Domain walls and deconfinement: a semiclassical picture of discrete anomaly inflow

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with Andrew Cox, Samuel Wong U of T

1909.10979, JHEP

based on earlier "pre-anomaly" work

with Mohamed Anber, Tin Sulejmanpasic
Lewis&Clark Durham 1501.06773, PRD

[related to talks by Anber and by Bandos]

Motivation

Recent exciting development in QFT: new ’t Hooft anomaly matching conditions - Gaiotto, Kapustin, Komargodski, Seiberg,… 2014-

We all thought anomaly matching was ‘set in stone’ since ca. 1980. Played major role in, say, Seiberg dualities in the 1990’s…

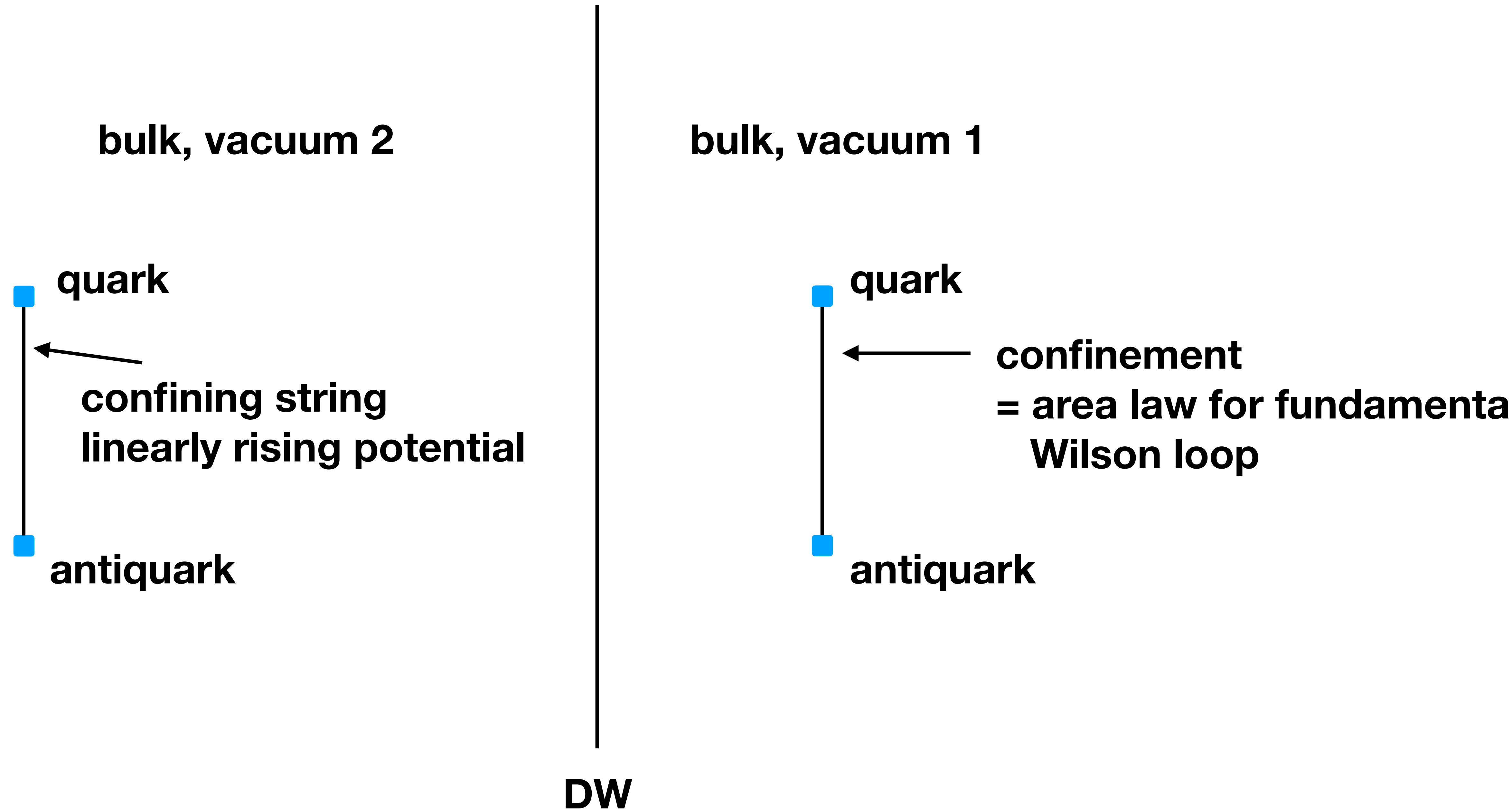
Turns out things have been missed; anomaly matching back to being an area of active research. Some striking consequences, e.g.

- **1.** “Dashen phenomenon” [~1960s!] in QCD at $\theta = \pi$ due to anomaly matching - CP breaking and domain walls (DWs); worldvolume “nontrivial” due to “discrete anomaly inflow”
- **2.** constraints on possible IR phases of various 4d gauge theories… see Anber’s talk

this talk:

