Confinement and strings in circle-compactified gauge theories: "same and different"

Erich Poppitz U of T

An overview and some recent results,
will mention *some aspects* of work with

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Larry Yaffe, U. of Washington 1707.08971

After a typical QFT course, you hear from grad students: confinement occurs because the beta function is negative and coupling becomes strong.

[But: asymptotically free theories exist that don't confine!]

... which is to say "it's complicated" and we won't think about it

... will leave it to experimentalists (=lattice people)

These attitudes have merit: indeed, in real-world YM (QCD) confinement occurs at strong coupling & is 'nonperturbative'.

It was realized within the past 10 years, that there are 'deformations' of YM theory, which allow for controlled analytical studies of confinement at weak coupling. It is believed that they are continuously connected - without phase transition - to the real-world YM theory.

↑

This is the subject of my talk.

Note: I cannot review all approaches to the confinement problem, see Jeff Greensite's book.

I want to explain, as nontechnically as I can, what are these 'deformations' of YM theory, which allow for controlled analytical studies of confinement at weak coupling,

what we learn, and what is the evidence that there is a continuous connection to YM theory on R^4.

The main tool is this: the 'deformation', which allows us to divorce 'nonperturbative' from 'strong coupling' is a judiciously chosen circle compactification of YM theory.

For this talk, I will focus on 4d YM theory with SU(N) gauge group and Nf adjoint Weyl fermions, massive or not.
My theory space:

Nf=1 Weyl, massless: SYM with four supercharges

6>Nf>1 Weyl, massless: QCD(adj)

6>Nf>1 Weyl, massive: dYM, d = "deformed"

Nf=1 Weyl, massless:	SYM with four supercharges
6>Nf>1 Weyl, massless:	QCD(adj)
6>Nf>1 Weyl, massive:	dYM, "d" = "deformed"

I will study these theories compactified on a circle of circumference L. It is crucial that this is a spatial circle and fermions are periodic (not finite T!). Thus, spacetime is really $R^{1,2}$x S^1, but I will usually call it R^3xS^1, for brevity.

Weak coupling is assured - will explain - if circle is small

$$NL\Lambda \ll 1$$

[Note: peculiar double-scaling large-N limit can be taken with L-> 0. Strange things happen then... Cherman EP 2016]

Nf=1 Weyl, massless: SYM with four supercharges

6>Nf>1 Weyl, massless: QCD(adj)

6>Nf>1 Weyl, massive: dYM, "d" = "deformed"

circle, with $\underline{N L \Lambda \ll 1}$ + periodic (around L) fermions

key features:

1. dynamical abelianization $SU(N) \to U(1)^{N-1}$ at 1/NL

2. weak coupling

3. relevant d.o.f. at distances >> NL: "dual Cartan gluons"

4. mass gap & confinement due to the proliferation (not really 'condensation') of nonperturbative semiclassical objects

-monopole-instantons - dYM

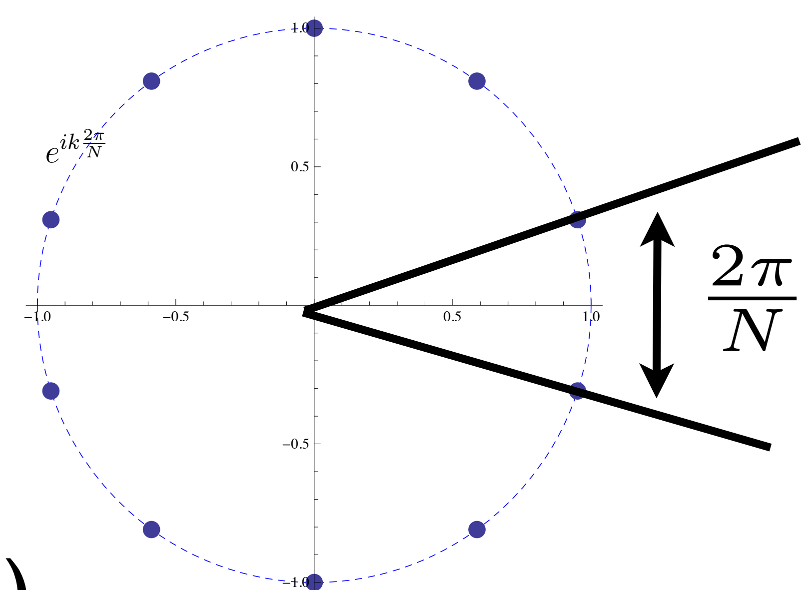
-magnetic bions - SYM/QCD(adj)

1. dynamical abelianization $SU(N) \to U(1)^{N-1}$ at 1/NL

2. weak coupling

on a circle, new gauge invariants:
traces of powers of noncontractible Wilson loop

$\Omega = \quad \mathcal{P}\exp[i\int_{S^1} A_4 dx^4]$ unitary NxN matrix, unit determinant



small-L, periodic adjoints:

vevs of $e^{iA_4 L}$ ~ $e^{ik\frac{2\pi}{N}}$
(k=1,...,N)

lightest W-bosons ~ 1/NL, only Cartan survive! 1.✓

weak coupling: W-boson mass ~ 1/NL >> Λ 2.✓

unbroken center symmetry ("zero form"): $\mathrm{tr}\,\Omega^p \to e^{\frac{2\pi i p}{N}}\,\mathrm{tr}\,\Omega^p$

√1. dynamical abelianization $SU(N) \to U(1)^{N-1}$ at 1/NL

√2. weak coupling: g = 4d gauge coupling frozen at 1/NL

3. relevant d.o.f. at distances >> NL: “dual Cartan gluons”

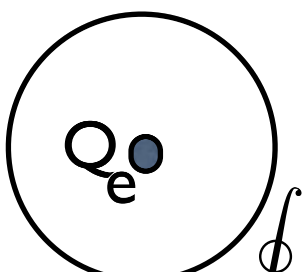
long distance theory of 3d Cartan gluons, dualize: “$e^2 d\sigma = *F$”

$$\partial_i \sigma \sim \frac{L}{g^2} \epsilon_{ij} E_j, \quad j = 1, 2 \qquad\qquad \partial_0 \sigma \sim \frac{L}{g^2} F_{12}$$

spatial gradient = 3d electric field

as we have N-1 Cartan dual photons

$$\oint_C d\boldsymbol{\sigma} = 2\pi \boldsymbol{\lambda}$$



$$\oint d\sigma = Q_e$$

monodromy of “dual photon” around a spatial loop = electric charge inside

weight vector of el. charge

long distance theory, perturbative: $L_{eff}^{pert.,3d} \sim \frac{g^2}{L} (\partial_\mu \vec{\sigma})^2 + \ldots$ 3√

√1. dynamical abelianization $SU(N) \to U(1)^{N-1}$ at 1/NL
√2. weak coupling
√3. relevant d.o.f. at distances >> NL: "dual Cartan gluons"
4. mass gap & confinement due to the proliferation (not really 'condensation') of nonperturbative semiclassical objects

-monopole-instantons - dYM
-magnetic bions - SYM/QCD(adj)

two scales:

W-boson mass $M \sim \frac{1}{L}$

dual photon mass $m \sim M e^{-\frac{\mathcal{O}(1)\pi^2}{g^2}}$

like Polyakov, 1977... but different - locally 4d! - extra instanton, theta...

Kraan van Baal; Lee Yi ~1997

$$L_{eff}^{dYM} = M \left[(\partial_\mu \vec{\sigma})^2 - \sum_{i=1}^{N} m^2 \cos \vec{\alpha}_i \cdot \vec{\sigma} \right]$$

Unsal Yaffe 2008

$$L_{eff}^{QCD(adj)/SYM} = M \left[(\partial_\mu \vec{\sigma})^2 - \sum_{i=1}^{N} m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i+1(\text{mod} N)}) \cdot \vec{\sigma} \right]$$

SYM: Seiberg Witten 1997...
QCD(adj): Unsal 2007

4.√

√1. dynamical abelianization $SU(N) \to U(1)^{N-1}$ at 1/NL
√2. weak coupling
√3. relevant d.o.f. at distances >> NL: "dual Cartan gluons"
√4. mass gap & confinement due to the proliferation (not really 'condensation') of nonperturbative semiclassical objects

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$$= M\left[\sum_{a=1}^{N} (\partial_\mu \sigma^a)^2 - m^2 \cos(\sigma^a - \sigma^{a+1(\mathrm{mod}N)})\right]$$

Z_N cyclic structure - crucial for string properties [vs Seiberg-Witten]

Anber Sulejmanpasic EP 2015
Anber EP 2016
Shalchian EP 2017

$\sigma^a \to \sigma^{a+1(\mathrm{mod}N)}$

= center symmetry on magnetic variables

$$L_{eff}^{QCD(adj)/SYM} = M\left[(\partial_\mu \vec{\sigma})^2 - \sum_{i=1}^{N} m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i+1(\mathrm{mod}N)}) \cdot \vec{\sigma}\right]$$

$$= M\left[\sum_{a=1}^{N} (\partial_\mu \sigma^a)^2 - m^2 \cos(\sigma^a + \sigma^{a+2(\mathrm{mod}N)} - 2\sigma^{a+1(\mathrm{mod}N)})\right]$$

Studies of dynamics at small-L, using these effective theories, have branched out in different directions: phase structure, theta-dependence, deconfinement transition, addition of fundamental flavors, etc; can't review all.

