$$\left| \frac{1}{c_s^2 \rho} \frac{Dp}{Dt} - \frac{1}{\rho c^2} \left(\frac{\partial p}{\partial s}\right)_\rho \frac{Ds}{Dt} \right| \ll \frac{U}{L}$$

for incompressibility

i.e.

$$\left| \frac{1}{\rho} \frac{D\rho}{Dt} \right| \ll \frac{U}{L} \text{ (typical gradients of } \vec{u}\text{)}$$

hence we need each term $\ll \frac{U}{L}$

(1) $$\Big| \frac{1}{\rho c_s^2} \left(\frac{\partial p}{\partial s}\right)_\rho \frac{Ds}{Dt} \Big| \ll \frac{U}{L} \qquad (1.1)$$

$$\frac{1}{\rho c_s^2} \left(\frac{\partial p}{\partial s}\right)_\rho = \frac{\beta T}{c_p} \qquad \beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \; ; \; v = \frac{\text{Volume}}{\text{particle}}$$

thermal expansion coefft.

fixed-p heat capacity

various TD equalities

<u>Now, see HW2</u>:

$$\frac{Ds}{Dt} = \frac{1}{\rho T} \varkappa_T \vec{\nabla}^2 T + \frac{2\mu}{\rho T} \left(e_{ij} - \delta_{ij} \tfrac{1}{3} e_{ll}\right)^2 + (\lambda\text{-term})$$

i.e

typical variation of T

$$\frac{Ds}{Dt} \sim \frac{\varkappa_T}{\rho T} \frac{\Delta T}{L^2} + \frac{\mu}{\rho T} \left(\frac{U}{L}\right)^2 \qquad (1.2)$$

(1.2) + (1.1.) =>

$$\left| \frac{\beta T}{c_p} \frac{\varkappa_T}{\rho T} \frac{\Delta T}{L^2} + \frac{\beta T}{c_p} \frac{\mu}{\rho T} \left(\frac{U}{L}\right)^2 \right| \ll \frac{U}{L}$$

$$\frac{\beta \varkappa_T \Delta T}{c_p \rho L} \ll U \quad \& \quad \frac{\beta \mu}{c_p \rho} \frac{U}{L} \ll 1$$

correcting the ones on p. 95.

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(2) $$\left| \frac{1}{c_s^2 \rho} \frac{Dp}{Dt} \right| \ll \frac{U}{L} \qquad (2.1)$$

we have by N.-S. equ. $\rho \frac{D\bar{u}}{Dt} = -\bar{\nabla} p + \ldots$

imagine flow w/ scales U, L, τ

where $\tau \neq \frac{L}{U}$ (example follows!)

then

$$\rho \frac{Du}{Dt} \sim \rho \frac{U}{\tau} \sim \frac{\Delta p}{L}$$

so $\frac{\Delta p}{\tau} \sim \frac{\rho L U}{\tau^2}$ ---> use $\frac{\Delta p}{\tau} \sim \frac{Dp}{Dt}$ in (2.1) →

$$\left| \frac{1}{c_s^2 \rho} \frac{Dp}{Dt} \right| \ll \frac{U}{L}$$

$$\downarrow$$

$$\frac{1}{c_s^2 \cancel{\rho}} \frac{\cancel{\rho} L \cancel{U}}{\tau^2} \ll \frac{\cancel{U}}{L}$$

$$c_s^2 \tau^2 \gg L^2$$

$$c_s \tau \gg L$$

or $\tau \gg \dfrac{L}{c_s}$

time it would take sound to pass a distance $\sim L$ (typical scale variation of flow)

typical time scale on which flow varies

(in other words, if $\tau \gg \dfrac{L}{c_s}$

effectively take $c_s \to \infty$ (incompressible))

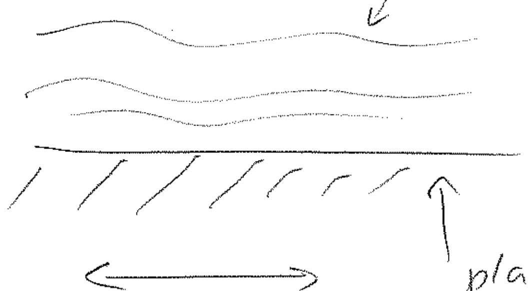
Note: if $\tau = \dfrac{L}{U}$ we have $\tau \gg \dfrac{L}{c_s} \to \dfrac{L}{U} \gg \dfrac{L}{c_s}$

(not new)

or $\dfrac{U}{c_s} \ll 1$

What flow can have $\tau \neq \frac{L}{U}$?

Example:



$u_{plate} = U \cos(\omega t)$

So: U - scale of velocity of fluid

$$\begin{cases} u_{fluid} \equiv u_{plate} & @ \text{ plate} \\ u_{fluid} \to 0 & @ \infty \end{cases}$$

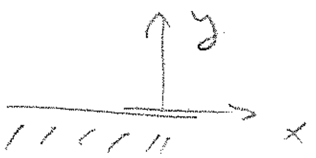
$\omega \approx \frac{1}{\tau}$: τ : scale of time change

? L ? $\longrightarrow$ dim analysis... $L = \sqrt{\frac{\nu}{\omega}}$

scale on which fluid velocity varies

Note, three scales not related by $\tau = \frac{L}{U}$!

In this example, $\exists$ exact solution ...



$\frac{1}{\tau} \sim \omega$

$$\vec{u} = (u_x(y), 0, 0)$$

$$u_x(y,t) = U e^{-y\sqrt{\frac{\omega}{2\nu}}} \cos\left(\omega t - y\sqrt{\frac{\omega}{2\nu}}\right)$$

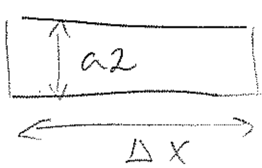
$$\frac{1}{L} = \sqrt{\frac{\omega}{2\nu}}$$

U.

//

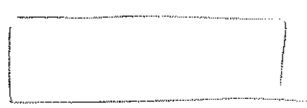
Another Ex: starting flow in a pipe

$t < 0$ $\quad p_L$



$p_R = p_L$

$t \geq 0$ $\quad p_L$



$p_R = p_L - p_1 \Delta x$

stationary flow reached @ time $\sim \frac{a^2}{\nu}$

$\left(\frac{a^2}{\nu}\right)$ - time scale

this is flow solved in class

$$u(r,t) = \frac{p_1}{4\mu}(a^2 - r^2) + \# \frac{p_1 a^2}{\mu} f(r/a)\, e^{-\# \frac{t}{(a^2/\nu)}}$$