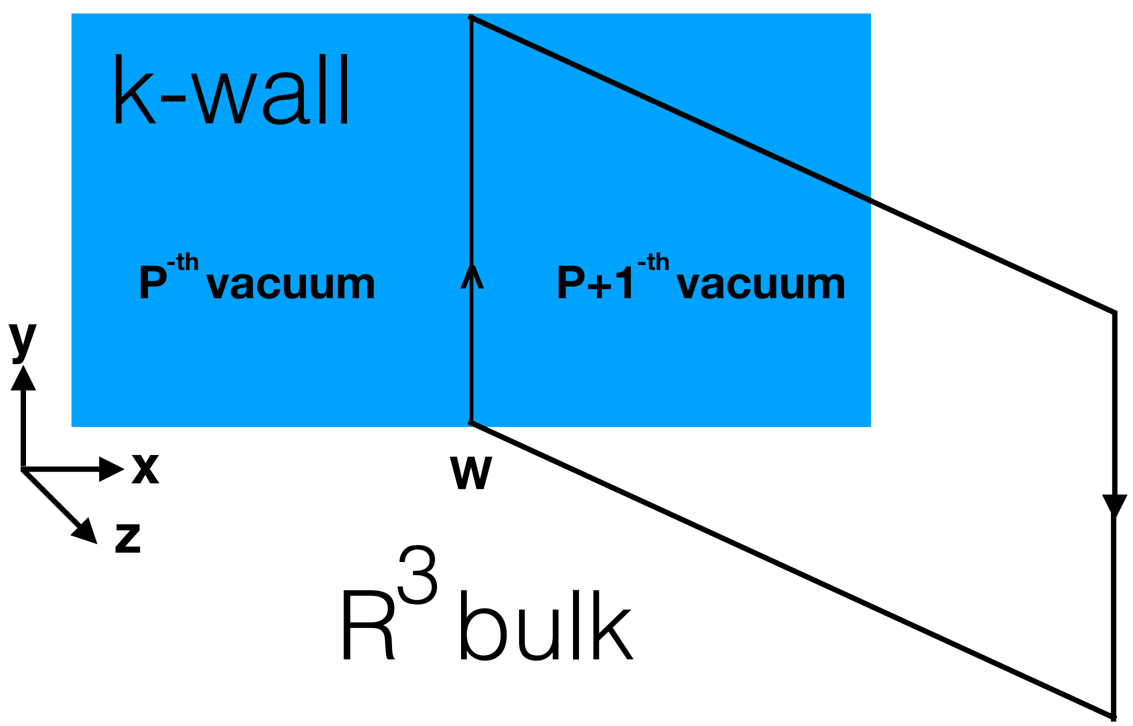
(story also generalizes to k-walls in high-T SU(N), so picture borrowed)

high-T SYM: $T \gg \Lambda$ k-wall

$Z_{2N}^{(0)}\, Z_N^{(1)}$ 't Hooft anomaly on worldvolume



1 fermion condensate on k-wall

2 quarks deconfined on k-wall

$Z_N^{(1)}$ broken (not in bulk)

first via holography: *F1* on *D1*

[Aharony, Witten 1999;…]

here, QFT: 2d YM with massless fermions screens

[Schwinger model - many; nonabelian - Gross, Klebanov, Matytsin, Smilga 1995; Armoni, Frishman, Sonnenschein 1997;…]

so we find “D-branes” and “strings”, once again, in QFT

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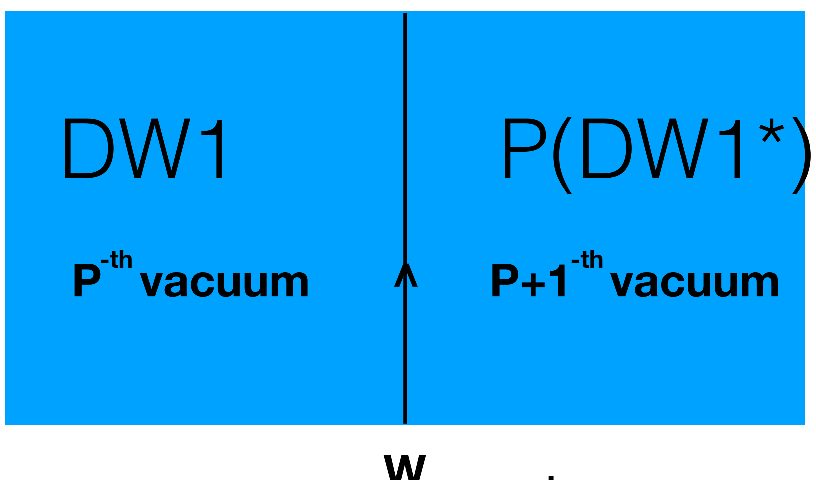
when first saw…experienced a flashback: low-T small-S1 SYM

[1501.06773 Anber, Sulejmanpasic, EP]