Will focus on confining string properties:

...mass gap & confinement

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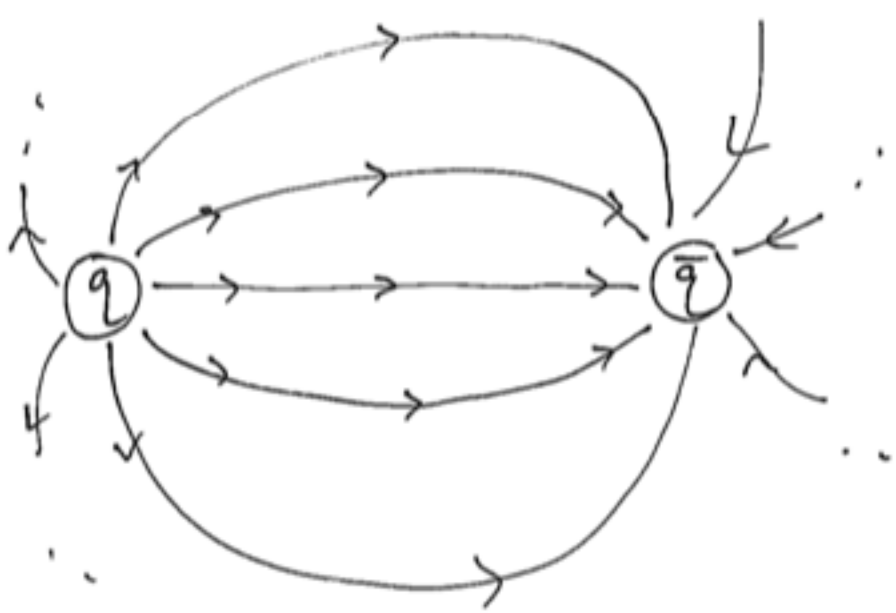
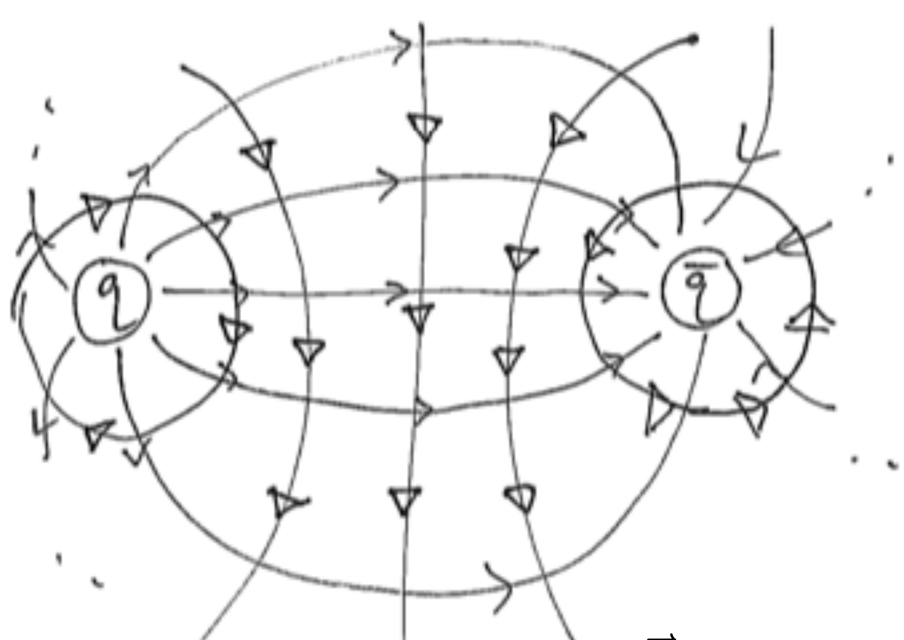


electric field lines

Will focus on confining string properties:

...mass gap & confinement



$\vec{\nabla}\sigma$ field lines - dual: $\partial_i \sigma \sim \frac{L}{g^2}\epsilon_{ij} E_j, \quad j = 1, 2$

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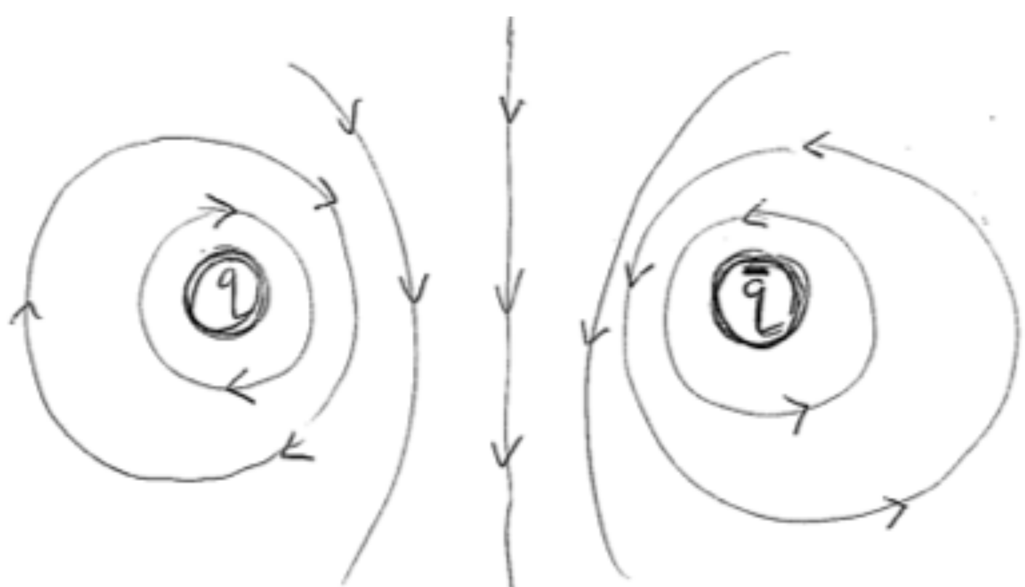


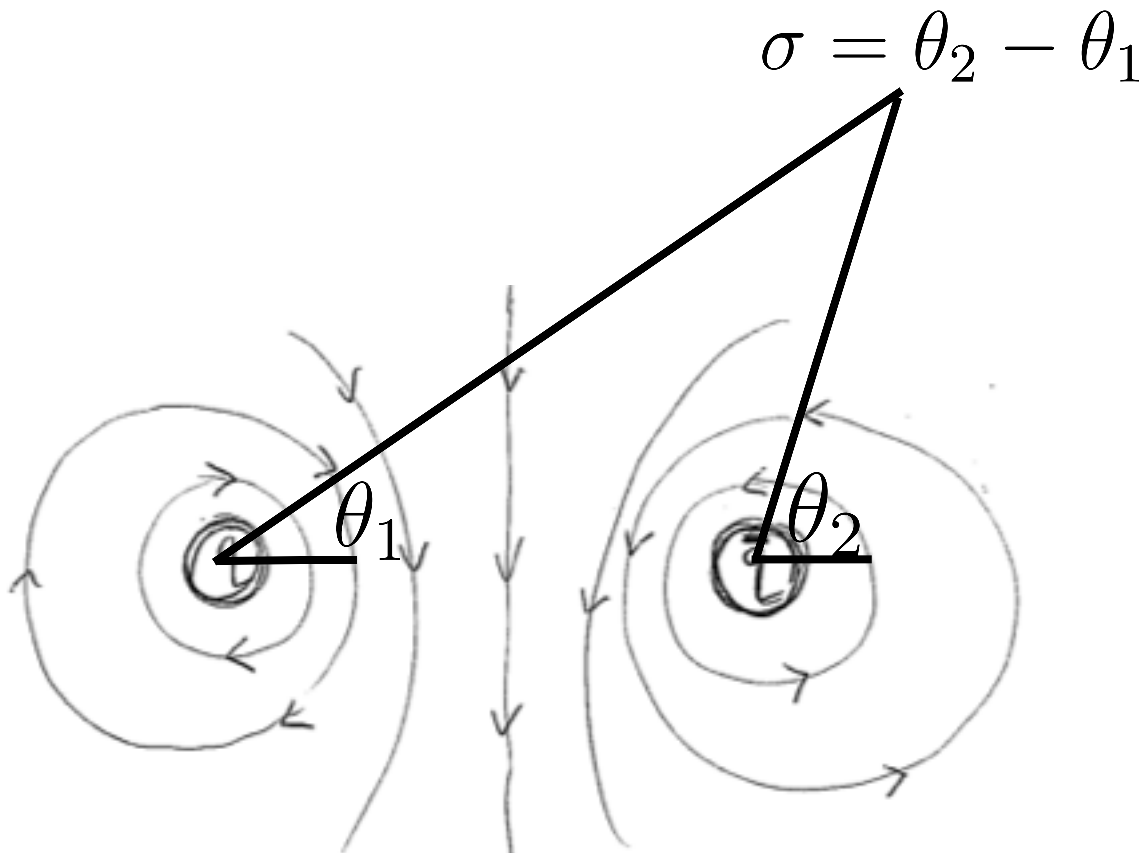
$\vec{\nabla}\sigma$ field lines (dual)

