Emergent "dimensions" and "branes" from large-N confinement

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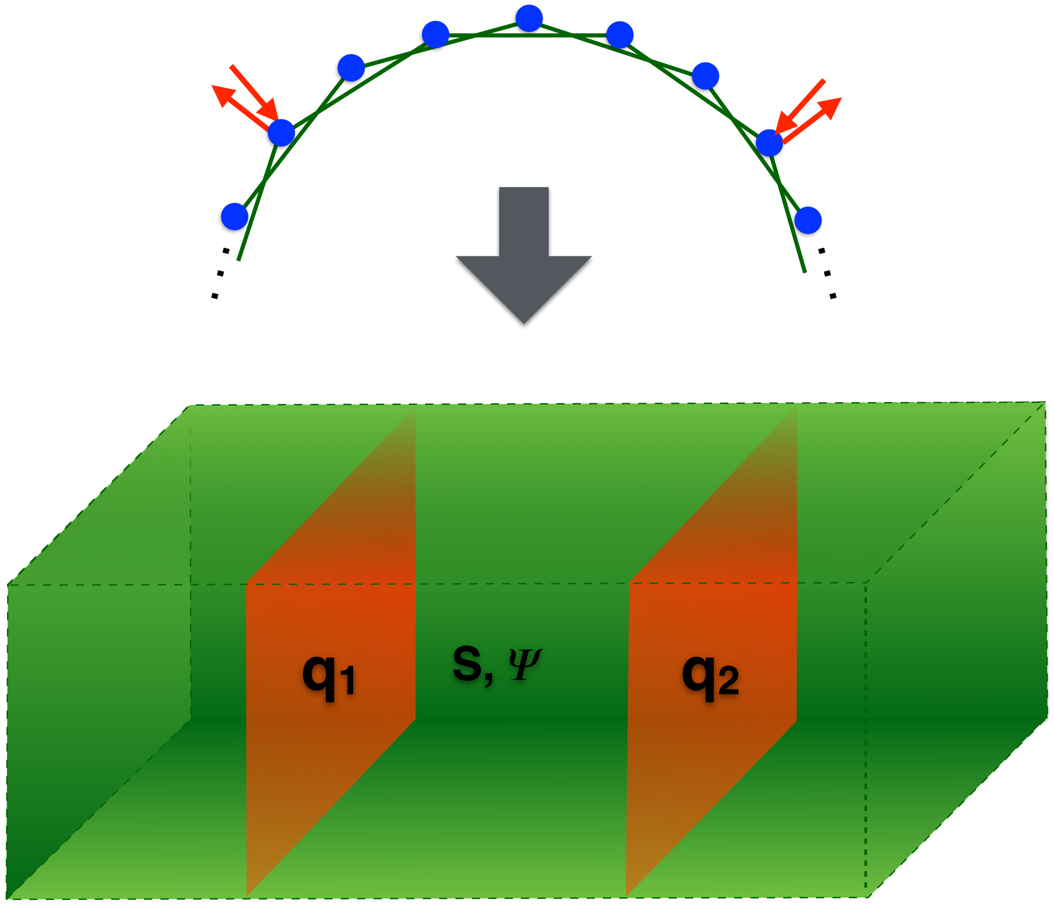
with **Aleksey Cherman** (University of Washington)



1606.01902 + many earlier related works,
by us and by others

This is talk is about the unusual behavior of a confining gauge theory, $\mathcal{N}=1$ SYM, and its $\mathcal{N}=0$ deformations, at large N, in a particular - but nonperturbatively calculable! - limit.

(N=number of colours; $\mathcal{N}$=number of supersymmetries).

SYM/QCD(adj) & deformed Yang-Mills (dYM) on $R^{1,2} \times S^1_L$, small L

most of the talk about SYM: pure YM + 1 massless adjoint Weyl

QCD(adj): pure YM + many massless adjoint Weyl

dYM: pure YM + many massive adjoint Weyl, w/ $m \sim \frac{1}{NL}$

mass gap, confinement, center stability in a controlled manner!

nonperturbative calculability is not common in locally 4d theory, much exploited in past 9 years; still on!

