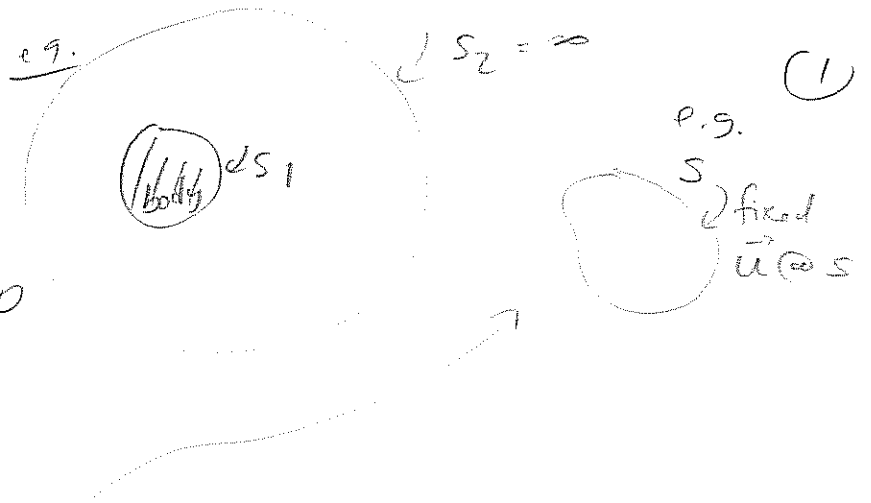


Stokes' eqns:

$$\begin{cases} \mu \Delta \vec{u} = \vec{\nabla} p \\ \vec{\nabla} \cdot \vec{u} = 0 \end{cases}$$



assume \exists two solutions $(\vec{u}_1, p_1) \neq (\vec{u}_2, p_2)$ obeying same b.c.; their difference,

also obeys eqns:

$$\begin{aligned} \vec{v} &= \vec{u}_1 - \vec{u}_2 \\ \vec{P} &= p_1 - p_2 \end{aligned}$$

i.p.

$$\mu \Delta \vec{v} = \vec{\nabla} P \quad / \cdot \vec{v}$$

$$\mu \vec{v} \Delta \vec{v} - \vec{\nabla} (\vec{v} P) = 0 \quad (\vec{\nabla} \cdot \vec{v} = 0)$$

$$0 = \mu \int_V dV \vec{v} (\vec{\nabla}^2) \vec{v} - \int_V \vec{\nabla} (\vec{v} P) dV$$

$\int_V \vec{\nabla} (\vec{v} P) dV = \oint_S \vec{v} P d^2S$
 $\vec{v} = 0$ (same b.c.)

$$\int_V \mu \nabla^2 (n_i e_j \cdot n_i) - (n_i e_j \cdot n_i) = \int_V \mu \nabla^2 (n_i e_j \cdot n_i) - \int_V \mu \nabla^2 (n_i e_j \cdot n_i) = 0$$

$$\frac{\delta}{\delta v} \int_{\partial S} v_i \cdot n_i \, dS = 0 \text{ as } \bar{v} = 0 \text{ @ } S$$

(2)

$$\Rightarrow 0 = \int_V \underbrace{v_i^2}_{\geq 0} \, dV$$

- a \sum of 9 positive terms (squares)

- where \int over V vanishes

\Rightarrow every single term $\equiv 0$.

$$\frac{\partial}{\partial x_j} v_i = 0 \rightarrow v_i = \text{const.}$$

$$\text{but } v_i = 0 \text{ @ } S$$

$$\Rightarrow v_i = 0 \text{ every where}$$

□