kinetic term $M \int d^2x (\vec{\nabla}\sigma)^2$

= electrostatic energy of dipole in dual picture

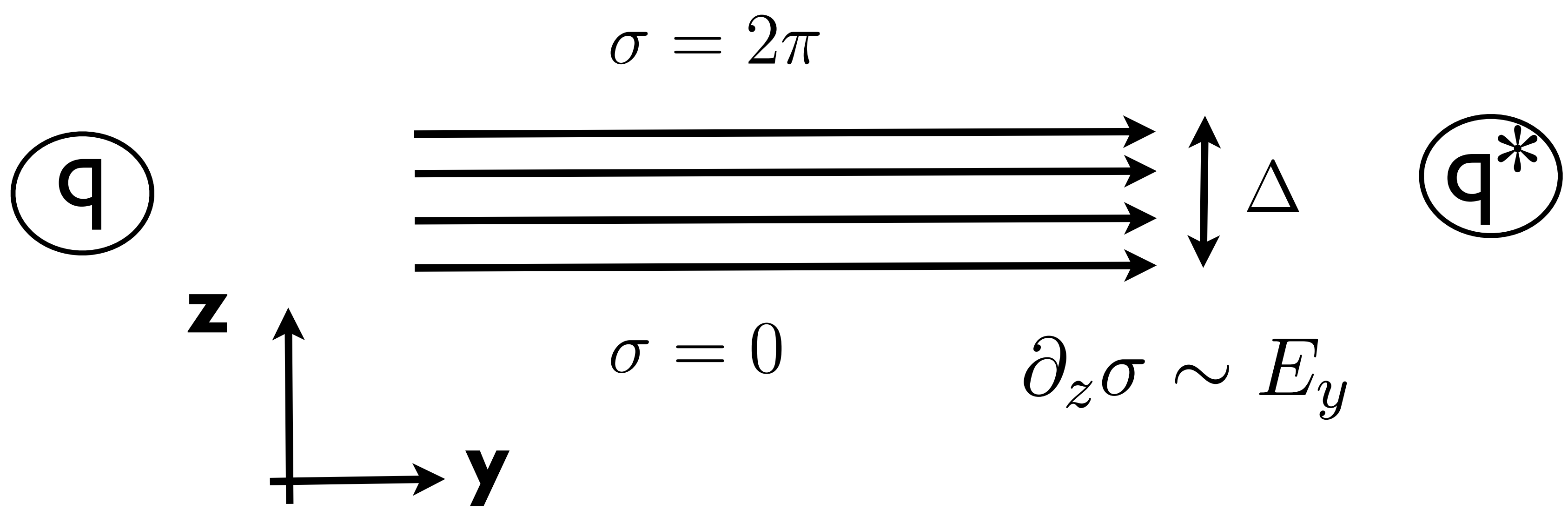
Will focus on confining string properties:

...mass gap & confinement



$\vec{\nabla}\sigma$ field lines (dual)

however, nonperturbative potential term $Mm^2 \int d^2x \cos\sigma$ abhors the spread of flux



$$M \int d^2x (\vec{\nabla}\sigma)^2 \sim \frac{(2\pi)^2}{\Delta}$$

(energies per unit length)

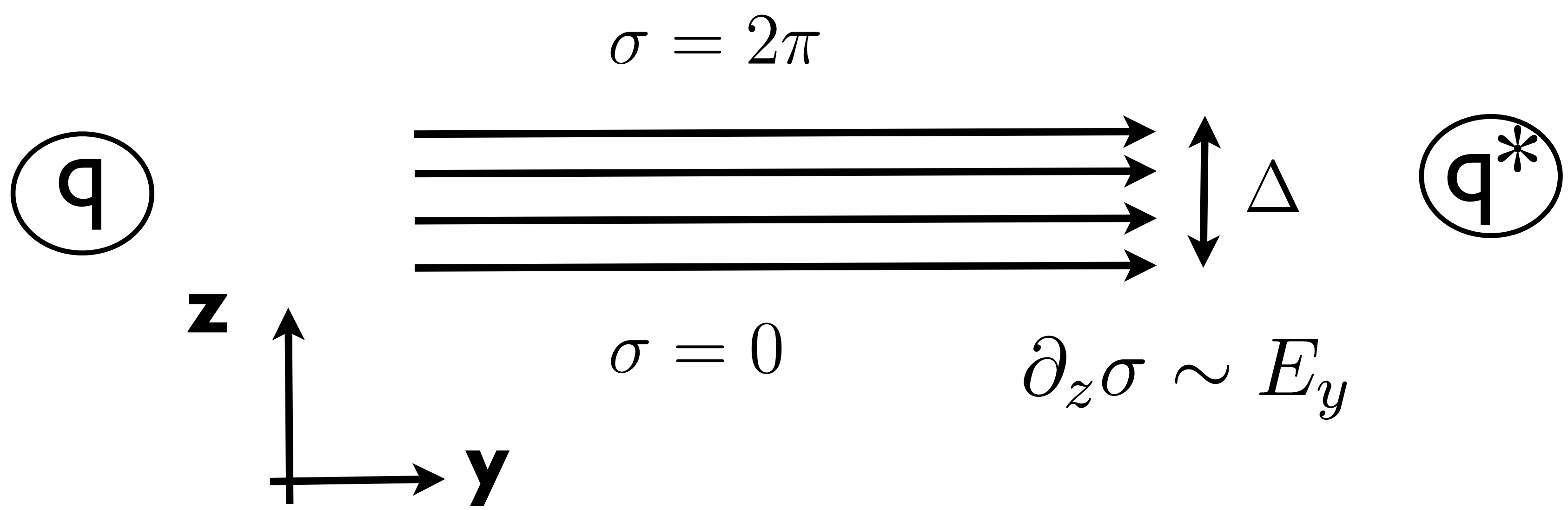
$$M m^2 \int d^2x \cos\sigma \sim m^2 \Delta$$

perturbative:
prefers the spread of flux

compromise

$$\Delta \sim \frac{1}{m}$$

nonperturbative:
abhors the spread of flux



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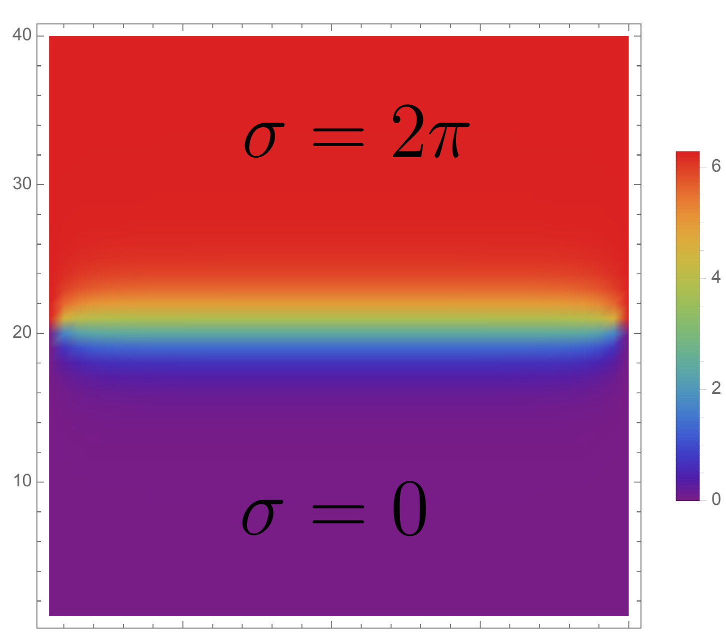
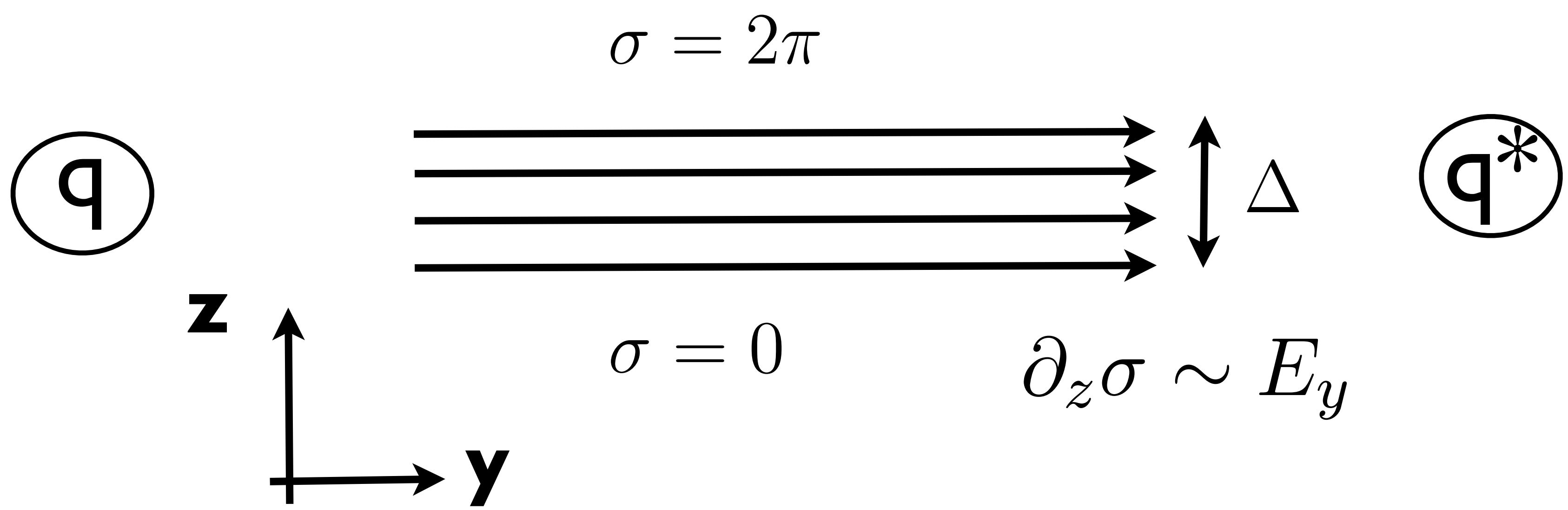


