Higher symmetry ’t Hooft anomalies, phases, and domain walls

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1805.12290, 1807.00093, 1811.10642

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1904.11640

related work by **Cordova-Dumitrescu; Bi-Senthil; Wan-Juven Wang**
1805.12290 1806.09592 1812.11955

Summary

Higher form symmetry $\supset$ 1-form symmetry $\supset$ $\mathbb{Z}_N^{(1)}$ center symmetry

this talk

1.

Gauging center symmetry *(nondynamical background fields)* **leads to new 't Hooft consistency conditions, due to new mixed anomalies involving center symmetry - missed in the 1980's!**

2.

These consistency conditions constrain IR phases of gauge theories to be "nontrivial."

3.

They also imply that, whenever domain walls exist, their worldvolume physics is quite nontrivial: discrete version of "anomaly inflow."

Examples in talk are from my work and the others' mentioned above.

(Many works on other anomalies involving 1-form symmetries.)

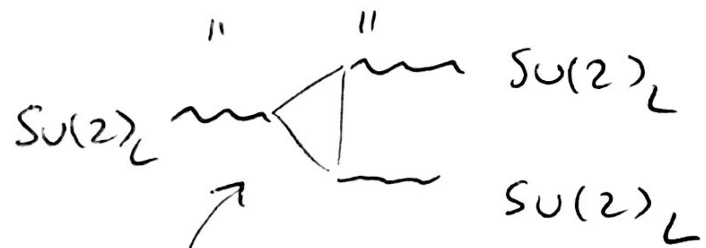
REMINDER: 't Hooft consistency conditions

SU(3)-color QCD with 2 massless fundamental flavors

$SU(2)_L \times SU(2)_R \times U(1)_V$

imagine, e.g. gauging $SU(2)_L$ L-quarks = 3 $SU(2)_L$ fundamentals

UV:



not a triangle, Witten anomaly, Z_2 valued)

't Hooft:
anomaly
RG invariant

IR:

massless baryons $\begin{pmatrix} p \\ n \end{pmatrix}$ 1 $SU(2)_L$ fundamental

or

massless pions π^+, π^-, π^0 chiral broken

MORAL: 't Hooft anomaly matching constrains any fantasy IR phase!

remarkably, discrete 0-form/1-form analogue, missed earlier

Gaiotto, Kapustin, Seiberg, Komargodski, Willett, 2014-… : "Dashen phenomenon"=mixed CP-center anomaly

$\cancel{CP}$ @ $\theta = \pi$

Higher form symmetry $\supset$ 1-form symmetry $\supset$ $\mathbb{Z}_N^{(1)}$ center symmetry

2D compact U(1) with charge-N massless Dirac

"charge N Schwinger model"

remarkably alike

4D SU(N) with n_f massless Weyl adjoints

$n_f = 1$ = SYM

" n_f *QCD(adj)*"

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}_+ (\partial_- - iNA_-)\Psi_+ + i\bar{\Psi}_- (\partial_+ - iNA_+)\Psi_-$$

$U(1)_V$ *and* $U(1)_A$: $\Psi_\pm \to e^{\pm i\chi}\, \Psi_\pm$

$$[\mathcal{D}\Psi] \to [\mathcal{D}\Psi]\, e^{i 2N\chi \cdot \int \frac{d^2x\, F_{12}}{2\pi}}$$

axial anomaly

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$U(1)_V$ *and* $U(1)_A$: $\Psi_\pm \to e^{\pm i\chi}\,\Psi_\pm$

$$[\mathcal{D}\Psi] \to [\mathcal{D}\Psi]\, e^{i\,2N\chi\cdot \underbrace{\int \frac{d^2x\, F_{12}}{2\pi}}_{\mathbf{Q_{top.}}}}$$

axial anomaly

$e^{i\,2N\chi\, Q_{top.}}$ $\hookrightarrow$ quantized $\in \mathbb{Z}$ ("1st Chern class")

phase is unity when $\chi = \frac{2\pi}{2N}$ $\mathbb{Z}_{2N}^{d\chi}$ *discrete chiral anomaly free*