- **3. focus on deconfinement of quarks on DWs**

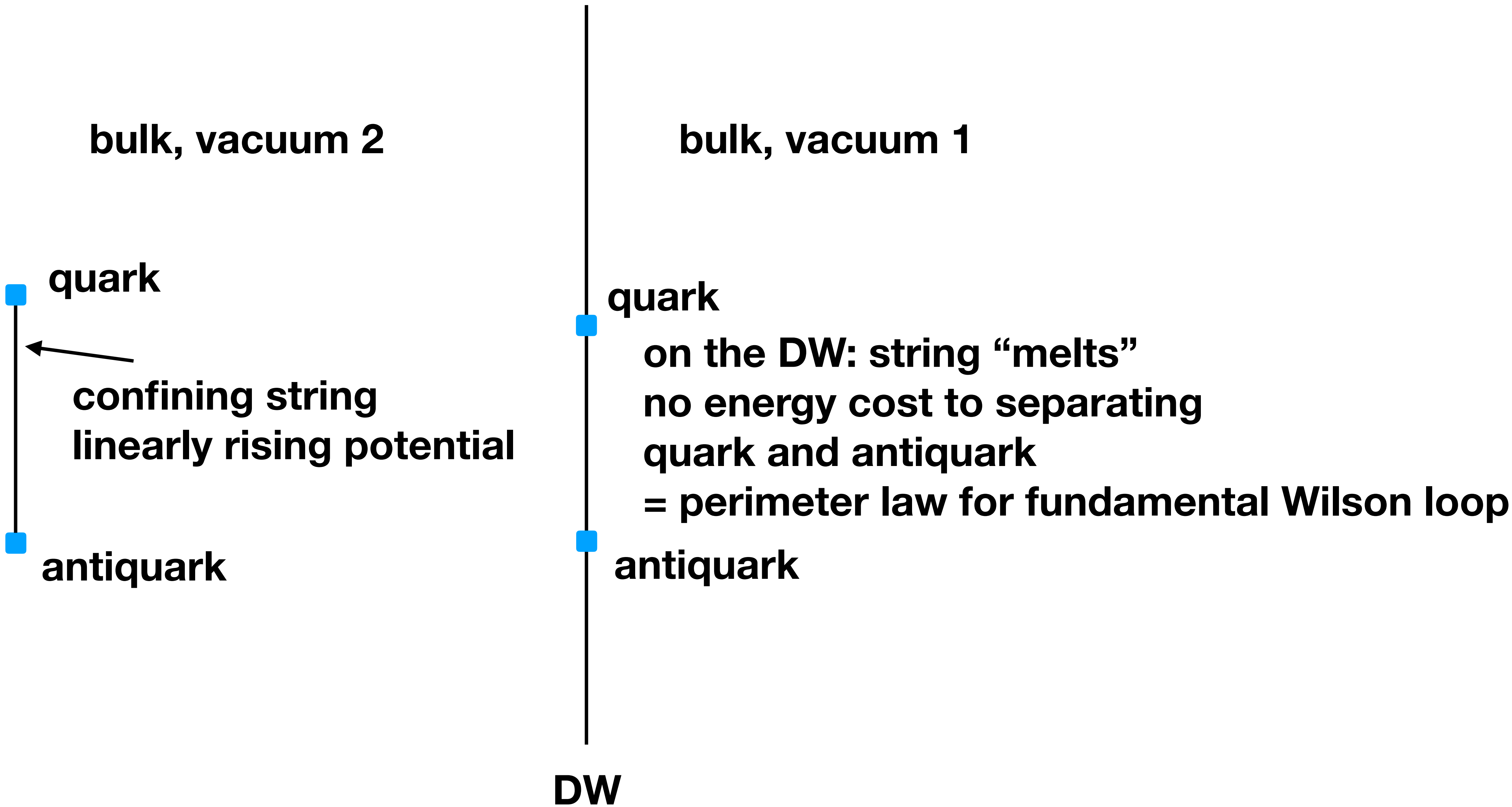
- focus on deconfinement of quarks on DWs



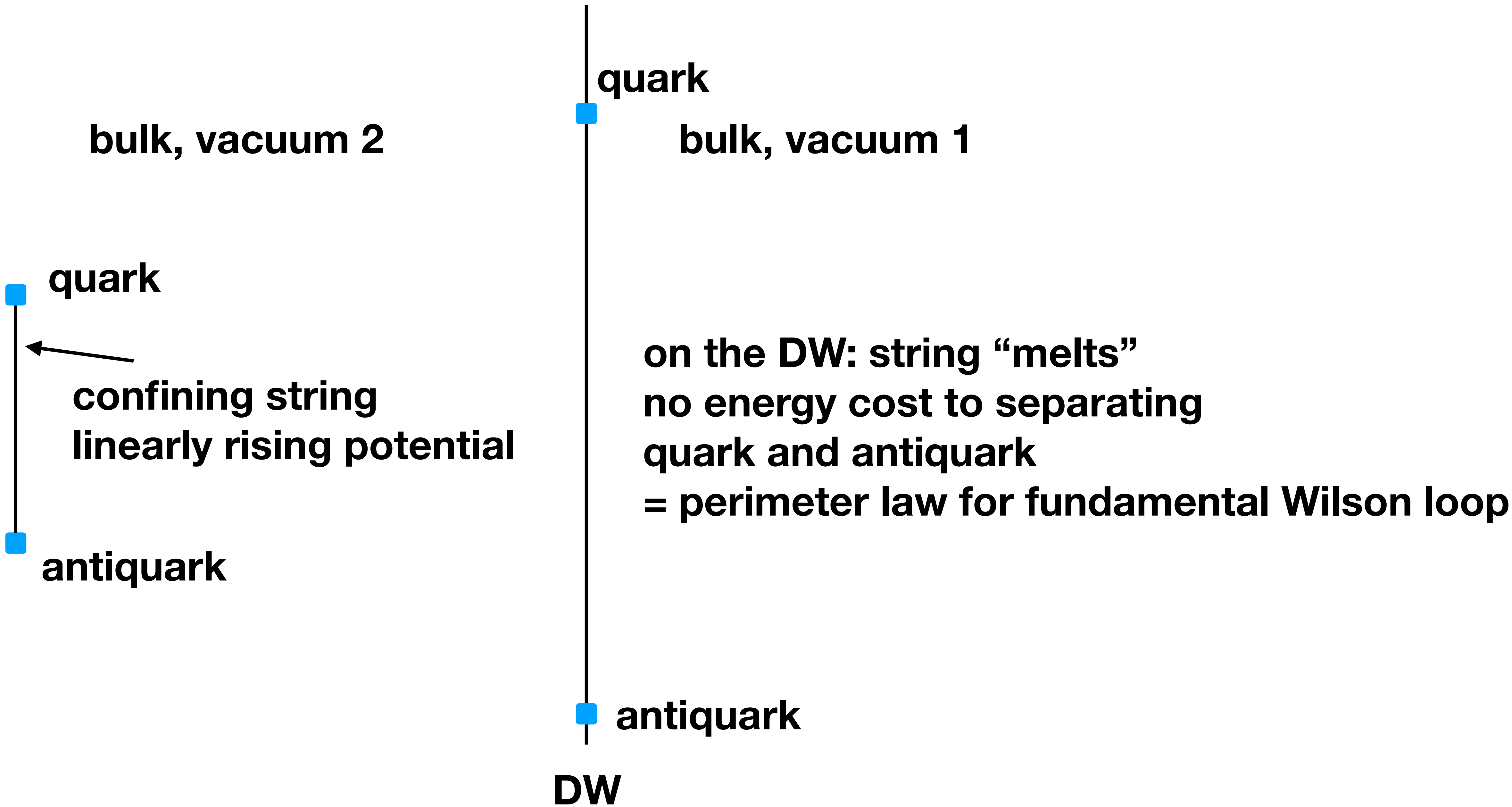
picture applies to a variety of theories with broken discrete symmetries:

