diversion - prompted by question: (27.1) ①

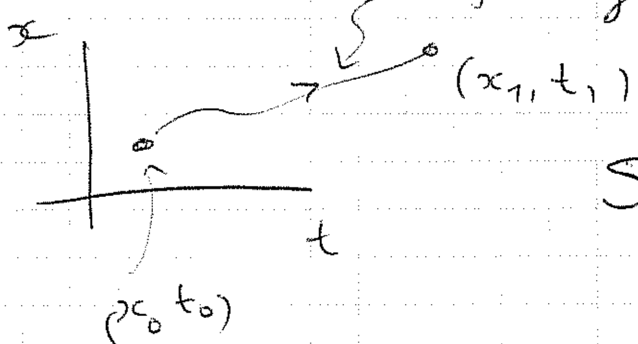
for a relativistic particle:

Galileo's principle →

→ Lorentz invariance principle

$S(\text{trajectory } x_0 t_0 \to x_1 t_1)$ should be same in any Lorentz frame (i.e. invariant under Lorentz transf's)

trajectory - must be timelike



$$S = \text{const} \int_{x_0 t_0}^{x_1 t_1} ds$$

ds = spacetime interval between two neighboring points

$$\Rightarrow \quad ds^2 = -c^2 dt^2 + d\vec{x}^{\,2}$$

$$ds = \sqrt{-ds^2} = \sqrt{c^2 dt^2 - d\vec{x}^{\,2}} =$$

remember, timelike

$$= c\, dt \sqrt{1 - \frac{d\vec{x}^{\,2}}{c^2 dt^2}}$$

$$= c\, dt \sqrt{1 - \frac{\vec{V}^2}{c^2}}$$

5. $$S(\bar{x}_0 t_0 \to \bar{x}_1 t_1) = \text{cont}\ c \int_{t_0}^{t_1} dt \sqrt{1 - \frac{\vec{v}^2}{c^2}}$$

$$\vec{v} = \frac{d\vec{x}(t)}{dt}$$

$$\vec{x}(t_0) = \bar{x}_0$$
$$\bar{x}(t_1) = \vec{x}_1$$

cont = ?

take nonrelativistic limit:

$$\sqrt{1 - \frac{\vec{v}^2}{c^2}} \approx 1 - \frac{1}{2}\frac{\vec{v}^2}{c^2} + \dots$$

$$\left(\sqrt{1-x} \approx 1 - \frac{1}{2}x + \dots\right)$$

$$S(\bar{x}_0 t_0 \to \bar{x}_1 t_1) = \text{const.}\ c \int_{t_0}^{t_1} dt \sqrt{1 - \frac{\vec{v}^2}{c^2}} \approx$$

$$\approx \underbrace{\text{cont}\ c \int_{t_0}^{t_1} dt}_{\text{indep. of } \vec{x}(t) \text{ - drop}} - \frac{\text{const}}{2} c \int_{t_0}^{t_1} \frac{\vec{v}^2}{c^2} dt + \dots$$

$$\approx -\frac{\text{const}}{2}\frac{1}{c}\int_{t_0}^{t_1} dt\ \vec{v}^2 = \int_{t_0}^{t_1} \frac{m\vec{v}^2}{2}\, dt$$

$$\Rightarrow \text{const} = -mc \quad \& \quad \boxed{S = -mc\int ds}$$

relativistic

$$S = -mc^2 \int_{t_0}^{t_1} dt \underbrace{\sqrt{1-\frac{\vec{v}^2}{c^2}}}_{L(\vec{v}^2)}$$

$L(\vec{v}^2)$ → {homogeneity + + isotropy}

→ {but NOT Gallilean relativity !!}

Equation of motion:

$$\frac{d}{dt}\frac{\partial L}{\partial \vec{v}} = 0 \Rightarrow \frac{d}{dt}\left(\frac{\vec{v}}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}\right) = 0$$

(relativistic acceleration = 0.)

For a charged particle:

$$S = -mc^2 \int dt \sqrt{1-\frac{\vec{v}^2}{c^2}} + q \int dt \left(\varphi(\bar{x},t) - \frac{d\vec{x}}{dt}\cdot \vec{A}(\vec{x},t)\right)$$

$$= -mc^2 \int dt \sqrt{1-\frac{\vec{v}^2}{c^2}} + q \int \underbrace{\left(dt\, \varphi(\bar{x},t) - d\vec{x}\cdot \vec{A}(\vec{x},t)\right.}$$

$(dt, d\vec{x})$: 4 - vector

$(\varphi, \vec{A})$) : 4 - vector

$dt \cdot \varphi - d\bar{x}\cdot \bar{A}$ = Lorentz invariant.

hw1#1: nonrelativistic limit forces!