(likewise, 4D QCD(adj) has $SU(n_f) \times \mathbb{Z}_{2Nn_f}^{d\chi}$ *global chiral symmetry)*

We want to know what
charge-N Schwinger model or QCD(adj) "do" in the IR?

assisted by **claim:**

There is a mixed anomaly between

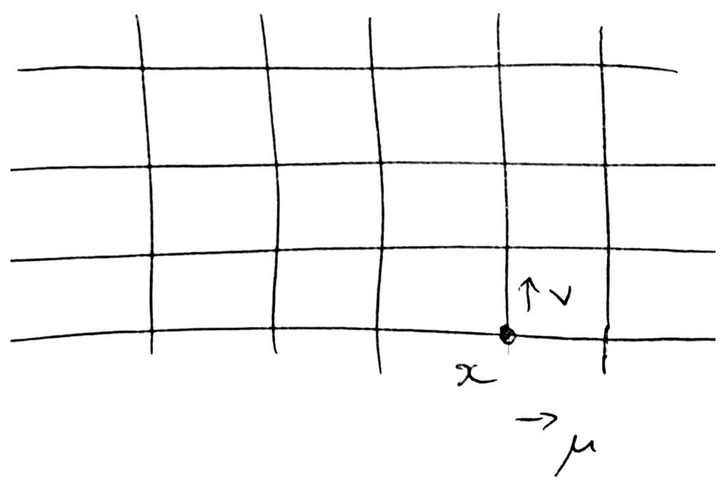
$\mathbb{Z}_{2N n_f}^{d\chi}$ discrete "0-form" chiral, present in both models
($n_f \to 1$ in 2D)

$\mathbb{Z}_N^{(1)}$ discrete "1-form" center, present in both models

This is especially easy to see on the lattice - and we're at a lattice meeting.

(N.B.: lattice is not required; i.e. entire story is not a lattice artifact!
Continuum version requires introducing gauge bundles
and transition functions on general manifolds.)

Take 2D lattice, charge-N matter, compact U(1):



$$\mathbb{Z}_N^{(1)}: \quad U_{x,\mu} \to e^{i\frac{2\pi}{N}k_\mu} U_{x,\mu}$$

$$k_\mu = (k_1, k_2)$$

parameters: mod N integers, x-independent

well known… new name: “global 1-form $\mathbb{Z}_N^{(1)}$ center symmetry”

does not act on local observables (plaquette  clearly invariant)

only acts on (topologically nontrivial) Wilson lines: “1-form” symmetry

$$e^{i\oint dx^1 A_1} \to e^{i\frac{2\pi}{N}k_1}\, e^{i\oint dx^1 A_1}$$

(same in 4D QCD(adj), except we have k_1, k_2, k_3, k_4)

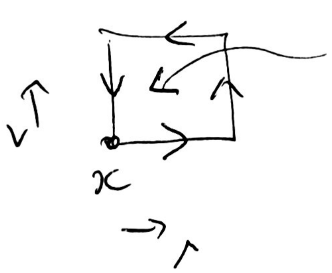
In the 2D charge-N matter, compact U(1), both discrete chiral and center are exact global symmetries, like the chiral symmetry of our QCD ex.

In the spirit of 't Hooft, let's now attempt to gauge the center.

$\mathbb{Z}_N^{(1)}$ acts on links $U_{x,\mu} \to e^{i\frac{2\pi}{N} k_{x,\mu}} U_{x,\mu}$

make parameter x-dependent

plaquette no longer invariant, need a $\mathbb{Z}_N$ gauge field on plaquettes



$e^{i\frac{2\pi}{N} b_{x,\mu\nu}}$ an integer (mod N)