QCD with broken CP, SYM with broken discrete chiral…

- focus on deconfinement of quarks on DWs



- focus on deconfinement of quarks on DWs



- focus on deconfinement of quarks on DWs

“explanation” of deconfinement on DW somewhat formal

- 3d CS theory (TQFT!) ‘lives’ on DW [e.g. Acharya-Vafa late 1990’s] Wilson loops in CS known to obey perimeter law (and to have nontrivial braiding); associating CS Wilson loops with fundamental Wilson loop in gauge theory then implies deconfinement

- MQCD picture of confining (F-) strings ending on (D-/M-) walls [Soo-Jong Rey, 1997; Witten, 1997]

- connection to mixed “CP/or other discrete/-center $(Z_N)^2$” anomaly

new [Gaiotto et al…]

- focus on deconfinement of quarks on DWs

predicted by formal anomaly inflow arguments in a variety of gauge theories with mixed discrete 0-form/1-form anomalies

prediction of deconfinement on DW somewhat indirect

based on DW worldvolume CS theories matching relevant anomaly

- it is nice to have a more physical picture
- difficult on R^4, where theory strongly coupled
  entails having a theory of confinement…
- possible on $R^3 \times S^1$, where a weak coupling realization of confinement and a nonperturbative semiclassical study of the vacuum is trustable! [Unsal et al, 2007-]

In fact, deconfinement on DWs was found in 2015 (Anber, Sulejmanpasic, EP) based on honest semiclassical analysis… before relation to anomaly inflow understood - ***which we explain and extend now:***

- focus on deconfinement of quarks on DWs

Here, consider SYM with discrete chiral symmetry

this story applies to nonSUSY YM (e.g. QCD at $\theta = \pi$) as well…

To achieve semiclassical calculability, rather than only rely on SUSY, compactly theory on S^1: control parameter $\underline{NL\Lambda \ll 1}$

key features:

1. dynamical abelianization $SU(N) \to U(1)^{N-1}$ at $\dfrac{1}{NL}$

2. weak coupling

3. relevant d.o.f. at distances >> NL: "dual Cartan gluons"

4. mass gap & confinement due to the proliferation of nonperturbative semiclassical objects

-monopole-instantons - (d)YM

-magnetic bions - SYM/QCD(adj)

CRASH COURSE: SYM on S^1 discrete chiral/center anomaly

"dual Cartan gluons" $NL\Lambda \ll 1$

fundamental weights!

$$x^a = \phi^a + i\sigma^a\,, \quad a = 1, \ldots N-1, \quad \text{with} \quad \sigma^a \simeq \sigma^a + 2\pi w_k^a\,, \quad k = 1, \ldots N-1\,.$$

compact scalars: "dual photons"

$$\frac{g^2}{4\pi L}\,\partial_\mu \phi^a = F^a_{\mu 4}, \quad \mu, \nu = 0, 1, 2, \quad \frac{g^2}{4\pi L}\,\epsilon_{\mu\nu\lambda}\partial^\lambda \sigma^a = F^a_{\mu\nu}$$

$$L = M\;\partial_\mu x^a g_{ab}\,\partial^\mu \bar{x}^b - M\,\frac{m^2}{4}\,\frac{\partial W(\boldsymbol{x})}{\partial x^a}\,g^{ab}\,\frac{\partial \bar{W}(\bar{\boldsymbol{x}})}{\partial \bar{x}^b}$$

Kahler metric (calculable!)

$$g_{ab} = \delta_{ab} + \ldots$$

superpotential

$$W(\boldsymbol{x}) = \sum_{a=1}^{N} e^{\boldsymbol{\alpha}_a \cdot \boldsymbol{x}} \qquad \boldsymbol{\alpha}_N = -\sum_{a=1}^{N-1} \boldsymbol{\alpha}_a$$

N vacua

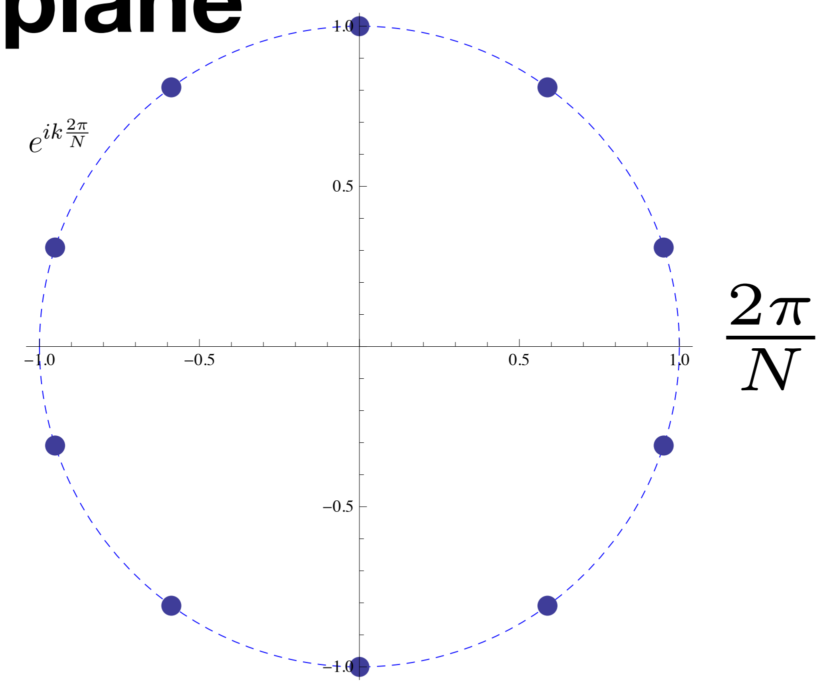
$$\langle W \rangle_k \equiv W_k = N e^{i\frac{2\pi k}{N}}$$