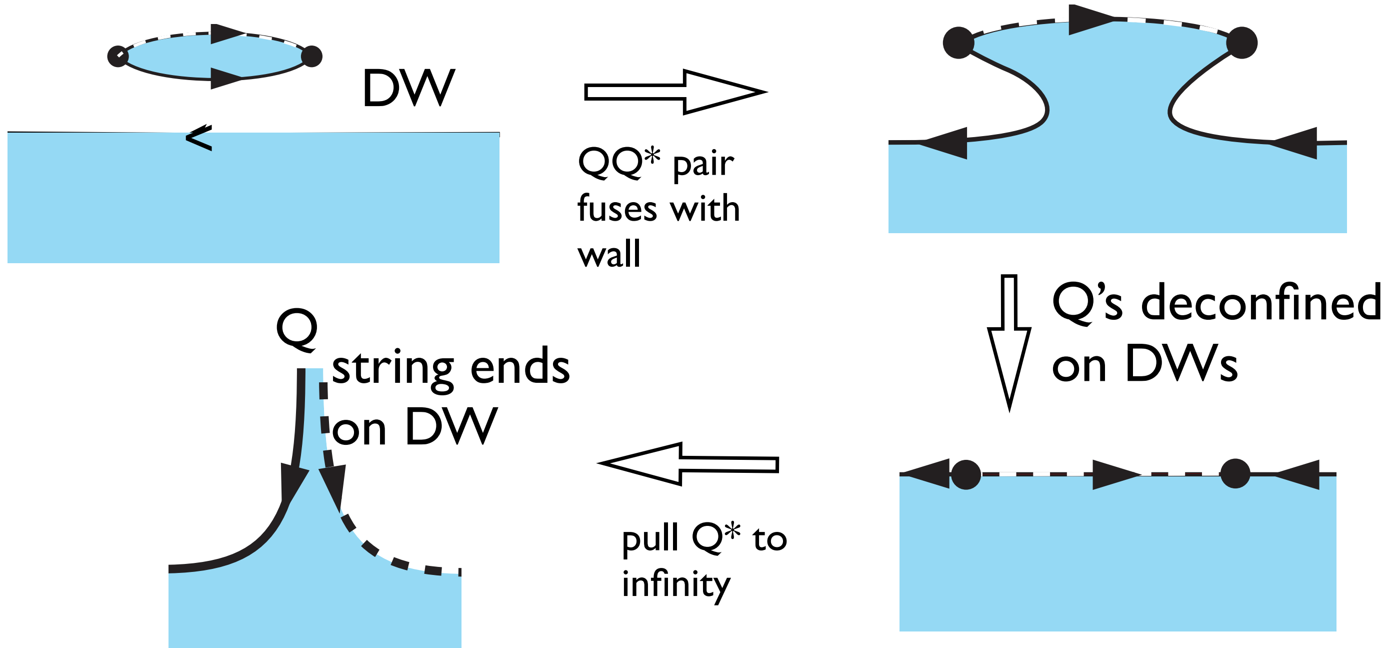
here: center unbroken $Z_N^{(0)C}$



quark

chiral broken $Z_{2N}^{(0)} \to Z_2^{(0)}$

further, as seen in MQCD [Witten, 1998] confining strings end on DW



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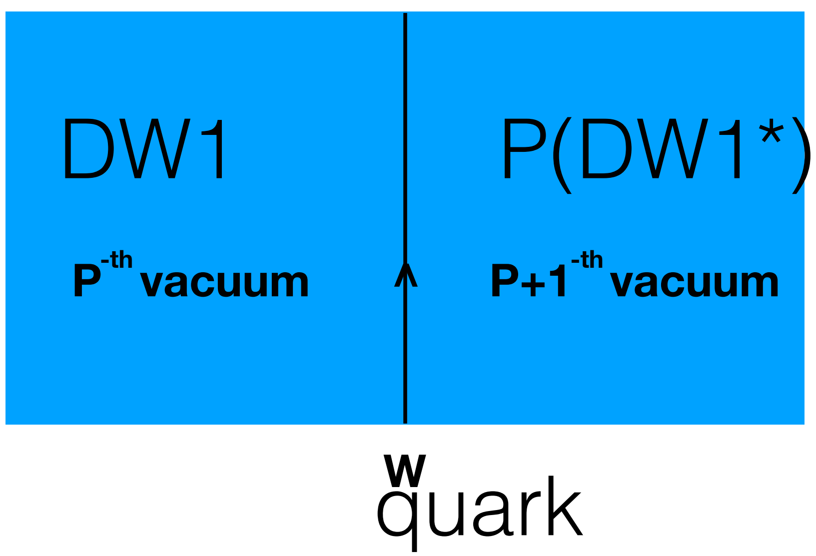

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reason for this similarity is that the 2d worldvolume IR TQFTs matching the relevant anomalies are identical in the high-T/large-L and low-T/small-L DWs [back in ’15 wasn’t aware that studying TQFTs…]

Time to wrap up…

- new higher-symmetry ’t Hooft anomalies imply a rich structure
- in some cases, they can be understood from different points of view
    - lattice is the natural habitat for discrete higher symmetries
    - however, lattice makes other ‘things’ harder, such as CS theory and anomalies
    - good to understand using variety of approaches - *operator in SYM/adj?*
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- **phases and symmetry realization of ‘bulk’ 4d (or <4d) theories can also be affected by discrete anomalies**
- didn’t dwell upon, but used, ordering of thermal phase transitions, also constrained by matching
- witness two-flavour QCD(adj) 1805.12290 Anber, EP + subsequent activity on non-spin backgrounds, TQFTs etc. Cordova, Dumitrescu; Bi, Senthil; Wan, Wang -

  can’t help but wonder how above anomalies, seen using complicated backgrounds, are reflected in the operator structure/representation; I don’t think level of understanding similar to that of usual 0-form continuum anomalies is reached yet!

- finally, other theories with such mixed anomalies can also be studied