Unsal w/ Yaffe, Shifman, EP, Argyres, Anber, Schafer, Cherman + others… (2007-)

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SYM: pure YM + 1 massless adjoint Weyl

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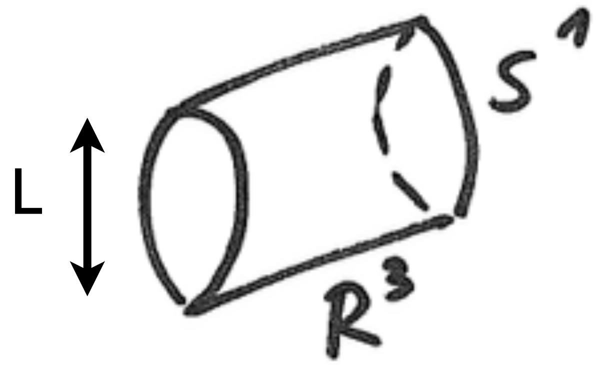

$\eta \equiv L\Lambda N \ll 1$ = calculable regime

with strong scale fixed, at large N,

size L has to shrink as $L \leq \frac{1}{\Lambda N}$

several unusual things happen at large N:

- mass gap goes to zero
- confining string tension, T_1, remains finite
- for a range of N, as **L becomes small** and N-large, the theory is that of a discretized **large emergent dimension** of size

$$\tilde{L} \sim aN \text{ with ``lattice spacing'' } a \sim \frac{1}{\Lambda \eta^2}$$

  with a Lifshitz scaling (spacelike, with z=2).

- in addition, we have that $L\tilde{L} \sim \alpha' \sim \frac{1}{T_1}$ … ?
- upon adding supersymmetry breaking deformations - mass for ‘gaugino’, or fundamental quarks, one obtains a “normal” scaling theory or a “braneworld”, respectively

PLAN OF TALK: explain how this comes about & discuss puzzles/questions…

FOUR facts about SU(N) SYM (gauge boson + massless gaugino).

1 Z_{2N} discrete chiral ("R") symmetry; on R^4 $\mathbb{Z}_{2N} \to \mathbb{Z}_2$, N vacua

- by gaugino condensation, not directly calculable.

2 On R^3 x S^1, calculability due to opening of "Coulomb branch", i.e. S^1 Wilson loop: $\mathrm{tr}\,\Sigma = \mathrm{tr}\,\mathcal{P} e^{i\int_{\mathbb{S}^1} A_3}$ can have vev.

S^1 is spatial, fermions are periodic (SUSY preserved): perturbatively "Coulomb branch," parameterized by the N-1 eigenvalues of Σ , not lifted.

3 When $\langle \Sigma \rangle \sim \mathrm{diag}(1, \omega, \ldots, \omega^{N-1})$ $\omega \equiv e^{2\pi i/N}$

we have (assuming weak coupling) that $\langle \mathrm{tr}\,\Sigma^n \rangle \;=\; 0, \forall n \;<\; N$

This special point in moduli space is called the "center symmetric point", with unbroken $\Sigma \to \omega\Sigma$ global center symmetry ("zero form")

($\langle \mathrm{tr}\,\Sigma^n \rangle$ are the order parameters for the center symmetry)

4 A rather **nontrivial** dynamical fact-we have to accept it!-is that nonperturbative effects on R^3xS^1 'lift' the Coulomb branch and stabilize the vacuum at the center-symmetric point!

(fascinating stuff, but not here:…**magnetic bions, neutral bions, complex saddles, Lefshetz thimbles**…)

SU(N) SYM (gauge boson + massless gaugino).

S^1 Wilson loop: $\mathrm{tr}\,\Sigma = \mathrm{tr}\,\mathcal{P}e^{i\int_{\mathbb{S}^1} A_3}$ can have vev.

