Discrete symmetries, ’t Hooft anomalies, and “adiabatic continuity”

Erich Poppitz U of TORONTO

An overview and some newer results.

Work with Mohamed Anber Lewis & Clark College, Portland

1805.12290[hep-th]
no time for ~~1807.00093[h~~ep-th]

QFTs, nonperturbatively, are hard to deal with.

Exact results: SUSY, often extended
...but real world: SM, BSM?

Lattice: global chiral symmetries-hard
gauged chiral symmetries-confusion reigns!

... any new analytic (trustable!) approach should be met with excitement and studied!

main message of this talk - combine two new (-ish) approaches

Around 2007, Unsal realized that a large class of 4d theories can be analytically studied using nonperturbative semiclassical means if compactified on a small circle:

$$\underline{NL\Lambda \ll 1}$$

number of colors

many studies over the past 10 years

incomplete alphabetical list

Aitken Anber Argyres Bergner Cherman Li Kanazawa Misumi Piemonte EP Simic Schaefer Shifman Shuryak Sulejmanpasic Tanizaki Thomas Vairhinos Voloshin Unsal Yaffe Zhitnitsky …

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Mechanism of confinement: abelian, locally 4d generalization of Polyakov's '77… but inherits much of 4d! [eg the anomalies I'll talk about!] Much closer to real world YM than Seiberg-Witten confinement, the other theory with calculable Abelian confinement!

- alas not this talk -

Anber, Sulejmanpasic, EP 2015
Shalchian, EP 2017

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discrete ’t Hooft anomaly matching

Upshot of talk:
discrete anomalies + small L results = suggest new interesting phases on R^4. Can be studied on the lattice.

More general moral:
- pay attention to new consistency conditions
- may mention some results on “hot” domain walls

Consider QCD(adj): SU(N) with n_f adjoint Weyl fermions (each one like a gaugino)

n_f < 6 for asymptotic freedom

can be solved nonperturbatively at small-L! - confinement and chiral symmetry breaking

n_f =1 is SUSY: one case where continuity for all L is guaranteed (Witten index)

n_f =2 is the one I will focus on mostly, for SU(2) gauge group

n_f =3,4,5… somewhere transition to conformal window? (unknown: 4?)

n_f=2 and 4 related to N=2 SUSY and N=4 SUSY, respectively

for those firmly rooted in real world, motivation is, for me, more of theoretical interest than applications - a rare example of nonperturbatively solvable QFTs! … never mind "MWT"

So, what is known from small-L?

QCD(adj) has SU(n_f) x U(1) classical chiral symmetry (U(n_f) rotates gauginos)

instanton has 2 N n_f gaugino zero modes: hence U(1) → Z_{2 N n_f}

anomaly free **discrete chiral symmetry**
(new feature as opposed to theories with fundamentals)

from small-L: SU(n_f) unbroken, Z_{2 N n_f} → Z_{2 n_f}, so N vacua Unsal, 2007

for n_f = 1 (SUSY) this is exactly what is known on R^4 (guaranteed)

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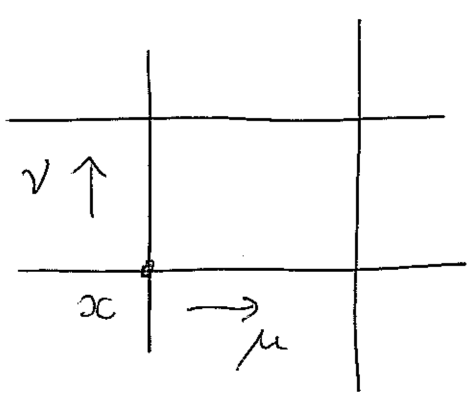
thus, we have believed that for n_f>1, there is a phase transition as L->infinity

in fact, QCD(adj) has SU(n_f) x Z_{2 N n_f} exact chiral symmetry

and a Z_N "1-form" center symmetry - not visible to the naked eye,

well known to the lattice folks, but thought - apart from some theoretical studies - largely irrelevant:



$$U_{x,\mu} \to z_\mu U_{x,\mu}, \quad \mu = 1,2,3,4$$

$$z_\mu = e^{i\frac{2\pi}{N} n_\mu} \leftarrow$$ mod N integer, one per spacetime direction

"1-form"!

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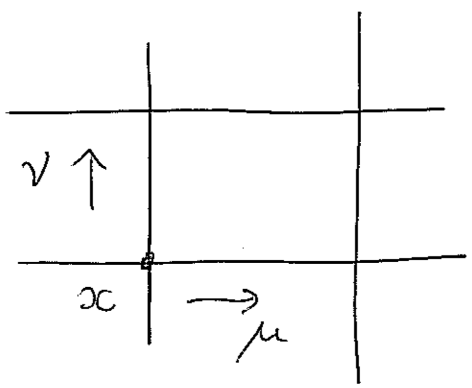

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Z_N "1-form" global center symmetry:

- only acts on fundamental representation Wilson line operators, infinite or wrapping around T^4

$$P e^{i \oint dx^1 A_1} \to e^{i \frac{2\pi n_1}{N}} \; P e^{i \oint dx^1 A_1}$$

- only preserved in theories with zero N-ality representations: pure YM, QCD(adj)
    - explicitly broken in theories with massless or light fundamentals (emergent if heavy)
    - in theories with two-index tensors only (AS, S) a Z_2 1-form center is exact, etc.

moral: QCD(adj) has $\underbrace{\text{SU(n\_f) x Z\_\{2 N n\_f\}}}_{\text{"0-form"}}$ x $\underbrace{\text{Z\_N}}_{\text{"1-form"}}$ exact global symmetry moral: QCD(adj) has $\underbrace{SU(n_f) \times Z_{2 N n_f}}_{\text{“0-form”}} \times \underbrace{Z_N}_{\text{“1-form”}}$ exact global symmetry

- whenever we have global symmetries, we can imagine gauging them
  if we fail to maintain gauge invariance, we say there's a 't Hooft anomaly
- 't Hooft anomaly is an anomaly w.r.t background gauge transforms

  it does not represent an inconsistency of the theory, but gives strong constraints on the possible IR dynamics:

- this is because the 't Hooft anomaly is an RG invariant - is the same at all scales - so we can compute it in the UV of an asymptotically free theory (using quark and gluon d.o.f.) and demand that it be the same in the IR (using whatever the IR d.o.f. are)

classic example: SU(n_f)_L x SU(n_f)_R in chiral limit of QCD
has 't Hooft anomaly; can be matched, in unbroken mode, by massless baryons, both L and R
or in Goldstone mode by pions (nature's choice)

reiterate crucial point: 't Hooft anomaly is an RG invariant

for our purposes best formulation:

not = to the variation of a 4d local CT (wouldn't be an anomaly)

but = to the 4d boundary variation of a 5d local term depending only on
background fields (the ones that gauge the global symmetry, nondynamical)

this term does not care about the scale and the gauge theory dynamics
(represents formally 't Hooft's weakly coupled anomaly cancelling sector)

moral: QCD(adj) has $\underbrace{SU(n_f) \times Z_{2 N n_f}}_{\text{"0-form"}} \times \underbrace{Z_N}_{\text{"1-form"}}$ exact global symmetry

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the non vanishing 't Hoof anomalies in QCD(adj) are

$[SU(n_f)]^3$	classic 't Hooft (gauginos = N^2-1 fundamentals)
$Z_{2 N n_f}\ [SU(n_f)]^2$ $[Z_{2 N n_f}]^3$ $Z_{2 N n_f}\ [\text{grav.}]^2$	Csaki-Murayama '97 think of $Z_{2 N n_f}$ as a $U(1)$ chiral subgroup

AND the new stuff:

$\underbrace{Z_{2 N n_f}}_{\text{"0-form"}}\ \underbrace{[Z_N]^2}_{\text{"1-form"}}$	Gaiotto et al '14-17

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Z_{2 N n_f} [Z_N]^2 | Gaiotto et al '14-17

"0-form" "1-form"

under chiral U(1): $\lambda \to e^{i\alpha}\lambda$ (adj.)

$$\mathcal{D}\lambda \to e^{i\alpha 2Nn_f \int F\tilde{F}}\mathcal{D}\lambda = e^{i\alpha 2Nn_f Q_{top.}}\mathcal{D}\lambda\Big|_{\alpha=\frac{2\pi}{2Nn_f}} = e^{i2\pi Q_{top.}}\mathcal{D}\lambda$$

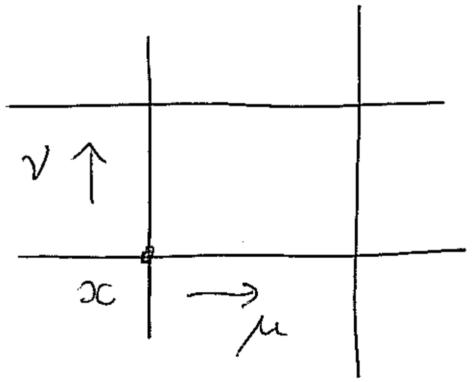
Z_{2 N n_f}

hence, Z_{2 N n_f} is a symmetry (as Q_top=integer)

but gauging Z_N center means (you have to "buy" this; can give lattice/continuum story, 2dU(1)…) **Q_top=k/N:**

Z_{2 N n_f}: $\mathcal{D}\lambda \to e^{i\frac{2\pi k}{N}}\mathcal{D}\lambda$

explain!