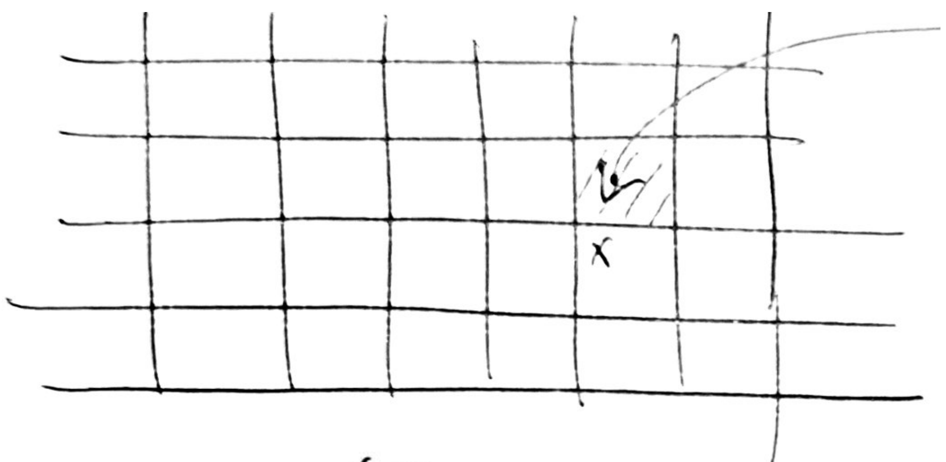
"2-form" $\mathbb{Z}_N$ gauge field

$$S \sim \sum_{x,\mu\nu} U_{\Box_{x,\mu\nu}} \Longrightarrow \sum_{x,\mu\nu} U_{\Box_{x,\mu\nu}} e^{i\frac{2\pi}{N} b_{x,\mu\nu}}$$

gauged 1-form center: r.h.s. has 1-form center gauge invariance in the theory with 1-form center gauge invariance

$$\sum_{x,\mu\nu} U_{\Box_{x,\mu\nu}} e^{i\frac{2\pi}{N} b_{x,\mu\nu}}$$

consider a simple background (suffices!)



$$e^{i\frac{2\pi}{N} b_{x,\mu\nu}} \in \mathbb{Z}_N$$

nonzero
phase on a single
plaquette only

aka "center vortex" or
"'t Hooft flux" background

this $\mathbb{Z}_N^{(1)}$ background explicitly breaks $\mathbb{Z}_{2N}^{d\chi}$ chiral: **anomaly!**

to see, recall:

$$1 = \prod_{\text{all } x} U_{\Box_x} = e^{i\int d^2x\, F_{12}} = e^{i 2\pi Q_{top.}} \Rightarrow Q_{top} = 1 \bmod \mathbb{Z}$$

by periodicity ↑ all x ... ↑ in continuum limit

in theory with gauged center, use $\mathbb{Z}_N^{(1)}$ gauge invariant def. of $Q_{top.}$

$$e^{i\frac{2\pi}{N}} = \prod_{\text{all } x} U_{\Box_{x,\mu\nu}} e^{i\frac{2\pi}{N} b_{x,\mu\nu}} = e^{i 2\pi Q_{top}} \Rightarrow Q_{top} = \frac{1}{N} \bmod \mathbb{Z}$$

in unit 't Hooft flux background

moral: gauge center -> fractional topological charge

recall measure transform under anomaly-free chiral:

$$[\mathcal{D}\psi] \xrightarrow[\mathbb{Z}_{2N}^{d\chi}]{} [\mathcal{D}\psi]\, \underbrace{e^{i 2\pi Q_{top}}}_{= e^{i\frac{2\pi}{N}}}, \text{ since } Q_{top} = \frac{1}{N}$$

in theory with gauged center

gauging $\mathbb{Z}_N^{(1)}$ explicitly breaks $\mathbb{Z}_{2N}^{d\chi}$: **mixed 't Hooft anomaly!**

- $e^{i\frac{2\pi}{N}}$ phase in chiral transform of partition function **IS** the anomaly
- the phase is independent on torus size, it is RG invariant, same in IR!

