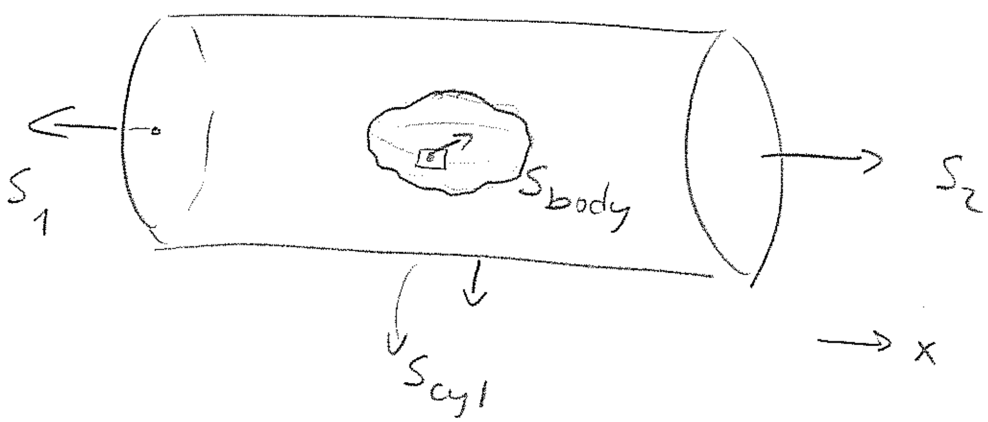
$$F_x = 0 \qquad F_y = -\rho U \Gamma$$

$\Downarrow$ no drag? ("D'Alembert paradox" — or theorem)

Also in 3d ... momentum equation

(steady flow) $\quad (\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla} p$

(1) $\quad (u_i \partial_i) u_j = -\partial_j p$, in index notation



$\int(1)$ over V: $\quad \rho \int dV (u_i \partial_i u_j) = - \int \partial_j p \, dV$

l.h.s $\qquad$ r.h.s

$\underline{\text{l.hs}} = \rho \int dV \, u_i \partial_i u_j = \rho \int dV \, \partial_i (u_i u_j) = \quad (\text{since } \partial_i u_i = 0)$

$$= \rho \oint_S d^2 s^i u^i u^j = \rho \int_{S_{cyl}} d^2 s^i u^i u^j +$$

$$+ \rho \int_{S_1} d^2 s^i u^i u^j + \rho \int_{S_2} d^2 s^i u^i u^j + \rho \int_{S_{cyl}} d^2 s^i u^i u^j$$

on both S_{cyl} & S_{body} $\vec{u}$ is $\perp$ $d^2\vec{S}$

so $d^2S^i u^i = 0$ on S_{cyl} & S_{body}

So we have

$$\underline{\underline{\text{l.h.s}}} = \rho \int_{S_1} d^2S^i u^i u^j + \rho \int_{S_2} d^2S^i u^i u^j$$

$$\underline{\underline{\text{r.h.s.}}} = - \int_V \partial_j p \, dV = - \int_V \partial_i (\delta_{ij} p) \, dV =$$

$$= - \oint_S d^2S^i \delta^{ij} p =$$

$$= - \int_{S_{cyl}} d^2S^i \delta^{ij} p - \int_{S_{body}} d^2S^i \delta^{ij} p$$

$$- \int_{S_1} d^2S^i \delta^{ij} p - \int_{S_2} d^2S^i \delta^{ij} p$$

Take

$\underline{\underline{(\text{r.h.s.})^{j=x} \text{ component}}}$: along channel

$$- \int_V \partial_x p \, dV = - \underbrace{\int_{S_{cyl}} d^2S^x p}_{\left(\begin{array}{l} d^2\vec{S}_{cyl} \text{ is } \perp \vec{1}_x \\ \text{so this} = 0 \end{array} \right)} - \underbrace{\int_{S_{body}} d^2S^x p}_{\downarrow \; (\text{force})^x \text{ on body} = (\text{drag})^x} - \int_{S_1} \overbrace{d^2S^x}^{-d^2S} p - \int_{S_2} \overbrace{d^2S^x}^{+d^2S} p$$

So we have

$$(\text{r.h.s.})^{j=x} = -(\text{Drag})^x + \int_{S_1} d^2S\, p - \int_{S_2} d^2S\, p$$

$$(\text{l.h.s.})^{j=x} = \rho \int_{S_1} \underbrace{d^2S^x}_{-d^2S} u^x u^x + \rho \int_{S_2} \underbrace{d^2S^x}_{+d^2S} u^x u^x$$

$$= -\rho \int_{S_1} d^2S\, (u_x)^2 + \rho \int_{S_2} d^2S\, (u_x)^2$$

equate the two:

$$(\text{Drag})^x = -\int_{S_1} d^2S\, (\rho\, (u_x)^2 + p)$$

$$+ \int_{S_2} d^2S\, (\rho (u_x)^2 + p)$$

take $S_2 \to +\infty$
$S_2 \to -\infty$

@ ∞'s flow is unidirectional & uniform, pressure $= p_0$

hence $\rho(u_x)^2 + p\Big|_{+\infty} = \rho(u_x)^2 + p\Big|_{-\infty}$

By Bernoulli along streamlines

& there's no drag.

⇒ a steady uniform flow of ideal fluid

⇒ no drag

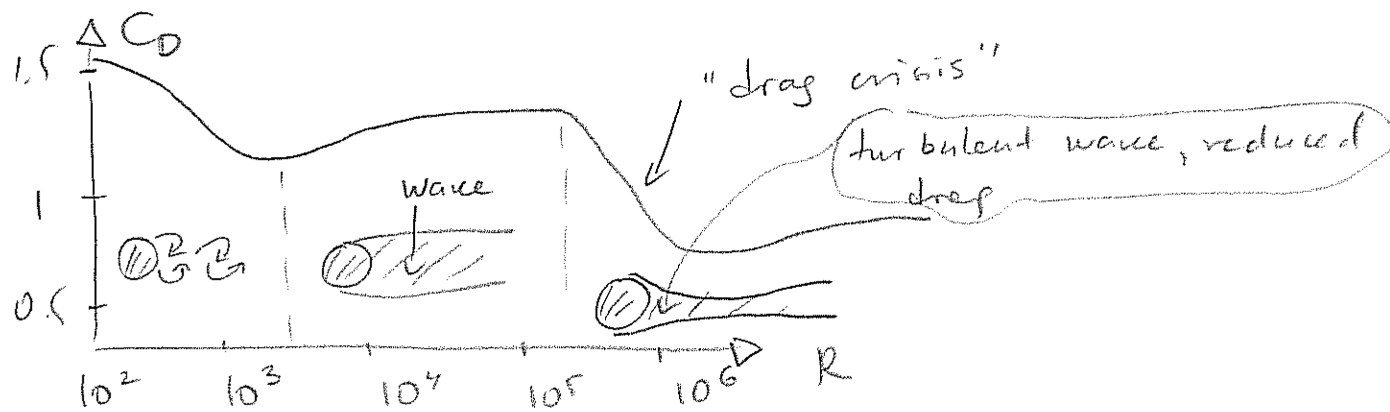
(D'Alembert)

<u>different from real flow is that</u>

<u>there is no wake</u>

recall $C_D = \frac{\text{Drag}}{\rho U^2 a}$ vs. $R = \frac{2aU}{\nu}$ a = radius

@ $R \to \infty$ expected $C_D \to 1$ ... but in reality



($C_D \not\to 1$ as $R \to \infty$ -- always ∃ wake)

$\boxed{\text{boundary layer \& its separation}}$