$$U_{x,\mu} \to z_\mu \, U_{x,\mu} \, , \quad \mu = 1,2,3,4$$

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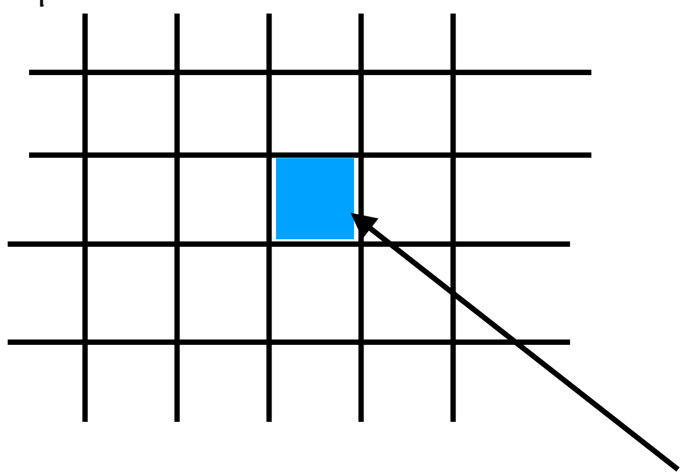
remember $U_{plaquette} = \prod_{link \in plaquette} U_{link} = e^{i a^2 F_{plaquette}}$



$$\prod_{all\ plaquettes} U_{plaquette} = e^{i flux\ thru\ T^2} = 1$$

$\longrightarrow$ $flux\ thru\ T^2 = 2\pi n, \ n \in Z$ 2d U(1) Q_top!

2-form Z_N gauge field = plaquette based Z_N field; ex.: $\prod_{all\ plaquettes} U_{plaquette} e^{i\frac{2\pi}{N}} = e^{i flux\ thru\ T^2} = e^{i\frac{2\pi}{N}}$

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Z_{2 N n_f}: $\mathcal{D}\lambda \to e^{i\frac{2\pi k}{N}} \mathcal{D}\lambda$

explained! (for 2d U(1)

(for 4d SU(N)… need 2 orthogonal planes…)

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under chiral U(1): $\lambda \to e^{i\alpha}\lambda$ (adj.)

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Z_{2 N n_f}

Z_{2 N n_f} is a symmetry if Q_top=1

but gauging Z_N center means (you have to “buy” this; can give lattice/continuum story, 2dU(1)…) **Q_top=k/N:**

Z_{2 N n_f}: $\mathcal{D}\lambda \to e^{i\frac{2\pi k}{N}}\mathcal{D}\lambda$

- mixed discrete chiral-center ’t Hooft anomaly = discrete Z_N phase in chiral transform
- phase independent on volume of T^4 (used to compute Q_top), so same at all scales
- the phase = the boundary variation of a 5d bulk local term [not a 4d one!]; dep. only on background discrete chiral and discrete center gauge fields [best on lattice/triangulation!] can be written; has to be matched in the IR along with the other ’t Hooft anomalies

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Z_{2 N n_f} or Z_N-center can be broken in the IR, or matched by a CFT, or some TFT

back to the IR spectrum of QCD(adj) - take N=2, n_f = 2:

for small-L: SU(2_f) unbroken, Z_{8} → Z_{4}, so 2 vacua, Z_N - center unbroken

we found a solution of all above 't Hooft matching condition on R^4 — Anber EP 1805.12290

for infinite-L: SU(2_f) unbroken, Z_{8} → Z_{4}, so 2 vacua, Z_N - center unbroken

by nonzero vev of 4-fermi operator $\mathcal{O}^{(1)} \equiv \left(\epsilon_{\alpha\beta}\lambda_i^{\alpha a}\lambda_j^{\beta a}\right)\left(\epsilon_{\alpha'\beta'}\lambda_{i'}^{\alpha' a'}\lambda_{j'}^{\beta' a'}\right)\epsilon^{ii'}\epsilon^{jj'}$

IR: single massless SU(2_f) Weyl doublet $\mathcal{O}^{(2)\,\gamma}_{[ij]k} \equiv \epsilon_{\alpha\beta}\lambda_{[i}^{\alpha\, a}\lambda_{j]}^{\beta\, b}\lambda_k^{\gamma c}\epsilon^{abc}.$

for N=2, n_f = 2 the lattice has been seeing strange things, inconsistent with SU(2)→SO(2)
they say will check above for "baryon"… — Athenodorou, Bennett, Bergner, Lucini, 2015

"adiabatic continuity" from title
= same symmetry realization at small and large L ? ? ?

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AND the new stuff:

Z_{2 N n_f} [Z_N]^2 “0-form” “1-form”	Gaiotto et al '14-17

AND more new stuff: - “non-spin manifold X” for N=2 n_f = 2 only (twisted N=2 SYM)

Z_2 X; Z_4 X; “1-form”	Dumitrescu-Cordoba 1806.09592 - last week!

claim our proposal 1805.12290 modified by adding an “emergent Z_2 gauge theory” in IR
not clear yet (to me!) how to probe for it (say on lattice… 't Hooft loop?) stay tuned!

“adiabatic continuity” from title
= same symmetry realization at small and large L ? ? ?

Upshot of talk:
discrete anomalies + small L results =
suggest new interesting phases on R^4.
Can be studied on the lattice.

More general moral:
 - pay attention to new consistency
conditions

no time to ~~- may~~ mention some results on “hot”
domain walls

(domain walls in one phase mimic bulk behavior in another:
high-T DW low-T bulk and v.v.)