The mixed 0-form/1-form anomaly in Hilbert space: pouring the new wine into old bottles

Erich Poppitz U of Toronto

work with **Andrew Cox, F. David Wandler**

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UV

$\Bigg\downarrow$ anomaly matching

IR **??**

limits fantasies about IR!

thought anomaly matching was set in stone since ca. 1980
“0-form” anomalies played major role in, say, “preon” models (1980’s), Seiberg duality (1990’s)

new “generalized ’t Hooft anomaly matching”
Gaiotto, Kapustin, Komargodski, Seiberg, Willett + … (2014-)

Topic of this talk:

4d gauge theory with arbitrary gauge group w/ center:

$$SU(N)\ (Z_N), \quad Sp(N)\ (Z_2), \quad Spin(N)\ (Z_2, Z_4, \text{ or } Z_2 \times Z_2), \quad E_6\ (Z_3), \quad E_7\ (Z_2)$$

pure YM: mixed anomaly 0-form parity at $\theta = \pi$ and 1-form center symmetry

QCD(adj), YM + n_f Weyl adjoints: mixed anomaly between 0-form discrete chiral and 1-form center symmetry

Upshot:

Anomalies' consequence in Hilbert space on T^3: exact degeneracies.

Explain how these come about and discuss implications.

For many, a trip back to the '80s - but with novel interpretation!

Plan:

1 Review Euclidean picture

2 Motivation: why Hilbert space?… 2d

3 Gauging $Z_N^{(1)}$ on T^3, quantization, and centrally-extended algebra

4 Consequences, discussion, old vs new

5 Discrete chiral/center anomaly

6 Summary & outlook

most of talk use example of $SU(N)$ and parity/center anomaly at $\theta = \pi$

Review Euclidean picture

gauge theory w/ center - pure YM or QCD(adj): 1-form symmetry

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parity at $\theta = \pi$ (or discrete chiral): 0-form symmetry

parity at $\theta = \pi$ (or discrete chiral): require 2π shift of θ angle

partition function only invariant if $Q_{\text{top.}}$ integer

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gauge theory w/ center - pure YM or QCD(adj): 1-form symmetry

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partition function only invariant if $Q_{\text{top.}}$ integer

mixed anomaly: gauge background for 1-form, observe 0-form violated

vacuum can't be "trivially gapped"

2 Motivation: why Hilbert space?…

- implications very immediate, as seen in 2d

$$L = -\frac{1}{4e^2}\, f_{kl} f^{kl} + i\bar{\psi}_+(\partial_- + iqA_-)\psi_+ + i\bar{\psi}_-(\partial_+ + iqA_+)\psi_-$$

$$\mathbb{Z}_{2q}^{d\chi}:\ \psi_\pm \to e^{\pm i\frac{\pi}{q}}\psi_\pm \qquad \mathbb{Z}_q^C:\ e^{i\oint A_x dx} \to \omega_q\, e^{i\oint A_x dx}, \quad \omega_q \equiv e^{i\,\frac{2\pi}{q}}.$$

$$X_{2q}\, Y_q = \omega_q\, Y_q\, X_{2q} \qquad (\omega_q = e^{i\frac{2\pi}{q}})$$

[Anber, EP 2018, in fermion formulation]

anomaly=centrally extended algebra

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$$X_{2q}\,|P,\theta\rangle = |P+1(\mathrm{mod}\ q),\theta\rangle$$

- discrete chiral broken

$$Y_q\,|P,\theta\rangle = |P,\theta\rangle\, \omega_q^{-P} e^{-i\theta}$$

- discrete E-field in each vacuum

- general interest, to understand phenomena from different angles

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

- to see extension of algebra, introduce 2-form center background on T^3

't Hooft twisted b.c.