(phase not a variation of a local 2D (4D) term, but of a 3D (5D) CS term same at all scales)

- if the IR theory is gapped and has a trivial (unique) ground state, nothing to transform under chiral, no way to match anomaly in IR hence IR theory must have "something" transform under chiral, so can not be trivial

Options for matching the mixed 0-form/1-form anomaly in the IR:

- IR CFT?
- breaking of the 0-form and/or 1-form symmetries

  anomaly is matched by a TQFT describing breaking [ex. follows]
- TQFT not related to breaking [Juven Wang…]

In the charge-N Schwinger model, one can show that:

Anber, EP 1807.00093
Armoni, Sugimoto 1812…

$Z_{2N}^{d\chi}$ broken to Z_2 fermion parity, so there are N vacua $|p\rangle$

$$\widehat{\mathrm{Chi}}\, |p\rangle = |p+1\rangle$$

$$\widehat{\mathrm{Center}}\, |p\rangle = |p\rangle\, e^{i \frac{2\pi}{N} p}.$$

center/chiral symmetry operators

center/chiral symmetry algebra:

$$\widehat{\mathrm{Center}} \cdot \widehat{\mathrm{Chi}} \cdot (\widehat{\mathrm{Center}})^{-1} = e^{i \frac{2\pi}{N}}\, \widehat{\mathrm{Chi}}$$

$e^{i \frac{2\pi}{N}}$ shows anomaly: if center gauged, chiral not invariant!

In the 2D charge-N Schwinger model, one can show that:

$\mathbb{Z}_{2N}^{d\chi}$ broken to Z_2 fermion parity, so there are N vacua $|p\rangle$

In each vacuum, the spectrum is gapped - a massive boson, as in in charge-1 massless Schwinger model. **So, what matches anomaly?**

A TQFT, a "chiral lagrangian" describing the N vacua. This is usually not trivial to get from the UV theory, but here it is (will not go through, just give flavor).

TQFT: N-dim Hilbert space, the N vacua - compact scalar and compact U(1)

$$S_{2-D} = i\,\frac{N}{2\pi}\int_{M_2} \varphi^{(0)} da^{(1)}$$

chiral $\phi^{(0)} \to \phi^{(0)} + \frac{2\pi}{N}$ center $a^{(1)} \to a^{(1)} + \frac{1}{N}\epsilon^{(1)}$

quantize: $a_0^{(1)} = 0$ find QM $S_{\mathbb{R}_t \times \mathbb{S}_1} = \frac{N}{2\pi}\int dt\, \varphi\, \frac{da}{dt}$

QM variables $\varphi(t)$ and $a(t) \equiv \oint_{\mathbb{S}_1} a^{(1)}$ $[\hat{\varphi}, \hat{a}] = -i\frac{2\pi}{N}$

Claim: upon gauging center, chiral transform shows anomaly; explicit…

Anber, EP 1811.10642

$$e^{i\hat{\varphi}} e^{i\hat{a}} = e^{i\frac{2\pi}{N}} e^{i\hat{a}} e^{i\hat{\varphi}}$$

- gauge invariant operators (same algebra)

So in 2D all seems nice and explicit (solvable model!), but we're at a "lattice-BSM" meeting… so let's go back to 4D.