So, let us assume

$$\langle\Sigma\rangle \sim \mathrm{diag}(1, \omega, \ldots, \omega^{N-1}) \qquad \omega \equiv e^{2\pi i/N}$$

and think about scales. Notice Σ transforms as an adjoint Higgs field under 3d gauge transforms and thus vev breaks $SU(N) \to U(1)^{N-1}$

The scale of the breaking is the lightest W-boson mass $m_W = \frac{2\pi}{NL}$ (use def. of Σ, fix gauge and map to $\langle A_3\rangle \neq 0$ to find this)

This explains our $\eta \equiv L\Lambda N \ll 1$ calculability condition stated earlier.

At long distances our theory is the one of the N-1 Cartan “photons”.

(the two lessons of this page)

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At long distances our theory is the one of the N-1 Cartan “photons”.

Cartan “photons” $F_k^{\mu\nu}$ $(k = 1,\ldots N-1,\ \mu,\nu = 0,1,2)$

Cartan “gluinos” ψ_k^α $(\alpha = 1,2)$

Cartan “scalars” ϕ_k from A_3L fluctuations (parameterize Coulomb branch)

$F^k_{\mu\nu} = \frac{g^2}{2\pi L}\,\epsilon_{\mu\nu\alpha}\partial^\alpha\sigma^k$ ← “dual photons” $\vec{\sigma}$

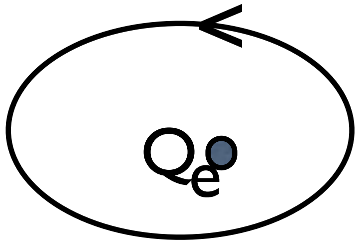
compact scalars: $\vec{\sigma}$ and $\vec{\sigma} + 2\pi\,\vec{w}_k$ identified in SU(N)

simple co-roots → $\vec{\alpha}_i^* \cdot \vec{w}_j = \delta_{ij}$

(magnetic charges)

fundamental weights (electric charges)

heuristically, from duality relation $\partial_i\sigma \sim \frac{L}{g^2}\epsilon_{ij}E_j,\quad j = 1,2$



$\oint d\sigma = Q_{\mathbf{e}}$

monodromy of dual photon related to electric flux

weight lattice periodicity of $\vec{\sigma}$ = fundamental Wilson lines well defined

SU(N) SYM (gauge boson + massless gaugino).

This explains our $\eta \equiv L\Lambda N \ll 1$ calculability condition stated earlier.

At long distances our theory is the one of the N-1 Cartan “photons”.

Will describe all in terms of the dual photons; the rest comes “for the ride” by SUSY.

$$F^k_{\mu\nu} = \frac{g^2}{2\pi L}\,\epsilon_{\mu\nu\alpha}\partial^\alpha\sigma^k$$

“dual photons” $\vec{\sigma}$

compact scalars: $\vec{\sigma}$ and $\vec{\sigma} + 2\pi\,\vec{w}_k$

$$\vec{\alpha}^*_i \cdot \vec{w}_j = \delta_{ij}$$

A technical detail: imagine group is U(N); the U(1) decouples. N-dimensional.

SU(N) SYM (gauge boson + massless gaugino).

perturbative
(dual of Cartan photons kinetic term)

$$S_\sigma = \int d^3x \left\{ \lambda m_W (\partial_\mu \vec{\sigma})^2 \right.$$

$$\lambda \equiv g^2 N \sim \frac{1}{\log \frac{1}{\eta}}$$

$(\eta \equiv L \Lambda N \ll 1)$

$$m_W = \frac{2\pi}{NL}$$

$$F^k_{\mu\nu} = \frac{g^2}{2\pi L} \epsilon_{\mu\nu\alpha} \partial^\alpha \sigma^k$$

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compact scalars: $\vec{\sigma}$ and $\vec{\sigma} + 2\pi\, \vec{w}_k$

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SU(N) SYM (gauge boson + massless gaugino).

perturbative
(dual of Cartan photons kinetic term)

non-perturbative :...magnetic bions...