't Hooft '81; van Baal '82,'84; Witten '82,'00; Gonzalez-Arroyo, Korthals Altes '88 …

$$A(L_1, y, z) = \Gamma_1 A(0, y, z) \Gamma_1^{-1}$$
$$A(x, L_2, z) = \Gamma_2 A(x, 0, z) \Gamma_2^{-1}$$
$$A(x, y, L_3) = \Gamma_3 A(x, y, 0) \Gamma_3^{-1}$$

torus = one coordinate chart

transition functions on overlaps = b.c.

cocycle condition on triple overlaps

$$\Gamma_k \Gamma_l = \Gamma_l \Gamma_k e^{i\frac{2\pi}{N}\epsilon_{klm} m_m}$$: constant twist matrices

$\vec{m}$ (mod N) … discrete magnetic flux

… center vortex … $\oint_{kl} C^{(2)} = \frac{2\pi}{N}\epsilon_{klm} m_m$ in Kapustin, Seiberg '14 formalism

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

space of fields w/ b.c.:

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quantization, $A_0 = 0$, Gauss' law:

$$U(L_1, y, z) = \Gamma_1 U(0,y,z)\Gamma_1^{-1}$$
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$$\mathcal{H}_\theta^{phys.} = \left\{ |\psi\rangle \in \mathcal{H} \; : \; \hat{U}\,|\psi\rangle = e^{-i\theta\nu}\,|\psi\rangle \,, \forall U \right\}$$

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1-form symmetries … "improper gauge trsfs.", "central conjugations" (Luscher)

$$C[\vec{k},\nu](L_1, y, z) = e^{i\frac{2\pi k_1}{N}}\ \Gamma_1 C[\vec{k},\nu](0,y,z)\Gamma_1^{-1}$$
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introduce some notation:

$$\hat{A}_k(x)\,|A\rangle = |A\rangle A_k(x)$$
$$g \circ A \equiv g(A - id)g^{-1}$$

functions $C[\overrightarrow{k},\nu]$ define operators: $\hat{C}[\overrightarrow{k},\nu]\,|A\rangle = |\,C[\overrightarrow{k},\nu]\circ A\rangle$

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

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$\longrightarrow$

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$\hat{T}_i$ generate global symmetries: act on Wilson loops in i-th direction

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

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$\hat{T}_i$, *(or their C's)* when $\vec{m} \neq 0$, have fractional $T^3 \to SU(N)$ winding number

$$Q[C] = \frac{1}{24\pi^2} \int_{\mathbb{T}^3} \mathrm{tr}\,(C dC^{-1})^3 \;\; = \ldots = \;\; \frac{\vec{m} \cdot \vec{k}}{N} + \nu$$

=…= the instanton number
of a 4d field configuration twisted by
Γ_i in space and $C[\vec{k}, \nu]$ in time…
fractional part only $\vec{m}$, $\vec{k}$ dependent

we define $\hat{T}_i$ s.t. $Q[T_l] = \frac{m_l}{N}$

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Q[C] = the instanton number of a 4d field configuration twisted by
Γ_i in space and $C[\vec{k}, \nu]$ in time; fractional part only $\vec{m}$, $\vec{k}$ dependent ['t Hooft '81; our appx.]

$$Q = \frac{1}{8\pi^2} \int \mathrm{tr}\, F \wedge F \qquad\qquad K_0(A) \equiv \frac{1}{8\pi^2}\, \mathrm{tr}\,(A \wedge F - \frac{i}{3} A \wedge A \wedge A)$$

b.c.!

$$Q[C] = \int_{\mathbb{T}^3} K_0\,(C \circ A) - K_0\,(A) \quad = \frac{1}{24\pi^2} \int_{\mathbb{T}^3} \mathrm{tr}\,(C dC^{-1})^3 + \cancel{\frac{1}{8\pi^2} \int_{\mathbb{T}^3} d\,\mathrm{tr}\,(iA\; dC^{-1} C)}$$

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boundary conditions on T^3

$\vec{m}$ (mod N) …

discrete magnetic flux

we define $\hat{T}_l$ s.t. $Q[T_l] = \dfrac{m_l}{N}$ take co-prime

then, in $\mathcal{H}_\theta^{phys.}$: $\hat{T}_l^N \, |\psi\rangle = |\psi\rangle \, e^{-i\theta m_l}$

eigenvalues of $\hat{T}_l$, generating 1-form Z_N

and $\hat{T}_l |e_l\rangle = |e_l\rangle \, e^{i\frac{2\pi}{N}e_l - i\theta\frac{m_l}{N}} = |e_l\rangle \, e^{i\frac{2\pi}{N}(e_l - \frac{\theta}{2\pi}m_l)}$

