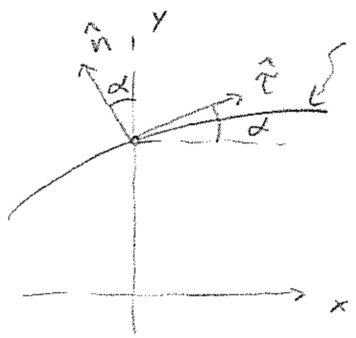
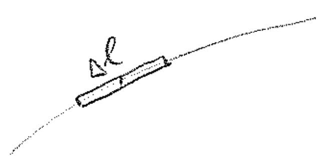
mass conservation & kinematic b.c.



$$\hat{\tau} = (\cos\alpha, \sin\alpha)$$

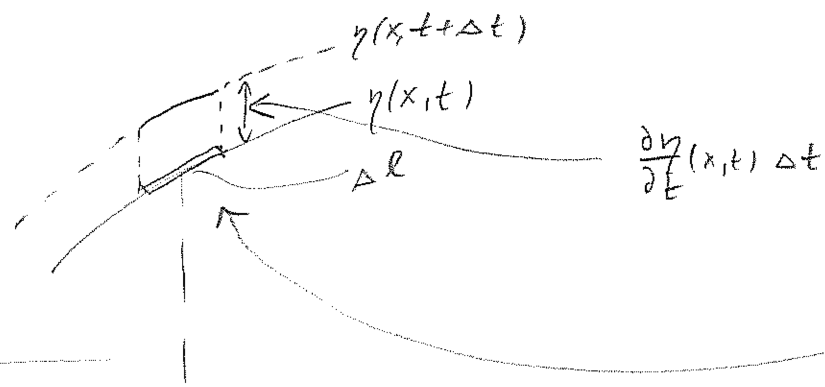
$$\hat{n} = (-\sin\alpha, \cos\alpha)$$

$\rho \vec{u} \cdot \hat{n} \, \Delta t \, \Delta l$ = mass flowing thru Δl in Δt

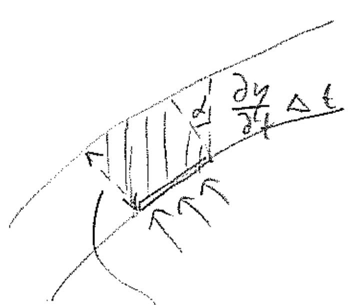


- if $\eta(x, \cancel{t})$ stationary, this should be $= 0$, as no fluid goes into air

- if $\eta(x,t)$ changes ... $\longrightarrow$ then fluid can fill extra volume



so $\rho \cdot \vec{u} \cdot \hat{n} \, \Delta t \, \Delta l = \frac{\partial \eta}{\partial t} \Delta t \, \Delta l \, \rho \times \cos\alpha$



$\frac{\partial \eta}{\partial t} \Delta t \cos\alpha$; extra volume $\frac{\partial \eta}{\partial t} \Delta t \, \Delta l \cos\alpha$.

cancel slstp:

$$\bar{u} \cdot \hat{n} = \frac{\partial \eta}{\partial t} \cos\alpha$$

$$-u_x\big|_\eta \sin\alpha + u_y\big|_\eta \cos\alpha = \frac{\partial \eta}{\partial t} \cos\alpha \qquad \Big| \times \frac{1}{\cos\alpha}$$

$$-u_x\big|_\eta \tan\alpha + u_y\big|_\eta = \frac{\partial \eta}{\partial t}$$

$$-u_x\big|_\eta \frac{\partial \eta}{\partial x} + u_y\big| = \frac{\partial \eta}{\partial t}$$

$$u_y\big|_\eta = \frac{\partial \eta}{\partial t} - u_x\big|_\eta \frac{\partial \eta}{\partial x}.$$

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same $\sim$ $\frac{D}{Dt}\left[y - \eta(x,t)\right] = 0.$

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also shows that on-surface particles remain there.

- otherwise violate mass conservation.