$$S_\sigma = \int d^3x \left\{ \lambda m_W (\partial_\mu \vec{\sigma})^2 \qquad + m_W^3 \underset{\overset{\uparrow}{e^{-\frac{8\pi^2}{g^2 N}}}}{e^{-2S_0}} \sum_{i=0}^{N-1} \sin^2\left[\frac{1}{2}(\vec{\alpha}^*_{i(\mathrm{mod}N)} - \vec{\alpha}^*_{i+1})\cdot\vec{\sigma}\right] \right\}$$

$$\lambda \equiv g^2 N \sim \frac{1}{\log\frac{1}{\eta}}$$

$$(\eta \equiv L\Lambda N \ll 1)$$

$$m_W = \frac{2\pi}{NL}$$

affine (lowest) root!

$$(\vec{\alpha}^*_i - \vec{\alpha}^*_{i+1})\cdot\vec{\sigma} = 2\sigma_i - \sigma_{i-1} - \sigma_{i+1}$$

$$\vec{\alpha}^*_1 = (1,-1,0,0\ldots0) \quad \vec{\alpha}^*_2 = (0,1,-1,0\ldots0) \;\ldots\; \downarrow \vec{\alpha}^*_N = (-1,0,0,\ldots.,1)$$

... can show there are N vacua etc., simplest way to proceed to our result:

expand around $\langle\vec{\sigma}\rangle = 0$ vacuum and rescale field; up to O(1) factors:

$$\boxed{\begin{array}{c} S_\sigma = \int d^3x \sum_{i=1}^{N} \{(\partial_\mu\tilde{\sigma}_i)^2 + \underset{\uparrow}{M^2}(2\tilde{\sigma}_i - \tilde{\sigma}_{i-1} - \tilde{\sigma}_{i+1})^2\} \\ \\ \frac{1}{a} := M = m_W e^{-S_0} = \Lambda\eta^2 \end{array}}$$

moral: a field in a latticized emergent dimension - with kinetic term $(\partial_y^2\tilde{\sigma})^2$

Where this came from?

use “power of holomorphy” **rather than** the fascinating stuff:

…it is much more widely known and accepted… magnetic bions, neutral bions, complex saddles, Lefshetz thimbles…

Cartan chiral superfields - define as fluctuation around vevs in one of the N vacua (k-th):

$$x^i = \delta\phi^i + i\delta\sigma^i + \theta^\alpha \ldots$$

Kahler potential superpotential: holomorphy+symmetry+limits

$$S = \int d^3x \int d^4\theta \, \underbrace{m_W g_{ij}^{-1}}_{K_{i\bar{j}}} x^i \bar{x}^j + \int d^3x \int d^2\theta \, \underbrace{\Lambda^2 \eta e^{\frac{2\pi i k}{N}} \sum_{a=1}^{N} e^{\vec{\alpha}_a^* \cdot \vec{x}}}_{W_k(\vec{x})} + \text{h.c.}$$

$$g_{ij} = \delta_{ij} \frac{16\pi^2}{3\lambda} + \ldots$$

(electric) Coulomb branch metric

$$\left. \frac{\partial W_k(\vec{x})}{\partial \vec{x}} \right|_{\vec{x}=0} = 0$$

no mention of center symmetry → Seiberg-Witten ‘97
Davies et al ‘99

instanton calculation completed only recently

Schafer, Unsal, EP, ‘12
Anber, Teeple, EP, ‘14

Where this came from?

$$S = \int d^3x \int d^4\theta \; \underset{K_{i\bar{j}}}{\underline{m_W g^{-1}_{ij}}} x^i \bar{x}^j \quad + \int d^3x \int d^2\theta \; \underset{W_k(\vec{x})}{\underline{\Lambda^2 \eta e^{\frac{2\pi i k}{N}} \sum_{a=1}^{N} e^{\vec{\alpha}^*_a \cdot \vec{x}}}} + \text{h.c.}$$

$g_{ij} = \delta_{ij} \frac{16\pi^2}{3\lambda} + \ldots$ (electric) Coulomb branch metric $\quad \left. \frac{\partial W_k(\vec{x})}{\partial \vec{x}} \right|_{\vec{x}=0} = 0 \quad x^i = \delta\phi^i + i\delta\sigma^i$

to see lattice derivative structure, rewrite W:

$$\sum_{a=1}^{N} e^{\vec{\alpha}^*_a \cdot \vec{x}} = \sum_a e^{-(x_{a+1} - x_a)} \equiv \sum_a e^{-\nabla_+ x^a} = 1 + \sum_{m=2}^{\infty} \frac{(-)^m}{m!} \sum_a (\nabla_+ x^a)^m$$