$SU(N)$ QCD(adj) w/ $n_f = 1, 2, 3, 4, 5$ Weyl

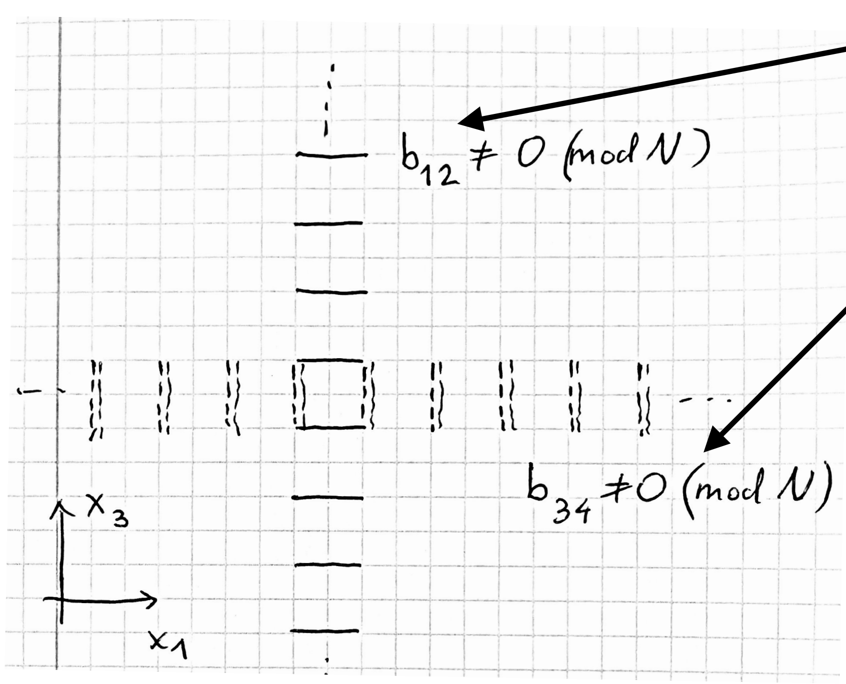
$SU(n_f) \times \mathbb{Z}_{2Nn_f}^{d\chi} \times \mathbb{Z}_N^{(1)} \leftarrow$ center

discrete chiral

to detect mixed anomaly, take $b_{x,\mu\nu} = 1$ on shown plaquettes

center v-x localized in x1,x2, along x3,x4

center v-x localized in x3,x4, along x1,x2



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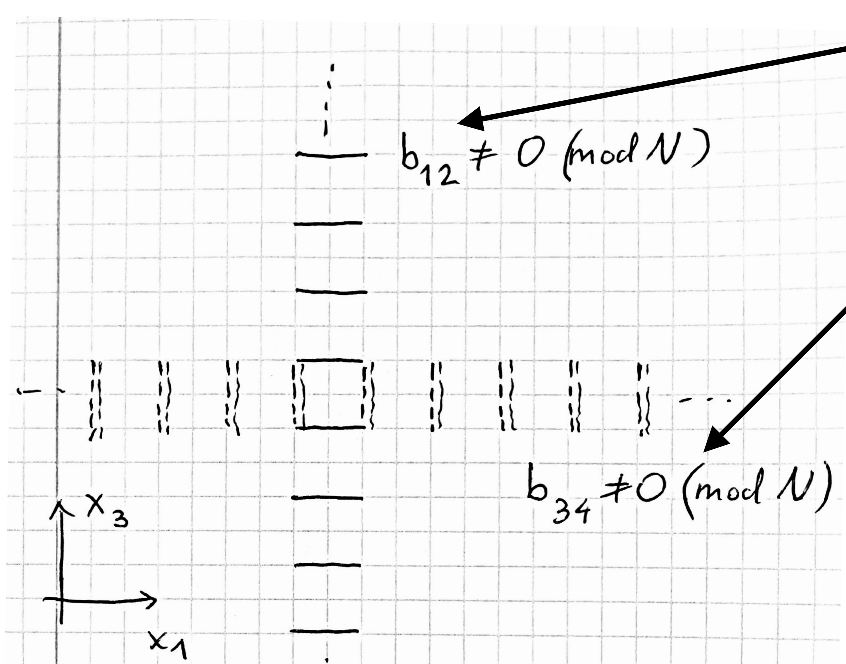

$\mathbb{Z}_N^{(1)}$ **center-gauge invariant**

$$Q_{top} \sim \sum_x \epsilon_{\mu\nu\lambda\sigma} \, \mathrm{Tr}\, U_{\mu\nu} U_{\lambda\sigma} \, e^{i\frac{2\pi}{N}(b_{\mu\nu} + b_{\lambda\sigma})} \quad - - -$$

