’t Hooft anomalies, high-T, and low-T domain walls in adjoint-fermion theories with (or without) supersymmetry

Erich Poppitz UofToronto

with **Mohamed Anber** **Lewis & Clark College, Portland, OR**

1807.00093, 1811.10642 (high-T) + many earlier related works, by us and by others

(+ revisit past work
1501.06773 (low-T) **M. Anber, Tin Sulejmanpasic, EP**)

Motivation:

general - improved understanding of nonperturbative dynamics of 4d gauge theories

immediate - new 't Hooft anomaly matching conditions

[Gaiotto, Kapustin, Komargodski, Seiberg, Willett; 2014-]

class of theories - SU(N) QCD(adj) nf massless Weyl

nf=0 pure YM (add mass to nf>0)

nf=1 super YM

nf=2, 3, 4, 5: asymptotic freedom, conformal, confining?

[Anber, EP; Cordova, Dumitrescu; Bi, Senthil;… 2018]

[lattice preliminary: Anthenodorou, Bennett, Bergner, Lucini 2014]

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all semiclassical at small spatial circle [R^3x S^1: Unsal+… 2008-]

Outline of talk: mostly nf=1 super YM

0. discrete 0-form/1-form 't Hooft anomalies imply rich domain wall dynamics

two semiclassical regimes both R^3x S^1 small S^1 size:

1. high-T domain walls

k-wall 2d QCD, depends on k,N
chiral and center breaking
strings ending on walls

2. low-T domain walls

semiclassical picture of worldvolume center breaking and strings ending on walls

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two semiclassical regimes both R^3x S^1 small S^1 size:

1. high-T domain ~~walls~~ lines k-wall 2d QCD, depends on k,N
chiral and center breaking
strings ending on walls

1 and 2
are similar

2. low-T domain ~~walls~~ lines semiclassical picture of worldvolume center breaking and strings ending on walls

obtain QFT picture of phenomena
first seen in MQCD or holography

Some further motivation to study domain wall (DW) worldvolume theories:

- some of these DW worldvolume theories are themselves the simplest study cases of QFTs with mixed 0-form/1-form 't Hooft anomalies - solvable 2d ex. here

- physics on the high-T DW (2d) shares features of the low-T theory, both bulk (4d) and DW (2d/3d)

- high-T DW are a semiclassical counterpart to "center vortices," field configurations thought to be responsible for area law of Wilson loop at low-T in pure YM

(not theoretically controllable; seen in lattice simulations)

[Greensite+…; 'D Elia, de Forcrand;… 1998-]

0. discrete 0-form/1-form 't Hooft anomalies

$$U(1)_R \times Z_N^{(1)} \longrightarrow U(x,\hat{\mu}) \to e^{i\frac{2\pi}{N}n_\mu}\, U(x,\hat{\mu})$$

$$\downarrow$$

$$\lambda^a_\alpha \to e^{i\omega}\lambda^a_\alpha$$

- closed loops invariant
- winding (F) loops transform

- nontrivial Jacobian

$$J = e^{i\omega 2N Q_{top.}}$$

$$\lambda^a_\alpha \to e^{i\frac{2\pi}{2N}}\lambda^a_\alpha$$

only $Z_{2N}^{(0)}$ survives

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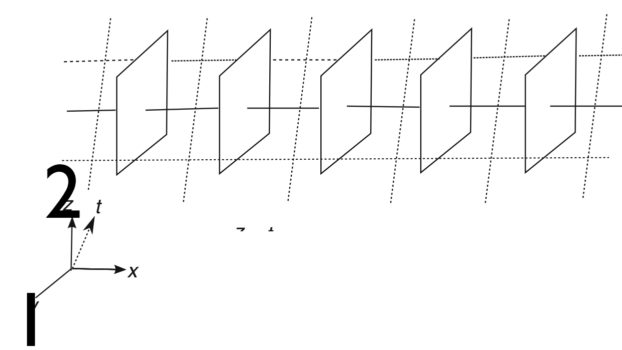
gauging 1-form center:



$n_{12} = 1$

topological background
(’t Hooft flux/thin “center vortex”)

$$Q_{top} = \frac{n_{12} n_{34}}{N}$$ [’t Hooft; van Baal 1980s]

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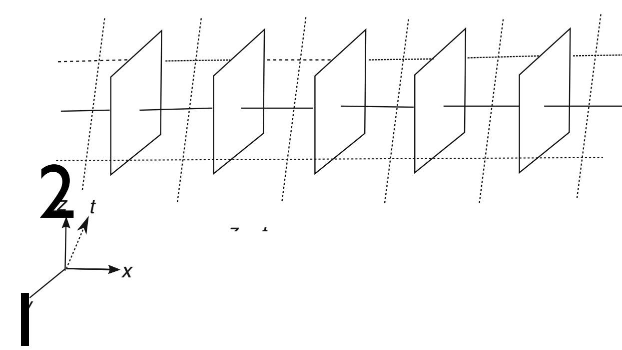

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gauge center = break discrete chiral

$$J_{Z_{2N}^{(0)}} = e^{i2\pi Q_{top.}} = e^{i\frac{2\pi}{N}n_{12}n_{34}}$$

$$\uparrow$$

't Hooft anomaly: must match… S_5d… nontrivial IR!

1. high-T domain walls/inflow, center…

low-T SYM: discrete chiral broken, center unbroken; anomaly saturated in “Goldstone” mode, DWs…

[Witten; Acharya, Vafa; late ‘90s…]

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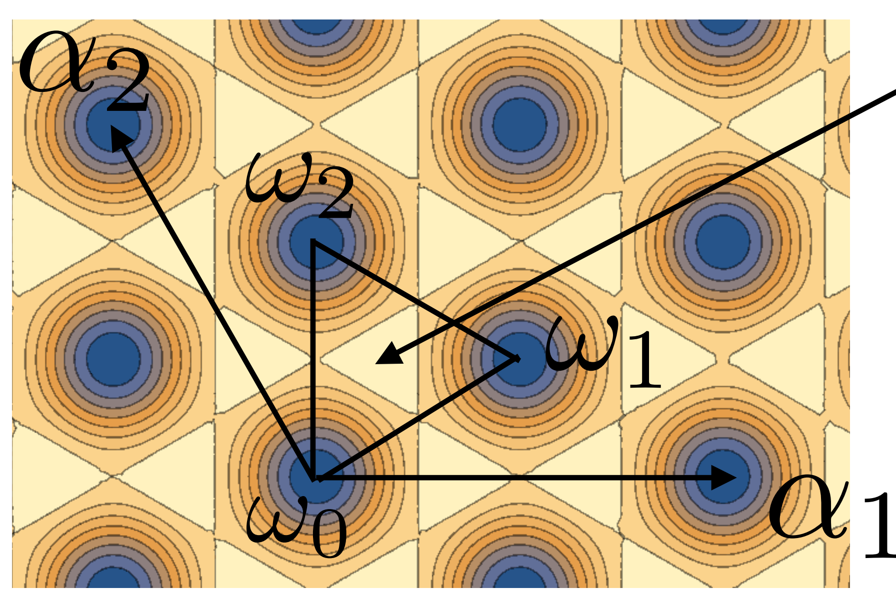
high-T SYM: $T \gg \Lambda$ [Gross, Pisarski, Yaffe 1980s]

$$V(\boldsymbol{A}_4) = \frac{4T^4}{\pi^2} \sum_{\boldsymbol{\beta}^+} \sum_{n=1} \frac{-1 + \overset{1}{\cancel{n_f}}(-1)^n}{n^4} \cos\left[\frac{n\boldsymbol{A}_4 \cdot \boldsymbol{\beta}}{T}\right]$$

SU(3)

T**A$_4$** -plane:



“Weyl chamber”

$Z_N^{(0)C}$ “0-form center”

(N=3: 60^o rotation)

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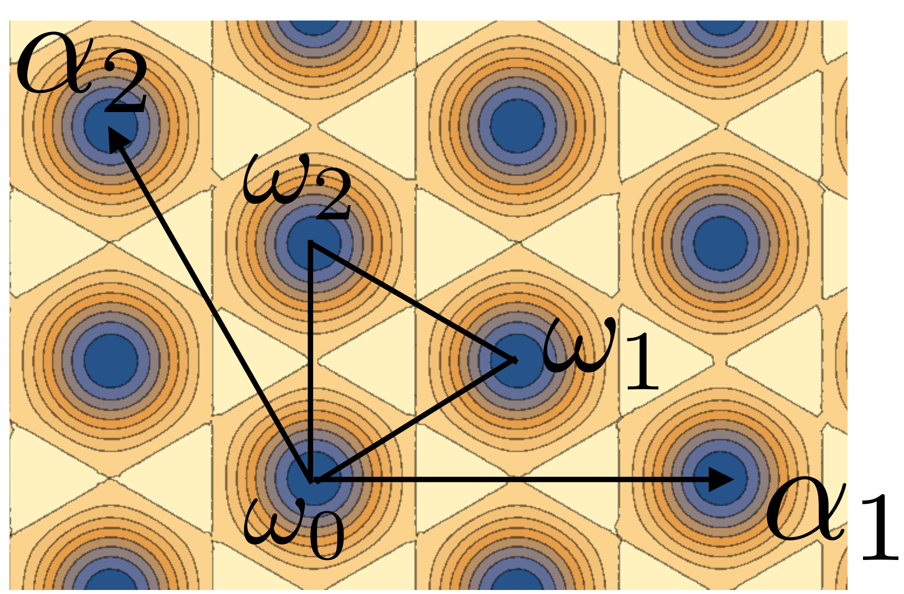

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$Z_N^{(0)C}$ "0-form center" broken

SU(N) unbroken

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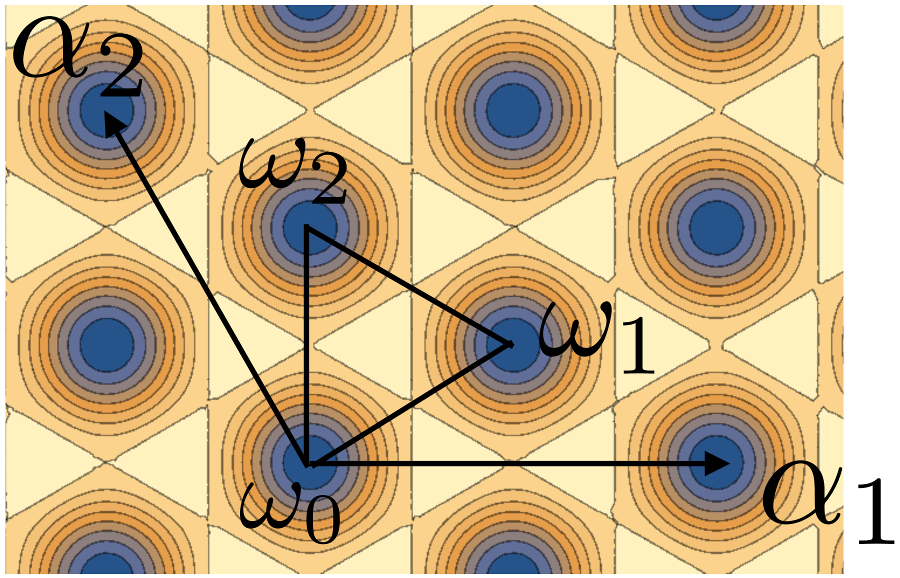
Polyakov loop vev: $\mathrm{tr}_F \left[e^{i\boldsymbol{\Phi}_0 \cdot \boldsymbol{H}}\right]\Big|_{\boldsymbol{\Phi}_0 = 2\pi \boldsymbol{\omega}_a} = N e^{-i\frac{2\pi a}{N}}$