broken discrete chiral

$$\mathbb{Z}_{2N}^{(0)} \to \mathbb{Z}_2^{(0)}$$

N vacua $\mathbb{Z}_{2N}^{(0)} \to \mathbb{Z}_2^{(0)}$

W-plane



$$\langle W \rangle_k \equiv W_k = N e^{i\frac{2\pi k}{N}}$$

k=1,…,N

k-th vacuum

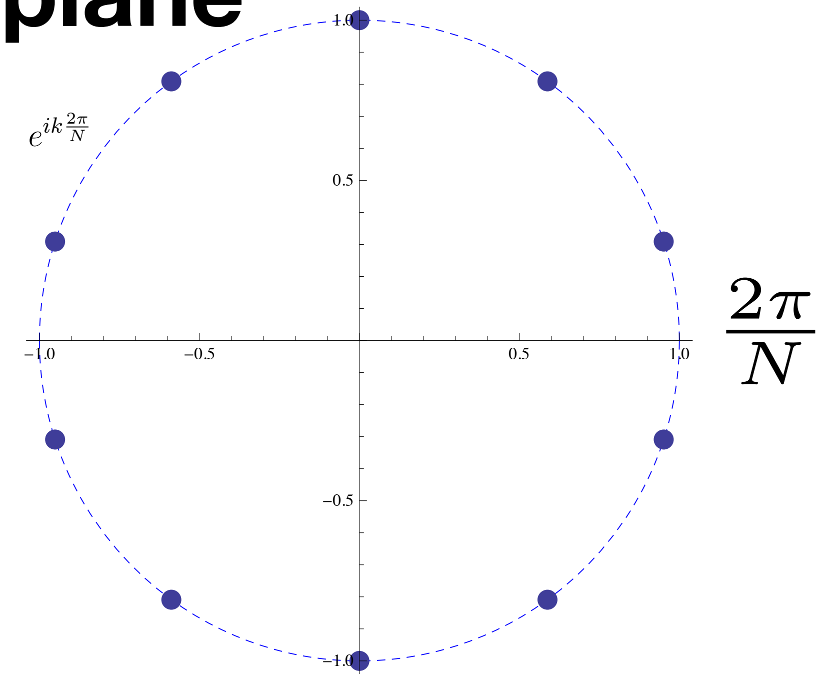
$$\langle \phi \rangle = 0 \quad \langle \boldsymbol{\sigma} \rangle_k = \frac{2\pi k}{N} \boldsymbol{\rho}$$

⟵ Weyl vector

k-walls: interpolate between vacua k units apart

N vacua $\mathbb{Z}_{2N}^{(0)} \to \mathbb{Z}_2^{(0)}$

W-plane



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k-th vacuum

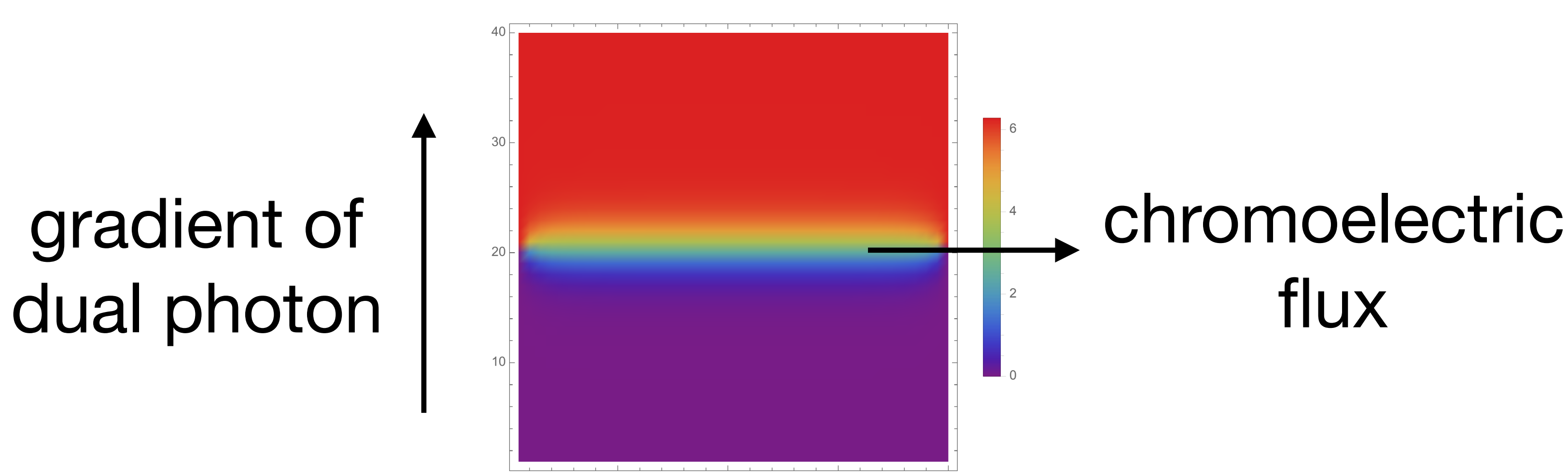
$$\langle \phi \rangle = 0 \quad \langle \boldsymbol{\sigma} \rangle_k = \frac{2\pi k}{N}\boldsymbol{\rho}$$ ← Weyl vector

k-walls: interpolate between vacua k units apart

k-walls carry electric flux:

gradient of dual photon ~ electric field $\partial_i \sigma \sim \frac{L}{g^2}\epsilon_{ij}E_j, \quad j = 1, 2$

space is 2 dimensional and DW are “domain lines” with flux:



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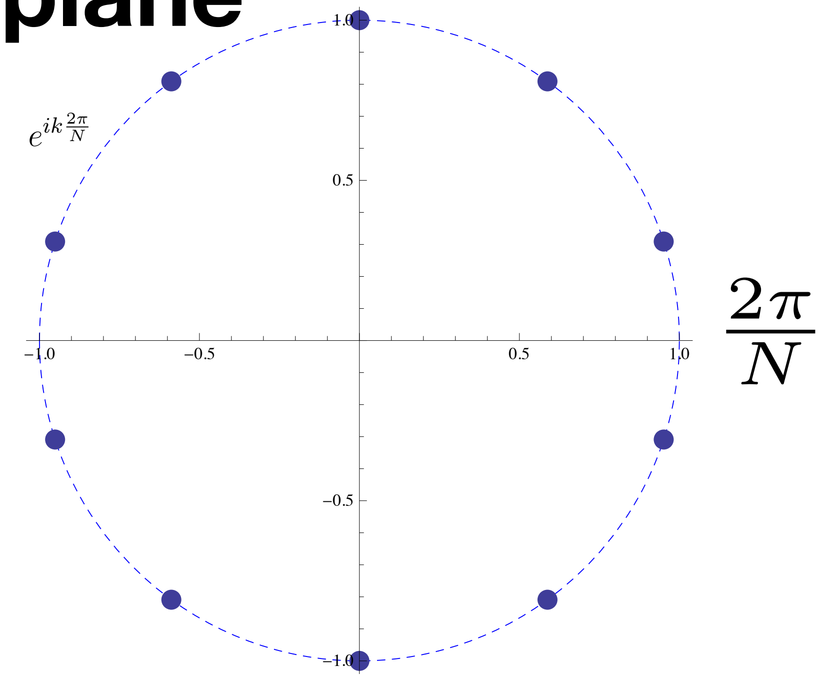

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k-walls: interpolate between vacua k units apart

What are the electric fluxes on the lowest tension (BPS) k-walls?

Question is nontrivial, since "dual photons" are compact scalars, can have extra monodromies across walls, in addition to that required by vacua they connect.

What are the electric fluxes on the lowest tension (BPS) k-walls?

ANSWER **1909.10979 with Cox and Wong**

there is a number of distinct BPS k-walls whose electric fluxes are given by:

$$2\pi\left(\boldsymbol{w}_{i_1} + \ldots + \boldsymbol{w}_{i_k} - \frac{k}{N}\boldsymbol{\rho}\right), \qquad \text{there are} \binom{N-1}{k} \text{ such walls,}$$

$$2\pi\left(\boldsymbol{w}_{j_1} + \ldots + \boldsymbol{w}_{j_{k-1}} - \frac{k}{N}\boldsymbol{\rho}\right), \qquad \text{there are} \binom{N-1}{k-1} \text{ such walls.}$$

$(i_1, \ldots, i_k)$ **and** $(j_1, \ldots, j_{k-1})$ **to be all taken different from 1…N-1**

$$\binom{N-1}{k-1} + \binom{N-1}{k} = \binom{N}{k}$$ **distinct BPS k-walls**

old story: **total number of distinct BPS k-walls is** $\frac{N!}{k!(N-k)!}$ **per**

Ceccoti-Vafa; Acharya-Vafa; Hori-Iqbal-Vafa…1990s-2000

new story: **the electric fluxes DWs carry, confinement in the bulk and deconfinement on the wall…**

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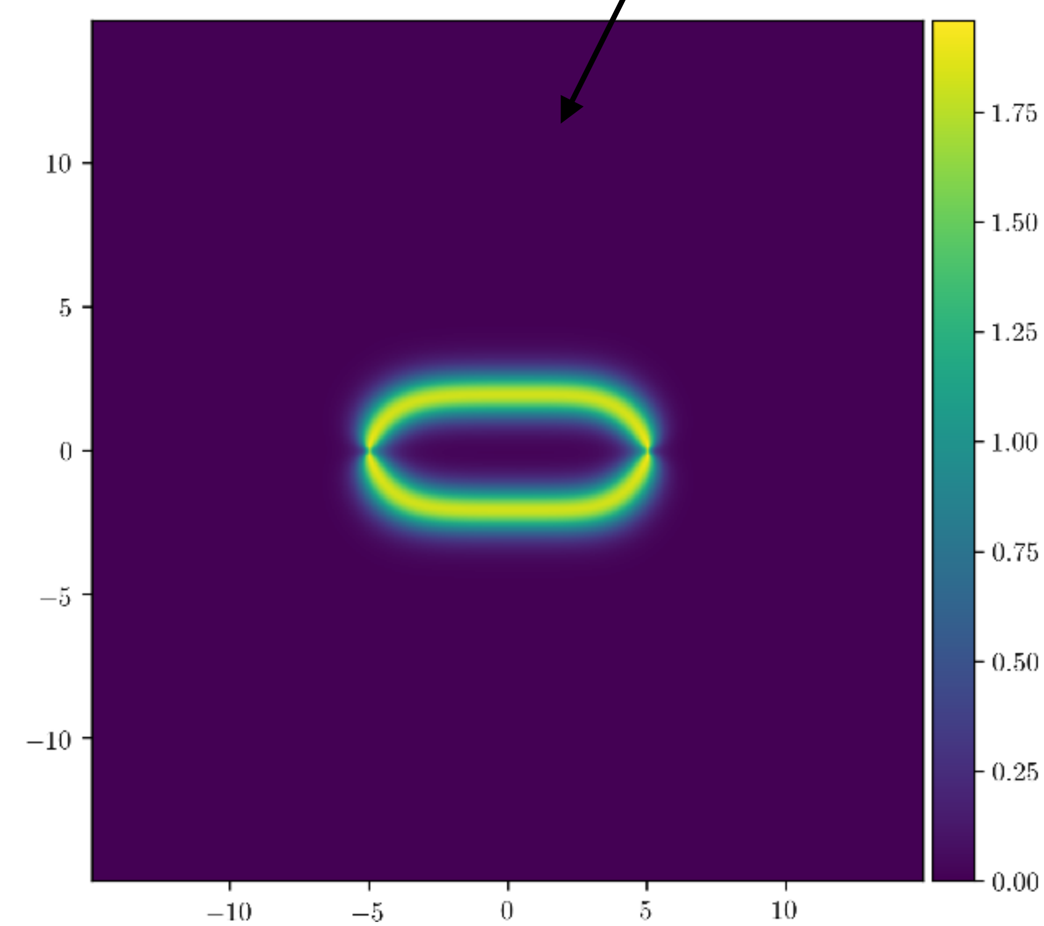
the combinatorics of fluxes given above is important for the general statements, but I won't bore you with that… but some pictures are due:

confinement in the bulk also in 1501.06773

DWs carry half the chromoelectric flux of quarks of weights w_k, highest weight of k-index antisymmetric (and all weights of fundamental rep)

vacuum 1; different vacuum inside



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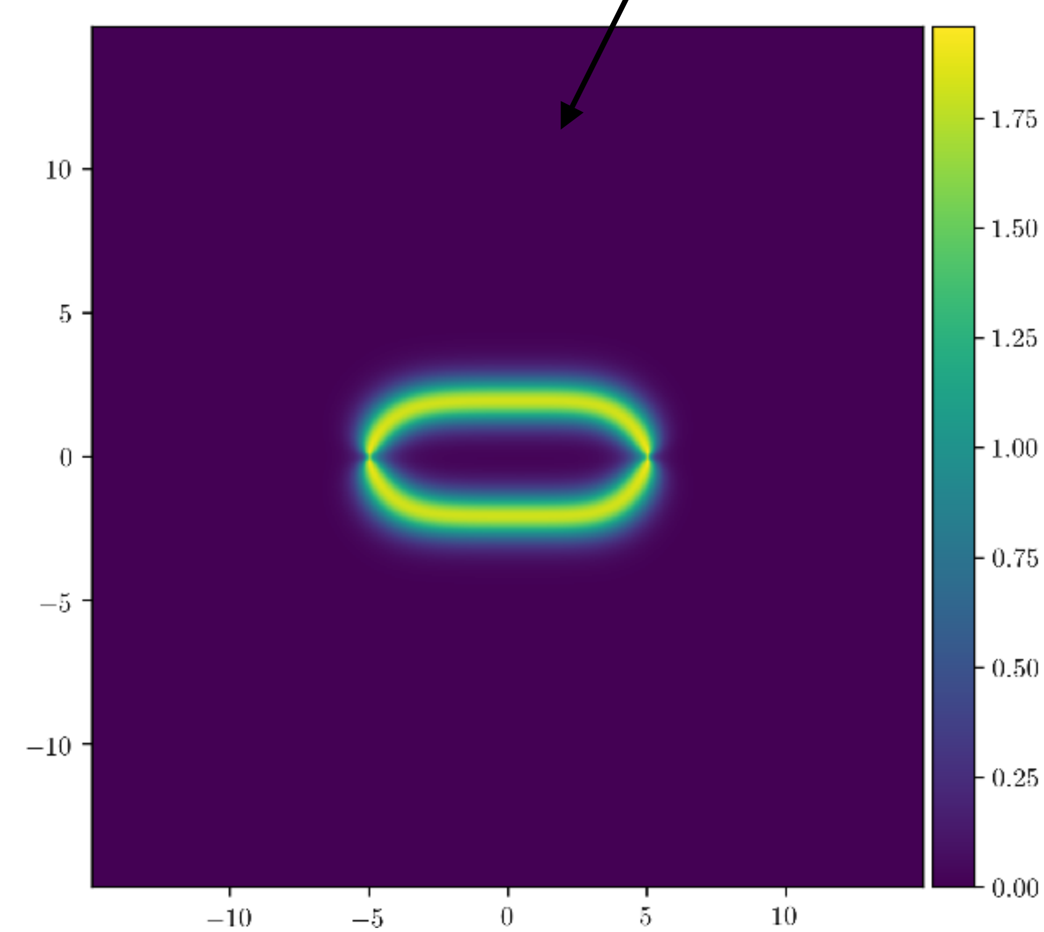


Thus, in the confining bulk, the quark's flux splits between two BPS DWs; inside the confining "double string", there is another vacuum.

Area law holds for all nonzero N-ality quarks due to vacuum degeneracy of inside and outside vacua.