$\vec{e}$ (mod N) …

discrete electric flux

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

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$\hat{T}_i$ generate global symmetries:
act by Z_N on Wilson loops wound in x^i

$$\hat{T}_l |\overrightarrow{e}\rangle = |\overrightarrow{e}\rangle \; e^{i\frac{2\pi}{N}e_l - i\theta\frac{m_l}{N}} = |\overrightarrow{e}\rangle \; e^{i\frac{2\pi}{N}(e_l - \frac{\theta}{2\pi}m_l)}$$

- $\hat{T}_l$ commute with Hamiltonian, generate 1-form $Z_N^{(1)}$
- all eigenvectors of $\hat{H}$ also labeled by Z_N electric flux $\overrightarrow{e}$
- states with different $\overrightarrow{e}$ related by winding Wilson loops

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

$$\hat{V}_\alpha[\hat{A}] = e^{i\alpha \int_{\mathbb{T}^3} K_0(\hat{A})}$$

$\oint K_0$ shifts by 1 under a unit winding gauge trf.

$\hat{V}_\alpha$ shifts θ angle, consider commutator with 1-form center:

$$\hat{T}_l \, \hat{V}_\alpha[\hat{A}] \, \hat{T}_l^{-1} = \hat{V}_\alpha[\hat{C}[k_i = \delta_{il}, 0] \circ \hat{A}] = e^{i\alpha \int_{\mathbb{T}^3} \left[K_0(\hat{C}[k_i=\delta_{il},0]\circ\hat{A}) - K_0(\hat{A})\right]} \, \hat{V}_\alpha = e^{i\alpha \frac{m_l}{N}} \, \hat{V}_\alpha$$

$$Q[T_l] = \frac{m_l}{N}$$

$$\hat{T}_l \, \hat{V}_{2\pi} = e^{i2\pi \frac{m_l}{N}} \, \hat{V}_{2\pi} \, \hat{T}_l$$

relation is behind the central extensions of all 1-form/0-form algebras, reflecting the anomaly in the $\vec{m} \neq 0$ Hilbert space on T^3

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

$$A(x,y,z) \to A^P(x,y,z) = -\Gamma_P A(L_1 - x, L_2 - y, L_3 - z)\Gamma_P \qquad \Gamma_P \Gamma_i \Gamma_P = e^{i\phi}\Gamma_i^{-1}$$

$\hat{P}_0 \hat{T}_i \hat{P}_0$ acts as a center trfm.: $T_i'(x,y,z) = \Gamma_P T_i(L_1 - x, L_2 - y, L_3 - z)\Gamma_P$

so on physical states T' is equivalent to T^{-1}

$$\hat{P}_0 \; \hat{T}_i \; \hat{P}_0 = \hat{T}_i^\dagger$$ = dihedral group of order 2N, at $\theta = 0$

at $\theta = \pi$, however, parity is $\hat{P}_\pi = \hat{V}_{2\pi} \hat{P}_0$

$$\hat{H}_\theta = \int_{\mathbb{T}^3} d^3x \left(\frac{g^2}{2} (\hat{\Pi}_i^a - \frac{\theta}{8\pi^2}\hat{B}_i^a)(\hat{\Pi}_i^a - \frac{\theta}{8\pi^2}\hat{B}_i^a) + \frac{1}{2g^2} \hat{B}_i^a \hat{B}_i^a \right) \qquad \hat{V}_{2\pi}\hat{\Pi}_i^a \hat{V}_{2\pi}^{-1} = \hat{\Pi}_i^a - \frac{1}{4\pi}\hat{B}_i^a$$

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

$\hat{P}_0 \, \hat{T}_j \, \hat{P}_0 = \hat{T}_j^\dagger$ = dihedral group of order 2N, at $\theta = 0$

at $\theta = \pi$, however, the parity generator is $\hat{P}_\pi = \hat{V}_{2\pi} \hat{P}_0$