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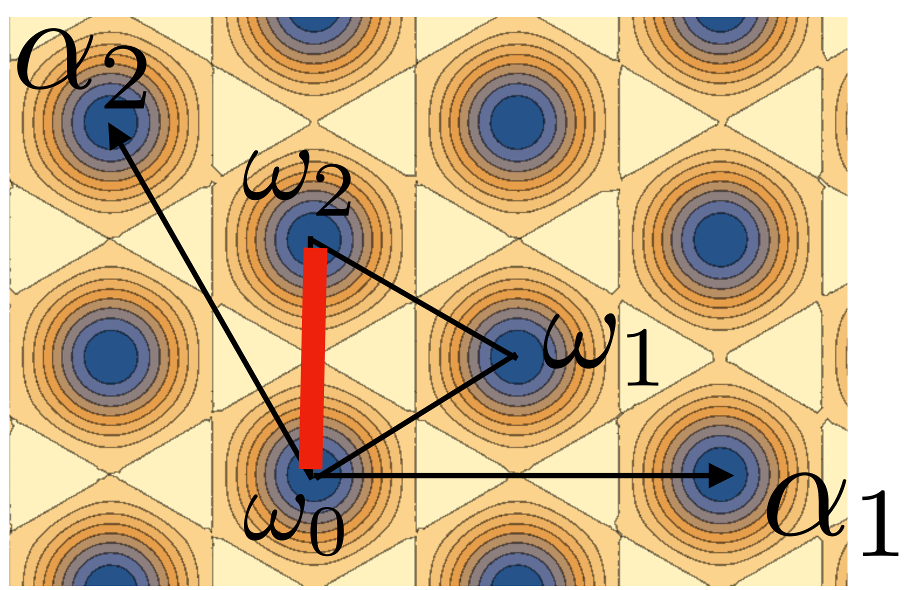
k-wall (line!) connects 0-th and k-th vacuum:

$$\mathbf{A}_4^{DW(k)}(z) = T\boldsymbol{\Phi}^{DW(k)}(z) \qquad \boldsymbol{\Phi}^{DW(k)}(\mp\infty) = \begin{cases} 0 \\ 2\pi\omega_{\mathbf{k}} \end{cases}$$



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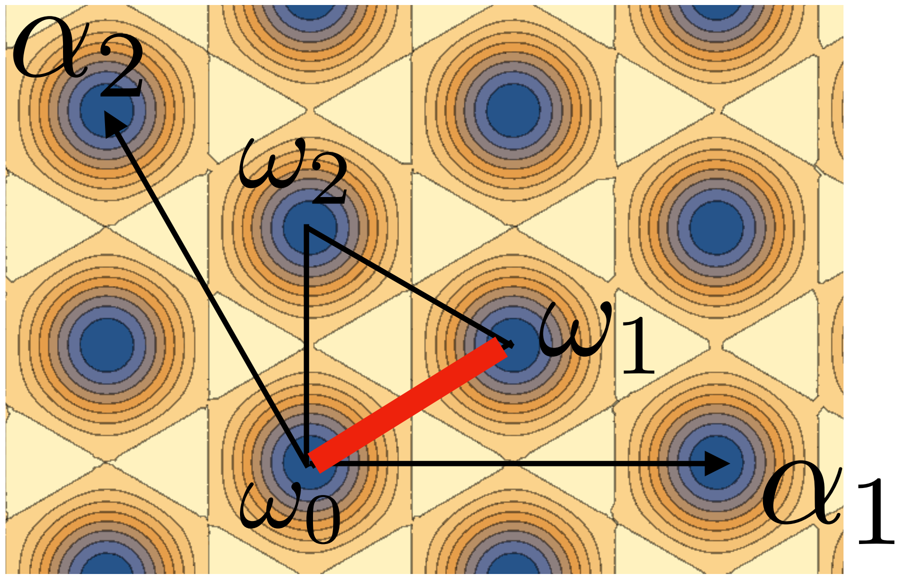

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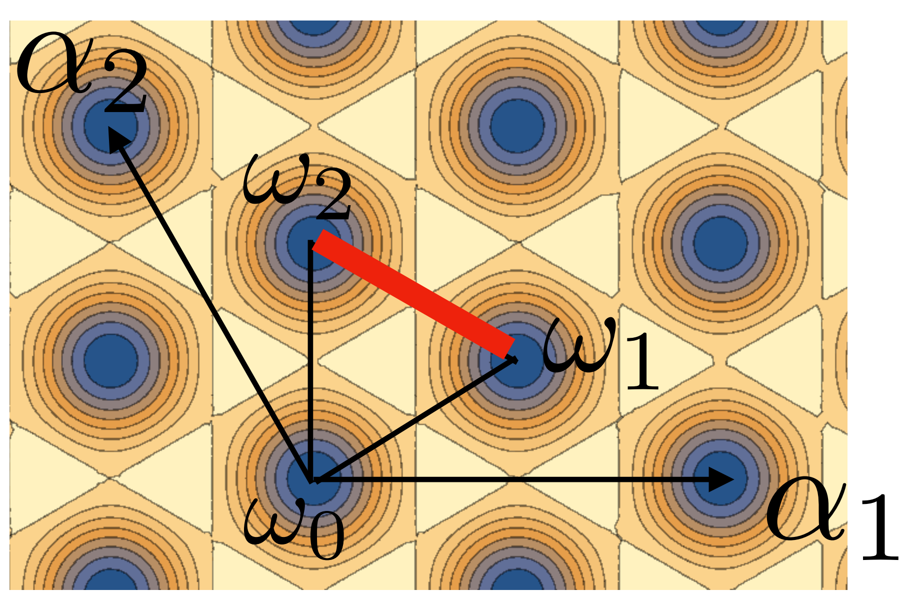

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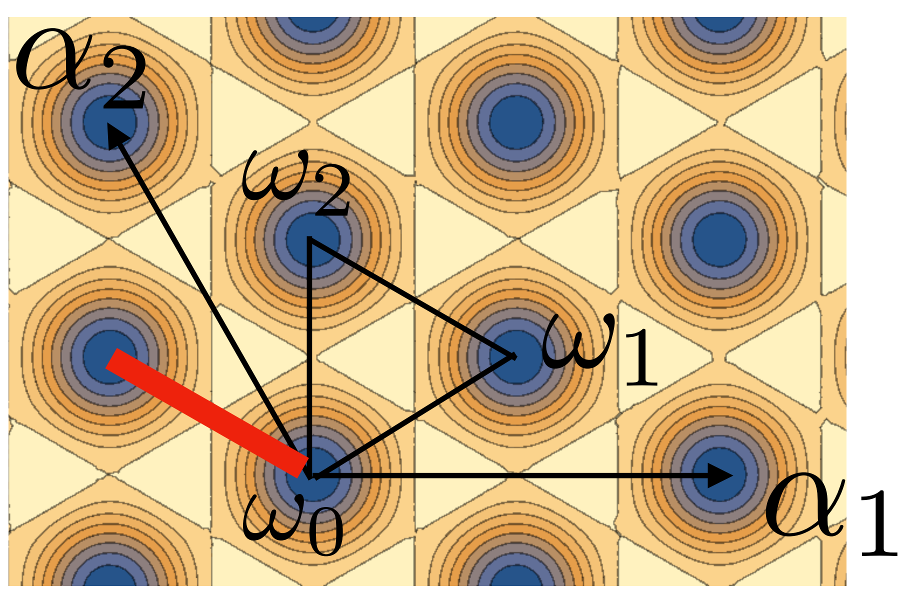

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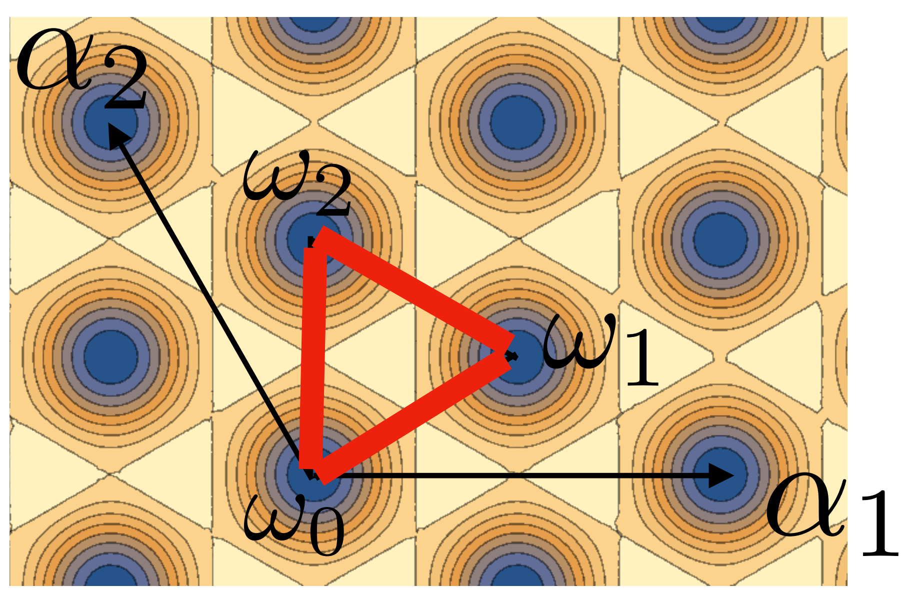

minimum action along sides of Weyl chamber SU(3)->SU(2)xU(1)

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high-T SYM: $T \gg \Lambda$ $\cancel{Z_N^{(0)C}}$ by Polyakov loop vev:

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[Bhattacharya, Gocksch, Korthals-Altes, Pisarski;+… 1992-]

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k-wall tension

$$T^2 \frac{(N-k)k}{\sqrt{g^2 N}}$$

k-wall width

$$\frac{1}{T\sqrt{g^2 N}} << \frac{1}{g^2 N T}$$

bulk "confining" scale

Casimir scaling down to ~T_C - lattice [… Bursa, Teper 2005]

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on k-wall: SU(N) —> U(1) x SU(N-k) x SU(k)

$$m_W \sim T\sqrt{\frac{N}{k(N-k)}}$$

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massless gauge bosons localized by bulk confinement, like [Dvali, Shifman 1996]

in purely bosonic theory, not much 'fun':
2d confinement, mass gap on wall

unless $\theta = \pi$ — or adjoint fermions, this talk

[Gaiotto, Kapustin, Komargodski, Seiberg; 2017]

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massless gauge bosons localized by bulk confinement, like [Dvali, Shifman 1996]

adjoint fermions: Matsubara mass, except some not commuting with wall profile

$$\mathrm{diag} \left[\underbrace{k, k, ..., k}_{N-k\,\mathrm{times}}, \underbrace{k-N, k-N, ..., k-N}_{k\,\mathrm{times}} \right]$$

- work out details:

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k-wall worldvolume theory

2d QCD gauge group

	$U(1)$ X	$SU(N-k)$ X	$SU(k)$	$U(1)_R$
ψ_+	$\gamma^{(N-k)} \equiv \sqrt{\frac{N}{k(N-k)}}$	$\square$	$\overline{\square}$	1
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$U(1)_R$

$\psi_\pm \to e^{i\chi}\psi_\pm$ Jacobian

$$\mathcal{J} \equiv \exp\left[i\, 2\chi(N-k)k\; \gamma^{(N-k)} \oint \frac{F_{12}^{N-k} dx^1 dx^2}{2\pi} \right]$$

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U(1) flux quantization?

$$\oint \frac{F_{12}^{N-k}dx^1dx^2}{2\pi} = \gamma^{(N-k)}\ n,\ \ n \in \mathbb{Z}$$

study b.c. on T^2, or constant flux backgrd

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$$= e^{2i\chi N n}$$

only $Z_{2N}^{(0)}$ survives, as in the 4d theory

1. high-T domain walls/inflow, center…

high-T SYM: $T \gg \Lambda$ k-wall

2d QCD gauge group U(1) x SU(N-k) x SU(k)

$Z_{2N}^{(0)}$ discrete chiral already seen

center symmetry? e.g. $q_1 \sim (\frac{k}{N}\gamma^{(N-k)}, \Box, \mathbf{1})$ part of SU(N) $\Box$

$$Z_N^{(1)} \quad W_{q_1} \to e^{i\frac{2\pi}{N}p} W_{q_1}$$

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gauging - unit 't Hooft flux in k-wall worldvolume

= k units of fractional U(1) flux $\displaystyle \oint \frac{F_{12}^{N-k} dx^1 dx^2}{2\pi} = k\, \frac{\gamma^{(N-k)}}{N}$

$$J_{Z_{2N}^{(0)}} = \exp\left[i \frac{2\pi}{2N} 2(N-k)k\gamma^{(N-k)} \oint \frac{F_{12}^{N-k} dx^1 dx^2}{2\pi}\right] = e^{i\frac{2\pi}{N}k}$$

mixed anomaly!