"Color field"
M. Rothko



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Comments:

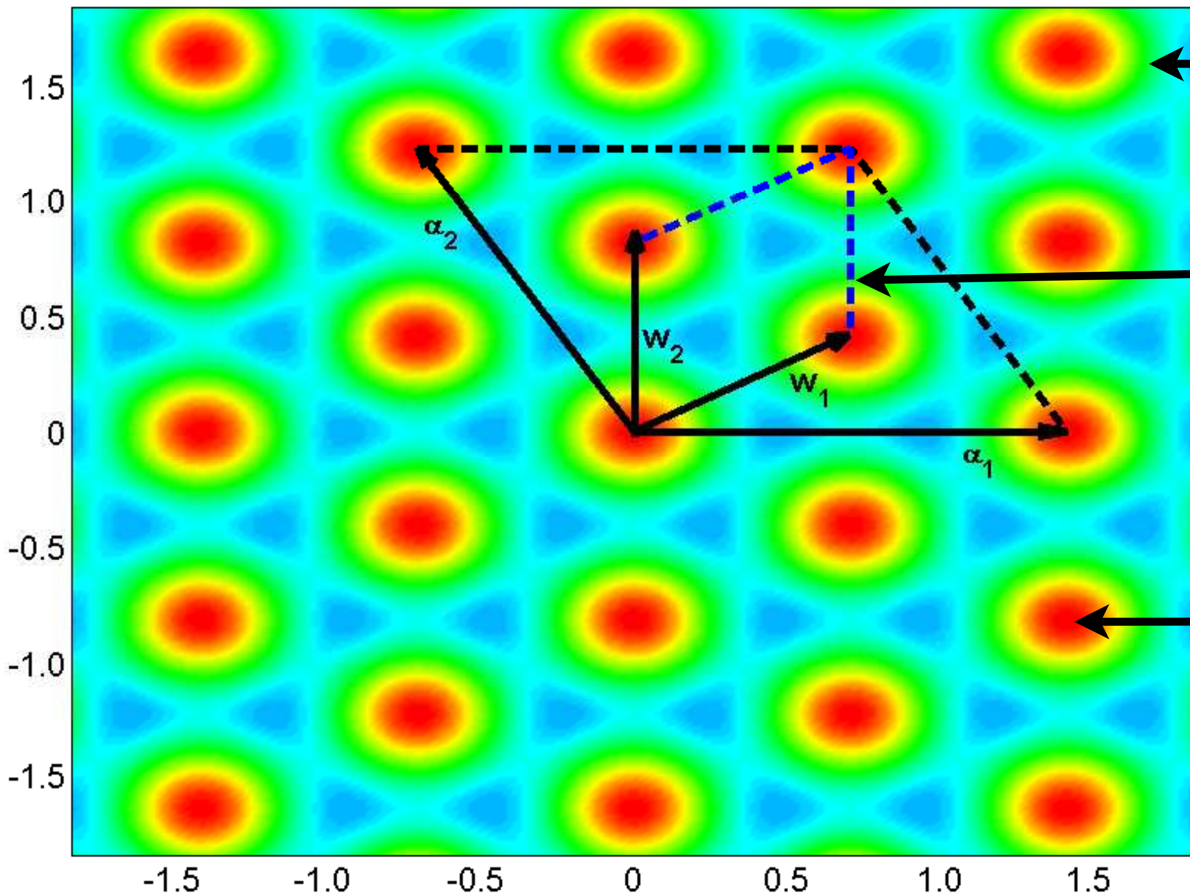
- this is of course the physics of the 3d Polyakov Model ← theoretically controlled! not a "model"...

- as well as that of the MIT Bag Model ← a model of the YM vacuum
  m ~ bag constant [relevant later!]

- novelty and interest due to locally-4d nature
  and the unbroken center symmetry of dYM, SYM, QCD(adj)
  [in Polyakov: no analogue of center & no vacuum angle: all due to locally 4d nature!]

The role of the unbroken center, “mesons” and “baryons”
dYM vs. Seiberg-Witten theory - the other theory with calculable abelian confinement:

contour plot of nonperturbative potential for dYM SU(3):



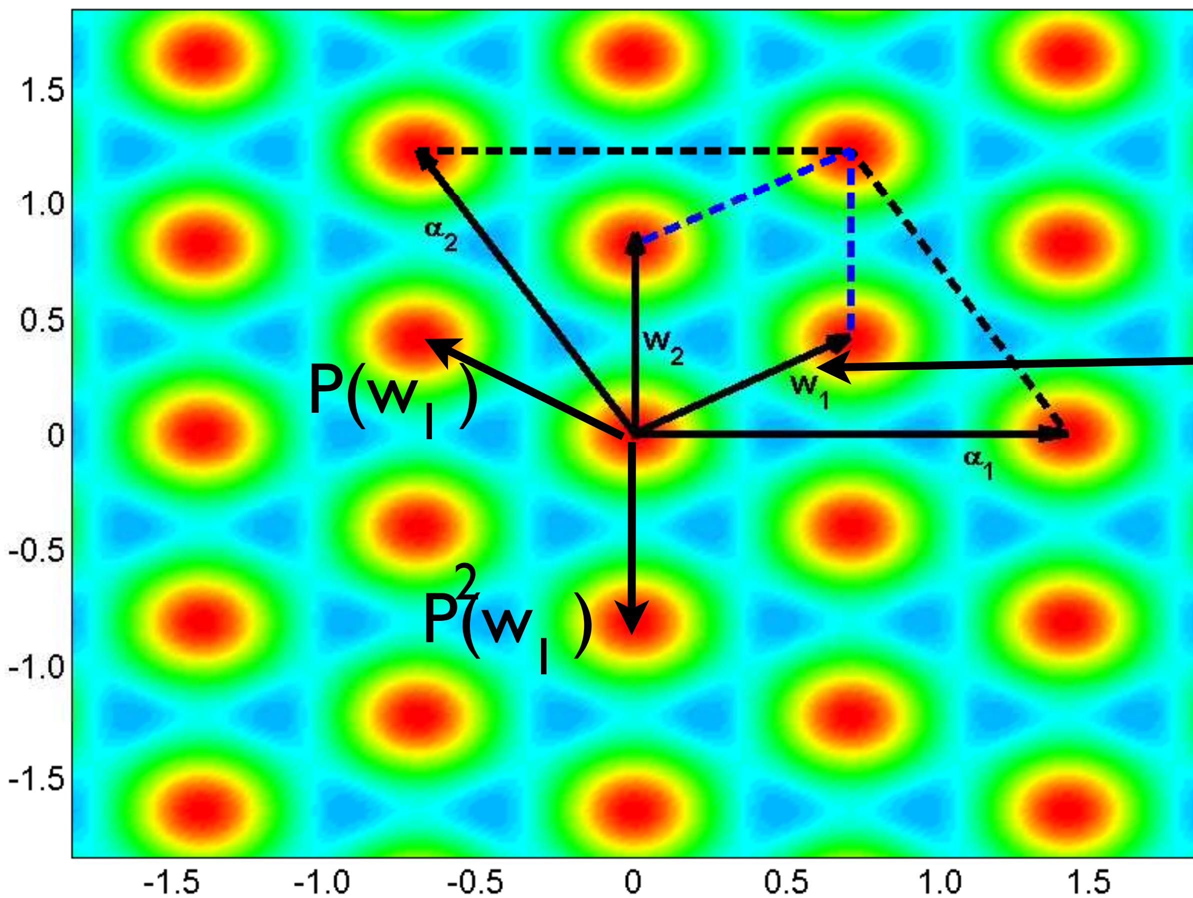
dual photon plane (2 dim basis)

unit cell of su(3) weight lattice - fundamental domain for SU(3) dual photons

minima of potential, degenerate

The role of the unbroken center, "mesons" and "baryons"

contour plot of nonperturbative potential for dYM SU(3):



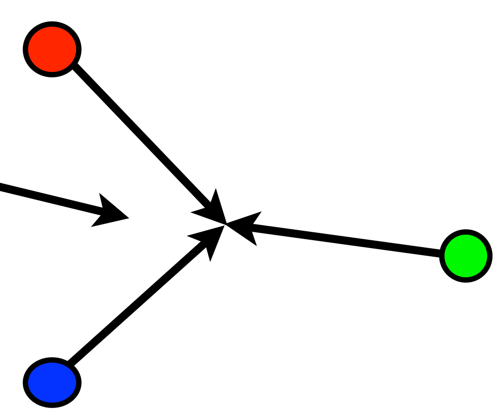
Z_3 symmetry clear = unbroken center

monodromy of dual photons confining quarks of weight w_1, etc.