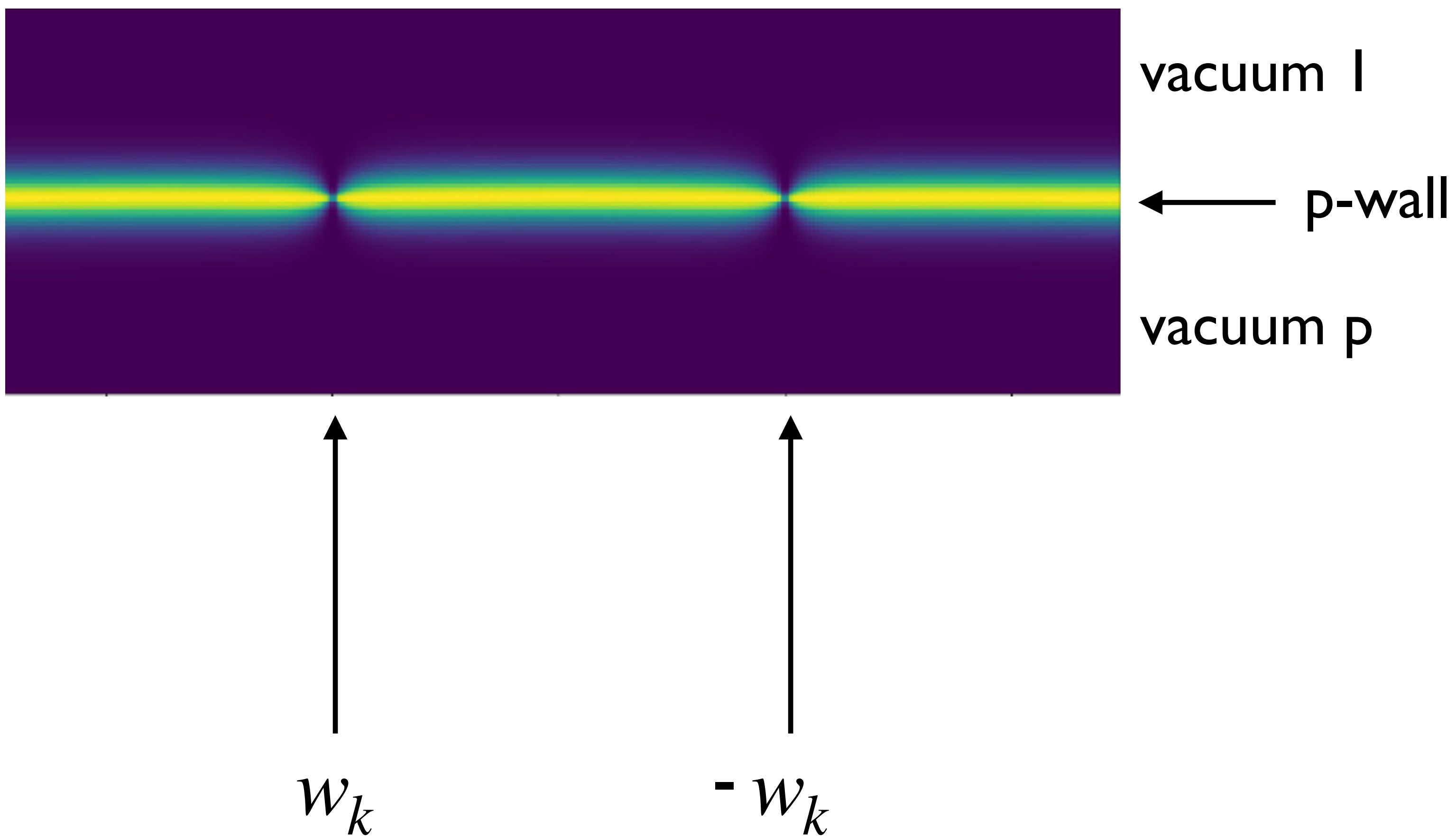
"Double-string" confinement also for nonSUSY YM at theta=pi, QCD(adjoint)….

new story: **the electric fluxes DWs carry, confinement in the bulk and deconfinement on the wall…**

deconfinement on the wall also in 1501.06773

Due to the DW multiplicity, the electric flux of a quark, w_k can be split to the left and the right.

Due to the BPS degeneracy of DWs, there is no tension difference and no force between the quark/antiquark.



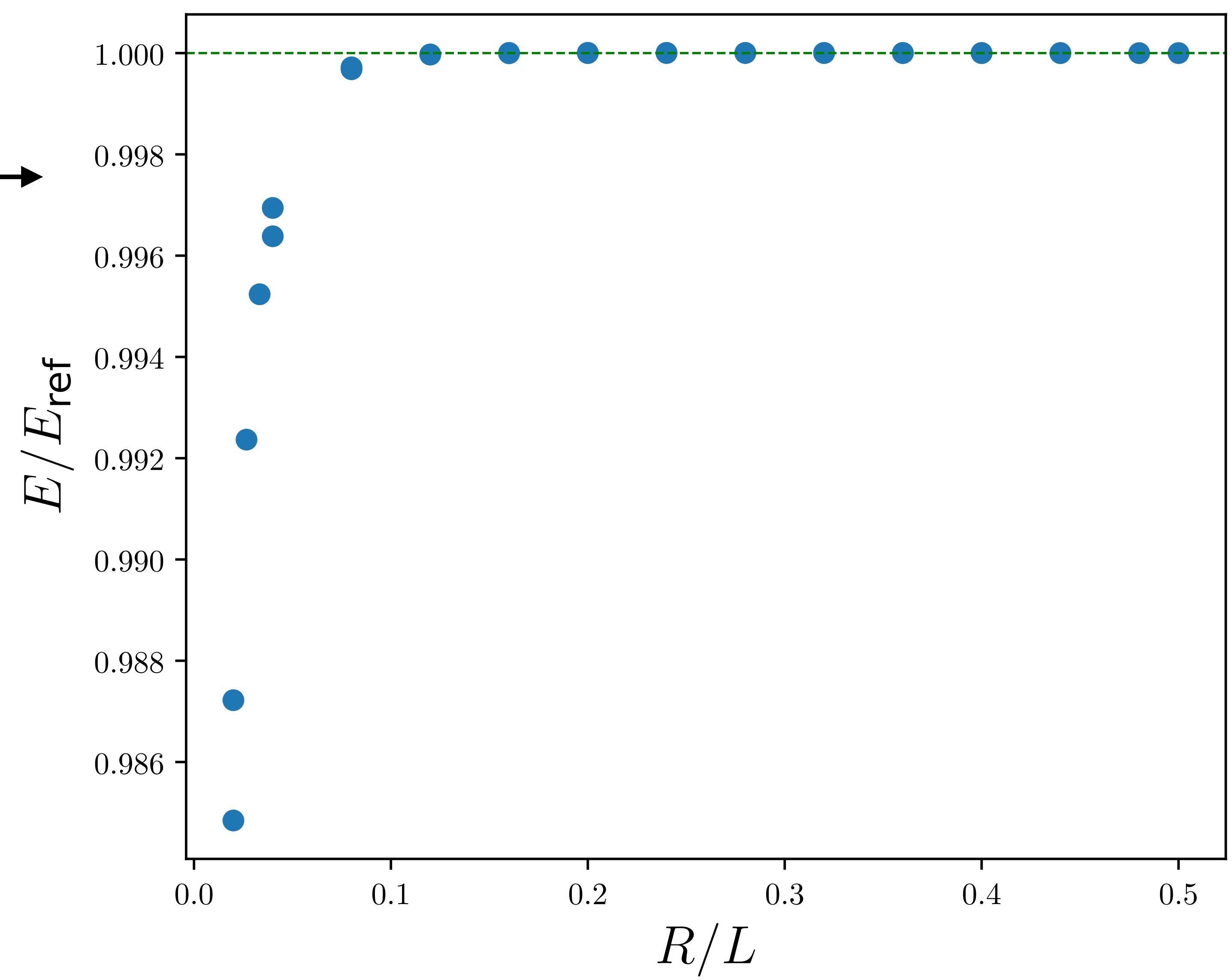
weights of quark/antiquark, any k=1,…N-1