topological charge

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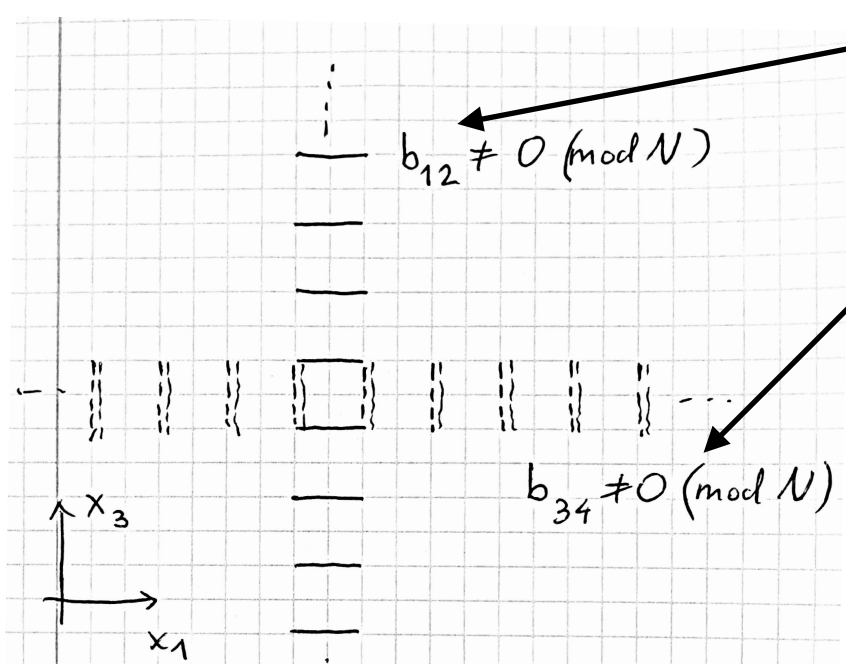


$$e^{i\frac{2\pi}{N}\left(b_{\mu\nu}(x) + b_{\lambda\sigma}(y)\right)}$$

$$\simeq e^{i\frac{2\pi}{N}\left(\delta_{\mu_1}\delta_{\nu_2}\, a^2\, \delta(x_1)\delta(x_2) + \delta_{\lambda_3}\delta_{\sigma_4}\, a^2\, \delta(x_3)\delta(x_4)\right)}$$

$$Q_{top} \sim \sum_x \epsilon_{\mu\nu\lambda\sigma} \operatorname{Tr} U_{\mu\nu} U_{\lambda\sigma}\, e^{i\frac{2\pi}{N}(b_{\mu\nu}+b_{\lambda\sigma})}$$

topological charge 1/N!

thus, upon gauging the center symmetry

SU(N) QCD(adj) w/ $n_f = 1, 2, 3, 4, 5$ Weyl

$SU(n_f) \times \mathbb{Z}^{d\chi}_{2Nn_f} \times \mathbb{Z}^{(1)}_N \leftarrow$ center

discrete chiral

discrete chiral lost:

$$\mathbb{Z}^{d\chi}_{2Nn_f} : [\mathcal{D}\psi] \to [\mathcal{D}\psi]\, e^{i\, 2\pi Q_{top}} \xrightarrow{\text{phase}} e^{i\frac{2\pi}{N}}$$ is "$\mathbb{Z}^{d\chi}_{2Nn_f}\left(\mathbb{Z}^{(1)}_N\right)^2$" anomaly

't Hooft anomalies for QCD(adj) to match

$[SU(n_f)]^3$

$\mathbb{Z}_{2Nn_f}[SU(n_f)]^2$

$[\mathbb{Z}_{2Nn_f}]^3$

$\mathbb{Z}_{2Nn_f}[G]^2$

$\mathbb{Z}_{2Nn_f}[\mathbb{Z}^{(1)}_N]^2$

(+ center-gravity subtlety for nf=2 - ask Juven Wang) various recent solutions + important studies and subtleties *clarified*

Anber-EP; ***Cordova-Dumitrescu***; Bi-Senthil; ***Wan-Wang,*** Ryttov-EP

the new features, for nf=2 and nf=3

"confinement without continuous chiral symmetry breaking, but with discrete chiral breaking"

*important **new** message re. anomalies*

in a theory with no gauge fields in IR, discrete chiral breaking needed to match chiral/center anomaly

- center unbroken (confinement)
- $SU(n_f)$ unbroken
- $\mathbb{Z}_{2Nn_f}^{d\chi}$ broken to $\mathbb{Z}_{2n_f}^{d\chi}$ - N vacua