- **arises from symmetries determining superpotential!**

- dual photon $2\pi \vec{w}_k$ periodicity means only $e^{i\vec{\alpha}^*_k \cdot \vec{\sigma}}$ can appear, as $\vec{\alpha}^*_i \cdot \vec{w}_j = \delta_{ij}$

- center symmetry $x^i \to x^{i+1(\text{mod}N)}$ leads to lattice translational invariance

- chiral symmetry, or discrete R, $e^{i\vec{\alpha}^*_k \cdot \vec{\sigma}} \to e^{\frac{2\pi i}{N}} e^{i\vec{\alpha}^*_k \cdot \vec{\sigma}}$ means only terms in W **(holom.!)**

- locality in the extra dimension due to weak coupling, not symmetry:

  symmetries alone would permit,e.g. $\text{Re} \sum_k e^{i\vec{\alpha}^*_k \cdot \vec{\sigma}} e^{-i\vec{\alpha}^*_{k+N/2} \cdot \vec{\sigma}}$ potential terms, but forbidden by holomorphy; nonlocal terms can arise from $K_{i\bar{j}}$ ~ kinetic terms in R^3

  from W-bosons, nonlocal in color space, $g_{ij} = \delta_{ij} \frac{16\pi^2}{3\lambda} - (1 - \delta_{ij}) \frac{\psi(\frac{|i-j|}{N}) + \psi(1 - \frac{|i-j|}{N})}{N}$

(can show nonlocal terms matter only for $N \geq N_* \sim 2\pi e^{\frac{8\pi^2}{3\lambda}}$, mass gap vanishing not altered)

Spectrum:

$$S = \int d^3x \int d^4\theta \; \underbrace{m_W g^{-1}_{ij}}_{K_{i\bar{j}}} x^i \bar{x}^j \quad + \int d^3x \int d^2\theta \; \underbrace{\Lambda^2 \eta e^{\frac{2\pi i k}{N}} \sum_{a=1}^{N} e^{\vec{\alpha}^*_a \cdot \vec{x}}}_{W_k(\vec{x})} + \text{h.c.}$$

$$g_{ij} = \delta_{ij} \frac{16\pi^2}{3\lambda} + \ldots$$ (electric) Coulomb branch metric $$\left. \frac{\partial W_k(\vec{x})}{\partial \vec{x}} \right|_{\vec{x}=0} = 0$$

$$\sum_{a=1}^{N} e^{\vec{\alpha}^*_a \cdot \vec{x}} = \sum_a e^{-(x_{a+1} - x_a)} \equiv \sum_a e^{-\nabla_+ x^a} = 1 + \sum_{m=2}^{\infty} \frac{(-)^m}{m!} \sum_a (\nabla_+ x^a)^m$$

to quadratic order, up to O(1) coeffts:

$$S \sim \int d^3x \int d^4\theta \; \lambda m_W \; x^i \bar{x}^i \; + \int d^3x \int d^2\theta \; \Lambda^2 \eta \; \sum_{a=1}^{N} (\nabla_+ x^a)^2 + \text{h.c.}$$

after a Z_N Fourier transform and rescaling $\Phi_p \sim$ Z_N FT of $x^j = \phi^j + i\sigma^j$

$$= \int d^3x \sum_{p=1}^{N} \left\{ |\partial_\mu \Phi_p|^2 + M^2 \sin^4\left(\frac{\pi p}{N}\right) |\Phi_p|^2 \; + \bar{\Psi}_p \partial\!\!\!/ \Psi_p + \frac{M}{2} \sin^2\left(\frac{\pi p}{N}\right) (\Psi_{N-p} \Psi_p + \text{h.c.}) \right\}$$

$$m_p = M \sin^2\left(\frac{\pi p}{N}\right) \qquad M \sim \frac{\Lambda^2 \eta}{m_W} = \Lambda \eta^2$$ = 1/a - the “lattice spacing”