new story: **the electric fluxes DWs carry, confinement in the bulk and deconfinement on the wall…**

deconfinement on the wall also in 1501.06773

Due to the BPS degeneracy of DWs, there is no tension difference and no force between the quark/antiquark:

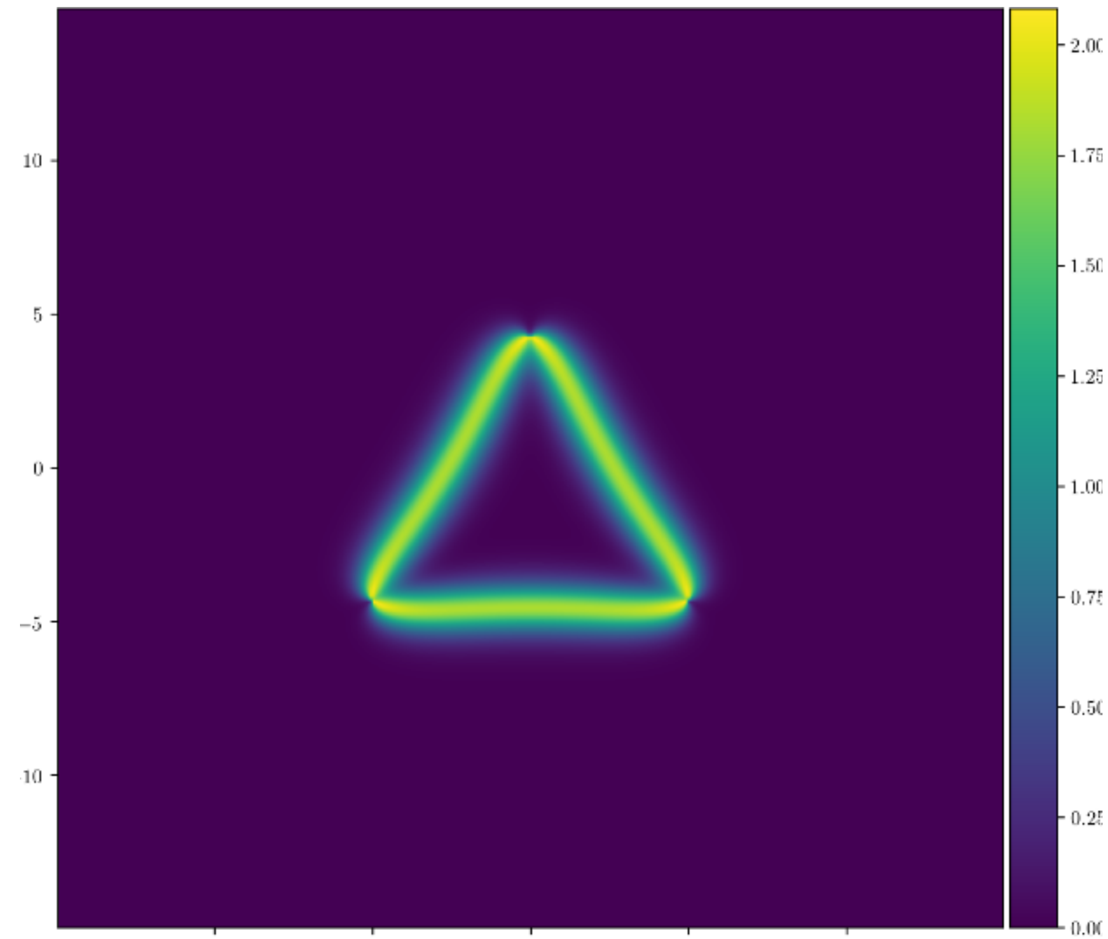
energy as a function of separation of quark and antiquark along DW →



new story: **the electric fluxes DWs carry, confinement in the bulk and deconfinement on the wall…**

if time permits one more picture… a static heavy baryon in SU(3) SYM is

(an N-polygon in SU(N))



notice marked contrast with Seiberg-Witten theory, where baryons are linear only [reason: unbroken 0-form center…]

Time to conclude:

Anomalies, vacuum structure, confinement and deconfinement on DWs quite intertwined, in nontrivial ways.

Studied a weakly-coupled semiclassically tractable example of the implications of anomaly inflow for the 0-form/1-form anomalies.

Physical picture appealing, based on our detailed understanding of the "double-string" confinement mechanism on $R^3 \times S^1$.

Applies also to various non-SUSY YM ($\theta = \pi$), QCD(adj).

Notice that symmetry/anomaly often not enough to fix the DW "worldvolume TQFT" [Cordova, Freed, Lam, Seiberg 2019].

Even in the case at hand, we only understand the TQFT on the k=1 walls (Anber, EP 1811.10642). For k>1 DWs on $R^3 \times S^1$ open…

For other theories on R^4 (e.g. ones with BCF anomaly, Anber's talk) DW inflow also still open …