$\hat{P}_\pi \, \hat{T}_j \, \hat{P}_\pi = e^{\frac{2\pi i}{N} m_j} \, \hat{T}_j^\dagger$ so the algebra at $\theta = \pi$ is extended

from now on $\vec{m} = (0,0,1)$ **ignore** T_1, T_2 **and labels** e_1, e_2

$$[\hat{T}_3, \hat{H}_{\theta=\pi}] = 0 \ , \quad [\hat{P}_\pi, \hat{H}_{\theta=\pi}] = 0 \ , \quad \hat{T}_3 \hat{P}_\pi = e^{i\frac{2\pi}{N}} \hat{P}_\pi \hat{T}_3^\dagger$$

$$\hat{H}_{\theta=\pi} |E, e_3\rangle = |E, e_3\rangle E$$

$$\hat{T}_3 |E, e_3\rangle = |E, e_3\rangle e^{i\frac{2\pi}{N} e_3}$$

$$\hat{T}_3 (\hat{P}_\pi |E, e_3\rangle) = (\hat{P}_\pi |E, e_3\rangle) e^{i\frac{2\pi}{N}(1-e_3)}$$

$$\hat{P}_\pi : |E, e_3\rangle \to |E, 1 - e_3 \ (\mathrm{mod} N)\rangle$$

3 Gauging $Z_N^{(1)}$ on T^3, quantization and centrally-extended algebra

$\hat{P}_0 : |E, e_3\rangle \to |E, -e_3\rangle$ $\theta = 0, e_3 = 0$ is parity invariant for all N

- "global inconsistency" for odd-N

$\theta = \pi$, odd-N: $e_3 = \dfrac{N+1}{2}$ invariant

$$\hat{P}_\pi : |E, e_3\rangle \to |E, 1 - e_3 \ (\mathrm{mod} N)\rangle$$

$\theta = \pi$, even-N all states doubly degenerate

- **mixed center/parity anomaly for even-N SU(N)**
- **mixed center/parity anomaly for all groups … if center is of even order (i.e., for all but E_6, where "global inconsistency")**

4 Consequences, discussion, old vs new

$$\vec{m} = (0,0,1) \qquad [\hat{T}_3, \hat{H}_{\theta=\pi}] = 0 \ , \quad [\hat{P}_\pi, \hat{H}_{\theta=\pi}] = 0 \ , \quad \hat{T}_3 \hat{P}_\pi = e^{i\frac{2\pi}{N}} \hat{P}_\pi \hat{T}_3^\dagger \ ;$$

- even N, exact parity degeneracy at any size torus (already seen in anomalies)

delicate cancellations of tunneling: semiclassics

phases due to $\vec{m}$/*contours/thimbles*... dYM ?

- as $L_i \to \infty$, expect lowest energy e-flux states => parity breaking vacua

other N-2 higher energy fluxes: metastable/unstable pairs of vacua... seen in dYM

old vs new: ... 1980 vs now:

interpretation as anomaly and the centrally-extended algebra new

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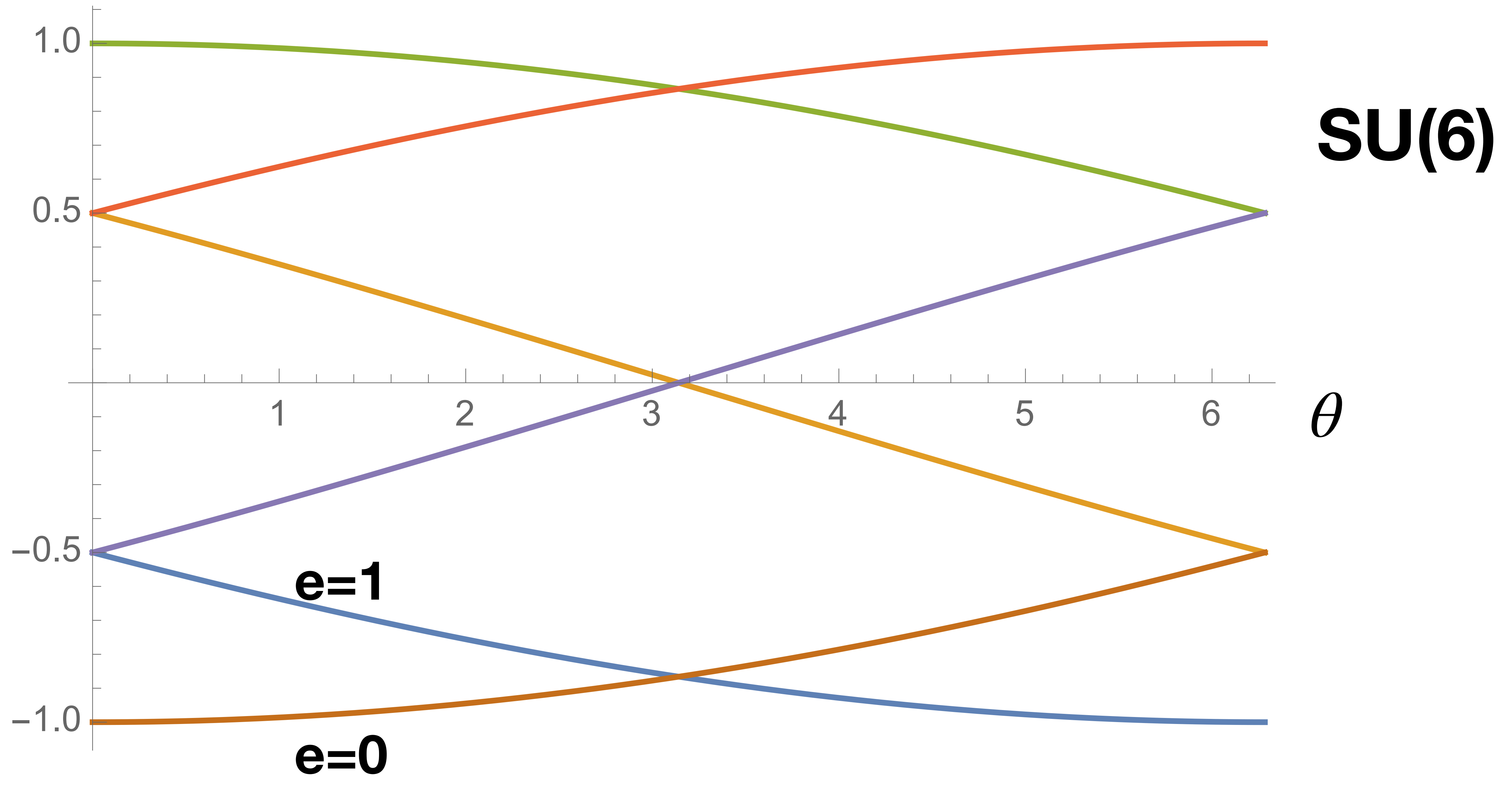
- **as $L_i \ll \Lambda^{-1}$ "femtouniverse"** [van Baal in 1999 review, $\vec{m} = (0,0,1)$]

$$E(\theta, e_3) = -\frac{C e^{-\frac{8\pi^2}{g^2 N}}}{L g^4} \cos\left(\frac{2\pi}{N} e_3 - \frac{\theta}{N} m_3\right)$$

fractional I's on $T^3 \times R$; no analytic soltns



for the experts, compare with dYM vacuum energies: $L_1, L_2 \to \infty, L_3 = L \ll 1/(N\Lambda)$

extended algebra seen in IR of dYM *[Aitken, Cherman, Unsal 2018]*: **no m … *precise* relation**

4 Consequences, discussion, old vs new

- contact with Euclidean, IR TQFT - double degeneracy constrains Z:

$$[\hat{T}_3, \hat{H}_{\theta=\pi}] = 0\ , \quad [\hat{P}_\pi, \hat{H}_{\theta=\pi}] = 0\ , \quad \hat{T}_3 \hat{P}_\pi = e^{i\frac{2\pi}{N}} \hat{P}_\pi \hat{T}_3^\dagger$$

$$Z[k,1] = \mathrm{tr}\,(e^{-\beta \hat{H}_{\theta=\pi}}\ \hat{T}_3^k)$$ trace over $\mathcal{H}^{phys.}_{\theta=0}$ with $m_3 \neq 0$

insert $\hat{P}_\pi^2 = 1$ in trace and use algebra: $Z[k, m_3] = Z[-k, m_3]\, e^{i\frac{2\pi k m_3}{N}}$

solution $Z[k, m_3] = e^{i\frac{\pi k m_3}{N}}\ \Xi$, with Ξ even wrt k

for a two-state IR TQFT $\Xi = 2\cos\frac{\pi k m_3}{N}$ - e.g. van Baal's e=0,1 states only, set E=0