dimension size $\tilde{L} = aN$ is much larger than both L and inverse strong scale:

$$\tilde{L}/L \sim N^2 \eta^{-3}, \tilde{L}\Lambda = N\eta^{-2}$$

Spectrum/scaling:

$$S \sim \int d^3x \int d^4\theta \, \lambda m_W \, x^i \bar{x}^i \; + \int d^3x \int d^2\theta \, \Lambda^2 \eta \sum_{a=1}^{N} (\nabla_+ x^a)^2 + \text{h.c.}$$

using a continuum notation (“a”-finite), at length scales ℓ

$$a \ll \ell \ll \tilde{L} \qquad \tilde{L} = aN$$

we can write action as

$$S = \int d^3x \, dy \, \{ |\partial_\mu \Phi|^2 + |\partial_y^2 \Phi|^2 + \bar{\Psi} \not{\partial} \Psi + \tfrac{1}{2} (\Psi \partial_y^2 \Psi + \text{h.c.}) \}$$

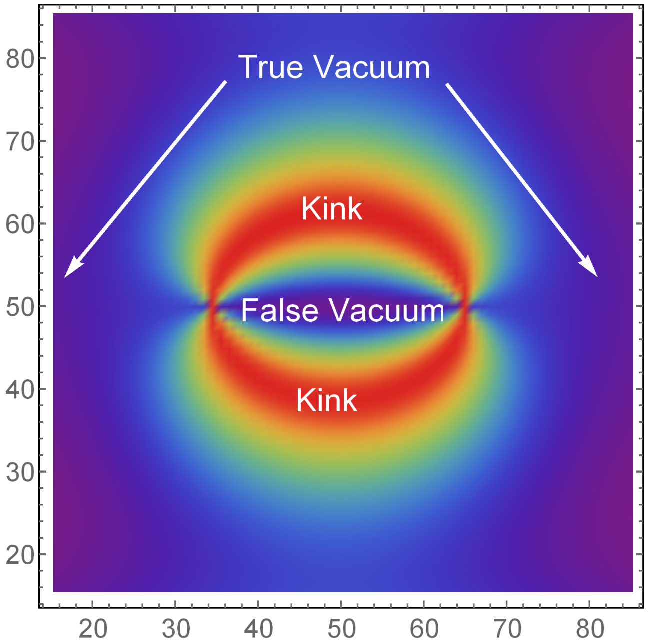
this has anisotropic spatial Lifshitz scale invariance with z = 2:

$$\begin{array}{cc} x_{0,1,2} \to \Omega \, x_{0,1,2}, & y \to \Omega^{1/z} \, y \\ \Phi \to \Omega^{-(1+1/z)/2} \, \Phi, & \Psi \to \Omega^{-(2+1/z)/2} \, \Psi \end{array}$$

up to irrelevant local + small, nonlocal in y, terms

z=2 —-> z=1 upon gaugino mass perturbation, also expected in dYM (maybe later)

Walls and strings:



… as a side remark, confining strings here (and in dYM) have rather distinct properties from those of other theories with abelian confinement, notably Seiberg-Witten theory; they are much closer to real QCD! - but that’s another talk …

- confining strings are composed of two domain lines

- tension (nonBPS) ~ tension of domain lines (BPS)

see Anber, Sulejmanpasic, EP, ‘15

$$\ldots + \int d^3x \int d^2\theta \; \overbrace{\Lambda^2 \eta e^{\frac{2\pi i k}{N}} \sum_{a=1}^{N} e^{\vec{\alpha}_a^* \cdot \vec{x}}}^{W_k(\vec{x})} + \text{h.c.}$$

BPS wall tension determined by values of superpotential at minima:

$$T_k = |W_k(\vec{0}) - W_0(\vec{0})| \; = \Lambda^2 \eta |N e^{\frac{2\pi i k}{N}} - N| \; = 2\Lambda^2 \eta \; N \sin \frac{\pi k}{N}$$

(finite T’s means even though chiral becomes U(1), no massless eta’ at infinite N)