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$Z^{(0)}_{2N}$ $Z^{(1)}_{N}$ 't Hooft anomaly on worldvolume

as implied by anomaly inflow

$$\left(S_{5-D} = i\,\frac{2\pi}{N} \int\limits_{M_5\ (\partial M_5 = M_4)} \frac{2NA^{(1)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi} \longrightarrow \oint_{M_{zx^4}} \frac{B^{(2)}N}{2\pi} = k \right.$$

$$\left. \longrightarrow S_{3-D} = i\,\frac{2\pi k}{N} \int\limits_{M_3\ (\partial M_3 = M_2)} \frac{2NA^{(1)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi} \longrightarrow Z^{(0)}_{2N}\ \ Z^{(1)}_{N} \text{ on worldvolume} \right)$$

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$Z_{2N}^{(0)}$ $Z_N^{(1)}$ 't Hooft anomaly on worldvolume

IR of worldvolume? matching?

N=2, k=1
= axial "charge-2"
Schwinger model
Anber, EP 1807.00093

on its own, simplest study case of QFTs with mixed 0-form/1-form 't Hooft anomaly

1. high-T domain walls/inflow, center…

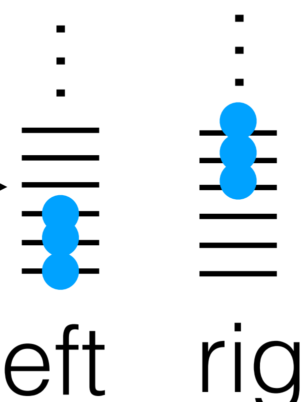
charge-q vector Schwinger model on spatial S^1

Anber, EP
1807.00093

1-form center + 0-form chiral; Hamiltonian [Manton '86; Iso, Murayama '89]

$$\oint A_x dx \equiv a \qquad Z_q^C : a \to a + \frac{2\pi}{q} \qquad (Z_q^C)^q$$ = G large gauge trf.

Dirac sea states $|n\rangle \longrightarrow$



$$Q_5 |n\rangle = |n\rangle \left(2n - \frac{qa}{\pi}\right)$$

$$G|n\rangle = |n + q\rangle$$

anomaly free Q_5

$$\tilde{Q}_5 \equiv Q_5 + \frac{qa}{\pi}$$ not G invariant $G : \tilde{Q}_5 \to \tilde{Q}_5 + 2q$

$$X_{2q} \equiv e^{i\frac{2\pi}{2q}\tilde{Q}_5}$$ G invariant, generates chiral $Z_{2q}^{(0)}$

1. high-T domain walls/inflow, center…

$X_{2q} \equiv e^{i\frac{2\pi}{2q}\tilde{Q}_5}$ G invariant, generates chiral $Z_{2q}^{(0)}$

$$X_{2q}|n\rangle = |n\rangle\, \omega_q^n \qquad (\omega_q \equiv e^{i\frac{2\pi}{q}})$$

Y_q generates 1-form center $(Y_q)^q = G$

$Y_q|n\rangle = |n+1\rangle$ because $G|n\rangle = |n+q\rangle$

q "theta vacua" $|\theta, k\rangle \equiv \sum_{n\in\mathbb{Z}} e^{i(k+qn)\theta}|k+qn\rangle,\ k = 0, 1, \ldots, q-1$

and their Z_q Fourier transfs:

$|P,\theta\rangle \equiv \frac{1}{\sqrt{q}}\sum_{k=0}^{q-1}\omega_q^{kP}|\theta, k\rangle,\ P = 0, \ldots, q-1,$ (need, clustering)

1. high-T domain walls/inflow, center…

$$\langle P', \theta | \bar{\psi}_+(x)\psi_-(x) | P, \theta \rangle = e^{-i\theta} \omega_q^{-P} \delta_{P,P'} C$$

from above formulae find action of symmetries

$$X_{2q} \, |P, \theta\rangle = |P + 1 (\mathrm{mod}\ q), \theta\rangle$$

$$Y_q \, |P, \theta\rangle = |P, \theta\rangle \, \omega_q^{-P} e^{-i\theta}$$

$$X_{2q} \, Y_q = \omega_q \, Y_q \, X_{2q} \quad (\omega_q = e^{i\frac{2\pi}{q}})$$

"central extension" manifestation of mixed discrete 't Hooft anomaly

1. high-T domain walls/inflow, center…

$$X_{2q}\, Y_q = \omega_q\, Y_q\, X_{2q} \quad (\omega_q = e^{i\frac{2\pi}{q}})$$

“central extension” manifestation of mixed discrete 't Hooft anomaly

relabel q->**N** $Z^{(0)}_{2N} \to Z^{(0)}_{2}$ **IR TQFT on N=2, k=1 DW:**

$$S_{2-D} = i\,\frac{N}{2\pi}\int_{M_2} \varphi^{(0)} da^{(1)}$$

$$e^{i\hat{\varphi}} e^{i\hat{a}} \;=\; e^{i\frac{2\pi}{N}} e^{i\hat{a}} e^{i\hat{\varphi}}$$

N=2, k=1,derivation from UV: $Z^{(0)}_{2N}$ $Z^{(1)}_{N}$ operators

1. high-T domain walls/inflow, center…

high-T SYM: $T \gg \Lambda$ k-wall

2d QCD gauge group

	$U(1)$ X	$SU(N-k)$ X	$SU(k)$	$U(1)_R$
ψ_+	$\gamma^{(N-k)} \equiv \sqrt{\frac{N}{k(N-k)}}$	$\square$	$\overline{\square}$	1
ψ_-	$-\gamma^{(N-k)}$	$\overline{\square}$	$\square$	1

$Z_{2N}^{(0)}$ $Z_N^{(1)}$ 't Hooft anomaly on worldvolume

IR of worldvolume? matching?

N=2, k=1
= axial "charge-2"
Schwinger model

Anber, EP 1807.00093

$Z_{2N}^{(0)} \to Z_2^{(0)}$

$Z_N^{(1)}$ broken (perimeter law)

screening of all charges in 2d massless-fermion theories

1. high-T domain walls/inflow, center…

high-T SYM: $T \gg \Lambda$ k-wall

$Z_{2N}^{(0)}\, Z_N^{(1)}$ 't Hooft anomaly on worldvolume

likely persists for all N, k=1 (perhaps all k):

$$Z_{2N}^{(0)} \to Z_2^{(0)}: \quad \langle \mathrm{tr} \psi_+ \psi_- \rangle \neq 0$$

all-N, k=1, gauged WZW

Affleck, '86

large-N, k=1

Zhitnitsky, '85; Burkardt '94

Glozman, Shifman…, '12

$$\langle \mathrm{tr} \psi_+ \psi_- \rangle \sim N \sqrt{\lambda}$$

gapped, IR TQFT

$$S_{2-D} = i\, \frac{N}{2\pi} \int\limits_{M_2} \varphi^{(0)} da^{(1)} \qquad e^{i\hat{\varphi}} e^{i\hat{a}} \;=\; e^{i\,\frac{2\pi}{N}} e^{i\hat{a}} e^{i\hat{\varphi}}$$

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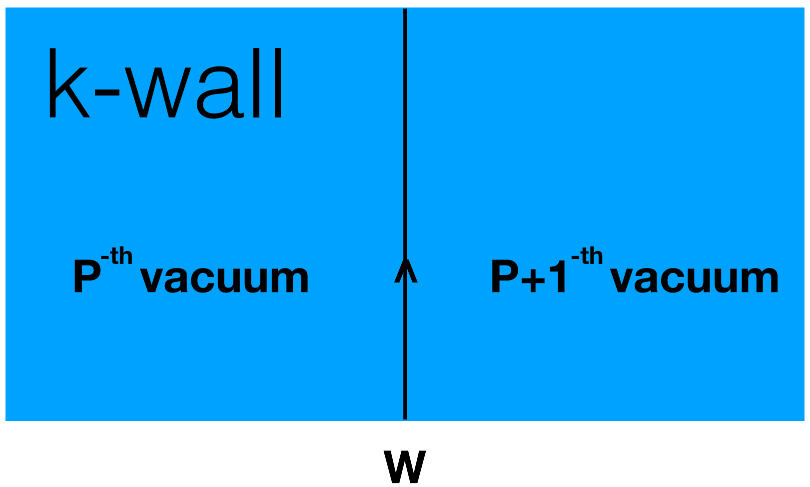

1 fermion condensate on k-wall

2 quarks deconfined on k-wall

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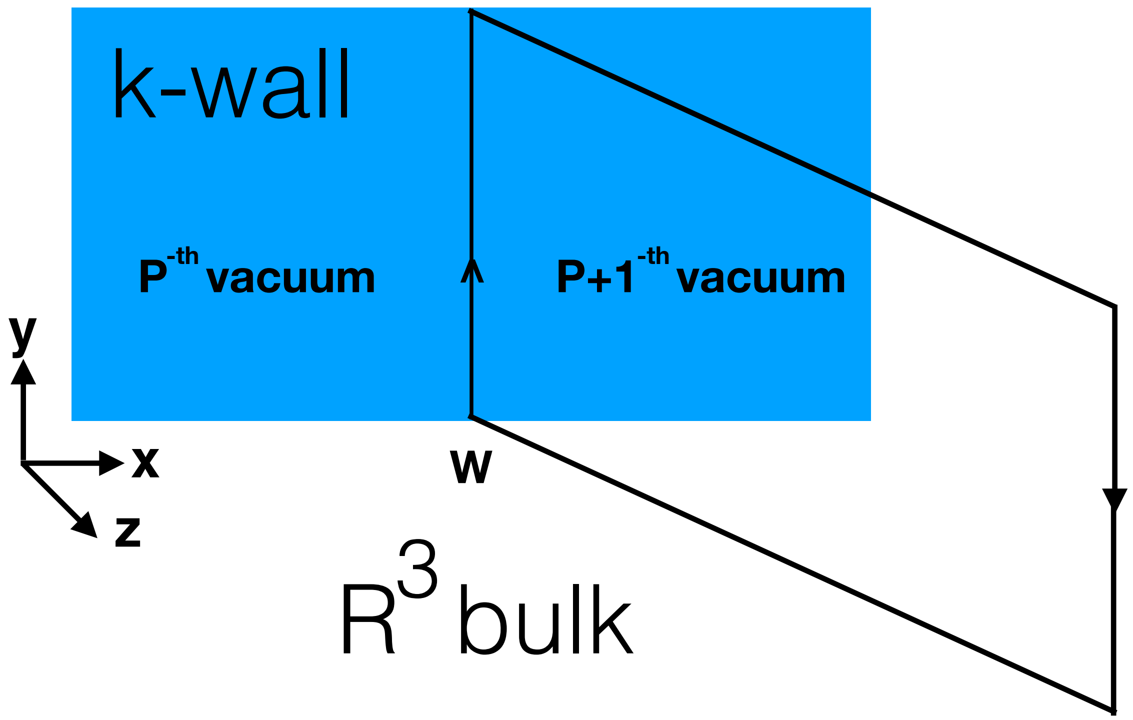