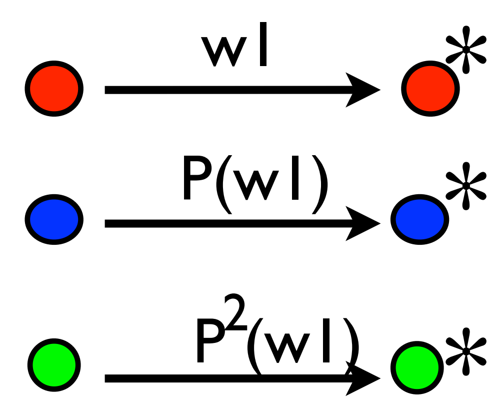
N monopoles "condense"
so non-composite strings (later)

$(1+P+P^2)w1=0$

"baryon vertex"
(DW junction)



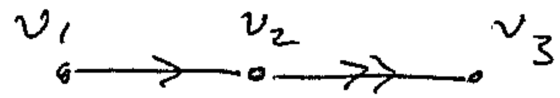
dYM:



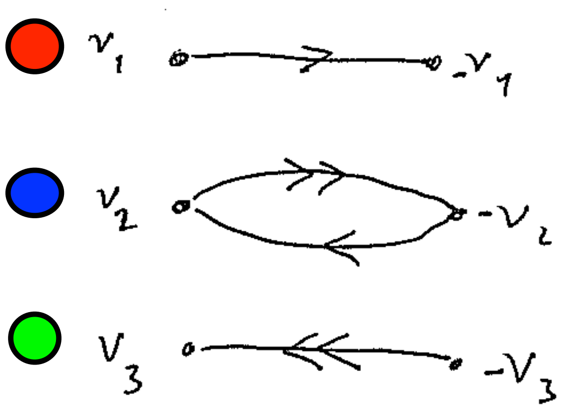
The role of the unbroken center, “mesons” and “baryons”
dYM vs. Seiberg-Witten theory - the other 4d theory with calculable abelian confinement:

Seiberg-Witten theory: N-1 monopoles condense

mesons



only linear baryons
(more dramatic for N>3)



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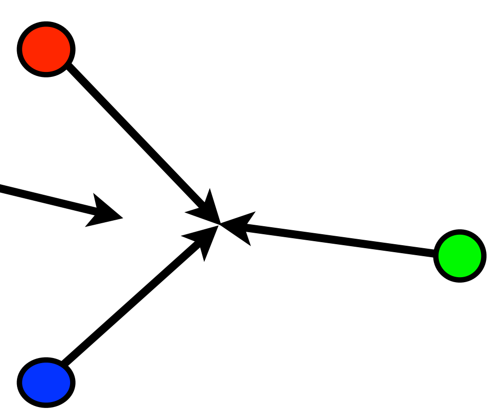

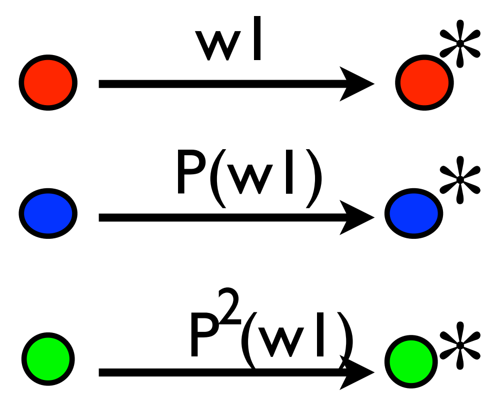

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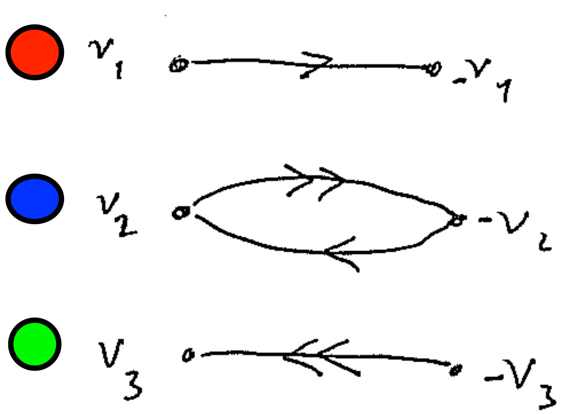
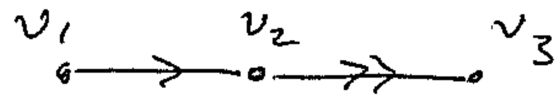




only linear baryons

(more dramatic for N>3)

study of "meson" and "baryon" spectra reveals some similarities... especially large-N counting

Aitken Cherman Yaffe EP 2017

N monopoles "condense"

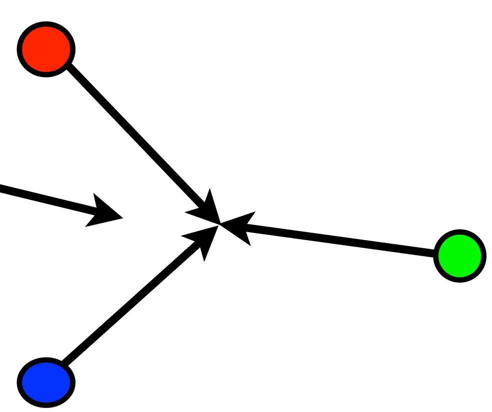
so non-composite strings (later)

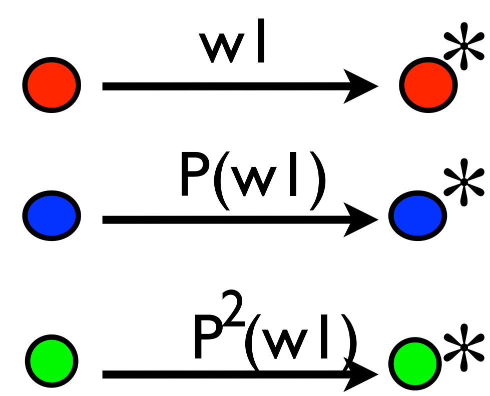
dYM:

$(1+P+P^2)w1=0$

"baryon vertex"

(DW junction)

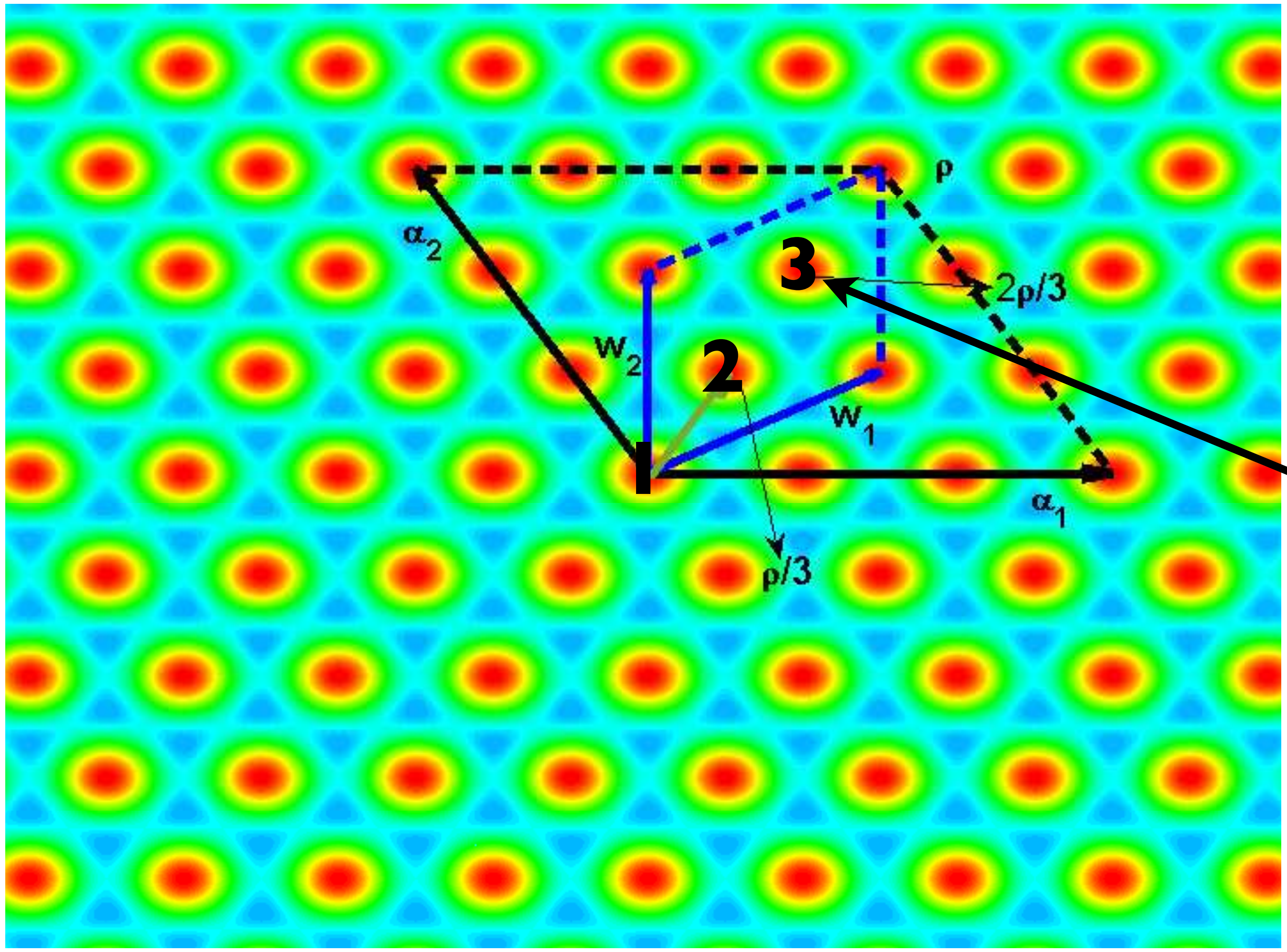




Switch to SYM/QCD(adj)!