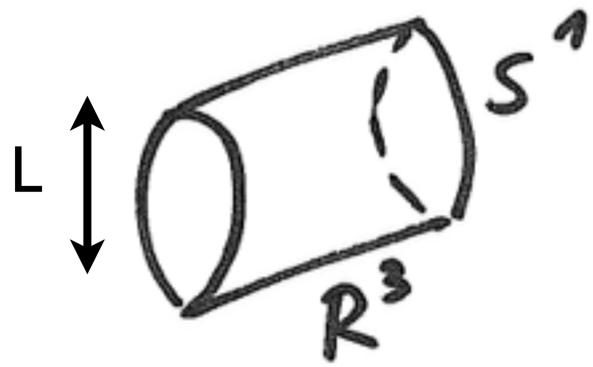
k=1 relevant for strings confining fundamental quarks

$T_1 \sim \Lambda^2 \eta$ compare with gap $M_1^2 \sim \dfrac{\Lambda^2 \eta^4}{N^4}$: suppressed by N and coupling

further, recall $\tilde{L} = aN \sim \dfrac{N}{\Lambda \eta^2}$ to obtain $\tilde{L} L \sim \dfrac{NL}{\Lambda \eta^2} = \dfrac{1}{\Lambda^2 \eta} \sim \dfrac{1}{T_1} \sim \alpha'$ “T-duality”?

Back to p. 2 and a summary of what I told you about so far:

SYM: pure YM + 1 massless adjoint Weyl



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  with a Lifshitz scaling (spacelike, with z=2).

- in addition, we have that $L\tilde{L} \sim \alpha' \sim \dfrac{1}{T_1}$ … ?
- adding SUSY breaking deformations - gluino mass, or fundamental quarks, one obtains a z=1 scaling theory or a “braneworld” [no time to give detail]

My presentation, for reasons I stated, stressed supersymmetry, but the important ingredient is semiclassical calculability - so I expect much of what I told you in dYM and QCD(adj). Much of the large-N studies remain to be done there, however.

↑

continuously connected to pure YM on R^4!

- dYM work to appear Shalchian, EP, ’16/‘17

Before discussing further questions, compare with abelian large-N in Seiberg-Witten theory on R^4:

Douglas, Shenker, ‘96

mass gap $m_n^2 \sim e_D^2 N m \Lambda \sin \frac{\pi n}{N}$ string tension $T_n \sim N m \Lambda \sin \frac{\pi n}{N}$

dual coupling, finite w/ N

$\mathcal{N}$=2 SUSY-breaking mass

(notice: no local extra dim. interpretation… no center unbroken)

lightest W-boson

$$m_W \sim \frac{\Lambda}{N^2}$$

calculability then requires large-N scaling as:

$$m_n \ll m_W \to \sqrt{m\Lambda} \ll \frac{\Lambda}{N^2} \longrightarrow m \ll \frac{\Lambda}{N^4}$$

—> mass gap and string tension vanish at same rate (dual coupling finite)

behaviour different from SYM discussion at small L, where we recall that

$$T_1 \sim \Lambda^2 \eta$$

gap $M_1^2 \sim \frac{\Lambda^2 \eta^4}{N^4}$: suppressed by N **and** coupling

All we did came from honest calculation. But, we'd like to better understand:

What is the reason for the vanishing of the large-N mass gap?

Broken symmetry?… *but which one ? obvious "one-form" center Z_N->U(1) unbroken in SYM*
an emergent one? Basar, Cherman, McGady… '14

Is the "extra dimension" a useful organizing principle?

light modes 'derivatively coupled'…

(Is the "T-duality-like" relation $L\tilde{L} \sim \alpha' \sim \frac{1}{T_1}$ **a coincidence?)**

Is there a large-N phase transition on the way from $\eta = \Lambda L N \ll 1$ **(small)**

to $\eta = \Lambda L N \gg 1$ **(large),**

or could the large-N mass gap vanish also on R^4? *(apart from holography we have no data)*

Another relation that'd be nice to understand - the one to Seiberg-Witten on R^4?

To conclude: the Abelian large-N limit displays many unusual features. It defies usual large-N rules and assumptions and challenges our understanding of large-N gauge dynamics…