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[Aharony, Witten 1999;…]

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[Schwinger model…;
Gross, Klebanov, Matytsin, Smilga 1995;
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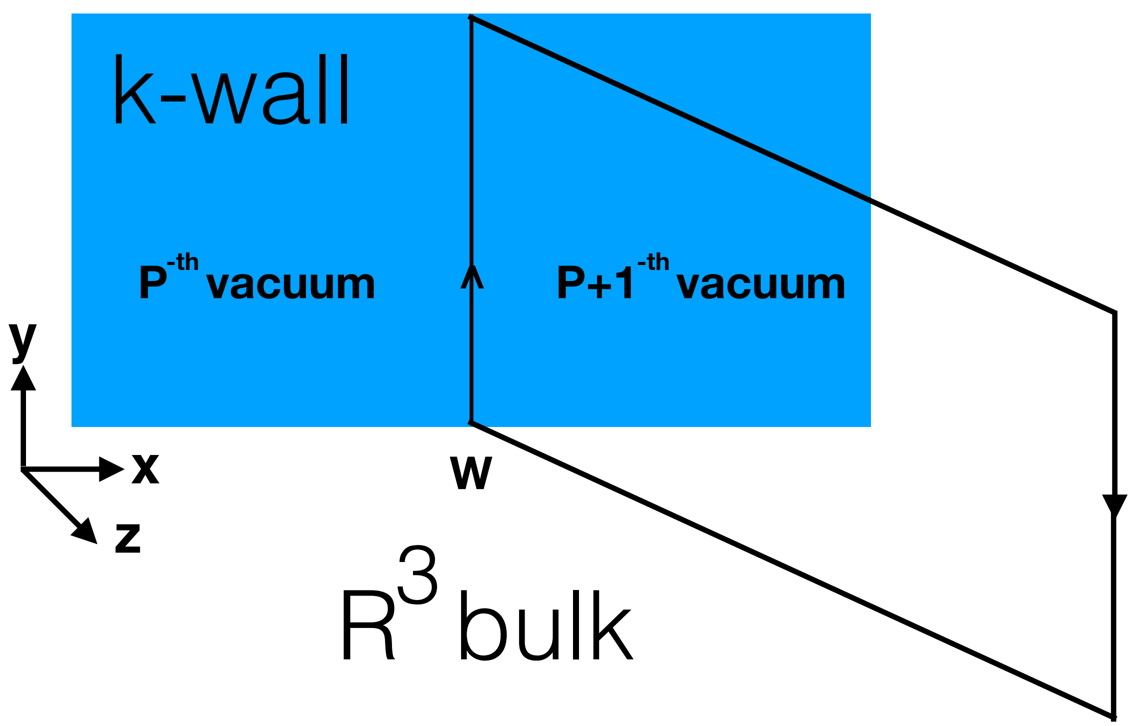

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… similar to **2. small-S^1 domain walls…**

2. small-S^1 domain walls…

[R^3x S^1: Unsal+… 2008-]

QCD(adj) semiclassical at small spatial circle $\Lambda N L \ll 1$

periodic fermions

here: SYM monopole + twisted ("KK") monopole superpotential

[Seiberg, Witten 1996;…]

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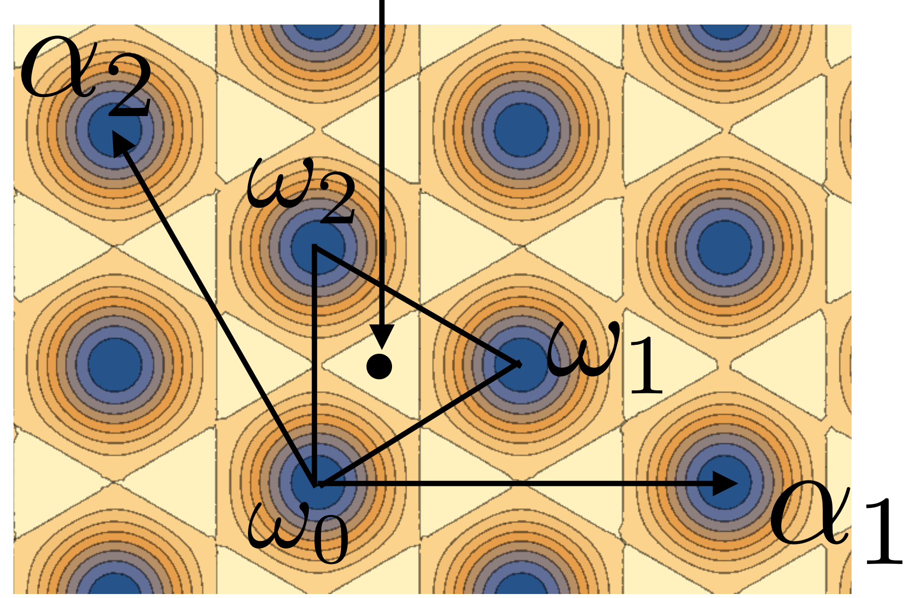
$\Lambda N L \ll 1$

now ***minimum*** at unbroken $Z_N^{(0)C}$ point - SU(N) abelianization

SU(3)
L$\mathbf{A}_4$ -plane:



but we also have N-1 “dual photons” and a ***“dual photon plane”*** —>

2. small-S^1 domain walls…

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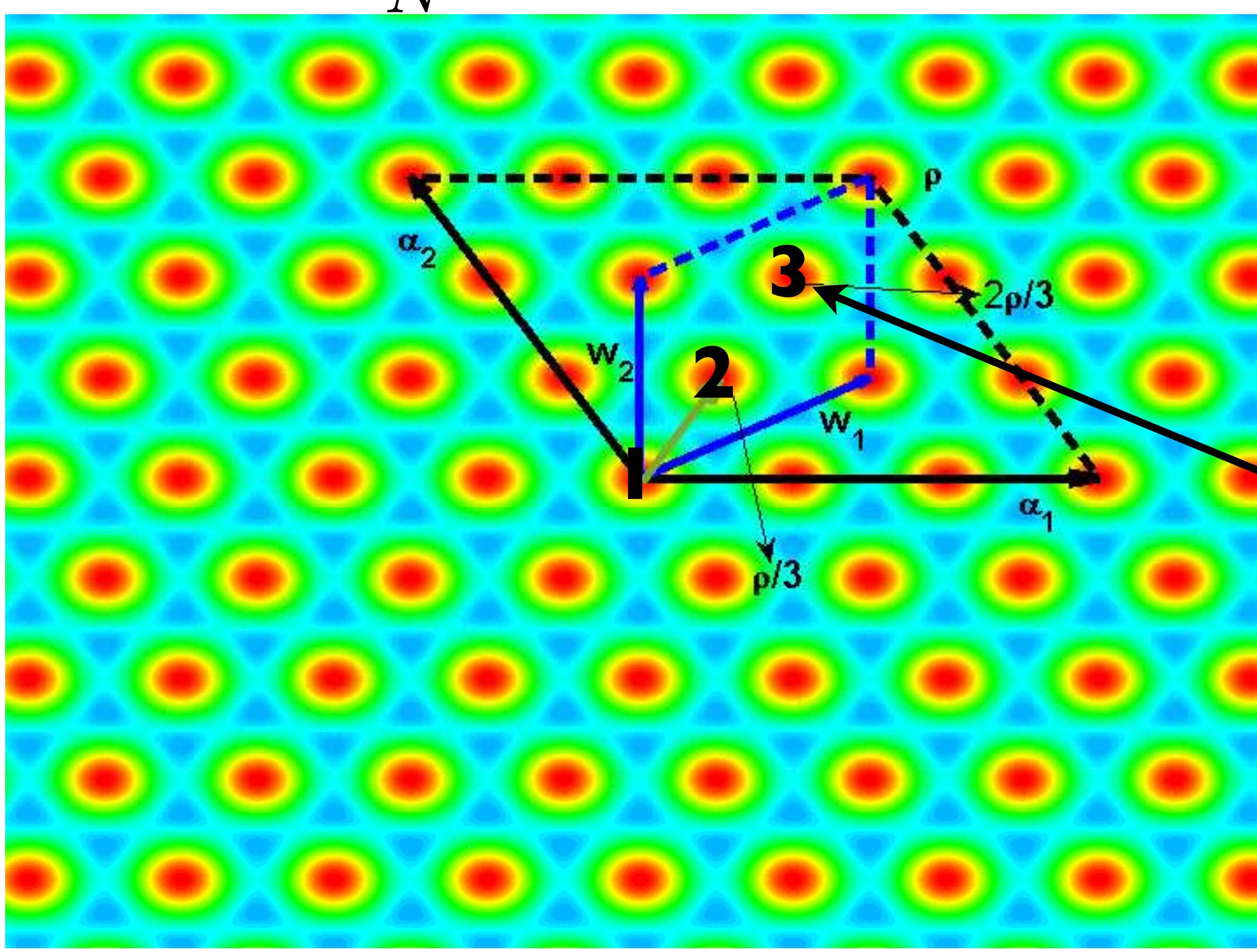
unbroken $Z_N^{(0)C}$



dual photon plane

periodicities:

w1, w2: weight vectors of SU(3)

3 vacua - **1,2,3**

broken discrete chiral symmetry

(preserve center symmetry

120 degree rotn + w_k shift)

2. small-S^1 domain walls…

[1501.06773 Anber, Sulejmanpasic, EP]