The role of the unbroken center, "mesons" and "baryons"

SYM vs. Seiberg-Witten theory - the other 4d theory with calculable abelian confinement:



← dual photon plane

periodicities:

w1, w2: weight vectors of SU(3)

3 vacua - **1,2,3**

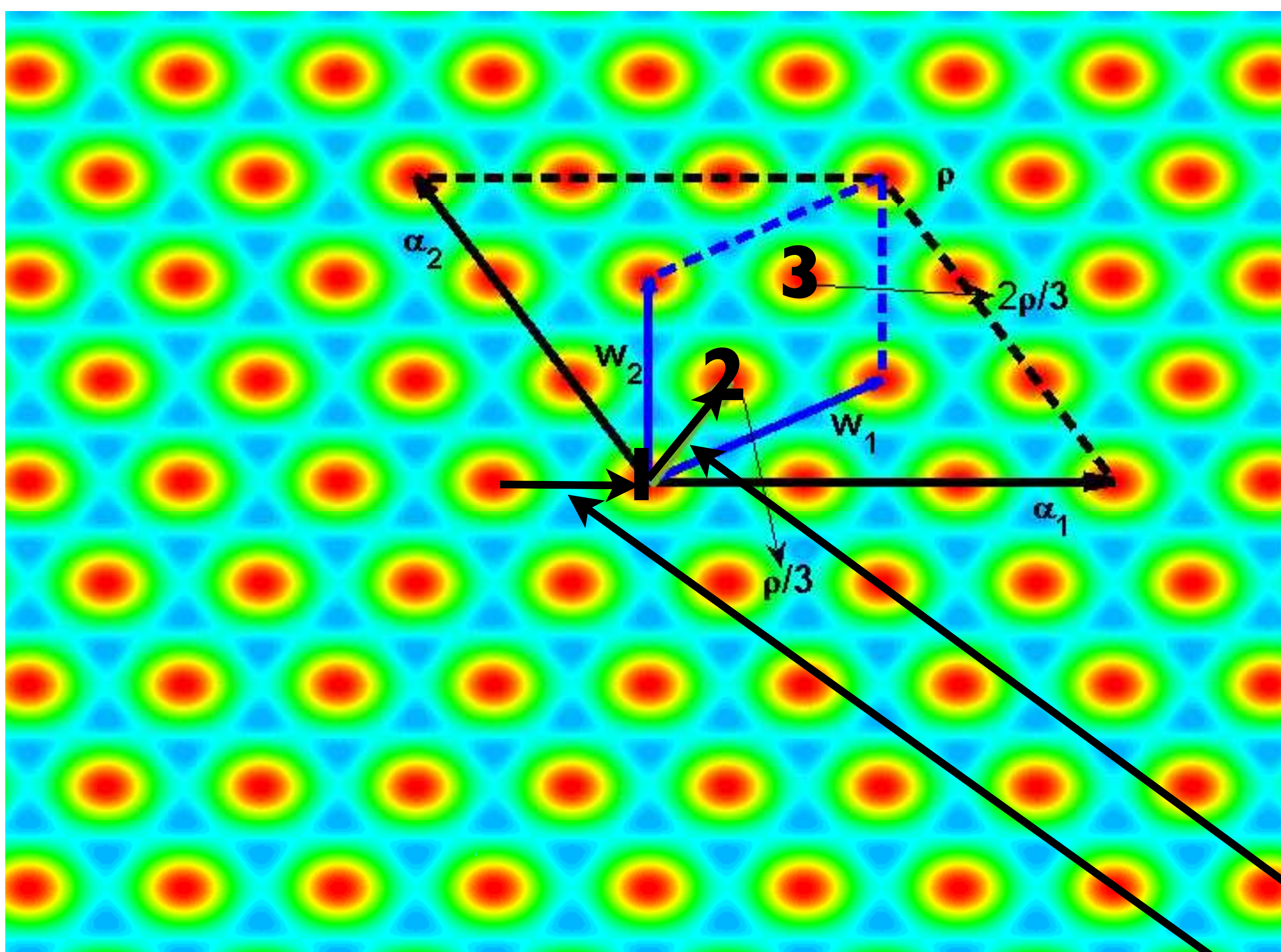
broken discrete chiral symmetry

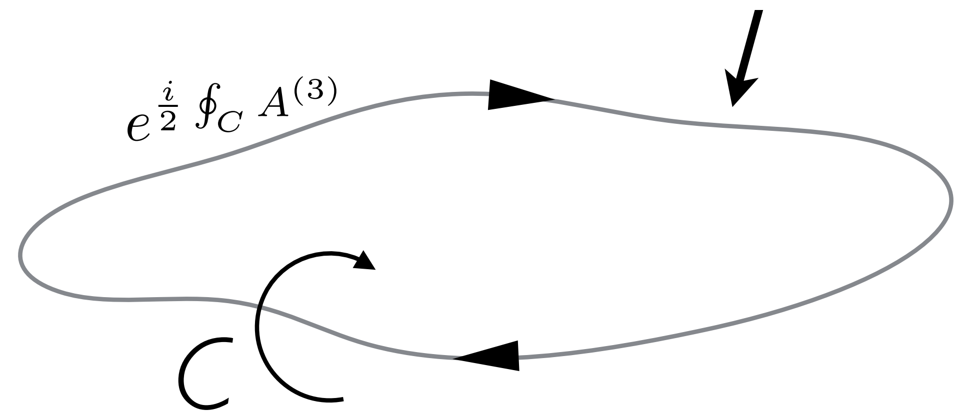
(preserve center symmetry

120 degree rotn + w_k shift)

The role of the unbroken center, “mesons” and “baryons”

SYM vs. Seiberg-Witten theory - the other 4d theory with calculable abelian confinement:





$$\oint_C d\boldsymbol{\sigma} = 2\pi\boldsymbol{\lambda}.$$

monodromy

for, e.g., a weight of the fundamental, say w1

DW1

P(DW1*)

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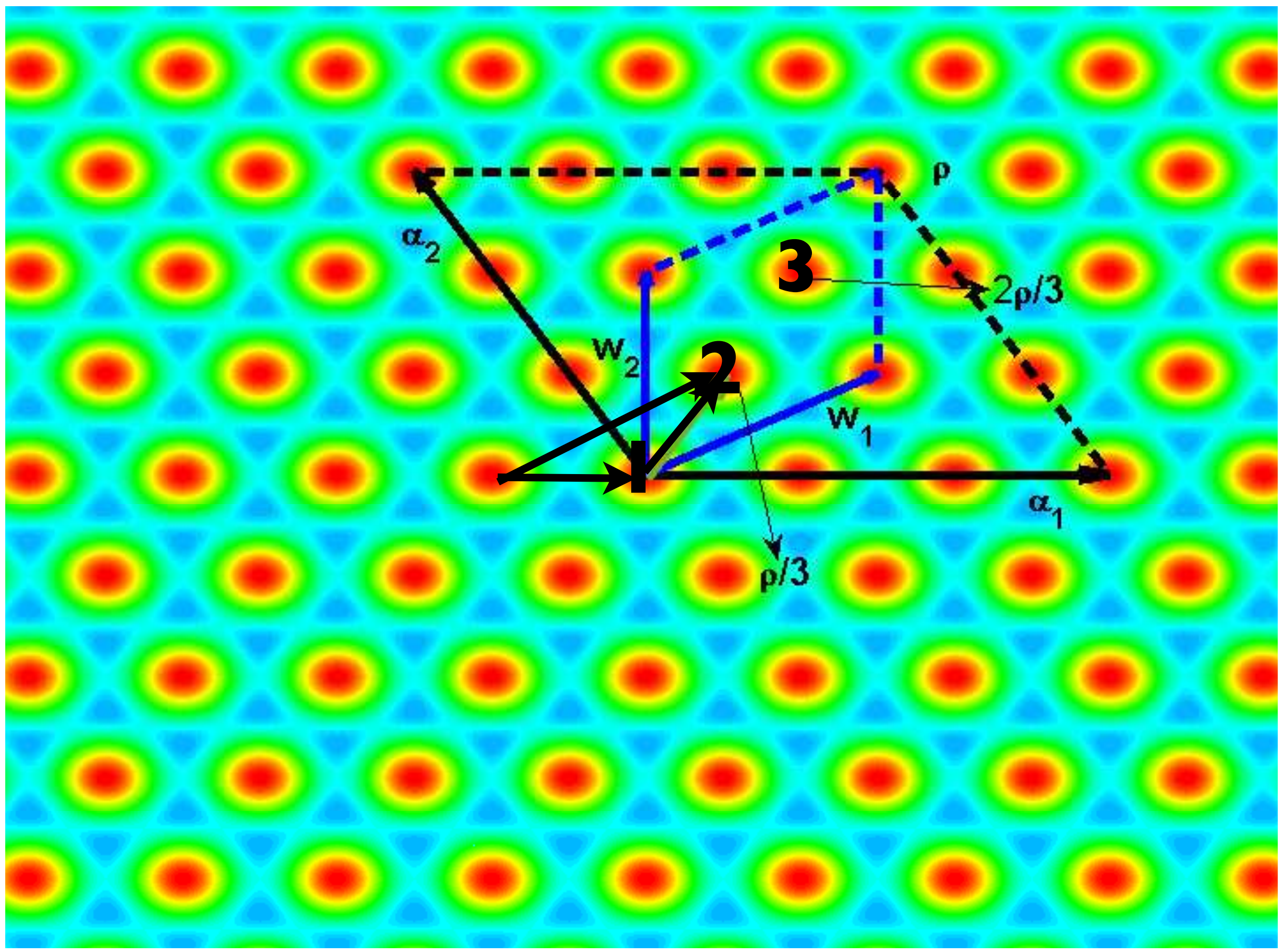
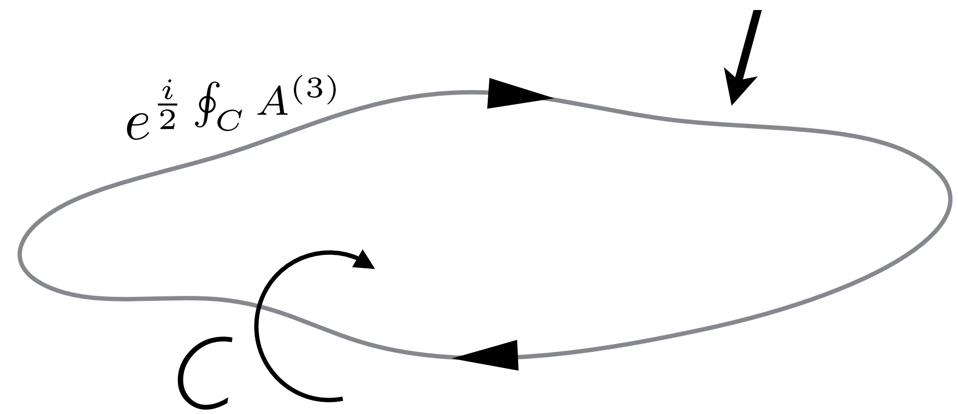
SYM vs. Seiberg-Witten theory - the other 4d theory with calculable abelian confinement:





$$\oint_C d\boldsymbol{\sigma} = 2\pi\boldsymbol{\lambda}.$$

monodromy

w1 = DW1 “+” P(DW1*)

(pictured the “w1”-confining string in vacuum 2)

for, e.g., a weight of the fundamental, say w1

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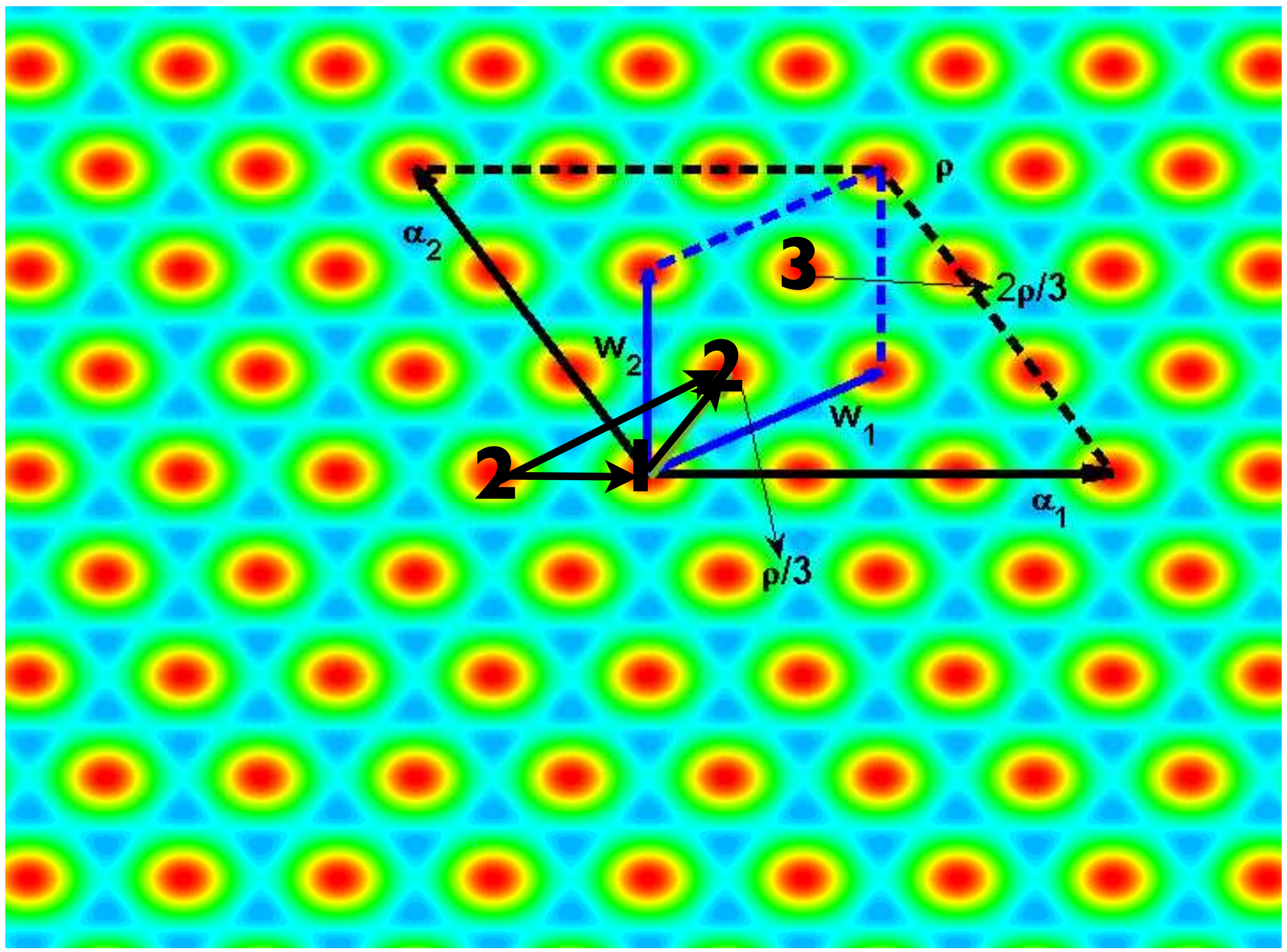

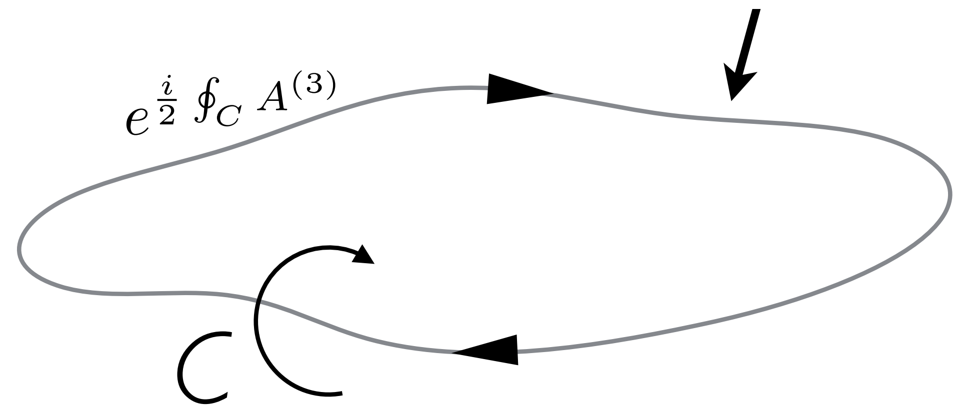

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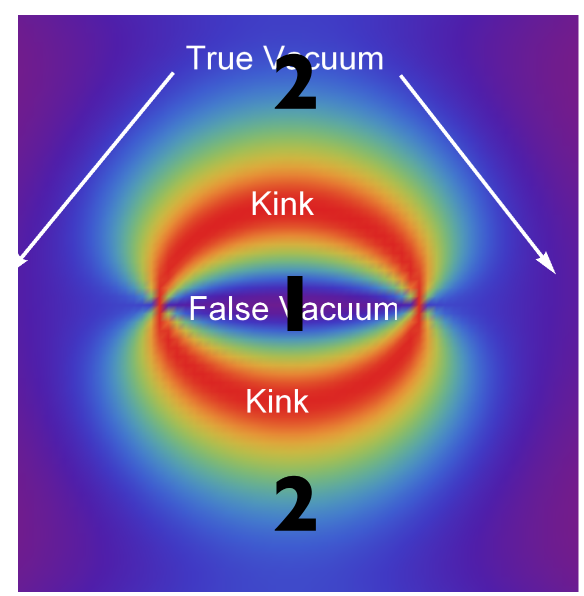
monodromy



w1-string
in vacuum 2

for, e.g., a weight of the fundamental, say w1

i.e. strings confining fundamental quarks have a "double-DW" structure...