... are these phases realized? are they "likely"? *.... we don't know*

n_f	IR Phase	Intact cχ sym.	Intact dχ sym.	Intact center sym.
≥ 6	Free	Yes	Yes	No
5	Fixed point	Yes	Yes	No
4	Fixed point	Yes	Yes	No
3	Confinement, massless composite fermions	Yes	No	Yes
2	Confinement	No	No	Yes
1	N =1 SYM	—	No	Yes
0	Pure YM	—	—	Yes

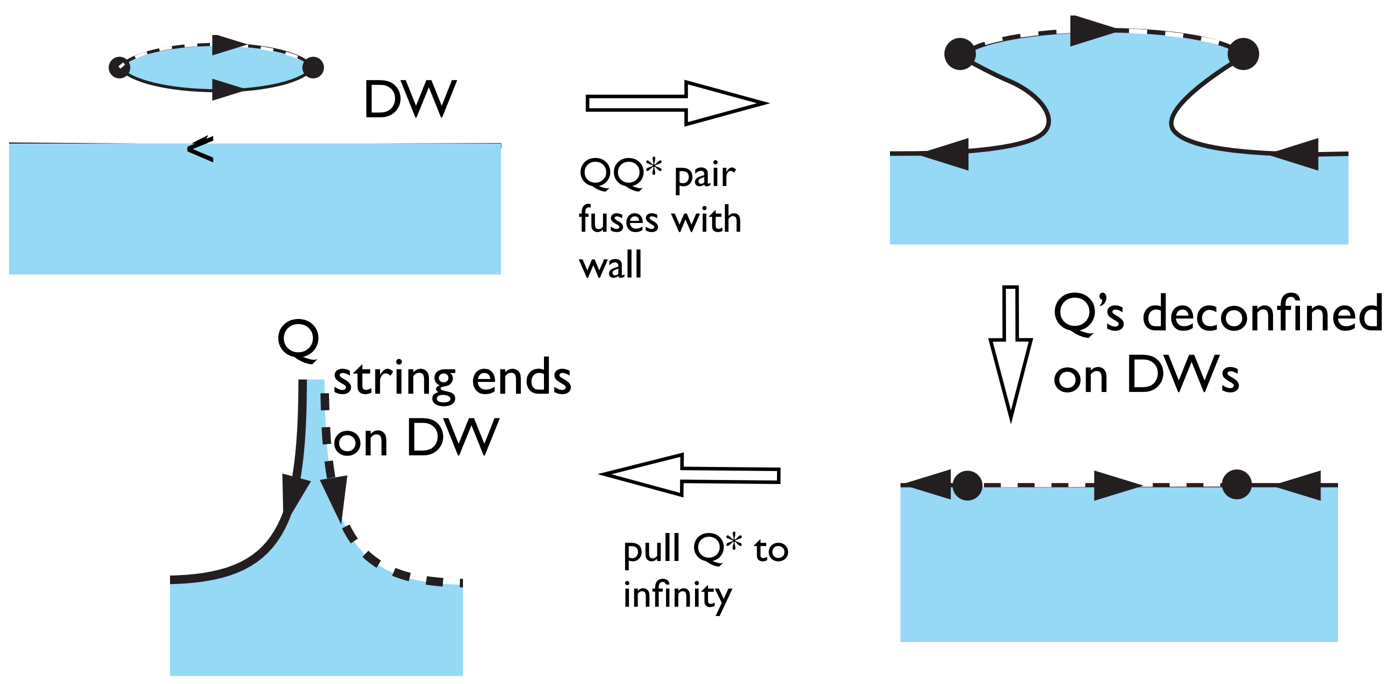
Notice, discrete chiral breaking also in "vanilla" phases with $SU(n_f)$ broken to SO(n)

Thus domain walls (DW) are a generic feature, no matter fate of SU(nf).

Turns out DW "worldvolume physics" is quite rich, due to "discrete anomaly inflow."