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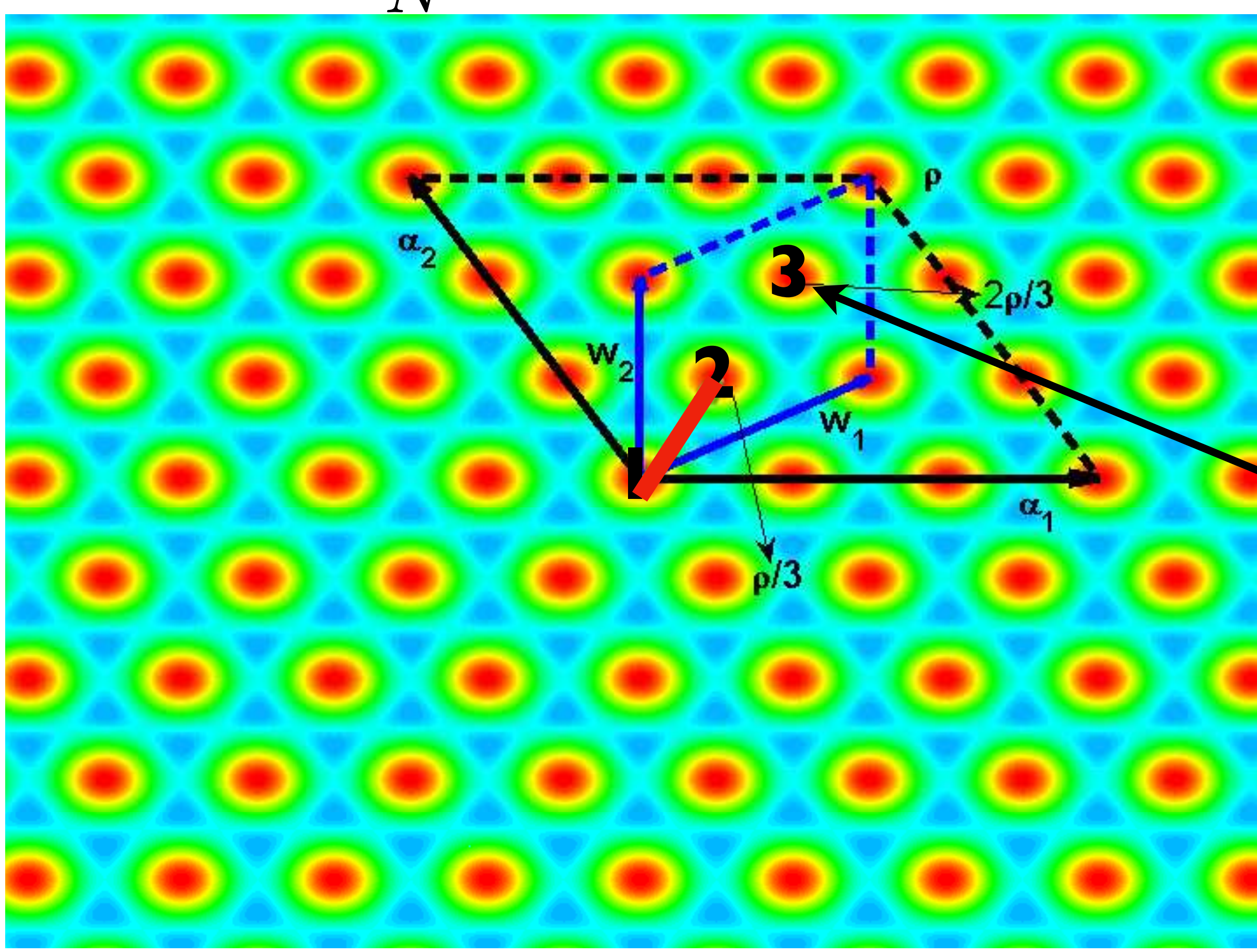

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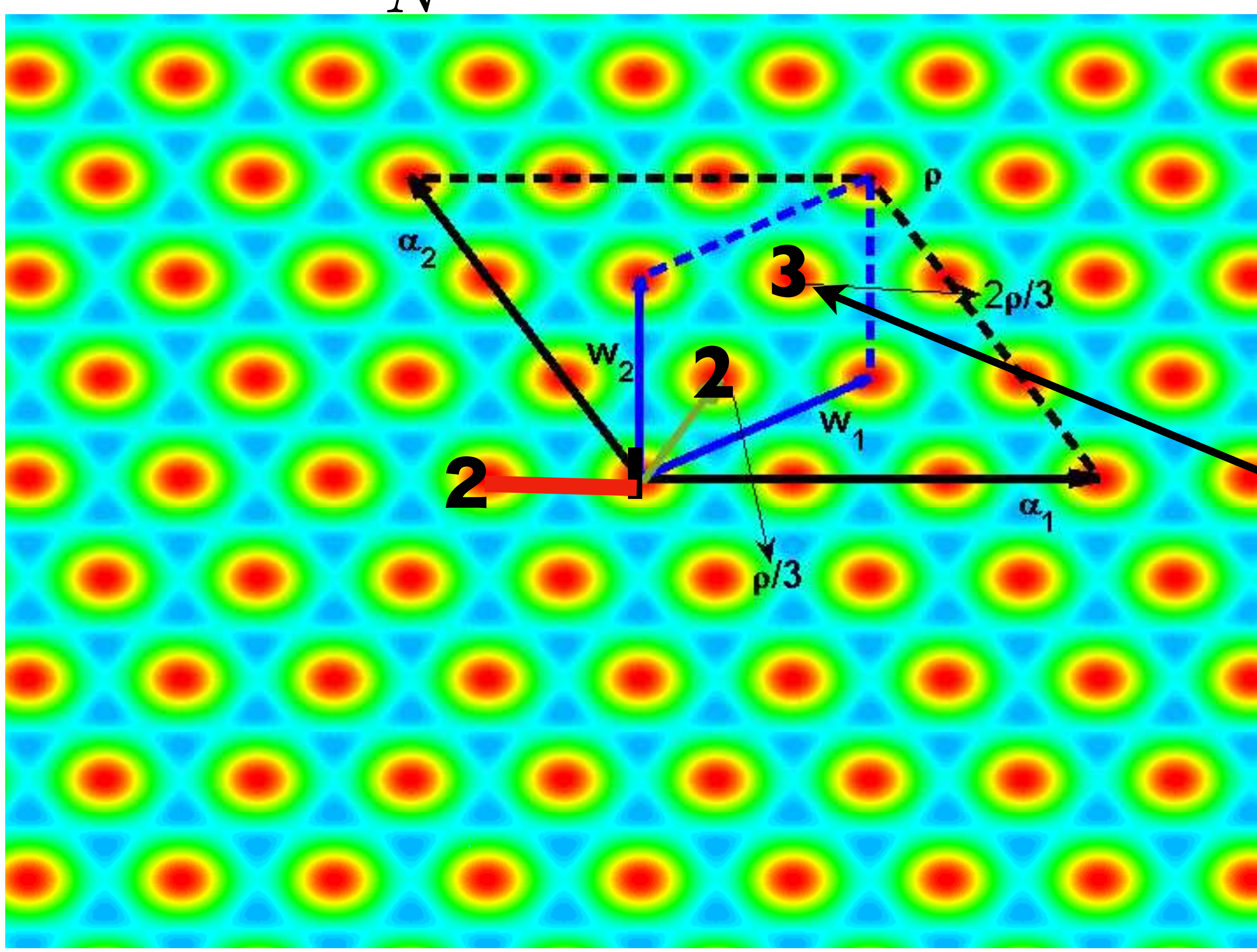

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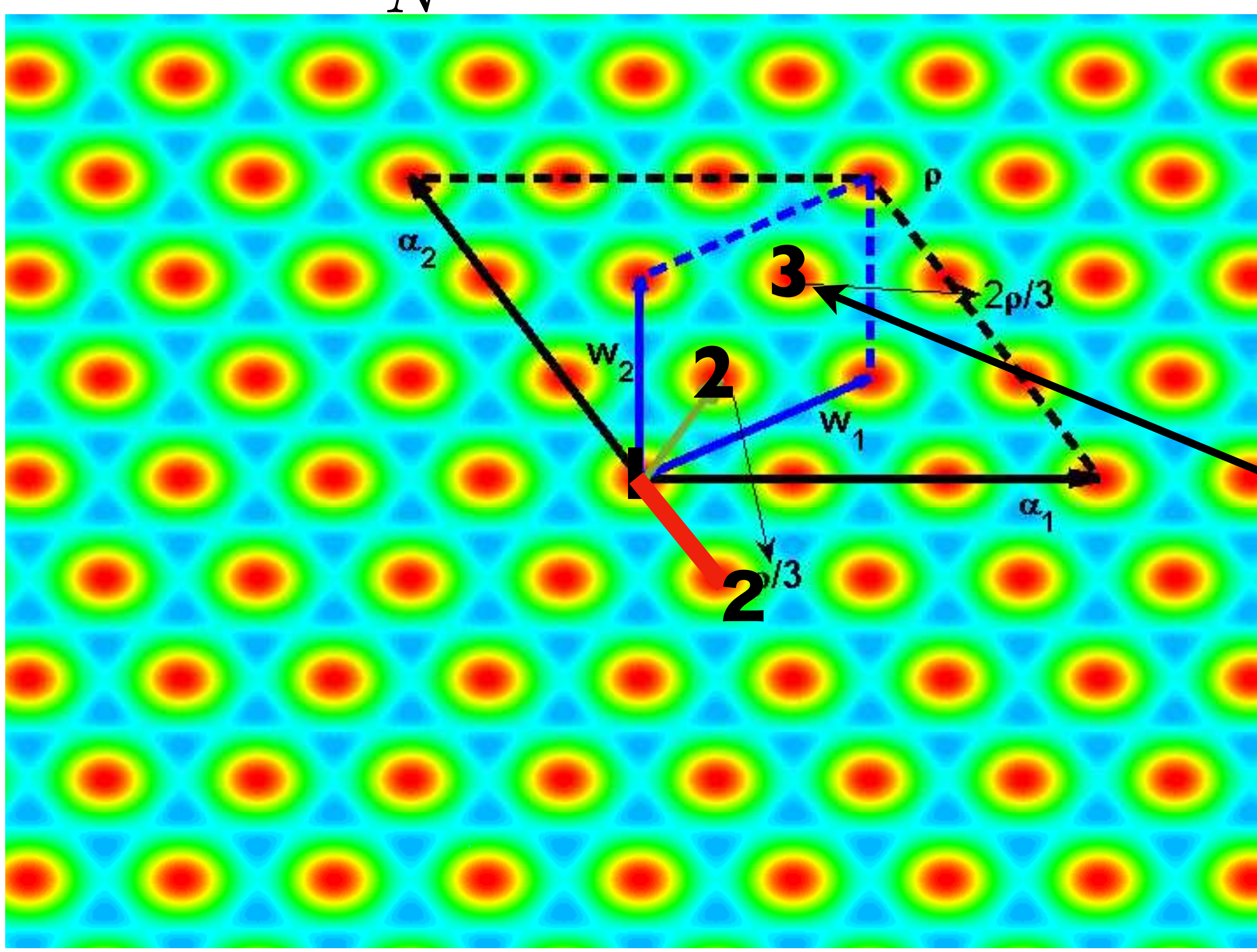