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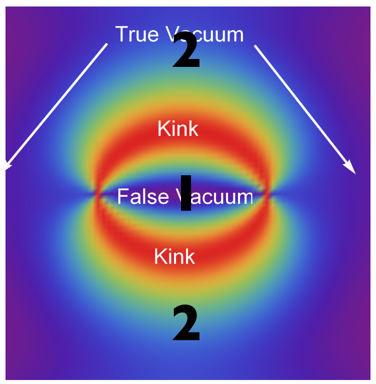
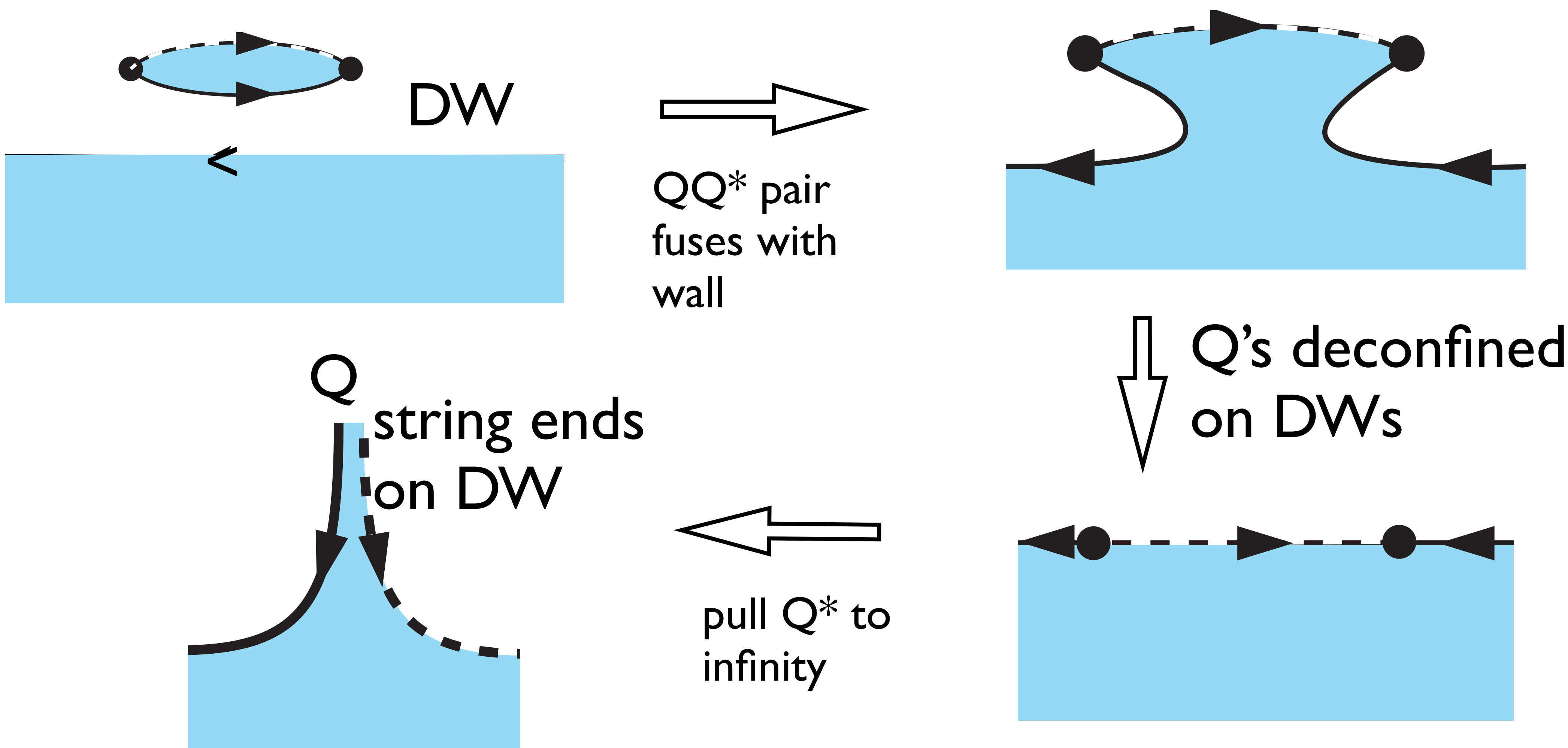


in SYM, QCD(adj), and dYM at theta=Pi (chiral ->CP)

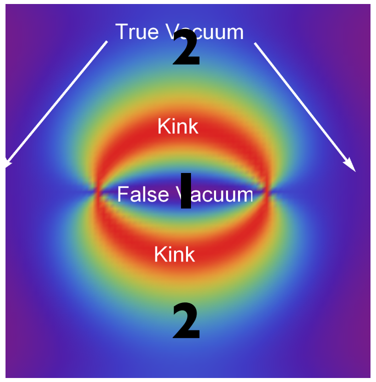
Anber Sulejmanpasic EP 2015

- strings can end on DWs
- quarks are deconfined on DWs



Originally advocated by S.-J. Rey/Witten 1997 using M theory (M2 ends on wrapped M5)...
here: QFT, semiclassical ("boring"!), no SUSY, no BPS,... explicit, not heuristic

i.e. strings confining fundamental quarks have a "double-DW" structure...



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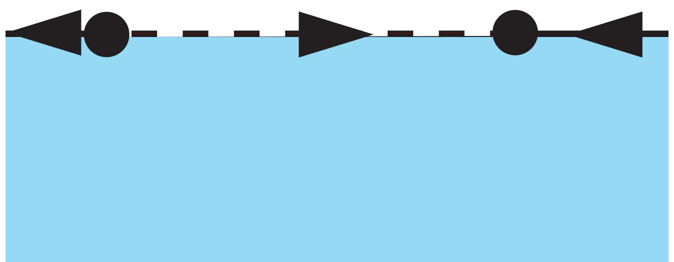


added bonus, 2 yrs later: turns out can be interpreted as due to discrete 1-form anomaly matching

Gaiotto, Kapustin, Komargodski, Seiberg, 2017

Z_N chiral/Z_N 1-form center mixed 't Hooft anomaly means that in a gapped theory the two symmetries can not be unbroken at the same time (unless TQFT)

in vacua: Z_N chiral broken, center preserved
on DW between Z_N vacua: Z_N chiral restored, center broken=deconfined quarks

- semiclassical construction makes the realization of this matching obvious...
[as usual with 't Hooft anomaly, one has to make assumptions on phase - not needed here]

- in SYM and QCD(adj) there is also a "baryon vertex", unlike Seiberg-Witten...skip

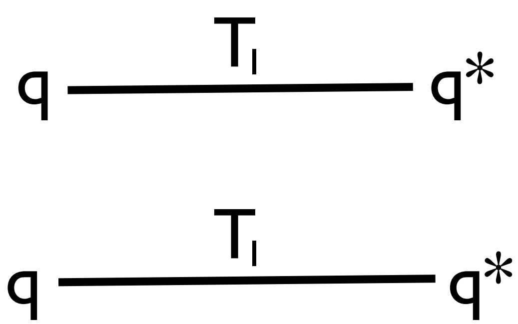
The final piece of 'data' I want to present is on the properties of k-strings in dYM and the large-N limit

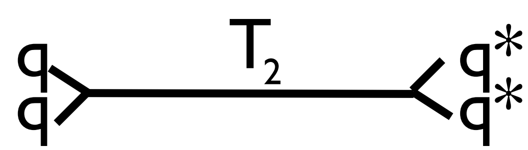
Shalchian EP 2017

Cherman EP 2016

Shalchian EP 2017

k-strings: take k pairs of fundamental quarks and antiquarks, e.g. k=2, and bring close together:





usually expect $T_2 < T_1$

and that T_k depends only on N-ality

at infinite-N $T_k = k\,T_1$ as glueball exchange suppressed

Results:

1. In dYM, for any k, the lowest tension k-string is the one sourced by quarks in the highest weight of the k-index antisymmetric [we gave detailed variational argument, as well as analytic, numeric results - separate talk].

Higher tension k-strings unstable to decay by creation of W-bosons.

2. We find in dYM a (sort-of) "square-root of Casimir" law Shalchian, EP 2017

$$\left(\frac{T_k}{T_1}\right)_{\rm dYM} \leq \sqrt{\frac{k(N-k)}{N}}$$

first (and last) seen in MIT Bag Model [Johnson Thorn 1974]

not an accident, as the physics of the Bag is repeated word-for-word in dYM, albeit for the Cartan components only: balancing "EM flux" energy and "Bag" energy gives square root of Casimir! [Johnson Thorn 1974 Hasenfratz Kuti 1978]

- at large-N: $\frac{T_k}{T_1} \leq \sqrt{k}$ - meaning k-strings are not free, but interacting at large N

3. This brings us to another aspect of the large-N story in dYM, SYM, QCD(adj):

$$L = M\left[\sum_{a=1}^{N}(\partial_\mu\sigma^a)^2 - m^2\cos(\sigma^a - \sigma^{a+1({\rm mod}N)})\right]$$

expanding cosine: an extra latticized dimension of N sites appears w/ spacing 1/m

mass gap ~ $\sin\frac{\pi q}{N}$ - vanishes at infinite N Cherman EP 2016

yet k-string tensions stay finite [vs Douglas Shenker 1996] Shalchian EP 2017

all in double-scaling large-N, small-L, fix $NL\Lambda \ll 1$: "like" T-duality???

Summary:

Studying dYM, QCD(adj), SYM on a small circle gives a theoretical laboratory elucidating many difficult to study nonperturbative effects: mass gap, confinement, (aspects of) chiral symmetry breaking; all within asymptotically free QFT.

The setup divorces "nonperturbative" from "strong coupling". It shows the importance of semiclassical configurations not appreciated before: composite magnetic bions, neutral bions etc. that I didn't talk about; also show need to complexify path integral...

Confinement in this regime is abelian, like in Seiberg-Witten theory, and unlike real QCD, where no scale separation between Cartan and non-Cartan components exists.

Nonetheless, many features come close to the real world, due to the unbroken Z_N center symmetry, the crucial difference with SW theory. I focused here on the confining strings, showing that they exhibit interesting properties and manifest different nontrivial phenomena in a calculable setup. The large-N limit and whether there is large-N transition as L is increased towards R^4 poses an intriguing question.

Continuity in L with real YM theory and QCD is difficult to establish analytically and lattice studies will be needed (some are underway); much evidence supports it.

Novel anomaly matchings may be useful to give further support of large-L continuity.

Cherman et al, Tanizaki et al 2017

Finally, if not yet made clear, my attitude to the story I told you is that it gives a rare theoretically controlled handle into nonperturbative physics and this alone makes it fun and worth exploring!

Thank you!