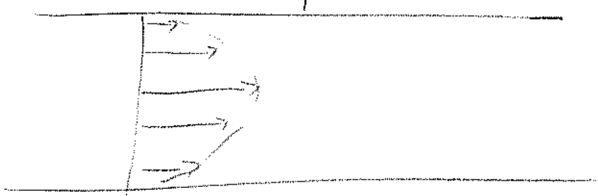
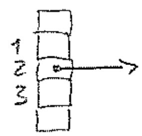
Can a fluid flow *against* $-\vec{\nabla}p$?

<u>pipe</u> : due to $\vec{\nabla}p$ ... (assume walls @ rest)

$$p = p_0 - p_1 x$$

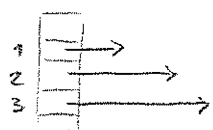
$\vec{\nabla}p \longleftarrow$ so force $= -\vec{\nabla}p \longrightarrow$





$\vec{f} = -\vec{\nabla}p$: pressure gradient force

but velocity distrib'n is : (assuming in upper half of flow)



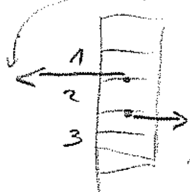
$v_1 < v_2 < v_3$

BUT

$v_3 - v_2 < v_2 - v_1$

so viscous forces are: (on 2)

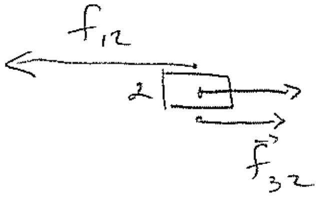
f_{12} : force from 1 on 2 : <u>larger</u>



f_{32} : force from 3 on 2 : <u>smaller</u>

(because 1st derivative is smallest in middle of flow)

$\Rightarrow$ hence

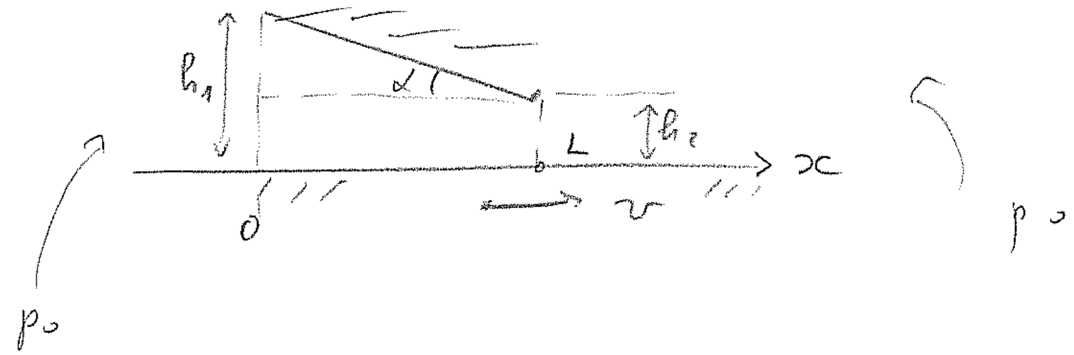


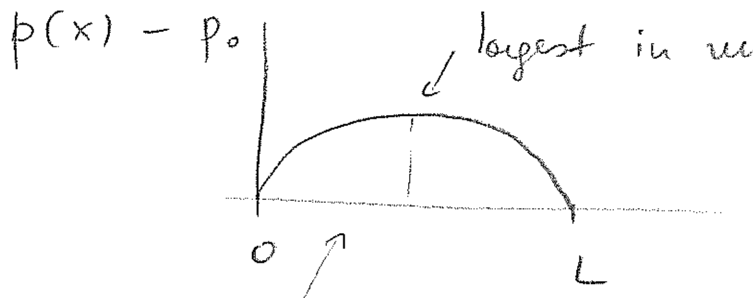
so that they balance out

$\mu\Delta\vec{u} = -\vec{\nabla}\vec{p}$ when $\vec{u} \,//\, -\vec{\nabla}p$.

<u>Example 1.</u>

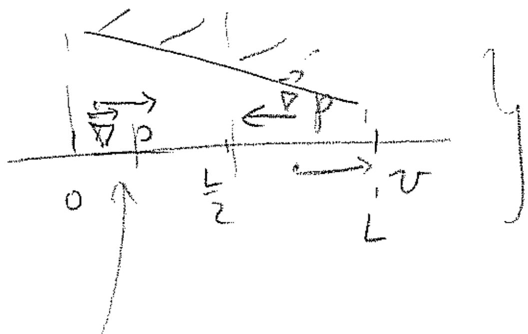
Example 2: thin channel flow

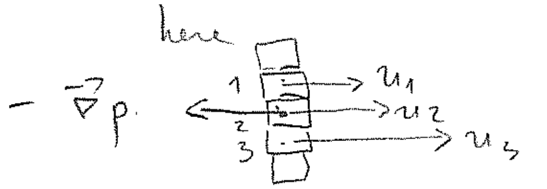




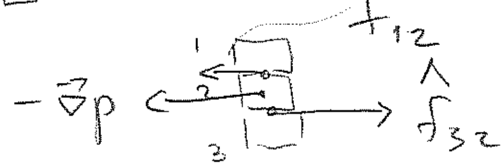
$$h(x) = h_2 - (h_1 - h_2)\frac{x}{L}$$

$$\frac{h_1 - h_2}{L} \ll 1$$





but here $u_3 - u_2 > u_2 - u_1$



flow arranges so that forces cancelled – 1st derivative increases going down (at $x < \frac{L}{2}$) & decreases @ $x > \frac{L}{2}$

So $\rightarrow$ $-\vec{\nabla} p$ & $\vec{u}$ can have opposite direction! in viscous flow

$\mu \Delta \vec{u} = -\vec{\nabla} p$ "Δ can have either sign"