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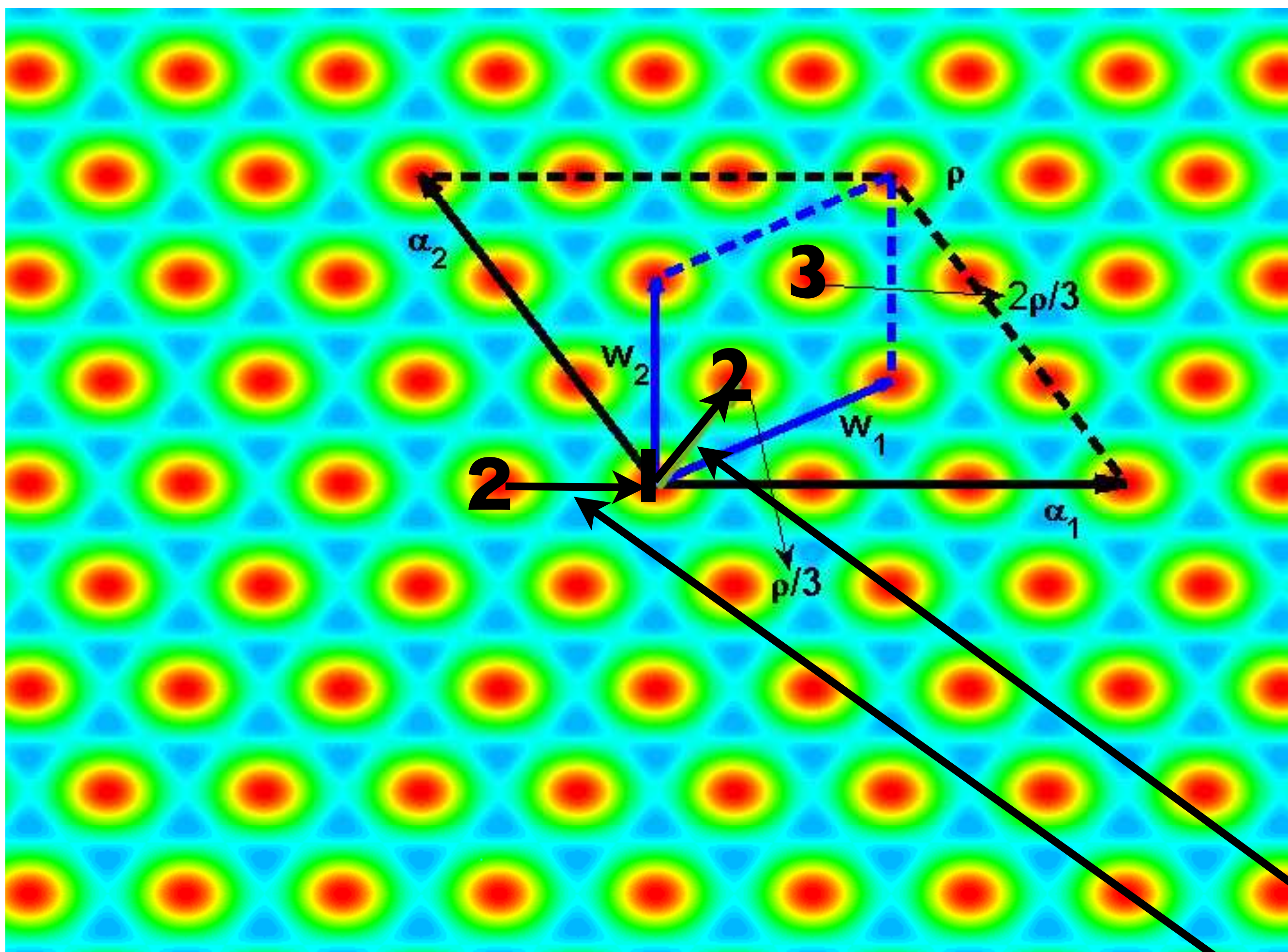
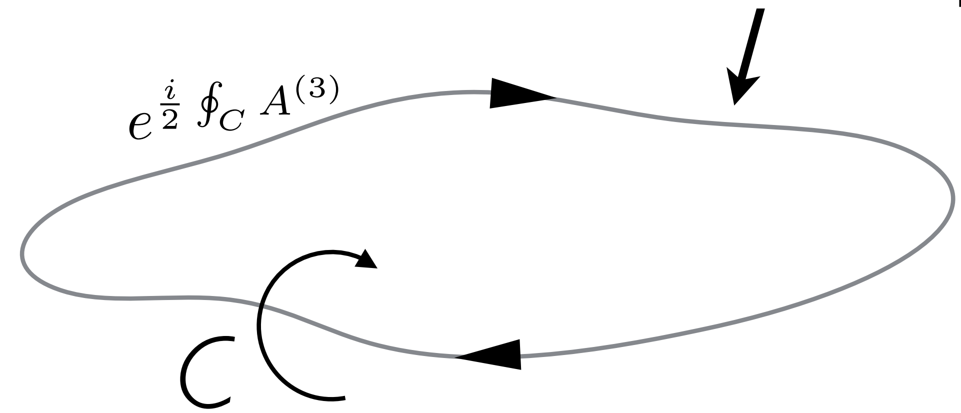


confinement: Wilson loop



$$\oint_C d\boldsymbol{\sigma} = 2\pi\boldsymbol{\lambda}.$$

monodromy

$$\partial_x \sigma \sim E_y$$

for, e.g., a weight of the fundamental, say w1

DW1

P(DW1*)

2. small-S^1 domain walls… [1501.06773 Anber, Sulejmanpasic, EP]

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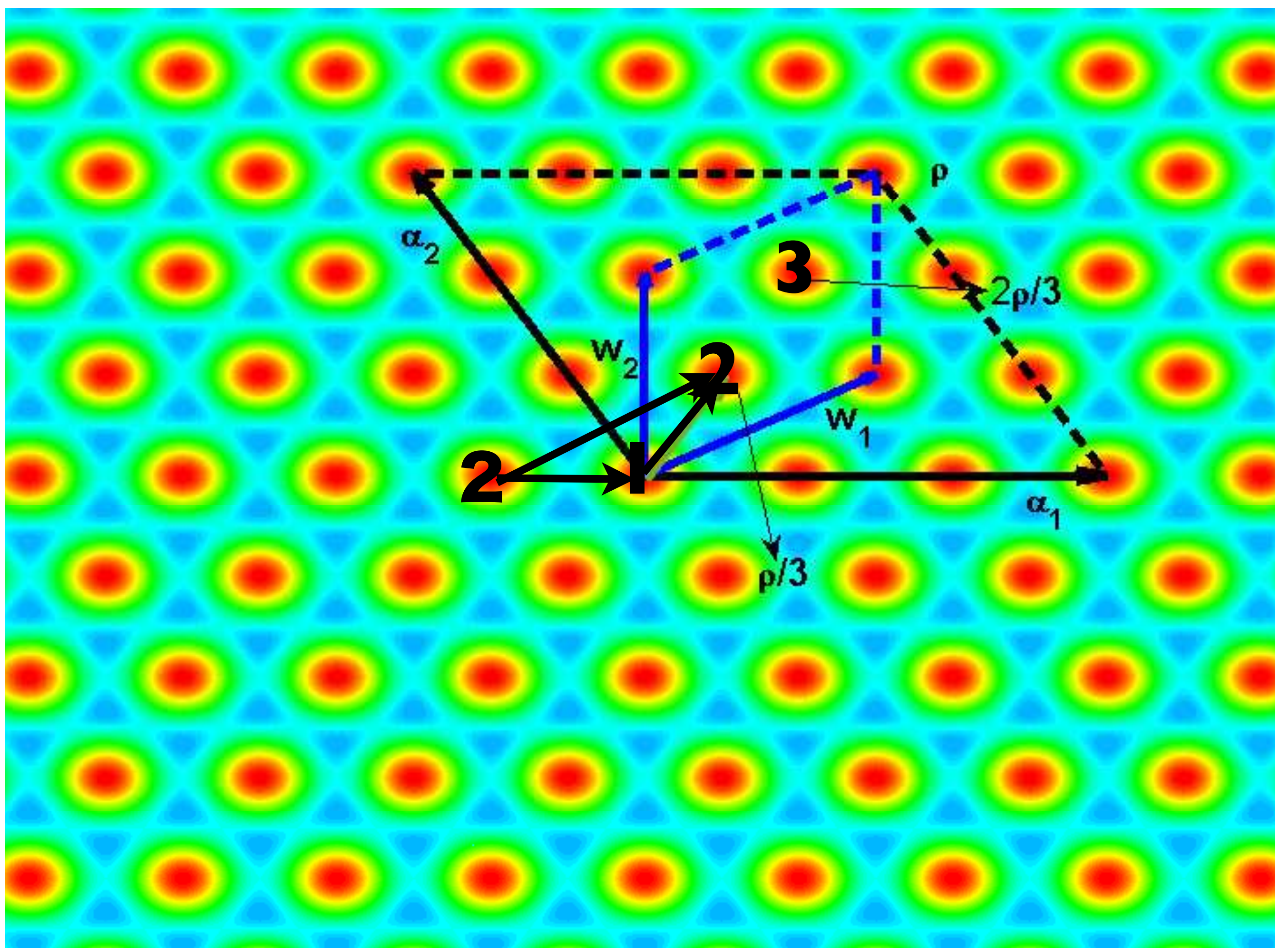

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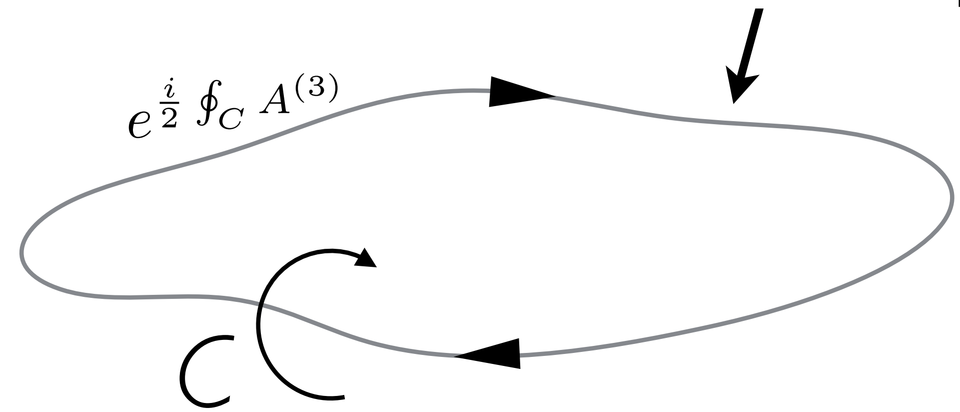

$$\oint_C d\boldsymbol{\sigma} = 2\pi\boldsymbol{\lambda}.$$

monodromy

w1 = DW1 “+” P(DW1*)

(pictured the “w1”-confining string in vacuum 2)

2. small-S^1 domain walls…

[1501.06773 Anber, Sulejmanpasic, EP]