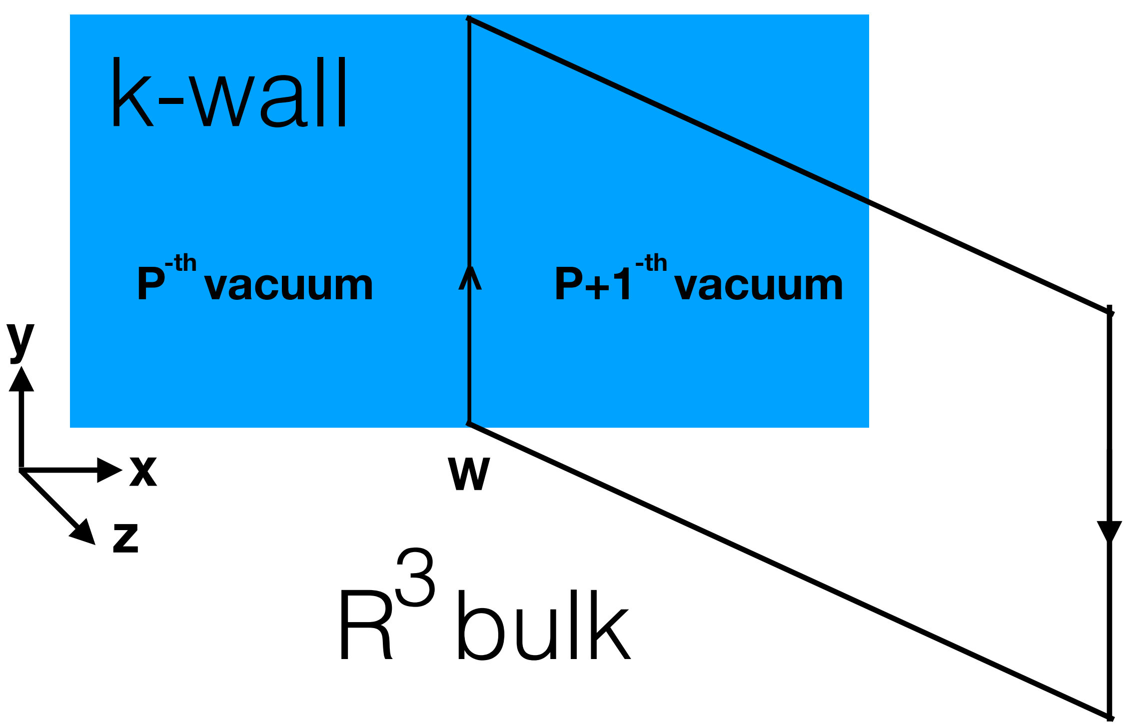
In particular, in confining theories, DW between chiral broken vacua deconfine probe quarks & confining strings end on DWs.

First seen on R^3 x S^1 *Anber-Sulejmanpasic-EP 2015* explicit semiclassics, *after Unsal 2007-* without relation to "anomaly inflow".



Also in high-T DW (semiclassical incarnation of center vortices!) between center broken vacua, similar story: "deconfine" probe quarks & confining strings end on DWs, **Anber--EP 2018**

$T \gg \Lambda$ $\qquad Z_{2N}^{(0)}\, Z_N^{(1)}$ 't Hooft anomaly on worldvolume



1 fermion condensate on k-wall

2 quarks deconfined on k-wall

first via holography: *F1* on *D1*

[Aharony, Witten 1999;…]

here, QFT: 2d YM with massless fermions screens

[Schwinger model - many; nonabelian - Gross, Klebanov, Matytsin, Smilga 1995; Armoni, Frishman, Sonnenschein 1997;…]

so we find "D-branes" and "strings", once again, in QFT

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Ex.: solvable 2D model-Hamiltonian, Eucl., bosoniz….

2.

These consistency conditions constrain IR phases of gauge theories to be "nontrivial."

Ex.: QCD(adj), rich… (also other 2-index reps)

3.

They also imply that, whenever domain walls exist, their worldvolume physics is quite nontrivial: discrete version of "anomaly inflow."

Ex.: high-T and low-T DWs worldvolume

Hopefully, you find some of this useful in the future!