4 Consequences, discussion, old vs new

Summary: **mixed parity/center anomaly in $\vec{m}$ leads to extended algebra**

$$[\hat{T}_3, \hat{H}_{\theta=\pi}] = 0 \ , \quad [\hat{P}_\pi, \hat{H}_{\theta=\pi}] = 0 \ , \quad \hat{T}_3 \hat{P}_\pi = e^{i\frac{2\pi}{N}} \hat{P}_\pi \hat{T}_3^\dagger$$

$\vec{m} = (0,0,1)$

anomaly: **SU(2k), Sp(2k+1), Spin(2k),** E_7

global inconsistency: **SU(2k+1),** E_6

neither: **Sp(2k), Spin(2k+1)**

the most unusual feature is the exact degeneracy at finite volume in the $\vec{m}$ background implied by the extension

would like to better understand in a calculable framework valid in a (partially) infinite volume, dYM

Chiral-center algebras: central extensions

SU(N) QCD(adj) with $n_f \leq 5$ massless Weyl adjoint fermions

$$\frac{\mathbb{Z}_{2n_f N} \times SU(n_f)}{\mathbb{Z}_{n_f}}$$ **anomaly free chiral** $$\hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} = e^{i\frac{2\pi}{2n_f N}\hat{Q}_5} = e^{i\frac{2\pi}{2n_f N}\int d^3x \hat{j}^0_f}\ \hat{V}^{-1}_{2\pi}$$

mixed chiral/center anomaly

in $\overrightarrow{m} = (0,0,1)$ leads to extended algebra

[more general twists possible - not here!]

$$[\hat{T}_3, \hat{H}] = 0, \quad [\hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}}, \hat{H}] = 0, \quad \hat{T}_3\ \hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} = e^{-i\frac{2\pi}{N}}\ \hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}}\ \hat{T}_3$$

$$\hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}}\ |E, e_3\rangle = |E, e_3 - 1\rangle$$ **- implies N-fold degeneracy**

Chiral-center algebras: central extensions

$\hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} |E, e_3\rangle = |E, e_3 - 1\rangle$ **- implies N-fold degeneracy**

- exact at any size T^3... at least $Z_{2n_f N} \to Z_{2n_f}$

- yes, for $n_f = 1$ (SYM)
- yes, for any $n_f < 6$ on $R^3 \times S^1$ Unsal 2007
- various R^3 proposals for $n_f > 1$

Cordova, Dumitrescu 2018
Anber, EP 2018
Ryttov, EP 2019
most recent lattice work
Athenodorou, Bennett, Bergner, Lucini 2021

- other groups: 2-fold , 3-fold, 4-fold degeneracies on T^3

whose center is $Z_2, Z_2 \times Z_2$ Z_3 Z_4

- min. breaking with multi-fermion condensates (or bilinear)

6 Summary & outlook

What I told you:

- **the mixed anomaly between 0-form parity/chiral/ and 1-form center can be seen as an extension of the symmetry operator algebra on T^3 with twisted b.c.** (= 2-form background for the 1-form center symmetry)

- **these central extensions imply exact degeneracies between appropriate electric flux states on T^3**

6 Summary & outlook

Lingering questions:

how is tunneling at finite volume avoided?
(learn more about semiclassics?)

may be useful for lattice studies ($\theta = \pi$, especially)?

what happens in theories (e.g. $n_f = 4{,}5$ QCD(adj))
thought to flow to CFTs in R^3 limit?

do more general anomalies involving 0-form and 1-form symmetries have Hilbert space implications?