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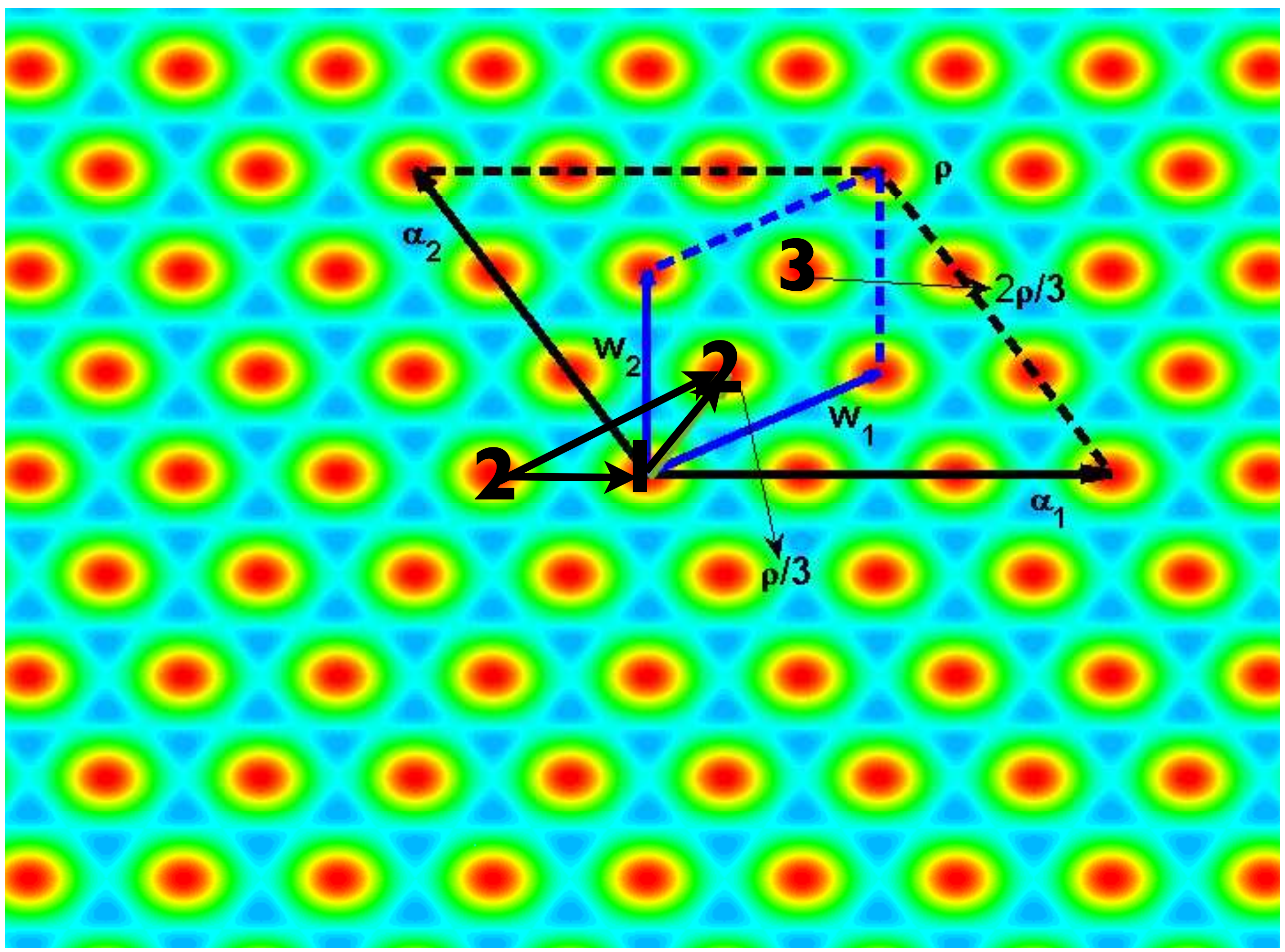

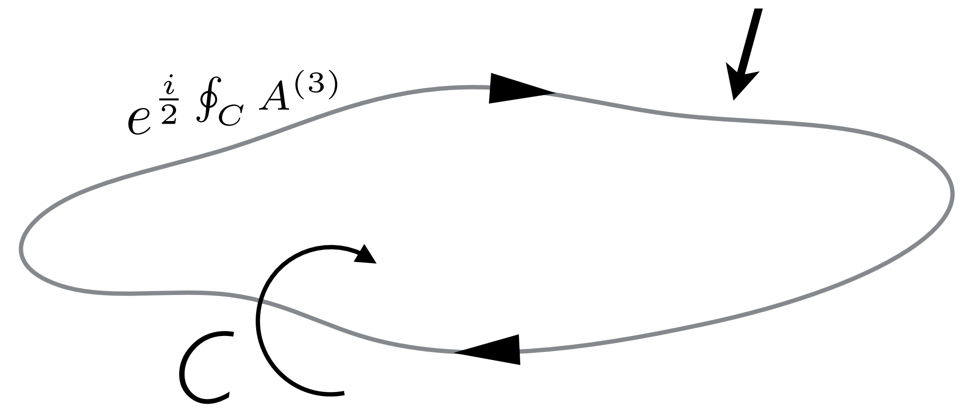

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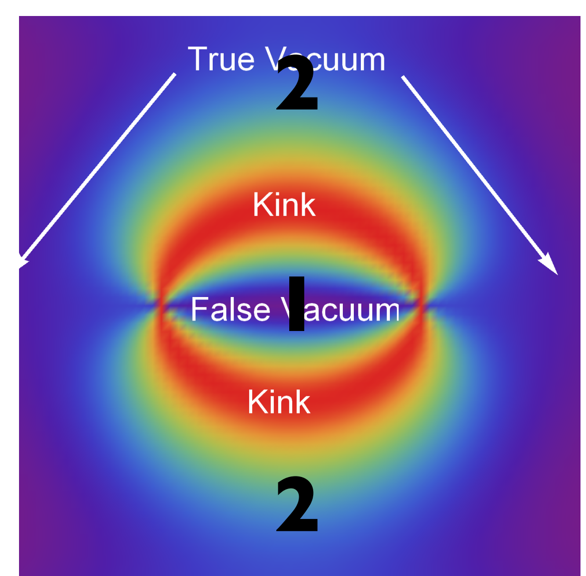

w1-string in vacuum 2

2. small-S^1 domain walls…

[1501.06773 Anber, Sulejmanpasic, EP]

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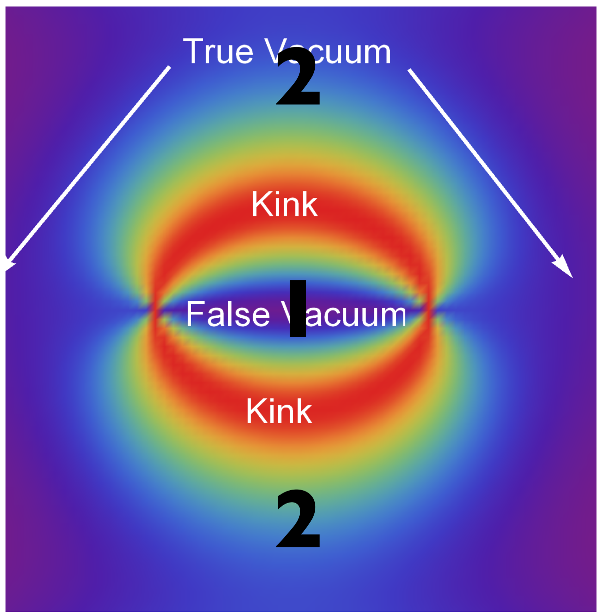

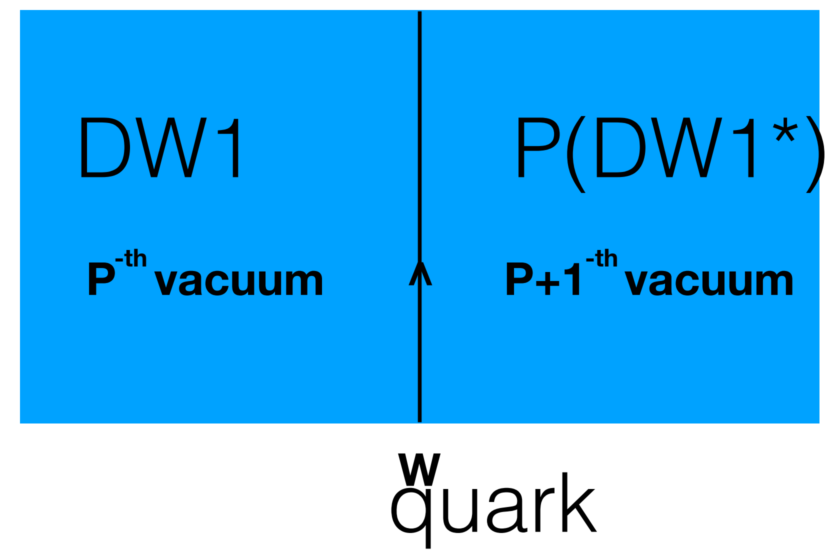

k=1 walls, at least, same “BF” 2d TQFT as high-T walls

semiclassical picture of confinement at small-L has some corollaries

1. baryons
2. strings ending on walls

2. small-S^1 domain walls…

[1501.06773 Anber, Sulejmanpasic, EP]

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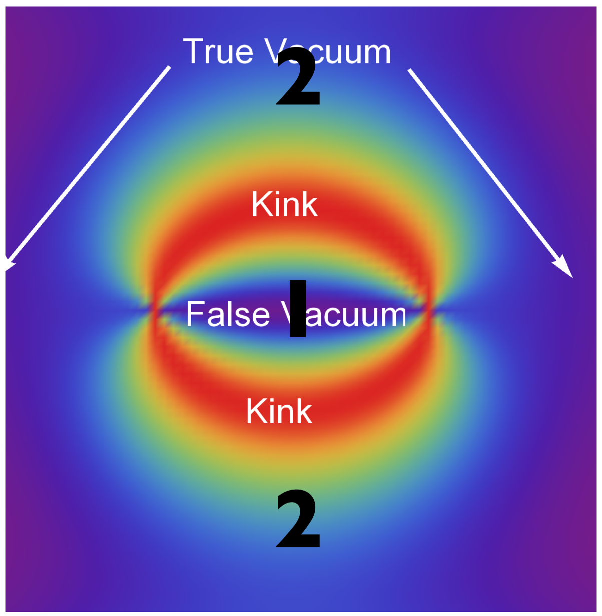
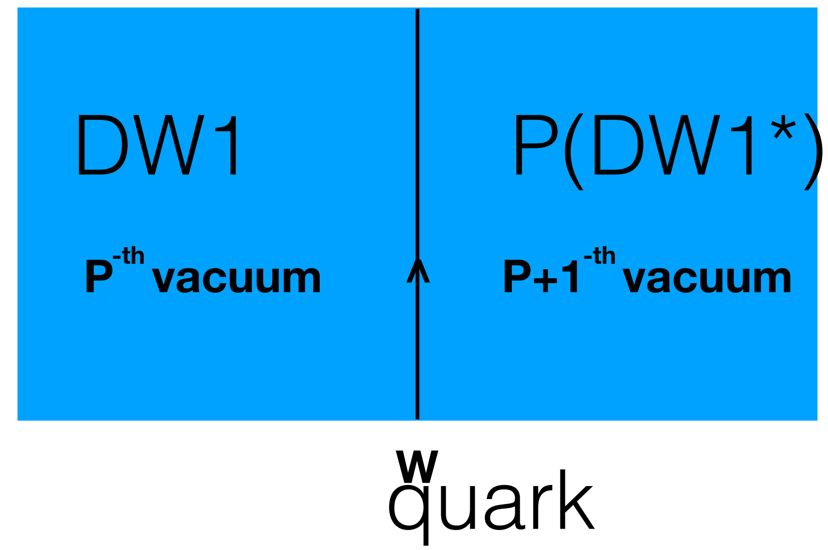


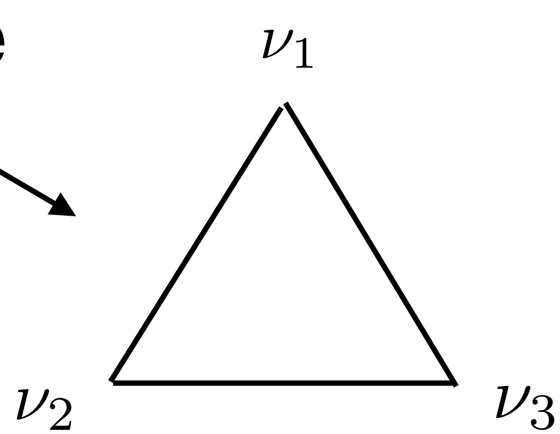


k=1 walls, at least, same "BF" 2d TQFT as high-T walls

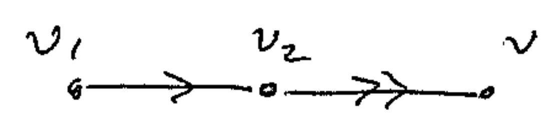
the three k=1 walls have the right fluxes

baryons:

Δ-like



cf SW theory:



2. small-S^1 domain walls... [1501.06773 Anber, Sulejmanpasic, EP]

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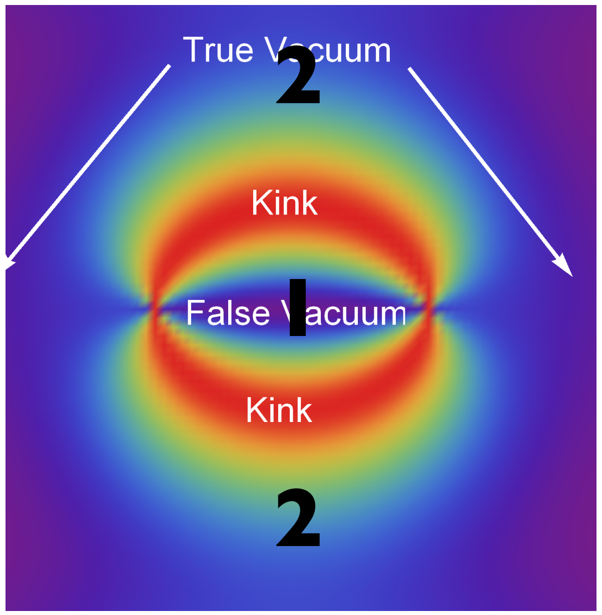

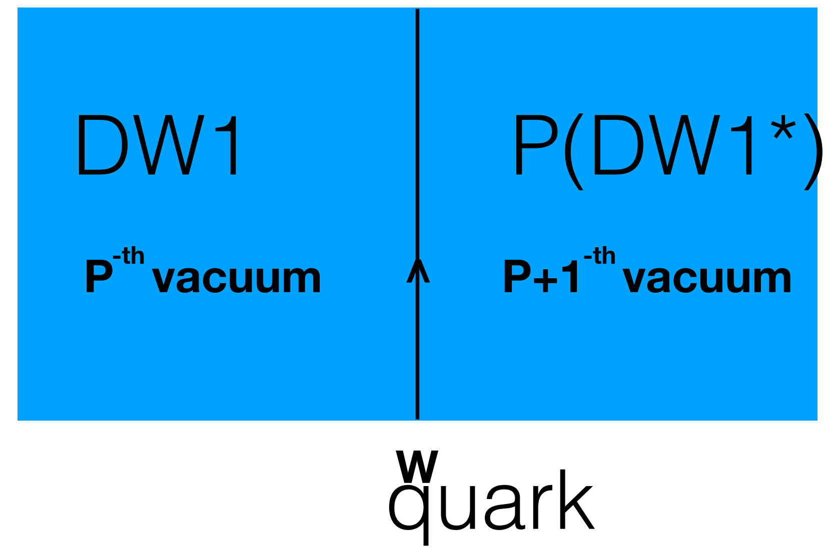

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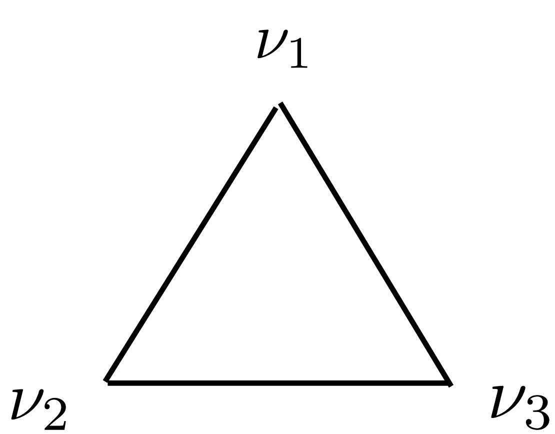
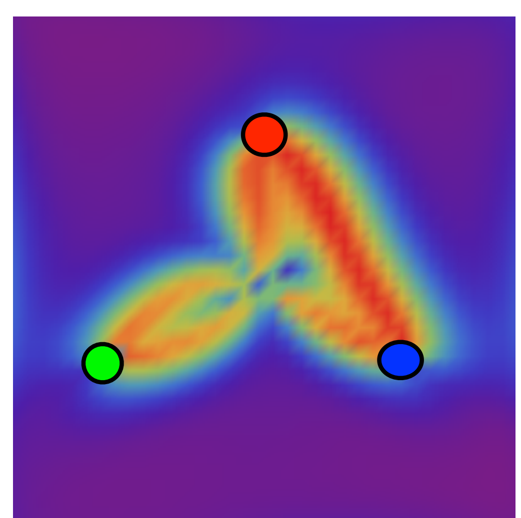
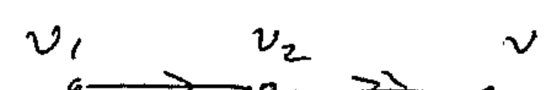




cartoon of a baryon in SU(3) QCD(adj)

cf SW theory:



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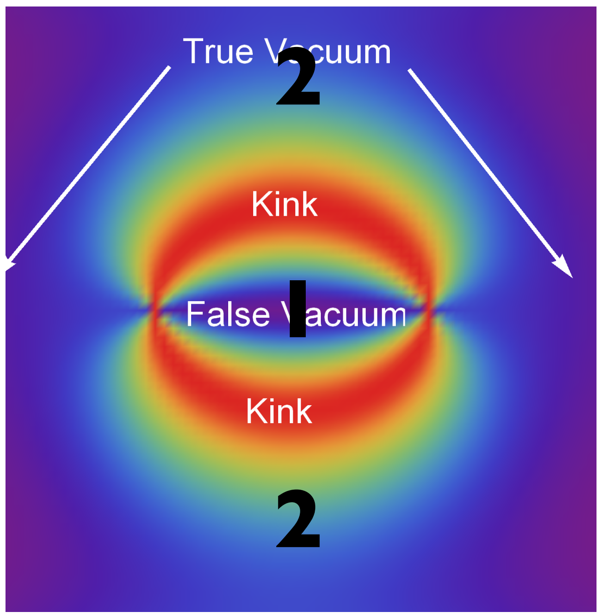
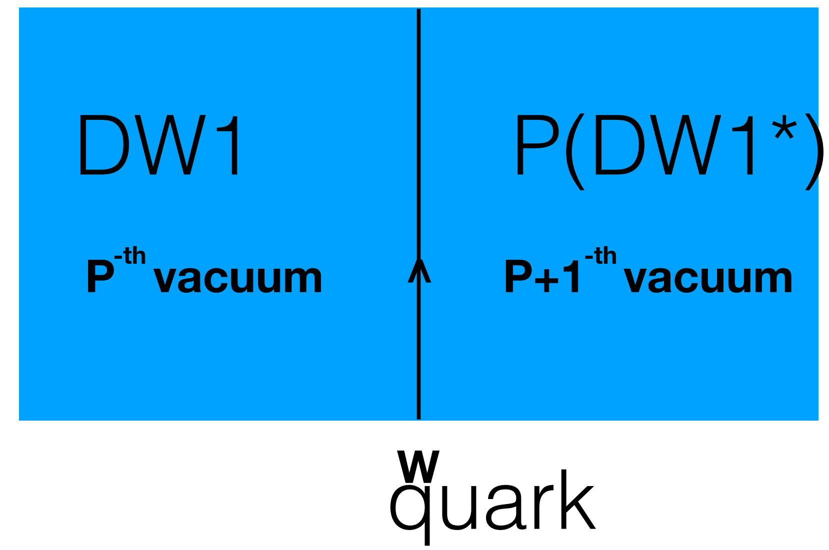
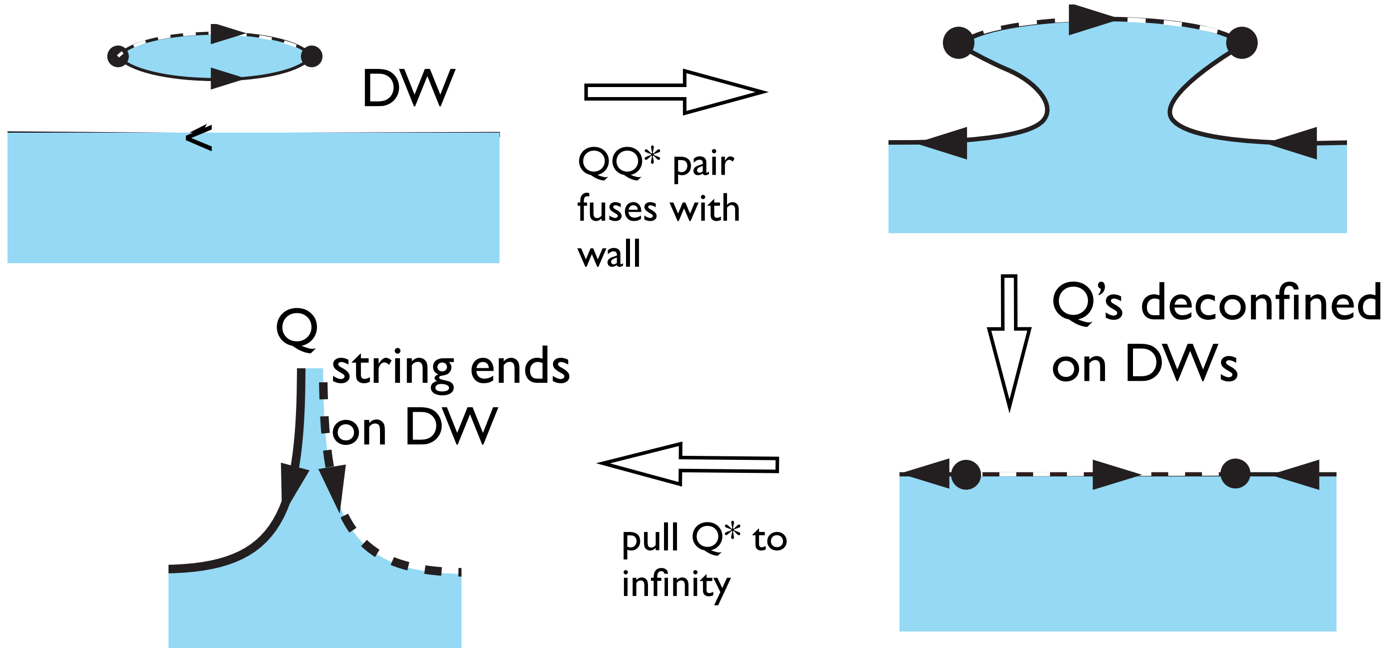




further, as seen in MQCD [Witten, 1998] confining strings end on DW



Summary/conclusions/…:

New ’t Hooft anomalies, missed before, have interesting implications; here: discrete chiral/center mixed anomaly

Example 1: k-walls between high-T center-broken vacua, rich worldvolume dynamics (SYM)

- 2d QCD (non-susy) on the k-wall
- screening (one-from center breaking) on wall
- confining strings ending on wall (~ F1/D1)
- high-T chirally restored phase, but $\langle \mathrm{tr} \psi_+ \psi_- \rangle \neq 0$ on wall

high-T walls share properties of zero-T bulk and zero-T walls

Summary/conclusions/…:

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Future:

- nf>1 high-T k-walls detail
- zero-T k>1 semiclassical (?) walls vs. Acharya-Vafa
- lattice and high-T k-walls

  & low-T “center vortices” and confinement; k-wall condense?

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- what other consistency conditions have been missed?